Microwave Reflectometry for Fusion Plasma Diagnostics

at

JET Joint Undertaking
4-6 March 1992

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Foreword

The technical reports in this collection of papers on "Microwave Reflectometry for Fusion Plasma Diagnostics" were presented at the International Atomic Energy Agency Technical Committee Meeting, with the same title, held at the JET Joint Undertaking in Abingdon, Oxon, the United Kingdom, March 4-6, 1992. The meeting was attended by 37 participants from ten countries (and two international organizations).

This collection of papers is prepared from direct reproductions of the authors' copies. It is hoped that publication of the document in this way will provide timely information to participants of the meeting, as well as to others who may have not been able to attend but are interested in the proceedings.

Inadequate, but special acknowledgements are recorded here to Dr. Peter Stott and Dr. Alan Costley, the technical organizers of the Meeting, and to Greta Blankenback and Carol Simmons, who so ably and efficiently handled administrative and secretarial details before, during, and after the meeting.
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A: MEASUREMENT OF DENSITY PROFILES
Fast sweep multiple broadband reflectometers on

ASDEX and ASDEX-Upgrade


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Abstract

Here we present the main characteristics of two reflectometric systems: i) the ASDEX system, in operation between December 1988 and August 1990; ii) the ASDEX Upgrade system presently under development.

1- ASDEX reflectometric system

1.1 General description

A microwave reflectometry system (O mode), was developed for the ASDEX Tokamak /1/ consisting of six channels covering the frequency range 18 to 60 GHz, and probing electron densities from $0.4 \times 10^{13}$ cm$^{-3}$ (see Fig 1).

Simultaneous operation of different channels was performed for the first time in broadband reflectometry, delivering complete density profiles ($-8 < r < 42$ cm), with three reflectometers; 24 density profiles with good temporal (2 ms) and spatial (1 - 2 cm) resolutions could be measured in each shot. Density fluctuations were detected simultaneously with four reflectometers probing 15 different plasma locations during each shot. Fluctuations up to 300 kHz could be resolved.

Several diagnostic techniques were used: (i) broadband operation (homodyne detection); (ii) fixed frequency operation (homodyne and heterodyne detection); (iii) toroidal, poloidal and radial correlation reflectometry.

1.2 System hardware

The system was provided with several hardware innovations that were the basis of successful measurements. Solid state YIG tuned oscillators were used for the first time in microwave reflectometry, ensuring low noise stable signals, and broadband operation in 2 ms (specific fast drivers were developed for this purpose). Focused hog-horn antennae were designed and constructed having high efficiency and narrow incident microwave beams. The
outcoming transmission line was decoupled from the incident one, in order to minimize the contributions to the plasma signals from spurious reflections (Fig. 2).

1.3 Data analysis techniques

Linear digital filtering and tunable filtering techniques were implemented in order to obtain density profiles; data from the three reflectometers was automatically fitted. Standard filtering techniques are not effective in cases of strong plasma fluctuations, and either faster sweepings or more sophisticated filtering techniques must be used. New intelligent signal processing techniques have been studied and are presently being tested on the ASDEX data; these new algorithms are also to be implemented on the ASDEX Upgrade data.

1.4 Plasma Physics studies

The combined measurements in broadband and fixed frequency operation provided valuable complementary information about both density profiles and density fluctuations, in a wide range of plasma regimes. Based on the experimental results, plasma physics studies have been carried out, with special emphasis on Lower Hybrid Current Drive scenarios, and H-mode regimes /2/.

2. A Reflectometry System for ASDEX Upgrade

2.1 Purpose of the diagnostic

The aim of the ASDEX Upgrade tokamak is to achieve a reactor-compatible open divertor, in contrast to the ASDEX closed divertor. A key issue for investigation on ASDEX Upgrade will be therefore the edge region. The development of a microwave reflectometric system meets the specific needs of measuring density (namely in the edge) with high spatial and temporal resolutions. The main purpose of the diagnostic is to measure density profiles and density fluctuations, aiming at the study of the transport properties in the plasma. The following tasks are foreseen:

* measurement of density profiles in the peripheral and scrape-off layer plasma, and comparison of high field and low field side profiles;
* study of magnetic modes and density fluctuations in the peripheral and scrape-off layer plasma;
* measurement of density profiles near the ICRH antenna;
* density measurements in the magnetic X-point region.

With the proposed reflectometry system for ASDEX Upgrade some of the R & D issues for the operation of reflectometry in next step devices can be addressed, namely: (i) fast sweep operation; (ii) methods of minimizing the effect of fluctuations on profile measurements; (iii)
methods to obtain quantitative physics information on fluctuations; (iv) full remote control, and a fast data acquisition system including data compression facilities.

2.2 Choice of the system

The main parameters concerning the ASDEX Upgrade are shown in Table 1. The full range of magnetic fields ($B_t < 4$ T) was considered in the study of the frequency domains of operation that are presented in Table 2.

| major radius $R_0$ | 1.65 m |
| minor radius $a$ | 0.5 m |
| plasma height $b$ | 0.8 m |
| plasma current $I_p$ | $< 2$ MA |
| toroidal magnetic field $B_t$ | $< 3$ MA |
| plasma density $n_e$ | $1.5 \times 10^{20}$ m$^{-3}$ |

Table 1

<table>
<thead>
<tr>
<th>Band (GHz)</th>
<th>Low field</th>
<th>High field</th>
</tr>
</thead>
<tbody>
<tr>
<td>K (16 - 25)</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>Ka (25 - 36)</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>Q (32 - 50)</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>V (50 - 72; W (72 - 108)</td>
<td>O / X</td>
<td>O</td>
</tr>
</tbody>
</table>

Table 2

With O-mode the probed densities are between $n_e -0.32\times10^{13}$ cm$^{-3}$ and $-14.5\times10^{13}$ cm$^{-3}$, corresponding to frequencies from 16 to $-110$ GHz (see Fig. 3). X-mode will be used on the low field side only, as the accessibility condition on the high field side would require very high probing frequencies (> 120 GHz); it will enable to probe the plasma edge from densities close to zero with frequencies above 60 GHz (for $B_t - 3$ T).

With O-mode operation, spatial resolutions typically of ~1 cm for the density profiles are expected at the edge ($x/a > 0.8$), and ~1.5 cm in the bulk plasma, considering $n_e(0)=5\times10^{14}$ cm$^{-3}$ and a parabolic profile. In the case of X mode, with the same plasma parameters, and for $B_t - 2$ T, the values expected are 0.5 cm at the edge and 1 cm in the bulk. The spatial resolution will be improved both for O and X-mode for higher density and magnetic field regimes (as it will be the case of ASDEX Upgrade operation foreseen beyond 1993, with $n_e -15\times10^{14}$ cm$^{-3}$, $B_t - 4$ T).

