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E. Suraud[1], S. Ayik[2], M. Belkacem[3†] and Feng-Shou Zhang[3,4]

[1]Groupe de Physique Théorique, Laboratoire de Physique Quantique,
Université Paul Sabatier, 118 route de Narbonne,
31062 Toulouse Cedex, France

[2]Tennessee Technological University, TN 38505 Cookeville, USA

[3]GANIL, BP5027, F14021 Caen Cedex, France

[4]Institute of Modern Physics, Academia Sinica, P. O. Box 31,
Lanzhou 730000, China

GANIL P 93 16

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Abstract

It is shown that the numerical simulations of the recently developed Boltzmann-Langevin model exhibit large dynamical fluctuations in momentum space during the early stages of heavy-ion collisions, which arise from an interplay between the nuclear mean-field and binary collisions. It is pointed out that this transient behaviour provides an initial seed for the development of density fluctuations, and could strongly influence the particle production cross-sections at subthreshold energies.

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†Present address: LNS, Viale Andrea Doria, Catania, Italy

1 Introduction

The Boltzmann-Uehling-Uhlenbeck (BUU) model has been extensively applied for describing a large variety of observables associated with nuclear collisions at intermediate energies [1]. The average description provided by the BUU model is well suited for describing processes involving small fluctuations. However, for processes involving large fluctuations, for example, near instabilities and bifurcations, such an average description is inadequate. In these situations the stochastic transport models provide a more appropriate starting point. The recently developed Boltzmann-Langevin(BL) model can be regarded as a semi-classical example of such a stochastic one-body transport theory [2, 3, 4], and it is therefore a promising model for describing catastrophic phenomena such as phase transitions and nuclear fragmentation. However, because of the numerical complexity, only a limited number of applications have been carried out so far [5, 6].

In this paper, we investigate the development of the momentum space fluctuations in heavy-ion collisions in the framework of the BL model, and discuss the role of these fluctuations on the dynamical evolution of the system. We calculate the time evolutions of the average value and the variance of the quadrupole moment of the momentum distribution, which provide a measure for dissipation and fluctuations in momentum space, respectively. In heavy-ion collisions with relative velocities comparable to the Fermi velocity, both the nuclear mean-field and the binary collisions play important roles. The numerical simulations of the BL equation in this energy regime indicate that large dynamical fluctuations are introduced into momentum space during the early, strongly dissipative stages of the collision process. This transient behaviour of the momentum distribution is consistent with the expectations from the fluctuation-dissipation theorem, according to which large fluctuations should occur at times when the dissipation rate is high. The large fluctuations in momentum space during the early stages provide the initial seed for developing density fluctuations in r-space, and also may greatly influence the pre-equilibrium processes such as particle production below threshold energies.

In section 2, we briefly describe the BL model and a projection method for obtaining numerical simulations of this model. In section 3, we present results of the calculations carried out for the collision of two carbon nuclei, and discuss the influence of the nuclear mean-field and binary collisions on momentum space fluctuations. In section 4, we present a simple model for illustrating the transient behaviour of the variance associated with the quadrupole moment of the momentum distribution. Finally, in section 5, we point out the role of the large dynamical fluctuations in momentum space on particle production below threshold energies and give some conclusions.

2 The Boltzmann-Langevin Description of Heavy Ion Collisions

Here, we briefly summarize the BL model and a projection method for its numerical simulations. For a detailed presentation of these topics, we refer the reader to refs. [2, 4, 5]. According to the BL model, the fluctuating phase space single-particle density $\hat{f}(\mathbf{r},\mathbf{p},t)$ is determined by

$$\left(\frac{\partial}{\partial t}+\frac{\mathbf{p}}{m}\cdot\nabla_r-\nabla_r U(\hat{f})\cdot\nabla_p\right)\hat{f}(\mathbf{r},\mathbf{p},t)=K(\hat{f})+\delta K(\mathbf{r},\mathbf{p},t) \qquad (1)$$

