

1894-11-13

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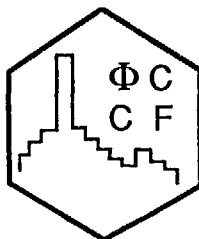
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**PATTERN RECOGNITION: INVARIANTS IN 3D**  
Application to tag the number of jets in  $e^+e^-$  events  
with a neural network

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Presented at the Second Workshop on Neural Networks:  
From biology to High Energy Physics  
Marciana Marina (IT), 18-26 June 1992

PCCF RI 9213

SECOND WORKSHOP ON NEURAL NETWORKS:  
FROM BIOLOGY TO HIGH ENERGY PHYSICS  
Marciana Marina (Isola d'Elba) June 1992

PATTERN RECOGNITION: INVARIANTS IN 3D  
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*Abstract.* We define invariants for 3D objects with a spherical symmetry. These new invariants are used to tag the number of jets in  $e^+e^-$  events.

#### 1-INTRODUCTION

In pattern recognition, some 2D invariants were used to solve some rotation invariant problems<sup>1</sup>.

But in  $e^+e^-$  events, the jets have a spherical 3D symmetry. The usual method to tag the number of jets is the clusterization using classical algorithms. An attempt to use a neural network was done by Jousset<sup>3</sup> using an Hopfield network.

In this paper we propose a set of 3D invariants which can be used to tag the number of jets in  $e^+e^-$  events.

#### 2-INVARIANTS IN 3D ROTATION

The angles used in 3D rotation are  $\vartheta$  and  $\phi$ ; we know the spherical orthogonal functions  $Y_{lm}(\vartheta, \phi)$ <sup>3</sup>.

A function  $f(\vartheta, \phi)$  can be developed using  $Y_{lm}(\vartheta, \phi)$  functions.

$$f(\vartheta, \phi) = \sum_l \sum_m \alpha_{lm} Y_{lm}^*(\vartheta, \phi) \dots$$

If the axis are rotated with  $\alpha, \beta, \gamma$  Euler angles the rotation angles become  $\vartheta'$  and  $\phi'$ . The new development is

$$f(\vartheta, \phi) = \sum_l \sum_m \alpha'_{lm} Y_{lm}^*(\vartheta', \phi').$$

But we know a relation between  $\vartheta, \phi$  and  $\vartheta', \phi'$  angles<sup>3</sup>.

$$Y_{lm}(\vartheta', \phi') = \sum_{m'} \mathcal{D}_{m', m}^l(\alpha, \beta, \gamma) Y_{lm'}(\vartheta, \phi),$$

where the  $\mathcal{D}$  functions are the usual orthogonal functions with Euler angles.

The relation between  $\alpha'$  and  $\alpha$  is then:

$$\alpha'_{lm} = \sum_{m''} \mathcal{D}_{m'', m}^l(\alpha, \beta, \gamma) \alpha_{lm''}.$$

Using the orthogonal relation<sup>3</sup>

$$\sum_m \mathcal{D}_{m'', m}^{l*}(\alpha, \beta, \gamma) \mathcal{D}_{m', m}^l(\alpha, \beta, \gamma) = \delta_{m', m''},$$

we see that the combination of the  $\alpha$  values

$$\beta(l) = \sum_m \alpha_{lm} \alpha_{lm}^*$$

is invariant for a 3D rotation.

The  $\alpha$  parameters are computed with the relation

$$\alpha_{lm} = \int f(\vartheta, \phi) Y_{lm}(\vartheta, \phi) d\Omega,$$

using the associated Legendre functions  $P_m^l(\cos\vartheta)$  we write for  $\alpha$

$$\alpha_{lm} = (-)^m \left[ \frac{(2l+1)(l-m)!}{4\pi(l+m)!} \right] \int P_m^l(\cos\vartheta) e^{im\phi} f(\vartheta, \phi) d\Omega.$$

The contributions to  $\beta(l)$  of opposite  $m$  values are identical.

### 3-SIMULATION OF 3 CLASSES

We have simulated 3 classes of different simple objects with dots on a sphere.

Classe 1:2 dots at  $\vartheta_1=0, \phi_1=0$  and  $\vartheta_2=\pi, \phi_2=0$ .

Classe 2:3 dots at  $\vartheta_1=0, \phi_1=0$  ;  $\vartheta_2=2\pi/3, \phi_2=0$  and  $\vartheta_3=2\pi/3, \phi_3=\pi$ .

Classe 3:4 dots at  $\vartheta_1=0, \phi_1=0$  ;  $\vartheta_2=\pi, \phi_2=0$ ;

$\vartheta_3=\pi/2, \phi_3=0$ ;  $\vartheta_4=\pi/2, \phi_4=\pi$ .

We have verified the invariance of  $\beta(l)$  values with random Euler angles. When we introduce a noise in the positions of the dots, the  $\beta(0)$  value is discriminating for the classes, because

$$\beta(0) = \frac{1}{4\pi} (\sum_{\text{dots}} P_0^0)^2,$$

this value is independent of the noise.

### 4-TAGGING THE NUMBER OF JETS IN $e^+e^-$ EVENTS

We give preliminary results on a method to tag the number of jets in  $e^+e^-$  events without external parameter.

In LEP detectors, the tracks of an hadronic event are given by the momentum  $p_i$ , the  $\vartheta_i$  and  $\phi_i$  angular positions.

We choose as  $f$  function the relation:

$$f(\vartheta, \phi) = \sum_{i \text{ tracks}} p_i \delta(\cos\vartheta - \cos\vartheta_i) \delta(\phi - \phi_i)$$

then

$$\alpha_{lm} = (-)^m \left[ \frac{(2l+1)(l-m)!}{4\pi(l+m)!} \right] \sum_{i \text{ tracks}} p_i P_m^l(\cos\vartheta_i) e^{im\phi_i}.$$

We have generated 2, 3 and  $n$  jets events with the ALEPH Monte-Carlo simulation. The jets were clustered with the JADE algorithm using  $ycut = (6/E)**2$ .

We have trained a (15,30,3)MLP network with 15  $\beta$  parameters and 5000 events in each class.

The test was done with a sample with 26654 2 jets events, 24122 3 jets events and 5154  $n$  jets events.

We consider as true jets the jets generated by the JADE algorithm. The classification given by the MLP network gives:

The purity of a 2 jets sample is 87.4%,

The purity of a 3 jets sample is 81.7%,

The purity of a n jets sample is 50.9%.

The percentage of true 2 jets tagged as 2 jets is 88.7%,

The percentage of true 3 jets tagged as 3 jets is 70.9%,

The percentage of true n jets tagged as n jets is 78.8%.

## 5-CONCLUSION

We have computed a new family of 3D invariants. We have given some preliminary results to tag the number of jets in  $e^+e^-$  events.

The tagging of jets can be improved, because we have supposed that the JADE algorithm is perfect; it is also possible to use with the invariants some shape variables as the aplanarity which describes the 3 jets events.

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