The WKB approximation for propagation in the plasma will be valid in most of the cases for the gradient and edge regions. For the lowest frequencies ($f = 16$ GHz, $\lambda = 1.875$ cm) the breakdown of the WKB shall occur for $L < 0.6 \lambda - 1$ cm; this might be the case of improved confinement H-mode regimes where the gradients at the edge are quite steep ($L - 2$ cm). In the X mode the lowest frequencies are $F - 60$ GHz, so $\lambda - 0.5$ cm; in this case the WKB approximation should be satisfied for the whole range of probed densities. In very flat profiles the central plasma region may only be accessible with X-mode reflectometry.
2.3 Experimental set-up

2.3.1 Configuration of the reflectometers

The ASDEX Upgrade reflectometry system will operate both in fixed frequency and broadband sweep, and will have the capability of performing correlation measurements. In order to minimize the influence of fluctuations during broadband operation fast frequency sweeping will be used (< 100 μs). The typical configuration of a reflectometric channel can be seen in Fig. 4, where the V-band reflectometer is schematically drawn.

For profile measurements the detection will be homodyne. In fixed frequency operation it will be used either an homodyne or an heterodyne detection. Heterodyne detection includes down conversion of both the reference and the plasma signals to an intermediate frequency (IF) of 1.5 GHz. The IF frequency is stabilized by a phase/frequency locking to a stable quartz reference enabling narrow band filtering. A coherent phase detector will eliminate phase ambiguities.

The system is provided with independent paths for the incident and reflected signals. The reference signal to be used in profile measurements, and the plasma reflected signal will follow the same outcoming path. In this way phase contributions from the microwave circuits will affect equally both signals, and so the relative phase between them will be mainly due to the propagation in the plasma.

2.3.2 Microwave sources

One critical item of the reflectometry system is the microwave source. As one of the main objectives of the measurements on ASDEX Upgrade is to understand the effect of the plasma fluctuations on the reflectometric signals, the sources should provide signals with high phase purity. So, our approach was to avoid oversized waveguides which could lead to phase mixing due to higher mode conversion.

Hyperabrupt Varactor-Tuned oscillators (HTO) were chosen. These are solid state devices whose behavior is not affected by magnetic fields and can therefore be placed close to the tokamak (some 5 meters away). Presently HTO oscillators are only available up to 18 GHz, and so the implementation of the sources for each frequency band (between 16 and 108 GHz) will require frequency multipliers as listed in Table 3. The doublers are active devices and in K and Ka bands they will produce a very "clean" output up to 40 GHz, with relative high level (+15 dBm) minimum. For Q, V and W bands additional multipliers will be required; the output power will be in this case > 1 mW (at 110 GHz). Improved drivers will be developed for the oscillators allowing full band sweeping in 10 μs.
Table 3

<table>
<thead>
<tr>
<th>BAND</th>
<th>1st FREQUENCY [GHz]</th>
<th>FREQUENCY MULTI.</th>
<th>FINAL FREQUENCY [GHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>8 - 12.4</td>
<td>× 2</td>
<td>16 - 24.8</td>
</tr>
<tr>
<td>Kα</td>
<td>12.4 - 18</td>
<td>× 2</td>
<td>24.8 - 36</td>
</tr>
<tr>
<td>Q</td>
<td>8 - 12.4</td>
<td>× 2 × 2</td>
<td>32 - 49.6</td>
</tr>
<tr>
<td>V</td>
<td>12.4 - 18</td>
<td>× 2 × 2</td>
<td>49.6 - 72</td>
</tr>
<tr>
<td>W</td>
<td>12.4 - 18</td>
<td>× 2 × 3</td>
<td>74.4 - 108</td>
</tr>
</tbody>
</table>

2.3.3 Antennae

(i) Characteristics

Focused hog-horn antennae of the same type developed for ASDEX shall be used in the reflectometers probing the low field and the high field sides of the plasma (Fig. 5 a); the hog-horns are being designed taking into account the results of a numerical study of the antennae radiation characteristics in the Fresnel region. For the two channels probing the plasma region in front of the ICRH antenna, pyramidal horns were designed in order to fit the reduced space available inside the ICRH antenna. For the X-point region strong refractive effects are expected to occur, and ray-tracing studies are needed in order to determine the antennae configuration.

(ii) Ray tracing studies

Ray tracing studies were performed aiming to study the more adequate positioning of the focusing region of the antennae and their localization in both single null and double null experiments. The main conclusions are: initial wavefronts should be focused beyond the reflecting layer (namely at the plasma center), otherwise refraction previous to the plasma cutoff would cause unwanted divergence; the antennae should face the plasma mid-plane in both plasma geometries. The second conclusion implies different positioning for single null and double null geometries. In Fig. 5 b it is shown schematically the localization of the antennae in both cases.

2.3.4 Data acquisition system

The reflectometric system shall be operated by remote control. Optical fibre links will ensure the transmission of the selected probing signals to the plasma, and of the measured signals to the data acquisition system.

Considering a number of 400 fringes per sweep (maximum value, foreseen for the upper band, W) and 20 samples per fringe (error < 5%), the following values are obtained for a sweep time of 100 μs:

<table>
<thead>
<tr>
<th>( f_b [\text{MHz}] )</th>
<th>( t_{aq} = 1 / (20 \times f_b) [\text{ns}] )</th>
<th>( f_s [\text{MHz}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>12.5</td>
<td>80</td>
</tr>
</tbody>
</table>
For a minimum sweep time of 10 μs (as provided by the microwave system), f\_b = 40 MHz, and the sampling frequency would therefore reach 800 MHz. A system is being implemented based on (i) commercially available CAMAC modules (100 MHz) for the lower reflectometer bands (K and Ka) to operate by mid 1992; (ii) the development of prototype acquisition channels at 200 MHz and 800 MHz sampling rate based on transputers, for the exploitation of the reflectometry system beyond 1992.

(i) CAMAC solution

An acquisition system composed by 100 M\text{samples/s}, 8 bit ADC modules, and 512K memory modules, will provide 100 MHz sampling rate and 1 Mb storage space per channel. In the case of Ka band (50 fringes, leading to 1000 points per sweep) the available memory will enable the acquisition of 1000 profiles per shot. This system, however, would not fully exploit the technical possibilities of the microwave system for the upper bands, due to the large number of fringes (< 400). The time window of measurements with high temporal resolution (< 100 μs) would be rather limited. Further extension of a CAMAC system in a later phase is limited by the large amount of data to be handled.