The left-hand-side describes the Vlasov propagation in terms of the fluctuating nuclear mean-field $U(\hat{f})$. On the right-hand-side, $K(\hat{f})$ is the usual collision term of the BUU form but expressed in terms of the fluctuating density $\hat{f}(\mathbf{r},\mathbf{p},t)$, and $\delta K(\mathbf{r},\mathbf{p},t)$ represents the fluctuating collision term, which arises from correlations not accounted for by the collision term. In analogy with the Brownian motion, it is assumed that eq. (1) describes a stochastic process, in which the entire single-particle density is a stochastic variable and the fluctuating collision term acts like a random force. In such a stochastic description the fluctuating collision term is characterized by a correlation function, which is assumed to be local in space-time, without memory effect,

$$\langle\delta K(\mathbf{r},\mathbf{p},t)\delta K(\mathbf{r}',\mathbf{p}',t')\rangle=C(\mathbf{p},\mathbf{p}')\delta(\mathbf{r}-\mathbf{r}')\delta(t-t') \qquad (2)$$

Here $\langle \cdots \rangle$ denotes the average performed over a set of single-particle densities generated during a short time interval, which corresponds to a local ensemble averaging [2, 5]. The reduced correlation function $C(\mathbf{p}, \mathbf{p}')$ can be explicity evaluated in the weak-coupling limit and it is completely determined by the locally averaged single-particle density, which is denoted by $f(\mathbf{r}, \mathbf{p}, t)$ in the following.

In principle, the numerical simulations of the BL equation can be carried out by employing standard methods for solving a typical Langevin equation. However, a direct simulation of this kind, for example employed in ref. [6], is not very practical and propagates too detailed information, which is not needed for describing the gross properties of density fluctuations. It may be sufficient to propagate the fluctuations associated to a few relevant variables. In ref. [5], a method has been proposed for obtaining approximate solutions of the BL equation by projecting the fluctuations on a set of .. w order local multipole moments of the momentum distribution,

$$\hat{Q}_L(\mathbf{r}, t) = \int d\mathbf{p} Q_L(\mathbf{p}) \hat{f}(\mathbf{r}, \mathbf{p}, t) \tag{3}$$

Where $Q_L(\mathbf{p})$ is the multipole moment operator of order L in momentum space. The fluctuations of the multipole moments are characterized by a diffusion matrix, which can be deduced from the microscopic correlation

$^{12}C + ^{12}C$, E/A=60 MeV/u, b=0 fm

function $C(\mathbf{p},\mathbf{p}')$ as

$$
\begin{aligned}
C_{LL'}(\mathbf{r},t) &= \int d\mathbf{p}d\mathbf{p}'Q_L(\mathbf{p})Q_{L'}(\mathbf{p}')C(\mathbf{p},\mathbf{p}') \\
&= \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 \Delta Q_L \Delta Q_{L'} \\
&\quad W(12,34) f_1 f_2 (1-f_3)(1-f_4) \qquad (4)
\end{aligned}
$$

where $\Delta Q_L = Q_L(\mathbf{p}_1) + Q_L(\mathbf{p}_2) - Q_L(\mathbf{p}_3) - Q_L(\mathbf{p}_4)$ and $W(12,34)$ denotes the same transition rates as in the collision term. The diffusion matrix $C_{LL'}$ contains a reduced information on the dynamics as compared to the microscopic correlation function $C(\mathbf{p},\mathbf{p}')$. However it is expected to provide a trustworthy description of the dynamics of the multipole moments $Q_L's$ and presumably of any observable related to them. Then, a single dynamical trajectory can be determined according to the following algorithm: (i) Starting from a definite density $\hat{f}(\mathbf{r},\mathbf{p},t)$ at a time t, its average evolution and the elements of the diffusion matrix are calculated during a time step Δt with the pseudo-particle simulations [7], yielding $f(\mathbf{r},\mathbf{p},t+\Delta t)$ and $C_{LL'}(\mathbf{r},t)$. (ii) In the second step, the fluctuations of the multipole moments are determined according to a multi-dimensional Langevin process,

$$
\hat{Q}_L(\mathbf{r},t+\Delta t) = Q_L(\mathbf{r},t+\Delta t) + \sum_{L'} \left(\sqrt{\Delta t C(\mathbf{r},t)}\right)_{LL'} W_{L'} \qquad (5)
$$

