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CHIRAL MODELS OF LOW ENERGY QCD

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CHIRAL MODELS OF LOW ENERGY QCD

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Abstract: Two processes may be distinguished when a hadron propagates in a dense baryonic medium. The polarization of the medium and the change in the quark structure of the hadron. The polarization of the medium is better described in terms of colorless mesons and nucleons while the intrinsic change of the hadron is better described by quark models. We show how to couple the two processes. We also relate the scaling of effective lagrangians, based on the QCD scale anomaly, to changes in the quark constituent masses.

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1. Ambiguity and compatibility in chiral quark models.

Today, quark models are fashionable and flourishing. They are used to describe *inter alias* nucleons, chiral symmetry breaking and the variation of hadron masses in dense and hot nuclear matter. And, of course, people are excited because it is all so successful. Some fifteen years earlier meson physics was flourishing. It was used *inter alias* to discuss PCAC, $\pi\pi$ scattering and the variation of nucleon masses in dense and hot nuclear matter. And, of course, people were quite excited because it was so successful. Still fifteen years earlier the hamiltonian was H = T + V, a sum of kinetic energy and 2-body interactions. It was used to describe *inter alias* nuclear binding, nuclear shapes and effective masses of nucleons propagating in nuclear matter. And, of course, people were very excited because it was so successful.

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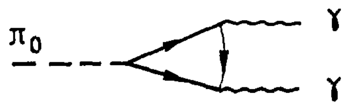
Are we simply restating the same physics in different terms? The question is more acute today than it was yesterday because the *dramatis personnae* seem to have changed. Although low energy QCD is, for the most part, practised without gluons, quarks are not only present, but they often replace nucleons without anybody really noticing or caring.

The π^0 electromagnetic decay rate.

One famous example is the pion electromagnetic decay rate usually described in your local field theory book [1] by the process:



(1.1)

It turns out that, whether you calculate it with quarks or nucleons in the fermion loop, you get the same answer which even fits experiment. Which of the two descriptions is correct? The quark one, obviously, since quarks are the fundamental constituents. Why, you even get an independent estimate of the number of colors (N_c=3 is obtained). The nucleon description is obviously better because the quark description is ambiguous: it is willing to admit quarks in the virtual intermediate states but it forgets that the pion itself is a $q\bar{q}$ excitation. The graph has the anomalous feature of not depending on the fermion (quark or nucleon) mass provided a Goldberger Treiman relation is used. The result is thus insensitive to quark confinement. Furthermore one cannot complain that one is making a perturbative QCD calculation for a low energy process because one never uses the quark-gluon running coupling constant. It is all buried in the π-nucleon coupling constant. In some way, the calculation of the pion decay rate is right in either description.

One might feel happy, if uneasy, about the fact that both descriptions of the pion decay give the same result and claim that the whole thing is a non-problem. But is it always so that one can replace nucleons by quarks and get the same answer? Suprisingly, it is often almost true, but never exactly so and sometimes downright wrong.

Spontaneous chiral symmetry breaking in the vacuum.

As a next example, consider spontaneous chiral symmetry breaking in the vacuum. The idea, originally due to Nambu and Jona-Lasinio [2] is deceivingly simple. Imagine quarks had a mass m usually called the constituent quark mass. The Dirac sea would then consist of quarks in negative energy plane wave states and its energy would be $-\nu \sum_{k<\Lambda} \sqrt{k^2 + m^2}$ where ν is the degeneracy of the plane wave states (ν=12 for u and d quarks) and Λ is a suitable cut-off of unknown origin, introduced to avoid the infinity in the sum. The Dirac sea energy decreases when the constituent mass increases and, were they free to do so, quarks would acquire infinite mass m and lower the vacuum energy indefinitely. Massive quarks in a Dirac sea break the chiral symmetry of the vacuum and this is the essence of the Nambu process. Since the Dirac sea energy decreases at most linearly with m, Nambu added a chiral symmetry restoring term to the energy, which is quadratic in m. The energy per unit volume is then:

$$\frac{E_{vacuum}}{\Omega} = -\frac{\nu}{\Omega} \sum_{k<\Lambda} \sqrt{k^2 + m^2} + \frac{a^2}{2} m^2 \tag{1.2}$$

and the vacuum constituent quark mass is the value of m which minimizes the energy (1.2) as shown by the full line of Figure 1.

In fact, this is not exactly what Nambu said. The important difference is not the setting. (He used a 4-fermion interaction which gener[illegible] m in a Hartree approximation but that is only a formal difference.) The difference is that he said all this before quarks were invented and so he filled the Dirac sea with nucleons instead of quarks. Today we use the same model with quarks instead of nucleons [3-7] and we call it low-energy QCD. The only change is that now the degeneracy is $\nu = N_c x4 = 12$ whereas Nambu used ν=4 for his nucleons. Accordingly, Nambu assumed that the vacuum value of m was the 938 MeV nucleon mass. But now that ν has increased from 4 to 12 we require the vacuum value of the quark constitutent mass m to be (very roughly) N_c times smaller, that is, 300 MeV, a third of the nucleon mass. Indeed it takes N_c=3 quark masses to substitute each nucleon mass.

Antiquarks in constituent quark models.