(ii) Dedicated solution

The minimum sweep time of 10 us should be considered as it will enable measurements on the background of 100 kHz fluctuations which are to be expected on ASDEX Upgrade with full heating power. In such cases the system should allow for sampling frequencies that might reach 800 MHz.

Modules with 200 MHz and 800 MHz sampling rates and fast real-time processing capabilities are not commercially available and have to be developed. The proposed transputer system will consist mainly of three components: (1) 200 or 800 MHz 8 bit ADC's as input channels; (2) a data storage and reduction computer per channel; (3) a UNIX VME-Bus host computer as frontend for the complete system.

The 200 MHz device will consist of one 8bit flash converter and a fast memory of 4 × 2 kilobytes, that is provided by an architecture based on 4 fast clocked FIFOs (see Fig. 6). Control operation of this input device including readout of memory and forwarding of data to the next stage is done by two T801 transputers which can be programmed to allow various modes of data acquisition (e.g. preprogrammed, triggered, posttriggered).

The 800 MHz devices for the upper channels can be developed in analogy to the 200 MHz devices. The four times higher sampling rate is achieved by using a sample and hold circuit with a demultiplexer which distributes the input signal to four 200 MHz ADC's. The 800 MHz acquisition channel is therefore designed to hold up to 32 k\text{B} data.

The storage and reduction computer is a multiprocessor of eight transputers in hypercube topology. It provides memory to store at least 32 Megabytes, i.e about 4000 bursts of data, and
offers a computing power of about 20 Mips/8 Mflops to obtain the density profiles from the reflectometry input signals.

References

Figure Captions
Fig. 1: Drawing showing the installation of the antennae inside the ASDEX vessel.
Fig. 2: Configuration of the microwave circuit for a reflectometry channel on ASDEX.
Fig. 3: Curves showing the radial dependence of the O mode cutoff (fpe), X mode upper cutoff (fco), the electron cyclotron resonance (fce) and the upper hybrid resonance (fuh), for two parabolic density profiles with (I) \( n_e(0) = 0.5 \times 10^{14} \text{ cm}^{-3} \), (II) \( n_e(0) = 2.0 \times 10^{14} \text{ cm}^{-3} \).
Fig. 4: Configuration of the microwave circuit for the V-band reflectometer channels on ASDEX Upgrade.
Fig. 5: (a) Section of a hog-horn antenna. (b) Schematic localization of hog-horn antennae in the high-field and low-field sides.
Fig. 6: Logic diagram for the 200 MHz data acquisition channel.
Fig. 2
Fig. 3

V BAND CIRCUIT

Fig. 4
200 MHz Acquisition Channel

Logic Diagram

Fig. 6
MEASUREMENT OF DENSITY PROFILES
USING THE MULTICHANNEL REFLECTOMETER AT JET

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Abstract
A reflectometer with twelve probing frequencies in the range 18 to 80 GHz is used on JET. In this paper we report on our experience in using the device to measure density profiles. In order to achieve routine and reliable operation modifications of the hardware were necessary, and a special mode of operation (sweep/dwell) had to be developed, in order to overcome the effects of density fluctuations. Also, accurate calibration techniques had to be developed. In general, these have been successful and the device now produces density profiles routinely on a wide range of JET pulses. Systematic comparisons have been made with profiles measured by other diagnostics, principally LIDAR Thomson scattering and the multichannel far infrared interferometer. In the paper, we present details of the developments and the results of the comparisons with other diagnostics.

The multichannel reflectometer for JET
With the twelve-channel reflectometer, that has been developed and constructed for JET [1,2,3,4], the plasma is probed along the midplane of the torus with radiation polarized in the O-mode. The frequencies lie in the range 18 to 80 GHz, corresponding to critical densities in the plasma between 0.4x10¹⁹ and 8x10¹⁹ m⁻³. Each channel of the system has two Gunn oscillators which are slightly separated in frequency (10.7 MHz) which is maintained by a phase-locked loop. In this way a heterodyne detection technique can be used. The reflectometer has two detection systems. Coherent detectors which measure both the wave amplitude and phase with a maximum time resolution of 2μs. Fringe counters which measure phase with accuracy of 1/128 of a fringe and a maximum time resolution of 10μs.
A basic advantage of this multichannel system is that in the fixed frequency mode the local variations in the density profile can be obtained at various different positions. The density profile can be measured by sweeping the sources of narrow ranges (typically 100 MHz). From the phase changes induced by sweeping the source frequencies the propagation delay in the plasma ($\tau_p(\omega)$) can be calculated at all centre frequencies $\omega$. An interpolation or fitting routine is used to get continuous information on $\tau_p(\omega)$ from the lowest to the highest frequency.

**Fig. 1:** Systematic representation of the multichannel reflectometer system.

**Measurement techniques**

The measurement of electron density profiles at JET, using reflectometry encountered two major difficulties. First, due to the effects of the fluctuations the radiation is reflected from the density layers in the plasma at various angles and can, in some cases, not be received by the antenna. Second, to obtain a profile measurement in the edge of the plasma with a uncertainty of only a few centimeters, accurate measurements of the phase excursions ($\approx 1\%$) during the frequency sweeps are required.
At JET the effect of the density fluctuations is reduced by positioning of the launch and receive antennas close to the plasma, by strong filtering of the input signals to the fringe counters (bandpass of 3 kHz) and by maximising the dynamic range of the system, using high gain amplifiers. After the data is collected software routines can correct for fringe jumps in the measurements. However, this is difficult to apply generally and difficult in cases were the signal loss, which causes the fringe jumps, is sustained for several milliseconds. The modifications to the system restrict the time resolution since all the fast transitions in the plasma, on a time scale of less than 0.3 ms, will be filtered out. As a result fast narrow-band sweeps are not possible due this filtering. A typical sweep time which can be applied is 3ms–6ms, with a frequency excursion of 100 MHz for each of the twelve sources. To compensate for possible changes to the density profiles during the sweep a special technique in which swept frequency intervals are followed by fixed frequency intervals has been developed (sweep/dwell mode of operation) [5]. In order to measure the phase excursions during the sweeps accurately enough the resolution of the fringe counters was improved to 1/128 of a fringe. Using the sweep–dwell mode a 1% accuracy for the propagation delay into the plasma can be obtained under most plasma conditions, provided an accurate calibration the optical path difference of reference and plasma arm (use of backwall of JET) can be made.

**Fig. 2:** Sweep–dwell mode of operation to measure the density profile. Displayed are the measured phase excursions for channel 1 (0.4 x10^{19} m^{-3}) and channel 3 (1.0 x10^{19} m^{-3}).
Fig. 3: Calibration of channel 1 of the reflectometer. Shown is the value for the propagation delay to the backwall of the Tokamak for 190 measurements. The pulse numbers vary from 24000 to 28000 covering the operation period of June 1991 to February 1992. At pulse number 26090 the new fringe counter were installed with a factor four improvement in the phase resolution.