Here $Q_L(\mathbf{r},t+\Delta t)$ is the multipole moment associated with the locally averaged density $f(\mathbf{r},\mathbf{p},t)$, and the quantity in the second term is the square-root of the diffusioin matrix [8], multiplied by the independent gaussian random

$^{12}C+^{12}C$, E=60 MeV/u, b=0 fm

numbers $W_{L'}$ with unit variance and zero mean, for each multipole moment. (iii) Finally, fluctuations are inserted into phase-space by scaling the local momentum distribution to the new values of $\hat{Q}'_L s$, $f(\mathbf{r}, \mathbf{p}, t + \Delta t) \longrightarrow \hat{f}(\mathbf{r}, \mathbf{p}, t + \Delta t)$. This procedure is repeated at each time step and for an ensemble of events.

In the practical applications of this method, the multipole space must be truncated to a reasonable size. We expect that the propagation of the fluctuations in the lowest order approximation by the quadrupole Q_2 scaling should provide a good approximation for the gross properties of the density fluctuations in r-space. This follows from the observation that in a fluid dynamical reduction of the BL eq. (1), the evolution of density is coupled to the momentum flow tensor, which is nothing but the local quadrupole moment of the momentum distribution [9]. Therefore, we expect that density fluctuations are mainly influenced by the quadrupole moment of the momentum distribution and not very sensitive to the higher multipoles. In this case eq. (5) reduces to a simple, one dimensional Langevin process for Q_2,

$$\hat{Q}_2(\mathbf{r}, t + \Delta t) = Q_2(\mathbf{r}, t + \Delta t) + \sqrt{\Delta t C_{22}(\mathbf{r}, t)} W_2 \tag{6}$$

Propagating fluctuations in the quadrupole mode alone does not provide a detailed description of the momentum space fluctuations, but accurately describes the fluctuations associated with the quadrupole moment of the momentum distribution. This has been tested by comparing the calculations performed with the quadrupole Q_2 scaling and those done with the quadrupole plus octupole Q_2+Q_3 scaling [5]. It was observed that: (i) The mixed diffussion coefficient C_{23} is much smaller than the diagonal elements C_{22} and C_{33}. Consequently, C_{23} can be neglected and the Langevin equations for Q_2 and Q_3 become uncoupled. (ii) It essentially gives the same result for the variance σ_2 of the quadrupole moment when one uses Q_2 scaling or Q_2+Q_3 scaling. Therefore, in order to describe the fluctuations of the quadrupole moment of the momentum distribution, it is sufficient to propagate the fluctuations associated within this mode. It is also independently verified by the calculations carried out in ref. [6], that the quadrupole scaling alone provides a good description of the fluctuations of the quadrupole moment.