We consider 300 MeV to be a "reasonable value" because it is close, although higher, to the value 220 MeV used in constituent quark models [8]. The latter confine quarks by making them interact with confining potentials and they are therefore incompatible with the idea of a Dirac sea of quarks. Why should quarks feel free to float around in plane-wave states ignoring confinement when they are in the Dirac sea? Could it be because, with a roughly 1 GeV cut-off Λ, they form a baryon soup about 64 times more dense than normal nuclear matter, which has a Fermi momentum $k_F = 1/4$ GeV ? Attempts to mine the Dirac sea have always produced mesons and anti-protons but never free anti-quarks. In fact the mini-hadronization process required to produce a meson or an anti-proton has not, to my knowledge, been studied much. In constituent quark models a Dirac sea cannot exist as such and anti-quarks are introduced as distinct particles which happen to have the mass (of unspecified origin) and other quantum numbers of Dirac sea quarks. But no Dirac sea is mentioned.

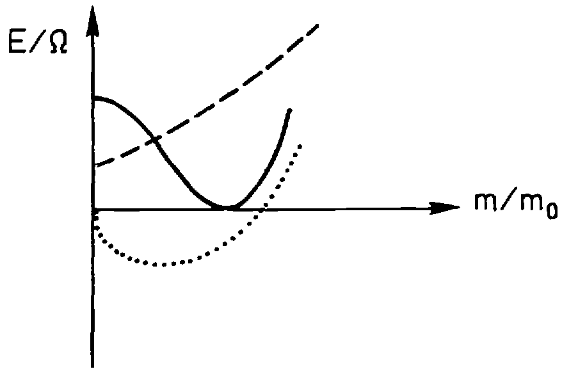


Fig.1: Contributions to the energy per unit volume. The full line is the Dirac sea energy (1.2) given by the Nambu Jona-Lasinio model, the lower dotted line is the vacuum energy (3.8) of the gluonium field which reproduces the QCD scale anomaly, and the upper dashed line is the Fermi sea contribution (1.3).

Partial chiral symmetry restoration at finite baryonic density.

The same quark/nucleon ambiguity is encountered when one considers partial chiral symmetry restoration at finite baryonic density

[9,10,5,11,12] . If a quark Dirac sea is used to describe the chirally broken phase of the vacuum, then certainly a Fermi sea of quarks should be used to describe nuclear matter at finite density. This has often been assumed [5,10]. However, who believes that nuclear matter is a Fermi sea of quarks? Does it matter whether we put nucleons or quarks in the Fermi sea? The process of partial chiral symmetry restoration is as simple as Nambu's spontaneous chiral symmetry breaking in the vacuum. It is the same process, whether you use the linear σ-model, the Nambu Jona-Lasinio model or Walecka's σ-ω model. Nuclear matter is assumed to be a Fermi sea of nucleons, but allow, for the time being, the theorist to choose quarks if he insists. Assume simply a Fermi sea of fermions of mass m. The Fermi sea contribution to the energy is then $\nu \sum_{k<k_F} \sqrt{k^2+m^2}$. Since k_F is about 1/4 GeV and m usually higher, the Fermi sea energy is roughly a linearly increasing function of m (dashed curve of Fig.1) which should be added to the vacuum energy (1.2). The result is that the minimum energy occurs for a value $m^* < m$ thus partially restoring the chiral symmetry of the vacuum. The vacuum energy (1.2) has different interpretations in various models, but the process of partial chiral symmetry restoration is always the same. Indeed, in the linear σ-model, the Nambu energy (1.1) is represented by the wine bottle shaped potential $\frac{\kappa^2}{8}\left(\sigma^2+\pi^2-f_\pi^2\right)^2$. In the Walecka σ-ω model, it is represented by the σ-meson mass term which is nothing but the quadratic approximation of the σ-model potential, namely $\frac{1}{2}m_\sigma^2(\sigma - f_\pi)^2$ with $m_\sigma = \kappa f_\pi$.

Models, like Nambu's, which use a Dirac sea of quarks, may be formulated in such a way as to allow for a Fermi sea of either quarks or nucleons and to avoid any double counting. This way, the partial chiral symmetry restoration produced by quarks or nucleons in the Fermi sea may be compared [7,13]. What is compared, in fact, is the difference between the two sums:

$$E_{Fermi} = \nu_q \sum_{k<k_F} \sqrt{k^2+m_q^2} \qquad \text{(in the case of quarks)} \qquad (1.3a)$$

and:

$$E_{Fermi} = \nu_N \sum_{k<k_F} \sqrt{k^2+m_N^2} \qquad \text{(in the case of nucleons)} \qquad (1.3b)$$

where the indices q and N refer to quarks and nucleons respectively. Notice that the same Fermi momentum k_F is required for either quarks or nucleons in order to obtain a given baryonic density. Again, the main difference is between the nucleon degeneracy $\nu_N=4$ and the quark degeneracy $\nu_q=12$. Notice also that, to the extent that the momenta k in the sums can be neglected compared to m, the quark and nucleon contributions to the energy become equal when:

$$m_N = N_c m_q \tag{1.4}$$

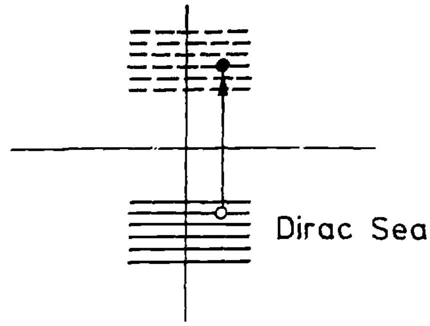
as in constituent quark models. Eq.(1.3) would imply constituent quark masses of about 300 MeV. A more precise evaluation shows that the quark and nucleon Fermi sea energies are about the same for constituent quark masses of about 400 MeV [13]. So our crude estimates are not so far off.