Density profile measurements
The propagation delay ($\tau_p$) into the plasma for each channel is calculated from the measured changes of the phase during the frequency sweeps. This propagation delay is a measurement, any resulting density profile an interpretation which depends on various assumptions. One technique to calculate the density profile is an Abel inversion of the measured $\tau_p$ values. The result of the inversion depends on the type of interpolation between the individual $\tau_p$ values at various different frequencies and an assumption for the edge profile. An second technique relies on the construction of a density profile from a class of functions, for which the propagation delays for the probing frequencies can be calculated. A comparison with the measured $\tau_p$ values is used to construct the best density profile. This method assumes a typical shape for the density profile.

The Abel inversion method is used for routine density profile calculations, because the second method is too restrictive in the class of density profiles which can be obtained. Using a proper interpolation for the $\tau_p$ values an error on a single channel of the reflectometer will remain clearly visible on the Abel inverted profiles. This enables us to take corrective action for following measurements.
At JET the electron density profiles are calculated routinely for every pulse, without manual intervention. A calibration is performed every so called 'dry run', at the start of each day. Two examples are given in Figs 4 and 5. The accuracy of the measurements is typically ± 4 cm.

Fig. 4: Typical example of the calculated electron density profiles from the reflectometer data during a 8 second time window of a JET discharge. In this discharge the density feedback to the plasma is switched off at 7.5 seconds and switched back on at 12 seconds. Note that only the part of the profile is plotted for which the reflectometer measures the density.
Fig. 5: An other example of the density profile measurements with the reflectometer. The measurements are made during an L to H-mode transition with the application of ICRF heating at 50.0 seconds.

The use of the fixed frequency intervals
In between sweeps the frequency of the sources is held constant to monitor the movements of the plasma. These measurements can also be used to check the profile calculations from the swept intervals; a calculated density profile should agree with the next density profile computing the movement of the reflection layers from the fixed frequency measurements. Using this technique a whole series of profiles can be compared which will reduce the statistical uncertainty of the profile calculations considerably. Secondly, density profiles are now also available in between the sweeps to a maximum of 32700 profiles each discharge (operation period of 1991 contained over 4000 discharges). This method is still under development. The results shown in this paper are not checked with this fixed frequency part of the data.
Comparison with other diagnostics

An important check on the accuracy of the calculated electron density profiles is a comparison with the data from other diagnostics. At JET the electron density is also measured by LIDAR Thomson scattering and a Far Infrared multichannel interferometer. An example of such a comparison is given in Fig 6, where three density profiles are compared at the same time. A technique to make routine comparisons between the diagnostics, computes the differences in the positions of several density layers between two of the three diagnostics. The density layers used correspond to the critical densities probed by the reflectometer. The differences in position between the density layers can be computed at every time in the discharge where the information from the diagnostics is available and for a large number of pulses. In Fig 7 the differences between the density layers of the reflectometer and LIDAR are plotted for all the pulses during a one day session.

![Comparison of three measurements of the electron density profile at the same time in an L-mode discharge.](image)
The main result of this comparison is that there is a general good agreement at the edge of the plasma (lower density), but that for this session LIDAR underestimates the value of the density near the center by \( \approx 10\% \), (a difference between the reflectometer and LIDAR which increases with density). A similar comparison between the reflectometer and the multichannel interferometer is made in figure 8 for the same session. There is a general good agreement in the profile shape between the diagnostics. However, there is a systematic difference in the profile position of \( \approx 7 \) cm, with the interferometer profile situated inside the reflectometer profile. This difference is generally observed, also between LIDAR and the interferometer. This shift may be due to the fact that both LIDAR and the reflectometer have a calibration of the profile position from the backwall of JET while the interferometer is computed on the magnetic flux surfaces.

The application of the comparison technique to the JET data is rather scarce yet. The technique needs to be improved and its full potentials to be explored.

![Graph](image)

**Fig. 7:** Systematic comparison between the reflectometer and LIDAR. Plotted are the differences in the position of the density layers up to \( 5.0 \times 10^{19} \text{ m}^{-3} \). The solid line at zero indicates a perfect agreement between the diagnostics.
Fig. 8: Systematic comparison between the reflectometer and the interferometer. Plotted are the differences in the position of the density layers up to $5.0 \times 10^{18} \text{ m}^{-3}$. The solid line at zero indicates a perfect agreement between the diagnostics.

Conclusions
Routine electron density profile measurements are obtained at JET with the multichannel reflectometer. In order to achieve this the effects of density fluctuations needs to be suppressed and accurate measurements of the propagation delay between plasma and reference arm for each probing frequency are required. This has led to substantial changes to the hardware, like position the antennas close to the plasma and strong filtering of the signals. A special measurements technique, the so called sweep/dwell mode is used. The required accuracy of the phase excursions during the sweeps is obtained by correcting for profile changes during the sweeps and by calibrating the system each day. Generally, good agreement is found with other measurements of the density profile at JET.
References


1. INTRODUCTION

Reflectometry experiment on TORE-SUPRA is designed to measure routinely the electron density profile of the plasma and to follow all change in the gradient /1/. Broadband technique is used, and, the frequency range is 25-75 GHz for the O mode allowing us to measure densities between 8.5 \times 10^{18} and 6.8 \times 10^{19} m^{-3}. X mode technique is under development with a frequency range of 75-110 GHz. It will be used for the determination of the density near the scrape-off layer according to previous experiments /2/ and, fixed frequency measurements for fluctuation studies. After a brief theoretical recall, an overview of the experimental set-up, detection technique, and signal analysis method is given, followed by some experimental results.

2. PRINCIPLE OF THE REFLECTOMETRY

According to the WKB approximation /3/-/4/, the phase variation \( \Phi \) of an ordinary wave of frequency \( F \) reflected at a critical density (\( x = x_c \)), measured at the edge of the plasma \( a \) is given by:

\[
\Phi(F) = \frac{4\pi F}{c} \int_{a}^{x_c} \mu(F,x) \, dx - \frac{\pi}{2}
\]  

(1)

Where \( \mu \) is the refractive index which is a function of frequency and space.

The reflection point location \( x_c(F) \) is deduced from \( \Phi(F) \) using an Abel inversion.
\[
  x_c(F) = \frac{c}{2\pi^2} \left( \frac{d\Phi}{dF} \right) \int_0^{F_2} \frac{1}{(F^2 - f^2)^{1/2}} \, df
\]  

One can see in equation 2 that the determination of \( x_c \) depends only on the phase variation \( \frac{d\Phi}{dF} \).