3 Fluctutions in Momentum Space: Transient Behaviour

We investigate the fluctuations introduced into momentum space in heavy-ion collisions by calculating the time evolution of the variance σ_2 of the quadrupole moment of the momentum distribution, which provides a good indicator for the magnitude of fluctuations [10]. In the calculations the BL events are determined by the quadrupole scaling, using so-called the global-soft prescription of ref. [5]. In this simplified treatment, eq. (6) is averaged over the vclume of the system and fluctuations are inserted globally into the momentum space, at each time step. As an example, Figure 1 shows the results of the calculations performed for central ^{12}C + ^{12}C collisions at a bombarding energy of 60 Mev/u. In this Figure, the ensemble averaged total quadrupole moment Q_2, the associated variance σ_2, and the diffusion coefficient $C_2 = C_{22}$ are plotted as a function of time. For completeness, the time evolution of the collision rate, and the average density defined as $<< \rho >>= \frac{1}{A} \int d\mathbf{r} \, \langle \int d\mathbf{p} \, \hat{f}(\mathbf{r}, \mathbf{p}, t) \rangle$ are also included in Figure 1. The averaged value of the quadrupole moment, shown in Figure 1a, exhibits a typical relaxation pattern. Also are plotted for further discussions the "one" and "two standard deviations" curves $Q_2 \pm \sigma_2$ and $Q_2 \pm 2\sigma_2$, which for a gaussian distribution would correspond to 84.3% and 99.5% of the number of events, respectively. The effects of the binary collisions do not show up in Q_2 immediately after touching, which occurs at about 5-10 fm/c in the Figure. Due to the effects of the mean-field, there is a time delay of about 15-20 fm/c, during which the averaged quadrupole moment does not show any damping, but in fact increases as a result of the initial compression and of the diabatic shift of the single-particle energies [11]. On the other hand, the variance σ_2 associated to the quadrupole moment, shown in Figure 1b, starts growing immediately after touching and reaches large values before the averaged quadrupole moment exhibits any sizeable damping. The diffusion coefficient

C_2 is contentrated during the early stages of the collision and its peak value is much larger than the asymptotic thermal background. As a result, the variance σ_2 exhibits a bumb during early stages of the collisions, which is a characteristic behaviour of a strong transient effect.

The timing and magnitude of momentum space fluctuations are strongly influenced by both the mean-field *and* the binary collisions. In order to investigate this interplay between mean-field and binary collisions, we have performed calculations by turning off either the mean-field, i.e. intranuclear cascade with fluctuations or the binary collisions i.e. Vlasov, and we have compared the results with those obtained with mean-field, two-body collisions and fluctuations included. Figure 2 shows a comparison between different calculations done for the time evolution of the averaged quadrupole moment Q_2, the associated diffusion C_2 and variance σ_2 and the average density $<< \rho >>$ of the system in central ^{12}C + ^{12}C collisions at a bombarding energy of 60 Mev/u. The calculations carried out in the BL model but without mean-field are indicated as INC + fluctuations. In addition, Figure 2 contains the result of calculations performed without incorporating the fluctuations. These calculations are performed at the mean-field level, with the mean-field plus collision term and in the framework of the intranuclear cascade without mean-field, and indicated as Vlasov, BUU and INC, respectively. As seen in Figure 2a, the averaged quadrupole moment in the Vlasov calculations exhibits monopole-type oscillations with a small damping. The results of BL and BUU calculations compare well, but in the INC calculations, the relaxation of the averaged quadrupole moment begins immediately after touching without any time dalay. In the INC calculations, since there is no mean-field, the system expands very rapidly without any initial compression, and hence the collision rate and the average density at any time, are much smaller than those obtained in the BL and BUU calculations. Consequently, the diffusion coefficients obtained in the INC calculations remain much smaller than those obtained in calculations with mean-field included. As a result the variance calculated in the INC framework even with fluctuations included are smaller than the variance calculated in the BL model, and it does not exhibit a pronounced transient behaviour.

One might argue that this behaviour only reflects trivial geometrical effects. A careful examination of the results plotted in Figure 1 shows that this argument is not founded and that no simple scaling exists between σ_2 and $<< \rho >>$ or the number of collisions per fm/c. The bump in σ_2 in the early times of the the collision hence reflects a subtle interplay between mean-field and two-body effects, provided fluctuations are properly included in the dynamics.

