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CALCULATION OF THE ODDERON INTERCEPT IN PERTURBATIVE QCD

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ABSTRACT

By using a variational method combined with conformal invariant techniques we show that the Odderon J-plane singularity in the leading logarithmic approximation of QCD lies above 1 ($\alpha_{Odd} \geq 1 + 0.28\ (g^2/\pi^2)$).

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There was and still there is a widespread belief in the equality of hadron-hadron and antihadron-hadron cross-sections at very high energies. This belief does not have rigorous, axiomatic foundations: it is mainly based upon the Regge pre-QCD phenomenology (see, e.g., ref. [1]). In 1973 this belief was questioned from the point of view of asymptotic theorems [2], generating the Odderon approach. The Odderon appears as a singularity of the t-channel partial wave with odd signature and $C = -1$, located near $J = 1$. QCD offers a natural dynamical framework for the occurence of this singularity as the result of the exchange of n reggeized gluons between colliding particles (in the simplest case, $n = 3$) with appropriate quantum numbers.

In a previous paper, we studied the conformal properties of the Odderon in QCD and we formulated a variational method for estimating the value of the bare-Odderon intercept [3]. However, till now, the question whether the Odderon intercept exceeds 1 or not was unanswered. This question is, of course, very important from theoretical, phenomenological and experimental points of view and, in particular, concerns the significance of the already present experimental signals in favour of the Odderon [4, 5]. In this paper we precisely tackle this question and we show that the Odderon intercept in the leading logarithmic approximation (LLA) of QCD exceeds 1. Therefore in the perturbative theory the difference between hadron-hadron and antihadron-hadron interactions grows with the energy.

The method in searching for the Odderon intercept was already described in ref. [3] (and references quoted therein, especially ref. [6]). We rely upon the conformally invariant technique developed in refs.[3] and [7] and use the functional depending on Odderon wave functions. Its maximal value is equal to the Odderon intercept [3]. For the Odderon, considered as a compound state of three reggeized gluons, the wave function $\psi_{m,\tilde{m}}$ in impact-parameter space depends on only one anharmonic ratio:

$$x = \frac{\rho_{12}\rho_{30}}{\rho_{10}\rho_{32}} \quad , \tag{1}$$

where $\rho_{ij} = \rho_i - \rho_j$, $\rho_i (i = 1, 2, 3)$ are complex coordinates of the gluons in the 2-dimensional impact-parameter space and ρ_0 is the coordinate of the Odderon with quantum numbers m and $\tilde{m}$ [3].

The function $\psi_{m,\tilde{m}}$ belongs to an irreducible representation of the conformal group with conformal weights m and $\tilde{m}$. This representation is unitary for $m = 1/2 + i\nu + n/2$, $\tilde{m} = 1/2 + i\nu - n/2$, where ν is real and the conformal spin n is integer.

The functional S of $\psi_{m,\tilde{m}}$, whose maximal value is proportional to $\omega = \alpha - 1$ (where α is the Odderon intercept) is given in our previous paper (see eqs. (12)-(16) of ref. [3]). For convenience of numerical calculations we perform the Fourier transform of the functions $\alpha(x, x^*)$ and $\beta(x, x^*)$ (see eqs. (13) and (14) of ref.[3])

$$\alpha_r(\sigma) = \int \frac{d^2x}{|x|}|x|^{-2i\sigma}\left(\frac{x}{x^*}\right)^{-r/2}\alpha(x, x^*)$$

$$\beta_r(\sigma) = \int \frac{d^2x}{|x|}|x|^{-2i\sigma}\left(\frac{x}{x^*}\right)^{-r/2}\beta(x, x^*) \tag{2}$$

and rewrite the functional in the following diagonalized form

$$S = \int_{-\infty}^{+\infty} d\sigma \sum_{r=-\infty}^{+\infty}\left[\frac{1}{2}|\alpha_r(\sigma)|^2 + \frac{1}{2}|\beta_r(\sigma)|^2\right]\chi_r(\sigma), \tag{3}$$

where the normalization condition for $\alpha_r(\sigma)$, $\beta_r(\sigma)$ is

$$\int_{-\infty}^{+\infty} d\sigma \sum_{r=-\infty}^{+\infty}\left[\frac{1}{2}|\alpha_r(\sigma)|^2 + \frac{1}{2}|\beta_r(\sigma)|^2\right] = 1 \tag{4}$$

and the functions $\chi_r(\sigma)$ are given below

$$\chi_r(\sigma) = -2Re\left[\psi\left(\frac{1}{2} + i\sigma + \frac{|r|}{2}\right) - \psi(1)\right] \tag{5}$$

in terms of the digamma function:

$$\psi(z) = \Gamma'(z)/\Gamma(z). \tag{6}$$

After integration by parts we can express $\alpha_r(\sigma)$ ($\beta_r(\sigma)$) (see eq. (2)) in the form:

$$\alpha_r(\sigma) = \int_0^{\infty}\frac{d|x|}{|x|^2}|x|^{-2i\sigma}\int_0^{2\pi}d\phi\, e^{-i\phi r}$$
$$\cdot\left[i\sigma + \frac{r}{2} + \frac{1}{2} + x\left(m - i\sigma - \frac{r}{2} + \frac{1}{2}\right)\right]\cdot\left(i\sigma - \frac{r}{2} + \frac{1}{2}\right)$$
$$\cdot\left(-\tilde{m} + i\sigma - \frac{r}{2} + \frac{1}{2}\right).Z(x, x^*) \tag{7}$$

where

$$Z(x, x^*) = [x(1-x)]^{\frac{m}{3}}[x^*(1-x^*)]^{\frac{\tilde{m}}{3}}\psi_{m,\tilde{m}}(x, x^*), \tag{8}$$

$\beta_r(\sigma)$ being obtained from $\alpha_r(\sigma)$ by the substitution $m \leftrightarrow \tilde{m}$, $x \leftrightarrow x^*$.