Modification of the nuclear medium by a propagating meson.

Mesons are $q\bar{q}$ excitations of the vacuum. If we have a model to build such excitations, we should be able to calculate how their structure is modified in a medium. Such calculations have been pursued in the framework of the Nambu Jona-Lasinio model for example [5,7,13,14]. It has also been speculated that all meson and baryon masses are reduced by the same factor at a given density [15,16]. Relative variation of the Φ and K masses could explain the relative production rate of these mesons as observed in dilepton production experiments [17]. The calculations run into the same nucleon/quark ambiguities we have discussed above. In the Nambu Jona-Lasinio model the meson is represented by propagating and interacting $q\bar{q}$ pairs:

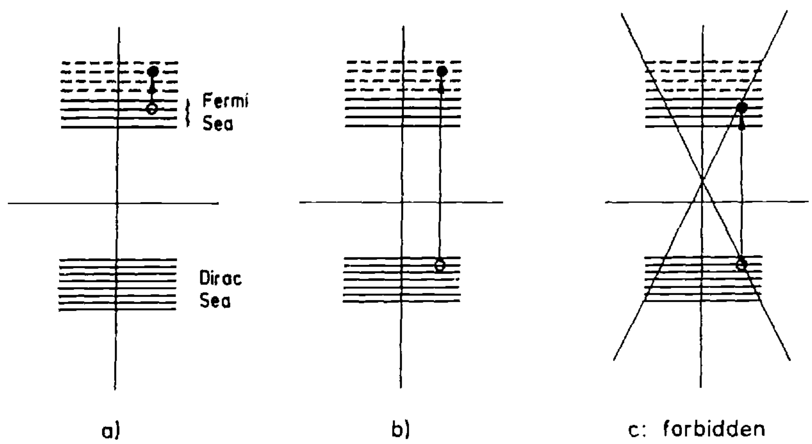
(1.5)

These processes are summed by a Bethe-Salpeter equation and, in the Nambu Jona-Lasinio model, they can desribe a *bound* $q\bar{q}$ state, but not a *confined* $q\bar{q}$ pair. In the vacuum, the $q\bar{q}$ pairs are quark-antiquark excitations in which Dirac sea quarks are excited into positive energy orbits:



(1.6)

In nuclear matter, there is a Fermi sea so that new excitations can occur as shown in (1.7). The first kind involves the excitations (1.7a) of the Fermi sea. The second kind involves excitations (1.7b) of Dirac sea quarks, but these excitations differ from the vacuum excitations (1.6) by a Pauli blocking which forbids the excitation (1.7c) of Dirac sea quarks into the Fermi sea:



(1.7)

We are immediately ·aced with more problems and ambiguities. The process (..7a) simply represents the excitation of the Fermi sea by the propagating meson. These excitations are better described when nucleons fill the Fermi sea (and are allowed to become deltas). The reason for this is that only a small energy gap separates filled and empty orbits in the Fermi sea. As a consequence, intermediate states occur which are degenerate with the energy transfered by the meson and the meson can be absorbed. This leads to an imaginary part in the meson mass operator. The threshold for meson absorbtion depends on the masses of the colorless real hadrons and these would be hopelessly wrong if the Fermi sea were represented by quarks. The process (1.7b) describes excitations of the Dirac sea by the meson. The process (1.7c) corrects for the $q\bar{q}$ excitations of the Dirac sea which normally occur in the vacuum but which are are forbidden in nuclear matter by the Pauli principle. It is referred to as *Pauli blocking*.

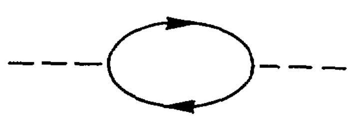
Processes such as (1.7) describe the modification of the nuclear medium by the propagating meson. They have already been extensively studied in the framework of meson-nucleon lagrangians [18-25] especially in Walecka's σ-ω model [26]:

$$\mathcal{L} = \bar{N}\left(i\partial_\mu\gamma^\mu - g_\sigma\sigma - g_\omega\omega_\mu\gamma^\mu\right)N + \frac{1}{2}(\partial_\mu\sigma)(\partial\sigma^\mu) - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \qquad (1.8)$$

Mesonic excitation modes and liquid-vapor transitions of nuclear matter have been calculated this way. Also, instabilitites have been found, both of the vacuum and of nuclear matter. Vacuum instabilities do not seem to appear in the Nambu Jona-lasinio model but they do occur in the σ-model [27,28] and in meson-nucleon calculations of nuclear matter [18,22]. They have been coined as *tachyon* modes [29] and it is also sometimes said that their occurence makes the theory *acausal*. These are unfortunate terms because the instabilities in no way imply that the model predicts particles travelling faster than light nor that causality is lost. They simply imply that, in a specific model, the assumed translationally invariant state vacuum state is unstable against external perturbations. This occurs whenever a meson self-energy turns from positive to negative [30].