4 A Simple Model

In order to illustrate even more clearly the transient behaviour of the fluctuations associated with momentum space, we present a model calculation based on the relaxation time ansatz of the collision term in the BL eq. (1). We consider the total quadrupole moment of the momentum distribution,

$$\hat{Q}_2(t) = \int d\mathbf{r}d\mathbf{p}Q_2(\mathbf{p})\hat{f}(\mathbf{r},\mathbf{p},t) \tag{7}$$

Approximating the collision term in eq. (1) by a relaxation time ansatz, $K(\hat{f}) = -\frac{1}{\tau}(\hat{f}(\mathbf{r},\mathbf{p},t) - f_\infty(\mathbf{r},\mathbf{p}))$, where f_∞ is a Fermi-Dirac distribution and with τ as the relaxation time, we can derive an equation for the variance σ_2 of the total quadrupole moment. This yields

$$\frac{d\sigma_2^2}{dt} = -\frac{2}{\tau}\sigma_2^2 + C_2 \tag{8}$$

where C_2 is the diffusion coefficient associated with the total quadrupole moment of momentum. In eq. (8), the time delay caused by the mean-field is assumed to be incorporated into the relaxation time. In order to obtain a simple analytical description of the qualitative behaviour of the variance σ_2, we parametrize the diffusion coefficient by a step-function

$$C_2 = \theta(t-t_0)[(C_{2,1} - C_{2,2})\theta(t_0+\Delta-t) + C_{2,2}] \tag{9}$$

The parameters $t_0, C_{2,1}, C_{2,2}$ and Δ of the diffusion coefficient as well as the relaxation time τ can be extracted from the dynamical calculations presented in Figure 2. The solution of eq.(8) is trivial in each time interval where C_2 is constant. It reads

$$\sigma^2_{\sigma_0,C_2}(t) = (\sigma^2_{0,C_2} - \frac{1}{2}C_2\tau)e^{-\frac{2t}{\tau}} + \frac{1}{2}C_2\tau \tag{10}$$

where σ^2_{0,C_2} is the initial value of σ^2 in the time interval with diffusion coefficient C_2. Integrating over time from zero to infinity with C_2 given as in eq. (9) leads to the solution

$$\sigma^2(t) = \theta(t-t_0)[\theta(\Delta-(t-t_0))\sigma^2_{\sigma_0,C_{2,1}} + \theta((t-t_0)-\Delta)\sigma^2_{\sigma_0,C_{2,2}}] \tag{11}$$

with the notations introduced above. The first phase corresponds to a delayed increase of σ_2. The second one is either a further increase of σ_2 towards the asymptotic value $\frac{1}{2}C_{2,2}\tau$ if $\sigma^2_{0,C_{2,1}}(t_0+\Delta) < \frac{1}{2}C_{2,2}\tau$, or a decrease towards this value, otherwise. The condition for obtaining a bump in σ_2 hence simply reads $\sigma^2_{0,C_{2,1}}(t_0+\Delta) > \frac{1}{2}C_{2,2}\tau$. Further examining this condition allows to rewrite it as

$$1 - \frac{C_{2,2}}{C_{2,1}} > e^{-\frac{2\Delta}{\tau}}$$

Typical values of $C_{2,2}, C_{2,1}$ can be extracted from any BUU or BL simulation as already stressed and show that $C_{2,2}$ is always small as compared to $C_{2,1}$ which suggests to replace the term $1 - \frac{C_{2,2}}{C_{2,1}}$ by an exponential, so that the criterion for existence of the bump in σ_2 may be finally written in the simple form

$$\frac{2\Delta}{\tau} > \frac{C_{2,2}}{C_{2,1}}$$

which is more convenient for qualitative discussion. Typical values of $\Delta \approx 20 - 30 fm/c$, $\tau \approx 20 - 40 fm/c$ and $C_{2,2}/C_{2,1} \approx 0.05$ as can be seen from Figure 2, as soon as mean-field and two-body collisions are both taken into account in the calculation. Fluctuations do not affect these values as discussd in the previous section (see also Figure 2). The bump in σ_2 should hence be present in any reasonable simulation of the BL equation. Note also that when mean-field is turned off the ratio $C_{2,2}/C_{2,1}$ becomes much closer to 1 and the bump would indeed disappear.