3. EXPERIMENTAL SET UP

The diagnostic system is located outside the vacuum vessel and is not linked to the tokamak. This solution allows to avoid DC breaks and flexible waveguides due to the high VSWR of these components that can disturb the quality of the received signal. However, the signal is very sensitive to any mismatches along the microwave line, and it becomes necessary to choose and calculate carefully all microwave components. Care has been taken to decrease the origins of spurious signal such as: the directivity leakage of the couplers, the VSWR of the antenna, the transmission of oversized waveguides and the parasitic reflection on the vacuum window.

The microwave set-up (see Fig.1), is composed of three single antenna reflectometers located in the equatorial plane. They are operated in O-mode within the following frequency ranges: 25-35 GHz, 35-50 GHz, and 50-75 GHz. All are made of fundamental waveguides excepted the transmission lines which are oversized waveguides (mainly WG 22).

The microwave sources are three SIEMENS backward wave oscillators (BWO). The sources are located 5 meters away from the machine because of the magnetic field (1 Tesla near the machine). At this location, iron/mumetal magnetic shielding is possible (\( B < 200 \) G) for the BWO and the ferrite isolators. Electrical isolation of BWO and detector is obtained by using a 50 \( \mu \)m mylar sheet. The linear frequency sweeping with respect to the time is obtained with the help of a programmable function generator. It delivers a low voltage ramp which is amplified and added to the high voltage line. The sweeping time for each BWO can be as low as 200\( \mu \)s without disturbing the wobulation. The voltage-frequency dependence has been measured with a Fabry-Perot cavity with an accuracy of \( 2 \times 10^{-5} \). To prevent strong amplitude modulation on the signal, the BWO are power-leveled by feedbacks on
PIN attenuators. Unfortunately, this technique limits the shortness of the sweeping time to 1 ms in order to preserve an efficient levelling. The reflectometers are swept one after another in order to avoid channel crosstalk. The whole profile is thus obtained in a time lag lying between 5 and 20 ms depending on the selected sweep time.

The non linear antennae had been calculated by Dr Thumm, (Stuttgart university). They have a low VWSR and keep a pure TE10 mode. The phase reference is taken at the aperture of the antenna with a dielectric plate made of mica, acting as a partial reflector. A 0.2 mm thick mica sheet (n=2.63) is compatible with mechanical and thermic constraints and the thickness is low enough to make the attenuation negligible.

The measurement is disturbed by the parasitic signals due to the reflection in the antenna, in the coupler and in the oversized transmission line. After a careful choice of the electrical lengths, all these parasitic reflections show up mainly outside the frequency range of the signal (see figure 2). The main problems came from reflections in the adapter flange, where the antennae are located. So, the quartz window (Ø = 280 mm - 20 mm thick) has been inclined at 20° and microwave absorbers are disposed all around.

The 3 dB directional couplers of high directivity (> 40 dB) allow the wave reflected from the mica (reference) and the one reflected from the plasma to be mixed in a Schottky diode, which is flat broadband detector (Hewlett-Packard for the ranges 25-35 GHz, 35-50 GHz, and ALPHA-TRG for the range 50-75 GHz).

In order to isolate the set-up from the reflection on the detectors (VSWR< 5/1), 10 dB directional couplers (VSWR = 1.1) or calibrated attenuators are used when the power level is high enough. This solution is better than the use of isolators, where the reflection is higher (VSWR > 1.3). The detected signal is then sent to a low noise preamplifiers (gain = 500, in 1 MHz) and a programmable band pass filters. Results are recorded in a Lecroy data acquisition system.

4. SIGNAL ANALYSIS

As the detected signal is generally disturbed by plasma turbulences, it is not always easy to measure exactly the 2π variation
corresponding to the maxima or the minima of the signal and it is suitable to determine the beat frequency by spectral analysis /5/.

The detected signal takes the form \( \cos \Phi(t) \). As we wobbleulate \( F = F_0 + A \ t \) it appears an instantaneous beat frequency:

\[
F_b = \frac{1}{2\pi} \frac{d\Phi}{dt}
\]

linearly related to the time delay \( \frac{1}{2\pi} \frac{d\Phi}{dF} \), between the reference and the reflected waves, used in equation 2.

A typical signal, \( V(t) \) coming out the detector is shown on figure (3a). The beat frequency (10 to 400 kHz) is then determined from Fast Fourier Transform of \( V(t) \) over time windows \( \Delta T \) shifted with \( \delta T \) intervals (either \( \Delta T/2, \Delta T/4, \ldots \)). The averaged beat frequency in the time interval \( \Delta T \) corresponds to the maximum of the modulus of the Fourier Transform. Actually, this method is valid if the beat frequency vary slowly with time \( t \) along the signal and can be considered as constant over \( \Delta T \). It allows, this way, to get rid of parasitic signals (see figure 3b). At the beat frequency \( F_b \), measured between \( \Delta T = t_1 - t_2 \) is attributed the time \( t = (t_1 + t_2)/2 \).

To calculate the Abel inversion, we make a linear interpolation (\( t \) versus \( F \)) between the experimental points. The accuracy of the inverted profile is less than 3 cm. We also have to assume an edge density profile for frequencies between 0 and 25 GHz.

Two assumptions for the edge profiles are commonly tested:

- Linear profile: \( n_e(x) = n_e(x_0) \frac{x - a}{x_0 - a} \) (density equal to zero on the last magnetic surface).

- Exponential profile: \( n_e(x) = n_e(x_0) \exp(-(x + a)/(x_0 - a)) \).

(\( x_0 \) corresponding to the cut-off layer at 25 GHz).

We estimate the effect of the uncertainty on the density profile by comparing different assumptions on edge profiles. The maximum error is less than 1.5 cm and is negligible at about 15 cm into the plasma for smooth profiles. For high curvature profiles (pellet injection, magnetic islands, effect of ergotic divertor), the accuracy of the inverted profile is typically less than 3 cm.
5. EXPERIMENTAL RESULTS

Comparing to other methods of density profile measurement (interferometry, Thomson scattering), the advantage of using the reflectometry technique is its sensitivity to the local density profile variation, density gradient change, and plasma edge displacement.

Figure 4, shows the different profiles obtained by reflectometry and interferometry assuming the same edge position. We can see that in the case of smooth density profile, the two methods are in good agreement. On figure 5, the profiles given by Thomson scattering and reflectometry experiments, show the complementarity between the two methods.

On figure 6, the density profile is given at different time during a shot in ohmic mode. The first one and the second one are during the current rising, and the others during the current plateau.

The effect of the ergodic divertor is observed on figure 7, as a decrease of the electron density.