Now if we calculate vacuum meson propagators by summing the $q\bar{q}$ excitations (1.6) of a quark Dirac sea, then the Pauli blocking correction is only consistently evaluated with a quark Fermi sea.

However Pauli blocking evaluations made with meson-nucleon lagrangians are also consistent. Indeed when the nucleon loop contribution to the meson propagator is evaluated in nuclear matter:

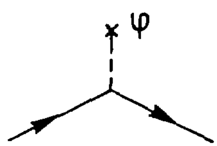


(1.9)

only the *change* in the propagator is actually evaluated, the vacuum contribution is included in the lagrangian mass term and it can be subtracted off by renormalisation.

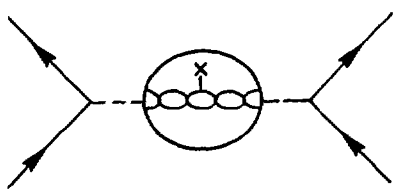
Modification of the meson structure in dense matter.

Another process contributes to the modification of the meson propagators. It is due to the reduction of the constituent quark mass in dense matter, as seen above. In the Nambu Jona-Lasinio model this mass reduction corresponds to a reduction of the quark condensate $\langle \bar{q}q \rangle$. Such processes are not included in the meson-nucleon lagragians used to calculate the modification of the nuclear medium. Indeed the quark propagators used in summing the $q\bar{q}$ excitations (1.5) are dressed by a scalar field insertion:



(1.10)

which produces the constituent quark mass and this constituent quark mass varies with density. Consider the process in which a meson is exchanged between nucleons:



(1.11)

The circle is a blow-up displaying the $q\bar{q}$ nature of the exchanged meson. The quark constituents of the meson feel the medium by interacting with the scalar field as in (1.10). Meson masses vary linearly with the constituent quark mass [13]. So we should modify the mass terms of meson-nucleon lagrangians accordingly. Lagrangians such as (1.8), possibly supplemented with other meson fields, then take the form:

$$\mathcal{L} = \bar{N}\left(i\partial_\mu\gamma^\mu - g_\sigma\sigma - g_\omega\omega_\mu\gamma^\mu - \ \ldots\right)N + \frac{1}{2}(\partial_\mu\sigma)(\partial\sigma^\mu) - \frac{1}{2}\left(\frac{\varphi}{\varphi_0}\right)^2 m_\sigma^2\sigma^2$$

$$- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_\omega^2\left(\frac{\varphi}{\varphi_0}\right)^2\omega_\mu\omega^\mu + \ \ldots \qquad (1.12)$$

where φ/φ_0 is the relative modification of the constituent quark mass in the medium. The modified meson-nucleon lagrangian (1.12) is similar to the one which is obtained by making the lagrangian scale-invariant. This is discussed in section 3. The process (1.11) is the physical process which is mocked up by the scaling.

In several of the examples discussed above, quarks actually complicate the physics more than they simplify it so is it really essential to use them? The fact that mesons are quark $q\bar{q}$ excitations (rather than $N\bar{N}$ and $\Lambda\bar{\Lambda}$) has been settled long ago. So we must face the fact that the quark and gluon structure of hadrons can be modified in a dense or hot medium. The problem is to avoid ambiguities and double counting when we take into account both the modification of the internal structure of the meson and the modification of the nuclear medium caused by the propagation of the meson. Perhaps we should endeavor to reconcile the quark structure of hadrons with the meson-nucleon lagrangians. But even there ambiguities remain. For example, any quark model will predict that quark exchange occurs [59] when the hadrons *overlap*. *How are such processes parametrized in effective meson-nucleon lagrangians? And should they?*

One sure way of *not* finding answers to these questions is to claim *urbi et orbi* that the model one happens to be using is dictated by QCD and $1/N_c$ expansions and to ignore all the other approaches on the basis that they are heretical. I was once privately told by an eminent theorist from CERN, that I was not even *allowed* to talk about quarks and mesons. I was only allowed to talk about quarks and gluons *or* about colorless hadrons, but not about both at the same time. Suprised, I asked him why and he told me that I should understand this as a frenchman, since in France we have an Academy which also tells us what we are allowed to ...

2.Chiral quark models of baryons in dense media.

Further problems appear when chiral models of baryons are considered. Bag models are discussed in other lectures. I wish to point out however that bag models offer an alternate mechanism to the Nambu process of spontaneous chiral symmetry breaking. One can speculate that the QCD ground state is a condensate of bound colorless gluon 0^{++} states called glueballs [31] , in which light quarks can be trapped forming bag-like cavity modes. The light quarks break the chiral symmetry in the vacuum by forming "shallow Dirac seas" in the glueballs [32]. I quote these works to stress that the Nambu process is far from being the only candidate, be it the simplest one. For example, it is not even obvious that the Nambu Jona-Lasinio lagrangian does a better job than, say, a σ-model lagrangian to describe quarks propagating in the QCD vacuum.

Others have speculated that the QCD vacuum is an instanton liquid [33]. Chiral symmetry is broken by the propagation of quarks in this medium. Other approaches have been suggested, based on a parametrization of the quark 2-point functions in the QCD ground state [34,35], or, still, on quarks propagating in a color-dielectric medium [36]. Our lack of knowledge of the nature of the QCD ground state (even in the absence of quarks) forbids us to be too doctrinary about effective theories and about the meaning of their parameters.