For completeness the solution of eq.(8) is shown in Figure 3b for two cases, in which the parameters are extracted from the BL simulations and from the INC simulations with fluctuations. These simple calculations clearly illustrate that as a result of the interplay between mean-field and binary collisions, large dynamical fluctuations are introduced into the momentum space during the initial stages of the collision process.

5 Conclusions

In this paper, we have investigated the temporal evolution of the single-particle momentum distribution in heavy-ion collisions using the BL model. Employing a projection procedure, we obtain numerical simulations of the BL equation and calculate the time evolution of the averaged quadrupole moment and the associated variance of the momentum distribution, which provide a measure for the dissipation and fluctuation properties in energy-momentum space. In heavy-ion collisions with initial velocities comparable to the Fermi velocity, both the nuclear mean-field and the binary collisions play equally important roles, and a host of new phenomena are expected to occur. In this energy regime, as a result of an interplay between mean-field and binary collisions, the dynamical evolution of the system exhibits an interesting transient behaviour, which can be summerized of follows. (i)Due to the combined effects of the initial compression and diabatic shift of the single-particle energies, there is a time delay associated with the damping mechanism.The averaged quadrupole moment of the momentum distribution retains its initial value, even increases, after touching for a period of time during the penetration of the colliding ions.(ii)On the other hand, the variance associated with the quadrupole moment begins to grow immediately after touching and reaches large values when the magnitude of the averaged quadrupole moment is still large. This transient behaviour of the momentum space provides an initial seed for the subsequent development of the density fluctuations, and could also make an important effect on the particle production mechanism below threshold energies. When the fluctuations are large, i.e., during the time window where the variance of the quadrupole of the quadrupole moment is going through a maximum, a fraction of the BL events may have a sufficient energy for producing mesons at below threshold energies, which is otherwise not possible in the extended mean-field description of the BUU model [12]. This effect is illustrated in Figure 1a. As a matter of fact, in a recent work [13], the K^+ production at subthreshold energies in ^{12}C + ^{12}C collisions has been investigated in the framework of the BL model by incorporating only the fluctuations associated with the quadrupole moment of the momentum distribution. The calculations indicate that the production cross-sections obtained in the BL model are nearly an order of magnitude larger than those obtained in the BUU model. It would be interesting to carry out the similar calculations for heavier systems and for the production of other mesons in heavy-ion collisions.

Acknowledgments

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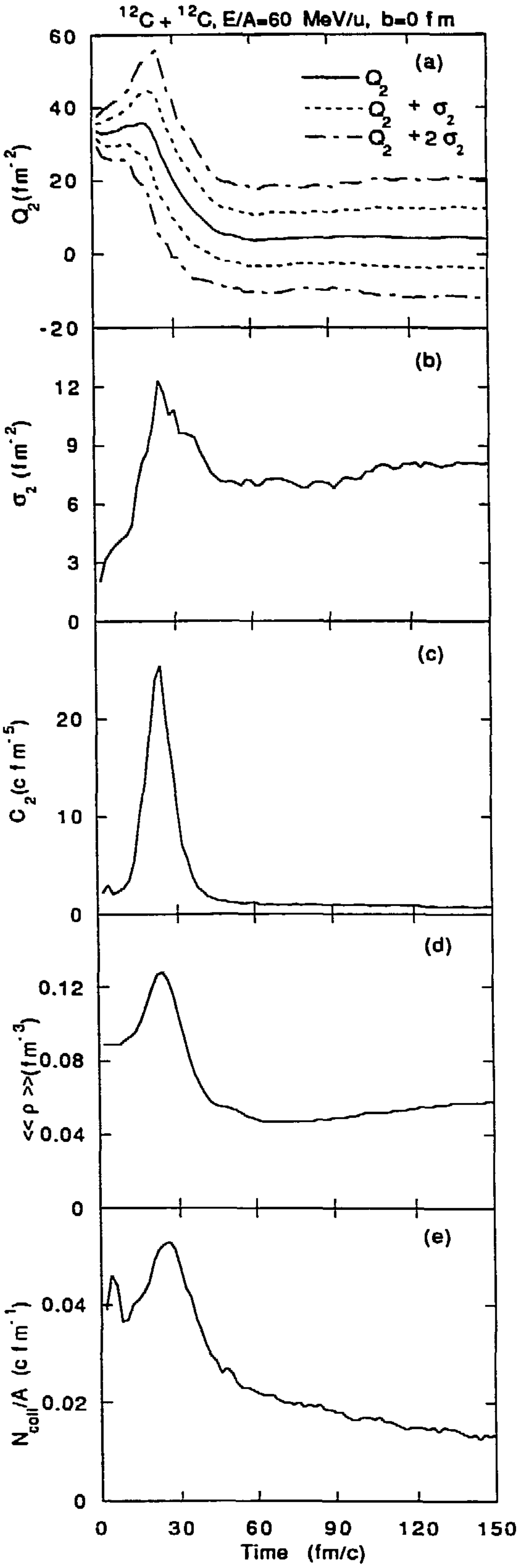
Figure Captions