6. CONCLUSION

The three reflectometers are routinely operated and we compute the whole density profile. Problems remain when the gradient is very strong at the edge, the waves are reflected and it is difficult to decorrelate the useful signal from spurious reflections. New investigations must be done in order to study the influence of the density fluctuations on the profile.

The X mode reflectometer should give a more precise and lower edge density profile. The measurement of fluctuations like M.H.D. activities, microfluctuations, can be also envisaged.

References
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Location of parasitic reflections due to the antenna, the coupler, and the oversized transmission line, in the spectrum.
FIGURE 3
Density profile at t = 34s

COMPARISON WITH INTERFEROMETRY
(ABEL INVERSION)
SAME EDGE ASSUMPTION

--- INTERFEROMETRY
--- REFLECTOMETRY

FIGURE 4

SHOT 6918

FIGURE 5
DENSITY PROFILES

Figure 6

DENSITY PROFILES SHOT 6829

With Ne(x) = 0

DENSITY (m$^{-3}$)

Figure 7

DENSITY PROFILES

With Ne(x) = 0

DENSITY (m$^{-3}$)
X-mode 37 GHz Reflectometry on the "Uragan-3M" torsatron

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1. Introduction

Measurements of electron density profile via microwave reflectometry using plasma probing by EM waves with ordinary (O) mode polarization are widely used for both tokamaks and stellarators. Extraordinary (X)-mode probing is less common because the refraction index of this wave depends on an additional parameter - magnetic field. In tokamaks the total magnetic field depends strongly on plasma current profile and is unknown a priori. It is not a case for stellarators where plasma influence on the total magnetic field distribution is weak (of order of beta). This allows to reconstruct the electron density profile from measurements of X-mode cut-off position versus probing frequency using data of the vacuum magnetic field calculation along the wave trajectory.

2. Experiment

This approach was used for the electron density profile reconstruction for RF heated plasma in the "Uragan-3M" torsatron[1]. Measurements were done for one of operational regimes \(B = 1.2\) T, \(i(0)=0.3\), \(P_{RF} =200KW\) and \(n_{e_{max}}(0)=5.10^{12}\) cm\(^{-3}\). The homodyne reflectometer with the BWO tuned in the frequency range of 36-40.5 GHz and X-mode launch was used. Single transmit/receive antenna was installed into vacuum chamber from the low field side (Fig.1). Simultaneously \(n_e\) profile information was obtained via 4 chord O-mode microwave interferometer (\(l=1.2\)mm). The electron density at plasma centre was measured by laser scattering.

Fig.2 shows the time behaviour of the line-averaged electron density and reflectometer signal for typical RF discharge (RF pulse duration-17 ms). Information on reflected wave phase shift was obtained from reflectometer signals by numerical signal
filtering which removed high frequency fluctuations. Computer simulation of experiment using data of average electron density measurements allowed to define the moment of wave reflection and to use this point as zero-point for phase shift determination.

Fig.3 shows results of electron density profile reconstruction from reflectometer data and vacuum magnetic field distribution; laser scattering data are shown also. The experimental points are well described by simple profile function

\[ n_e(r/a) = n_e(0) \left[ 1 - (r/a)^2 \right]^{0.67} \]

For conditions of experiment (electron density and magnetic field values, probing frequency range) microwave reflection took place at the near-axis part of plasma column \((0.1 < r/a < 0.6)\) where the density gradient was small. In this region the theory predicts rather large uncertainty \(\Delta r_c\) for O-mode cut-off layer position\cite{2}:

\[ \Delta r_c = \left| \frac{\lambda^2}{4\pi^2} \cdot \frac{n}{\sqrt{\nu n}} \right|^{1/3} \]

For X-mode a corresponding expression for \(\Delta r_c\) was calculated using the same approach as in \cite{2} and resulted in formula:

\[ \Delta r_c = \left| \frac{\lambda^2}{4\pi^2} \cdot \left( 1 - \frac{(u^4 - u^2)}{(1 - \nu^2)} \right) \cdot \frac{\nu n}{n} - \frac{(\nu^2 - \nu) u^2}{(1 - \nu^2)^2} \cdot \frac{\nu B}{B} \right|^{1/3} \]

where \(\nu = \omega^2/\omega, u = \omega_b/\omega\). One can see from (2) that the magnetic field gradient account diminishes the X-mode cut-off layer position uncertainty. Fig.4 shows the comparison of cut-off layer position determination uncertainties for X- and O-mode reflections calculated for experimental \(n\) and \(B\) profiles. It is seen that X-mode reflectometry results in rather small position errors \((\Delta r_c = 3-4\text{mm})\) even in a region with a zero density gradient.

Typical reflectometer signal showed existence of phase fluctuations. An analysis of a high frequency component of reflectometer signal was made using the presentation of the reflected wave phase fluctuations \(\Delta \phi\) as a monochromatic one having a frequency of \(\omega\) \cite{3}:

\[ \Delta \phi = \Delta \phi_0 + \Delta \phi \cdot \sin \omega t \]

The amplitude of phase fluctuation of \(\Delta \phi\) was determined from the ratio of harmonics in the spectrum of reflectometer signal.
fluctuations. Electron density fluctuations were determined from relation
\[
\frac{\delta n}{n} = \left( \frac{\Delta \Phi}{2\pi} \right) \cdot \lambda_0 \cdot \frac{\nu n}{n}
\]
and it was ascertained that the density fluctuation amplitude is increasing at the plasma periphery (Fig.5). Spectrum of fluctuations is concentrated in the region of 4-5 KHz. This value corresponds to the frequency of azimuthal \((m=3)\) mode of drift instability
\[
\omega^* = \kappa_\theta \cdot \nu^* \quad ; \quad \kappa_\theta = \frac{m}{\lambda_0} \quad ; \quad \nu^* = \frac{c}{\varepsilon B} \cdot n'
\]

3. Conclusion

We have shown that the upper X-mode reflectometry can be used for electron density profile determination in stellarators. The upper X-mode needs larger probing frequencies than O-mode but it gives evident advantages in comparison with O-mode reflectometry:
- it needs smaller range of frequency tuning (of order of \(2a/R\));
- it results in lesser errors in determination of cut-off layer position (due to lesser wavelengths and magnetic gradient influence).

References

Fig. 1
Fig. 2
Ne profile for U-3M experiment

Figure 3
**O- and X-mode position uncertainty for U-3M experiment**

![Graph](image)

Fig. 4

**Ne fluctuation level for U-3M experiment**

![Graph](image)

Fig. 5
PROFILE EVALUATION TECHNIQUES FOR O-MODE
BROADBAND MICROWAVE REFLECTOMETRY ON ASDEX†

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I - Abstract

Density profiles from reflectometry can be obtained, in principle, with phase or time delay measurements. In the first case frequency-modulated continuous waves (FM-CW) are launched into the plasma, and in the second one different types of signals, namely pulses, are used. Whereas in the ionosphere density profiles are normally obtained with pulsed radar techniques, in fusion plasmas FM-CW reflectometry has been mostly used.