We consider in the following the most important case $n = 0$ and $\nu = 0$, i.e. $m = \tilde{m} = 1/2$. which corresponds to the rightmost singularity in the J-plane. The intercept of the Odderon is expressed in terms of the maximum value of the functional S:

$$\omega_{Odd} = \frac{g^2}{8\pi^2} 9 S_{max} \tag{9}$$

over the class of functions which are normalized according to eq. (4) and are invariant under the modular transformations: $x \to 1 - x$, $x \to 1/x$. We chose the simplest trial function, which satisfy these constraints and has the correct small-x asymptotics in agreement with formulae (20) and (21) of ref. [3]:

$$\psi_{m,\tilde{m}}(x, x^*) = C \Big\{ [a(x, x^*)]^{\frac{1}{3}(m+\tilde{m})} + C_1 [a(x, x^*)]^{1-\frac{1}{6}(m+\tilde{m})} [C_2 - 2 \ln a(x, x^*)] \Big\} \tag{10}$$

for the case $m = \tilde{m} = 1/2$. Here

$$a(x, x^*) = \frac{|x|^2 |1 - x|^2}{(|x|^2 + 1)(|1 - x|^2 + 1)(|x|^2 + |1 - x|^2)} \quad . \tag{11}$$

The constant C is fixed by the normalization condition (4). The constants C_1 and C_2 are determined from the condition of the maximality of the functional S. Our best numerical result corresponds to the following values: $C_1 = -1/3$ and $C_2 = 1$. For these values of the constants the function $|\alpha_r(\sigma)|^2$ is strongly peaked around $\sigma = 0$, $r = 0$ and is very small everywhere beyond $|\sigma| = 0.4$. The corresponding numerical value of S is approximately $S \simeq 0.25$ and therefore from eq. (9) we get:

$$\alpha_{Odd} \geq 1 + 0.28 \frac{g^2}{\pi^2} \quad . \tag{12}$$

Thus, the Odderon intercept exceeds 1.

By taking into account that $\alpha_{Pom} = 1 + (g^2/\pi^2) 3 \ln 2$ (see eq. (19) of ref. [3]), we rewrite the inequality (12) in the form:

$$\alpha_{Odd} - 1 \geq 0.13 (\alpha_{Pom} - 1). \tag{13}$$

For example, for the typical value of the perturbative QCD Pomeron intercept $\alpha_{Pom} = 1.5$ we get $\alpha_{Odd} \geq 1.07$.

Details concerning the numerical procedure in obtaining the value of S and the study of its stability will be given elsewhere.

Strictly speaking, our result is valid only at large t (say $|t| \geq 5\ GeV^2$), where the asymptotic freedom in QCD is applied. In LLA the Odderon is a fixed square-root singularity in the J-plane. The effect of running the QCD coupling constant leads to a change in the form of this singularity: one obtains a sequence of Regge poles condensing to the point $J = 1$. The average distance between two such poles is determined by the coefficient in front of ν^2 in the expansion of $\omega(\nu)$ near $\nu = 0$ [6]. We estimated numerically this coefficient for our trial function and obtained

$$S \simeq 0.25 - 13\nu^2 \tag{14}$$

which should be compared with the Pomeron case, where

$$S \simeq 4\ln 2 - 14\zeta(3)\nu^2, \tag{15}$$

$\zeta(z)$ being the Rieman function.

It seems that the excess of the Odderon intercept over 1 is much less that the one for the Pomeron intercept (see (13)) and therefore it could be assumed that the unitarization effects for the Odderon are less important that for the perturbative Pomeron. Hence the renormalized singularity in J-plane could be the maximal Odderon [2,8].

The theoretical status of the Odderon is now very firm not only in the perturbative QCD theory but also in the non-perturbative approach: it was recently shown that the maximal Odderon approach is consistent with the Quantum Field Theory constraints [9].In any case, we see no fundamental reason for the decoupling of the Odderon in the forward direction.

On experimental level one expects, in the immediate future, an important result indicating the presence or the absence of Odderon effects in the non-perturbative region: a high-precision measurement of the ρ-parameter (the ratio of the real part over the imaginary part of the forward hadron amplitude) at $\sqrt{s} = 546$ GeV [10]. A recent almost model-independent analysis of the existing data shows that ρ-values in the range $0.15 - 0.23$ would be a signal of Odderon effects [11].

Our result showing that in the perturbative theory the difference between hadron-hadron and antihadron-hadron interactions grows with the energy points towards the possibility of a large number of new experimental effects at very high energy. Therefore there is urgency of adopting both p and $\bar{p}$ options at future accelerators like LHC or SSC.

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