Figure 1 Time evolution of the ensemble averaged quadrupole moment Q_2 (a) of the momentum distribution, the associated variance σ_2 (b) the diffusion coefficient C_2 (c), the average binary collision rate (d), the average density $<< \rho >>$ (e) for central $^{12}C + {}^{12}C$ collisions at a bombarding energy of 60 Mev/u. Calculations are performed in the BL model. In Figure (a) the two doted lines respectively correspond to $Q_2 \pm \sigma_2$ while the dashed-dotted lines correspond to $Q_2 \pm 2\sigma_2$.

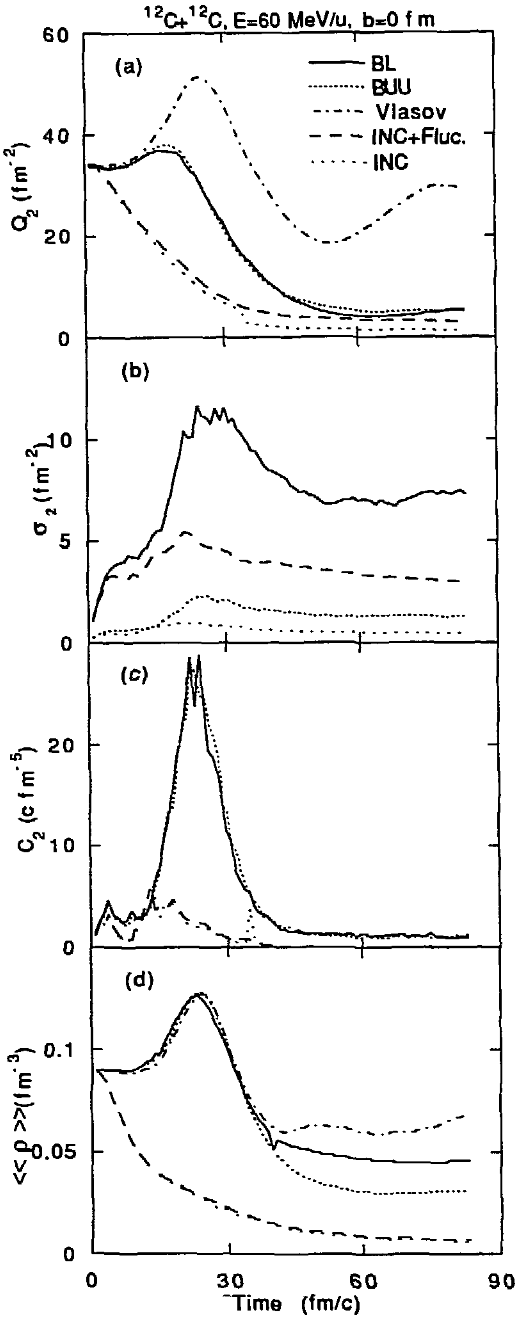
Figure 2 Same as Figure 1, but the calculations are performed in various models. The calculations performed in the BL, BUU, Vlasov, INC with and without fluctuations are indicated by solid lines, dotted lines, dashed-dotted lines, dashed lines and widely spaced dotted lines, respectively.