In both techniques the localization of each reflecting layer cannot be deducted from single measurements as, for the same measured phase shift or time delay, the location depends on the density of the plasma that the waves have encountered in their propagating path. So, in order to determine the correct position of each layer all the layers with lower densities have to be probed.

As microwaves are very sensitive to plasma modes and broadband turbulence the resulting phase or time delay perturbations may lead to the incorrect interpretation of the data, causing large errors in the evaluated profiles. Also, in some cases, it is not possible to probe the complete plasma and deviations may occur due to the missing information. The evaluation of the profiles must, therefore, include data analysis procedures that take into account both the effect of plasma fluctuations and the limitations of the diagnostic.

Here we present the techniques developed to analyse the ASDEX data, and discuss their potentialities for the routine evaluation of the density profiles from broadband reflectometry.

II - Basic principles of profile measurements from reflectometry

With the ASDEX O-mode system the density profiles were obtained in 2 ms with the simultaneous broadband operation of three reflectometers. The frequency range from \( F_1 = 18 \) to \( F_N = 60 \) GHz was covered, probing densities from \( 0.4 \) to \( 4.5 \times 10^{13} \) cm\(^{-3} \), [1].

Profiles were evaluated by Abel integration of the measured phase shift, \( d\phi(F)/dF \), using eq. (2.1). \( X_0 \) is the position of the first plasma layer, \( X_j(F_j) \) is the cut-off layer for the probing frequency \( F_j \) and \( \phi(F) \) is the phase shift undergone by the wave of frequency \( F \). In broadband measurements the plasma is continuously probed enabling to obtain the beat frequency \( f_B(t) = (d\phi/dF)(dF/dt) \) or the phase derivative characteristic, \( d\phi(F)/dF \).
for frequencies between $F_1$ and $F_N$. Since the plasma is only probed for frequencies above $F_1$ the integral $I_1(F_j)$ in eq. (2.1) and the localization of the first reflecting layer $X_1(F_1)$ were estimated by assuming a given shape for the density profile between $X_0$ and $X_1$, as discussed in section III.

$$X_0 - X_j(F_j) = \frac{c}{2\pi^2} \left[ \int_0^{F_1} \frac{d\varphi}{df} \frac{df}{\sqrt{F_j^2 - f^2}} \right] + \frac{c}{2\pi^2} \left[ \int_{F_1}^{F_j} \frac{r^{F_j} d\varphi}{df} \frac{df}{\sqrt{F_j^2 - f^2}} \right], \quad j = 2, 3, \ldots, N. \tag{2.1}$$

For standard profile evaluation the local phase shift derivative, $d\varphi(F_j)/dF$, can be replaced, in small intervals, $[F_j, F_j+1]$, by the differential $(\Delta \varphi/\Delta F)_j = [\varphi(F_{j+1}) - \varphi(F_j)]/(F_{j+1} - F_j)$, and the Abel integral equation can be accordingly written as

$$X_0 - X_j(F_j) = I_1(F_j) + \frac{c}{2\pi^2} \sum_{i=1}^{j-2} \left( \frac{\varphi(F_j) - \varphi(F_{j-1})}{F_j - F_{j-1}} \right) \int_{F_i}^{F_{i+1}} \frac{df}{\sqrt{F_j^2 - f^2}} + \frac{c}{2\pi^2} \frac{r^{F_j} \varphi(F_j) - \varphi(F_{j-1})}{F_j - F_{j-1}} \int_{F_{j-1}}^{F_j} \frac{df}{\sqrt{F_j^2 - f^2}}. \tag{2.2}$$

From eq. (2.2) it can be concluded that, in profile measurements, the phase effects of the plasma on two probing waves, with close frequencies $F_{j-1}$ and $F_j$, are subtracted and the phase shift difference, $\varphi(F_j) - \varphi(F_{j-1})$, is attributed to the effect of the plasma in the small region between $X_j(F_j)$ and $X_{j-1}(F_{j-1})$. From this point of view profile measurements are differential interferometric measurements.

### III - Sensitivity of the profile to the non-probed part of the edge plasma

As referred previously, the integral $I_1(F)$ in eq. (2.1) cannot be obtained from reflectometric measurements. In order to study the sensitivity of the measured profiles to the non-probed part of the plasma, several shapes of the profile between $X_0$ and $X_1$, were assumed according to the following equation, [2]

$$n_e(x) = n_e(X_1) \left( \frac{X_0 - x}{X_0 - X_1} \right)^s, \quad s > 0, \tag{3.1}$$

where the parameter $s$ defines the profile shape. For $s > 1$ and $s < 1$ convex and concave profiles are obtained, respectively, (see Fig. 1). The case $s = 1$ represents a linear profile.

It can be proven that the density profile of eq. (3.1) yields

$$\frac{d\varphi}{df} = \frac{4\pi^{3/2} \Gamma(\frac{1}{2})(X_0 - X_1)}{cs\Gamma(\frac{1}{s} + \frac{1}{2})} \left( \frac{f}{F_1} \right)^{2/s}, \quad f \leq F_1, \tag{3.2}$$
where $\Gamma(\cdot)$ is the Gamma function.

By making $f = F \sin u$ and using eq. (3.2) the integral $I_1(F)$ is transformed into

$$I_1(\beta; s) = \frac{2\Gamma(\frac{1}{2})}{\sqrt{\pi} s} \Gamma(\frac{1}{2} + \frac{1}{2}) \beta^{2/s} \int_{0}^{\arcsin \beta} \sin^{2/s} u \, du,$$  \hspace{1cm} (3.3)

where $\beta = \beta(F) \equiv F_1/F \leq 1$ is the normalized frequency.

As expected, for $\beta = 1$ eq. (3.3) gives $I_1(1; s) = X_0 - X_1$, $\forall s > 0$. In most cases, the integral can only be evaluated numerically. However, for $s = 1/q$, with $q$ integer, solutions may be obtained in closed form.

The value of $I_1(\beta; s)$ converges to zero when $\beta \to 0$, since the influence of the plasma outer part in the computation of the density profile is negligible for frequencies $F \gg F_1$.

Note: Eq. (3.2) can be applied to estimate the position $X_1$, if the measured phase derivative $d\varphi(F_1)/dF$ is used and a given shape $s$ is assumed.

When the proposed shape of the outer part does not fit the correct one the position of the reconstructed profile, (evaluated by eq. (2.1)), will exhibit errors due to wrong values taken by $I_1(F)$.