We have less trouble in predicting the modifications a nucleon undergoes in a dense medium than in understanding the nature of the medium or the existence *of* the baryon. This *is true to the extent* that we can represent the medium by a scalar field σ. It is usually assumed that it is the prevailing average value $\langle\sigma\rangle$ of the scalar field in the medium that determines the modification of baryons in a nuclear medium. It is well to remember that this is a low density approximation. As discussed above most models predict a reduction of $\langle\sigma\rangle$ in dense nuclear matter. The baryon mass and the inverse baryon size scale as $\langle\sigma\rangle$ so that baryons are expected to decrease their mass and to increase their size as the density increases [9]. In fact this modification of baryon mass is the exact analogue of the meson mass modification discussed in section 1. It is entirely determined by the curve (Fig.1) representing the energy per unit volume as a function of $\langle\sigma\rangle$, and, at densities not too close to the critical density at which chiral symmetry is restored, only the curvature of this curve at its minimum counts. A more significant parameter is the constituent mass $g\langle\sigma\rangle$ where g is the coupling constant of the σ-field to the fermions.

Quarks bound by hedgehog shaped chiral fields.

In chiral quark models, the nucleon is usually represented as a *bound* (and *not* confined) state of quarks in a chiral field of hedgehog shape. From the very first calculations [37,38], using the linear σ-model lagrangian [39]:

$$\mathcal{L} = \bar{q}\left(i\partial_\mu\gamma_\mu + g\left(\sigma + i\gamma_5\vec{\pi}.\vec{\tau}\right)\right)q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}\left(\partial_\mu\vec{\pi}\right)^2 - \frac{\kappa^2}{8}\left(\sigma^2 + \vec{\pi}^2 - f_\pi^2\right)^2 \quad (2.1)$$

it became clear that a coupling constant $g \gtrsim 3.5$ was required to form a bound state [37,38]. This means that constituent quark masses $gf_\pi \gtrsim 325$ MeV are required to form bound states. The soliton ressembles more a Skyrmion than a Friedberg-Lee soliton [40] in that the chiral field stays close to the chiral circle, making the last term of (2.1) not important. Solitons were also obtained in the presence of the ω, ρ, and a_1 fields [41] using the Lee and Nieh lagrangian [42] . In this case a soliton is obtained with a slightly weaker coupling of the scalar mesons to the quarks because considerable attraction is obtained from the ρ and a_1 fields. The coupling constants of the ρ and ω fields to the quarks is of the order of 4-5, considerably larger than those deduced from the Nambu Jona-Lasinio model [43,44,13].

Problems arise when the Dirac sea orbits are included. These are related to the vacuum instability of the renormalized σ-model against the formation of high gradients [27,28]. When the Dirac sea is included, the renormalized soliton energy has no lower bound. This prevents one from making a variational calculation of the soliton energy and makes it difficult to evaluate small changes in the shape of a soliton embedded in a dense medium.

The high gradient instabilities do not occur in the Nambu Jona-Lasinio model [2], described by the lagrangian:

$$\mathcal{L} = \bar{q}(i\partial_\mu\gamma_\mu - \varphi U_5)q - \frac{a^2\varphi^2}{2} \qquad (2.2)$$

with $U_5 = \exp\left(i\gamma_5\vec{\theta}.\vec{\tau}\right)$. (See the lectures of Ref.[3] for an introduction). However, the model forms only weakly bound states of quarks (solitons) with hedgehog shaped fields [45-47]. The reason is that, in this model, the Dirac sea yields a greater repulsion to the hedgehog field than is suggested by the kinetic term of the σ-model lagrangian (2.1). Soliton sizes are too small for the gradient expansion to be valid [48,49] so that the assimilation of the NJL model to a linear σ-model [14] is dangerous. The weak binding makes it unreliable to evaluate distortions of the soliton embedded in a dense medium so that we are not much better of than in the σ-model with the Dirac sea included.

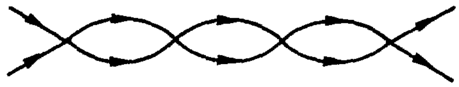
The main difference between the σ-model (2.1) and the Nambu Jona-Lasinio model (2.2) is the absence of a kinetic term for the meson fields which are pure $q\bar{q}$ excitations. The potential term of the linear σ-model keeps the fields on the chiral circle and does not affect significantly the nucleon properties [50] so that nucleon properties do not change much in the limit $\kappa \Rightarrow \infty$ of the non-linear σ-model. This also means that κ is not a parameter on which nucleon properties can be fitted. If the pion decay constant f_π is fixed to its measured value 93 MeV, then the nucleon properties depend only on one parameter, the coupling constant g. It is found [50] that with values $g \approx 5$-6 nucleon masses of about 1 GeV are obtained and that the axial coupling constant is $g_A \approx 1.5$, closer to the non-relativistic quark model value than to the Skyrme type model prediction of about 0.7-0.9 [51]. But g_A may be adjusted by adding an extra term to the lagrangian [52,46].