Figure 3 A schematic parameterization of the diffusion coefficient C_2 (a) and time evolution of σ_2 (b) as described by eq.(8) for central $^{12}C + {}^{12}C$ collisions at a bombarding energy of 60 Mev/u. Solid lines and doted lines correspond to the BL and INC with fluctuations, respectively. Parameters of the calculations are the following : $C_{2,1} = 28fm^{-4}$, $C_{2,2} = 1fm^{-4}$, $\Delta = 30fm/c$, $\tau = 38fm/c$ and $t_0 = 10fm/c$ in the case of BL ; $C_{2,1} = 3fm^{-4}$, $C_{2,2} = 1fm^{-4}$, $\Delta = 20fm/c$, $\tau = 23fm/c$ and $t_0 = 0fm/c$ in the case of INC with fluctuations.



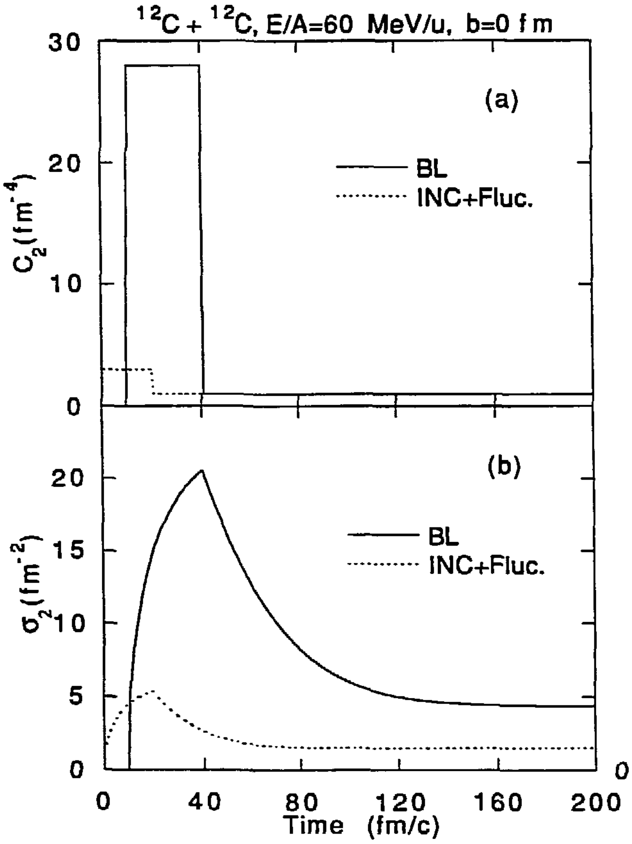
$^{12}C + ^{12}C$, E/A=60 MeV/u, b=0 fm
(a)
Q_2
$Q_2 + \sigma_2$
$Q_2 + 2\sigma_2$
$Q_2 (fm^{-2})$
(b)
$\sigma_2 (fm^{-2})$
(c)
$C_2 (c\,fm^{-5})$
(d)
$<<\rho>> (fm^{-3})$
(e)
$N_{coll}/A\ (c\,fm^{-1})$
Time (fm/c)

Fig. 1



$^{12}C+^{12}C$, E=60 MeV/u, b=0 f m
(a)
BL
BUU
Vlasov
INC+Fluc.
INC
Q_2 (f m^{-2})
(b)
σ_2 (f m^{-2})
(c)
C_2 (c f m^{-5})
(d)
<<ρ>> (f m^{-3})
Time (fm/c)

Fig. 2



$^{12}C + {}^{12}C$, E/A=60 MeV/u, b=0 fm
(a)
BL
INC+Fluc.
$C_2\,(fm^{-4})$
(b)
BL
INC+Fluc.
$\sigma_2\,(fm^{-2})$
Time (fm/c)

Fig.3