The Nambu Jona-Lasinio model has fewer parameters than the linear σ-model because each field has only one associated mass parameter but no coupling constant because the fields have arbitrary normalisation. However, a cut-off Λ needs to be introduced to regularize the Dirac sea constribution and this adds one parameter, making nucleon properties depend on the same number of parameters as in the σ-model. However, fewer parameters occur when vector fields are introduced [4,5,43,13].

We do not yet know whether the inclusion of vector fields in the Nambu Jona-Lasinio lagrangian will yield more binding for the quarks. The only available calculation [53] does show attraction obtained from the ρ and a_1 fields, as expected from previous gauged σ-model calculations [41], but it omits the repulsion of the ω-meson.

Baryons as bound quark-diquark composites.

The weak binding obtained with hedgehog fields in the Nambu Jona-Lasinio model need not be a weakness of the model. Indeed alternative consfigurations may turn out to be more bound. For example, diquarks may form sufficiently bound states [54,55,34,35] for the nucleon to prefer a quark-diquark configuration [55,34] . The original argument in favor of quark-diquark configurations comes from the observation that mesons and baryons have parallel Regge trajectories. Indeed a pair of quarks, coupled to form a color triplet, can be seen from afar by another quark as a color source identical to an anti-quark and, together, they may be bound by gluons as quark-antiquark pairs are in mesons. Nucleons and mesons formed by light quarks would then appear more like color strings [56] and would yield parallel Regge trajectories. The calculation of a nucleon is then achieved in two steps: the calculation of a bound diquark and the subsequent coupling of the diquark to a third quark. Diquarks are usually calculated by solving a Bethe-Salpeter equation which sums the ladders:



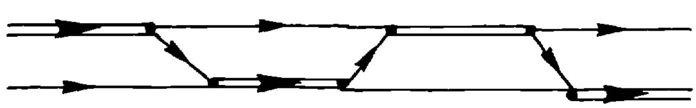
(2.3)

(the same which occur in the study of pairing vibrations) and complex poles of the diquark propagators indicate an instability of the assumed vacuum againt condensation of diquark pairs as in superconductivity [30].

Diquarks have been calculated with Nambu Jona-Lasinio type lagrangians [54] as well as with lagrangians originating from gluon exchange [55,34,44]. Considerable binding of the order of 300 MeV is obtained for a scalar color-triplet diquark in the Nambu Jona-Lasinio model when a 300 MeV constituent mass is used [54]. A diquark bound state can be simply expressed in terms of an auxiliary field which is used to bosonize the Nambu Jona-Lasinio action in the usual way. Auxiliary fields can be used to represent diquark qq pairs as well as quark-antiquark $q\bar{q}$ pairs. Condensation of the $q\bar{q}$ pairs leads to the usual chiral symmetry breaking and to the associated self-energy giving constituent quarks their mass. Condensation of the diquark pairs leads to spontaneous breaking of baryon number and color in the vacuum.

However the method of auxiliary fields suffers from an ambiguity related to the choice of either qq or $q\bar{q}$ pairs with which one decides to linearize the 4-fermion interactions. In Ref.[54] the scalar interactions are bosonized in terms of $q\bar{q}$ pairs and the vector interactions in terms of (diquark) qq pairs. Such ambiguities may however be avoided by a resummation of Feynmann diagrams, in which one is free to choose the self energies [34].

Whether the quark-diquark representation of the nucleon [55,35] is an improvement remains to be seen. Indeed several problems must be faced. The third quark may interact with the diquark by processes such as [35]:



(2.4)

and such interactions may well destroy the correlations which bind the diquark. Furthermore, it is not at all clear that the Nambu Jona-Lasinio model will yield sufficient binding for the quark-diquark system describing the nucleon. The binding could depend on the confining forces which are absent in the lagrangian. Finally it is also possible that, once the quark-diquark interactions are taken into account, the difference between this representation and any other way of coupling three quarks to form a nucleon, will be smeared out in the nucleon ground state.

3. Scale invariance and the nature of scalar mesons.

Because the vacuum is a 0^+ state, scalar fields are usually introduced to represent vacuum expectation values. Quite often, we only use the vacuum expectation value and not the full dynamics of the corresponding scalar meson. For example, it is the vacuum expectation value $\langle \sigma \rangle = f_\pi$ which is used in the σ-model for pion and soliton calculations and it makes little difference whether they are made with the linear or non-linear σ-model. As long as the fields remain on the chiral circle, meaning that $\sigma^2 + \pi^2 = f_\pi^2$, we are not sensitive to the σ-meson mass parameter κ. Similarly, in the standard model, only the average value ($\approx$ 250 GeV) of the Higgs field is used and the Higgs mass is not predicted. However the hunt for the Higgs is open and, similarly, we would like to identify the meson associated to the scalar field occuring in low-energy chiral models. This is important for understanding what process controls partial or complete chiral symetry restoration in dense and hot matter. In the linear σ-model (2.1) the nature of the scalar field σ is not specified. In the Nambu Jona-Lasinio model (2.2) the scalar field is $\varphi = -\langle \bar{q}q \rangle / a^2$ and it represents a quark condensate. It is an order parameter measuring the amount of chiral symmetry breaking. However this may not be the whole story.

Scalar fields need not be order parameters associated to a spontaneously broken symmetry. They may, for example represent quadratic fluctuations of some field in the vacuum. It has been suggested by the Syracuse group [57-59,50] to associate the scalar field appearing in low-energy chiral lagrangians to the QCD scale anomaly [60]. This is done done in by introducing a new scalar field which represents quadratic fluctuations of the gluon field (gluonium) in the vacumm. This idea has recently become rather fashionable [16,61].

The discussion is made easier by rewriting the chiral field in the form:

$$\sigma + i\vec{\pi}.\vec{\tau} \equiv \varphi U \qquad U = e^{i\vec{\theta}.\vec{\tau}}$$

$$\sigma + i\gamma_5\vec{\pi}.\vec{\tau} \equiv \varphi U_5 \qquad U_5 = e^{i\gamma_5\vec{\theta}.\vec{\tau}} \tag{3.1}$$

Consider the linear σ-model lagrangian (2.1) written with this form:

$$\mathcal{L} = \bar{q}(i\partial_\mu\gamma_\mu - g\varphi U_5)q + \frac{\varphi^2}{4}\,\mathrm{Tr}(\partial_\mu U)(\partial^\mu U^+)$$
$$+ \frac{1}{2}(\partial_\mu\varphi)(\partial^\mu\varphi) - \frac{\kappa^2}{8}\left(\varphi^2 - f_\pi^2\right)^2 \tag{3.2}$$

In the limit $\kappa \Rightarrow \infty$ the field φ acquires the value f_π at all points x and the lagrangian (3.2) reduces to the the non-linear σ-model represented by the first line in which φ is replaced by f_π.

We now follow Refs.[57,58,50], where a more detailed discussion can be found, and we consider the QCD trace anomaly. In a scale transformation the quark and gluon fields transform as follows:

$$A_{\mu a}(x) \Rightarrow \lambda A_{\mu a}(\lambda x) \qquad q(x) \Rightarrow \lambda^{3/2}q(\lambda x) \tag{3.3}$$

The scale transformation leaves the QCD action invariant. However the divergence of the current s_μ, associated to the scale transformation (3.3), does not vanish because an anomaly occurs. One finds that the divergence is equal to [60]:

$$\partial_\mu s^\mu = \frac{\beta(g)}{g}\langle F_{\mu\nu}F_{\mu\nu}\rangle \tag{3.4}$$

where $\langle\ \rangle$ means a vacuum expectation value. We introduce a scalar field χ to represent the right-hand side of (3.4):

$$\chi^4 \equiv \langle F_{\mu\nu}F_{\mu\nu}\rangle \tag{3.5}$$

The field χ is called the *gluonium field* and it is assumed to have scaling dimension 1 so that it tranforms as:

$$\chi(x) \Rightarrow \lambda\chi(\lambda x) \tag{3.6}$$

The idea is to write down a lagrangian for χ, such that the divergence of the current associated to the scale transformation (3.6) satisfies the equation:

$$\partial_\mu s^\mu = \chi^4 \tag{3.7}$$

Such as lagrangian is:

$$\mathcal{L} = \frac{a}{2}(\partial_\mu \chi)(\partial^\mu \chi) - \chi^4 \ln\frac{\chi}{\Lambda} \tag{3.8}$$

where Λ is a mass parameter.

The lagrangian (3.8) represents pure gluonium and it should describe the vacuum in the absence of quarks. The potential term has a minimum at $\chi_0 = \Lambda/(e)^{1/4}$ as shown by the dotted line of Fig.1. Small amplitude vibrations about this minimum represent a glueball with squared mass $m_\chi^2 = 4\chi_0^2/a$. It has been estimated using QCD sum rule methods [62] that $\chi_0 \approx 350$ MeV. If the scalar glueball is expected to have a mass in the range 800-1600 MeV [61] then the dimensionless constant a lies in the range 0.2-0.75.

Consider the scale invariance of the σ-model action obtained from the lagrangian (3.2). Assume that the field φ has a scaling dimension 1 and that the chiral angle θ, appearing in U, has a scaling dimension zero:

$$\varphi(x) \Rightarrow \lambda\varphi(\lambda x) \qquad \theta(x) \Rightarrow \theta(\lambda x) \tag{3.9}$$

Then all the terms of (3.2) yield a scale invariant action, except for the potential term $(\kappa^2/8)\left(\varphi^2 - f_\pi^2\right)^2$. The latter can be made scale invariant if we replace f_π by a term proportional to the gluonium field χ, thus yielding a potential term of the form $(\kappa^2/8)(\varphi^2 - R\chi^2)^2$ where R is a constant. This yields a coupling between the gluonium field χ and the field φ, which we call the quarkonium field. Adding the pure gluonium lagrangian (3.8), we obtain the "scaled" σ-model lagrangian:

$$\mathcal{L} = \bar{q}(i\partial_\mu\gamma_\mu - g\varphi U_5)q + \frac{\varphi^2}{4}\mathrm{Tr}(\partial_\mu U)(\partial^\mu U^+)$$
$$+ \frac{1}{2}(\partial_\mu\varphi)(\partial^\mu\varphi) - \frac{\kappa^2}{8}(\varphi^2 - R\chi^2)^2 + \frac{a}{2}(\partial_\mu\chi)(\partial^\mu\chi) - \chi^4\ln\frac{\chi}{\Lambda} \tag{3.10}$$

Note that, in this form, chiral symmetry breaking is driven by the gluonium potential term $\chi^4 \ln(\chi/\Lambda)$. This will also be the case in the examples described below.

In limit $\kappa \Rightarrow \infty$ of the non-linear σ-model, we have $\chi = \varphi/R$ and $f_\pi = R\chi_0$. Thus $\varphi = f_\pi \chi/\chi_0$ and we obtain a "scaled" non-linear σ-model [50]:

$$\mathcal{L} = \bar{q}\left(i\partial_\mu\gamma_\mu - gf_\pi \frac{\chi}{\chi_0} U_5\right)q + \frac{f_\pi^2}{4}\frac{\chi^2}{\chi_0^2} Tr(\partial_\mu U)(\partial^\mu U^+) + \frac{a}{2}(\partial_\mu\chi)(\partial^\mu\chi) - \chi^4 \ln\frac{\chi}{\Lambda} \tag{3.11}$$

This scaled non-linear σ-model is in fact quite similar to the linear σ-model. Indeed, the only difference is the shape of the potential term for χ in (3.11) and for φ in (3.2). They both have a minimum at a non-vanishing value of the respective scalar fields, so that, provided the scalar fields do not deviate too much from their value at the minimum, the dynamics are similar. Actually this is the whole idea of the form (3.11). In the non-linear σ-model, the fields are not allowed to deviate from the chiral circle. The lagrangian (3.11) is a model in which it is assumed that deviations from the chiral circle are controlled by the potential term of the gluonium field. Whether this is strictly true or whether actually both the quarkonium and gluonium fields φ and χ exist is interesting physics and it is discussed in Ref.[50,57,59]. In many instances, the scaled non-linear σ-model, as well as the scaled Skyrme model, discussed below, yield results which are similar to the linear σ-model.

All but the last term of (3.11) may be obtained by modifying the non-linear σ-model terms with a field χ, carrying scale dimension 1, so as to make each term scale invariant. This is exactly how the Skyrme lagrangian or any other lagrangian can be modified, so as to saturate the divergence of the scaling current s_μ with a single gluonium field. For example, the scaled Skyrme lagrangian has the form [58]:

$$\mathcal{L} = \frac{f_\pi^2\chi^2}{4\chi_0^2} tr(\partial_\mu U)(\partial^\mu U^+) + \frac{1}{32e^2} tr \left[(\partial_\mu U)U^+, (\partial_\nu U)U^+\right]^2 + \frac{a}{2}(\partial_\mu\chi)(\partial^\mu\chi) - \chi^4 \ln\frac{\chi}{\Lambda} \tag{3.12}$$

Current quark mass terms can of course be added. The form (3.12) is then the one used in the more recent applications where chiral symmetry restoration [61] and the variation of meson masses [16] are studied.

Scaled Skyrme-like lagrangians have been considered as a possible explanations of bag formation. The measure of bag formation is the ratio χ/χ_0 which should vanish or at least decrease inside a bag. The bag formation depends on the vacuum expectation χ_0, which is estimated to be 340 MeV [62]. Shallow bags are formed with this value and deeper bags are formed for smaller values. Solitons and bag formation with scaled Skyrme-like lagrangians with vector fields have also been calculated [63]. Rather shallow bags and a better, although still too small value $g_A \approx 0.9$ are obtained. It should be stressed that this bag formation does not mean confinement. Indeed, the quarks still have a finite mass outside the soliton.

There is another way of understanding the scaled Skyrme lagrangian (3.12). A soliton with winding number B=1 (such as a nucleon), constructed from this lagrangian, has an energy proportional to $(f_\pi \chi)/(e\chi_0)$. This means that the nucleon has a mass which is proportional to the gluonium field χ. This is exactly what is expressed by a scaled meson-nucleon lagrangian such as (1.12). The question raised here is: at which level should we scale the lagrangians? At the quark level as suggested in Refs.[50,57,59] or at the meson-nucleon (or Skyrme) level? If we decide to scale at the meson-nucleon level, then we can effectively reconcile the Nambu process of spontaneous chiral symmetry breaking with the scaling. The scaling of the meson mass terms simply mocks up the change in the meson mass due to the change in the constituent quark mass, as illustrated in (1.11). There remains however one difference. In the Nambu Jona-Lasinio lagrangian, the chiral symmetry is regulated by the Dirac sea quark mass. In the scaling lagrangians it is regulated by the term $-\chi^4 \ln(\chi/\Lambda)$ designed to saturate the scale anomaly with the single field χ. Is this important? Notice also that in the scaled lagrangians, the coupling of the meson fields to the fermions is unaltered and does not change with density. This should be checked.

Conclusion.

Nothing of what I have said is really profound. On the contrary I feel that much progress can be achieved in the study of low energy hadronic processes once it is realized that even the best physicists are more inclined to do what they can than what they ought to do. The reason is that they do not, in spite of so many claims to the contrary, really know what they should be doing. In such a situation it is better for Koestler's *sleepwalkers* to regain consciousness and to face squarely the fact that low energy hadronic physics is difficult and ambiguous. This is also true of experimental hadronic physics. Thank God because, there at least, the source of Query Confusion and Doubt about low energy QCD is not in our mind but is embedded somewhere in the apparatus. Where it should be.

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