In order to understand how these errors affect the position of the reconstructed profile the correct profile is assumed to be parabolic with $n_e(x) = n_e(0)(1-(x/X_0)^2)$, $0 \leq x \leq X_0$. In this case, the phase derivative is given by

$$\left(\frac{d\varphi}{df}\right)_{par} = \frac{4\pi X_0 f}{cF_0} \log \frac{1 + f/F_0}{1 - f/F_0},$$  \hspace{1cm} (3.4)

with $F_0 = 8.979\sqrt{n_e(0)}$. Let the reconstructed profile be computed from the phase derivatives given by eq. (3.4), for $f > F_1$ and by eq. (3.2), (with a fixed value of $s = 1/2$), for $f \leq F_1$. $X_0$ and $X_1$ are considered to be known exactly. If $X_c(F)$ and $X_c'(F)$ denote, respectively, the correct position and the position obtained with the outer part shape given by (3.1), the error in the profile location for the probing frequency $F$ is given by $X_c'(F) - X_c(F) = I_1(F) - I_1'(F)$, as $I_2(F) = I_2'(F)$.

Fig. 1
Fig. 2(a) sketches three parabolic profiles with $X_0 = 0.42 \ m$, (which is typical for ASDEX plasmas), and different values of the maximum density: $n_e(0) = 5 \times 10^{19} \ m^{-3}$, (solid line), $n_e(0) = 2.5 \times 10^{19} \ m^{-3}$, (dashed) and $n_e(0) = 1 \times 10^{19} \ m^{-3}$, (dash-dotted). In the figure is also indicated, by a dotted line, the minimum density probed by the reflectometry in ASDEX, ($\approx 4 \times 10^{18} \ m^{-3}$ corresponding to $f_r = 18 \ GHz$). The positions of $X_1$ are 0.325 m, 0.385 m and 0.403 m, for the low, medium and high density profile, respectively.

The errors in the location of these profiles are shown in Fig. 2(b). For the three graphs the errors are zero when $x = X_1$.

From this study the following conclusions can be drawn. The assumption of an incorrect shape for the outer edge profile produces errors in the position of the reconstructed profile which: (i) are small, (typically $< 1 \ cm$ in the case of ASDEX plasmas); (ii) are less significant in high density profiles than in low density ones; (iii) decrease from the outer part to the center of the plasma.

It can also be concluded that the linear model, ($s = 1$), is a good approximation to the outer part of the profile, (and has been widely used in the ASDEX profile measurements), except for steep profiles where a more realistic nonlinear convex shape must be used.

IV - Effect of modes and broadband turbulence

*Magnetic modes* produce localized periodic perturbations of the plasma density profile resulting in amplitude and phase modulations of the reflectometric signals. In Fig. 3 four raw data signals ($28 - 39 \ GHz$) are shown, measured with the Ka-band reflectometer during the ELMy phase of an H mode plasma (ASDEX # 32060). Time interval between samples is 10 ms and each signal was obtained in $\approx 1.8 \ ms$. Localized amplitude and phase
modulations with low frequency can be observed in the detected signals indicating that a mode should be present in the plasma. The density profiles evaluated from the broadband data shows that perturbations should be localized at $r \approx 35 cm$.

From the signals the temporal evolution of the mode can be observed and its frequency can be estimated ($f \leq 9 kHz$).

Drift wave activity and broadband turbulence leads to the scattering of the incident microwaves. The scattering process causes phase mixing and the decrease of the incident electric field. In Fig. 4 this effect is illustrated, where two broadband raw data signals (42 - 49 GHz) from the U band reflectometer are shown. The signals were measured during the steepening of the edge density profile in H-mode plasma of ASDEX #32060. As can be seen from the corresponding density profiles, depicted in Fig. 5, the signals are reflected respectively from the low gradient plasma (a) and from the steep gradient plasma regions (b). Signal (a) has a rather low amplitude and phase perturbations can be seen indicating that plasma fluctuations are present in the probed plasma region.

Signal (b) exhibits an increased amplitude and a very low level of phase perturbations, evidencing that plasma fluctuations have been significantly reduced following the steepening of the plasma profile. The above described effects of plasma modes and fluctuations can lead to large errors in the profile evaluation as shall be discussed in the next section.
V - Density profile measurements

Density profiles are obtained from the integration of the basic phase shift information $d\varphi/dF$ according to eq. (2.1). In standard profile measurements the phase shift characteristic is obtained from the frequency minima $F_j$, and $d\varphi/dF$ is replaced in the small interval $[F_j - F_{j-1}]$ by $2\pi/(F_j - F_{j-1})$.

The phase shift characteristics obtained respectively in a low density plasma with lower hybrid heating (ASDEX #27294), and during the quiescent phase of an H-mode plasma (ASDEX #32268) are depicted in Figs. 6(a) and 8(a). It should be noted the great increase of $\Delta \varphi/\Delta F$ with frequency in the first case and the small increase observed in the second one, indicating respectively a flat and a steep density profile.

A localized phase perturbation can be observed in the low density plasma and two regions of perturbations can be identified in the H-mode plasma. The density profiles evaluated from the measured phase shift characteristics are presented in Figs. 7(a) and 9(a). It is also shown for comparison the filtered characteristics (as discussed in section VI), and the corresponding density profiles, Figs. 7(b) and 9(b).

From the figures it can be observed that the density profile of the LH plasma is not significantly distorted by the density perturbations whereas in the H-mode plasma fluctuations originate important deviations of the profile.

The main difference between these two results is that in # 27294 the plasma density perturbations affect only waves in a narrow frequency range, whereas in # 32268 waves in a broader frequency range are disturbed. As each measured differential phase shift $\Delta \varphi(F_j)/\Delta F$ is used both to evaluate to position of the plasma layer $X_j$, and to estimate the phase effect of the plasma layer $n_e(X_j)$ on waves with higher frequencies ($F > F_j$), (see eq. (2.2)), the phase shift errors will "propagate" to the higher density regions of the evaluated profiles. For this reason evaluation procedures of plasma profiles from reflectometry should include filtering algorithms that minimize the effect of plasma fluctuations.
VI - Beat frequency estimation techniques

In low density ASDEX plasmas, \( n_e(0) < 3.5 \times 10^{13} \text{cm}^{-3} \), density profiles could be evaluated directly from the reflectometric raw data. For higher densities the phase shift characteristic is always affected by plasma modes and fluctuations, and several methods were developed to evaluate the plasma profiles.

1 - Methods using minima detection

The two methods described below use the minima of the detected signals, each local minimum being identified through an amplitude criterion.

1.1 - Signal linear filtering techniques

Digital bandpass filtering with different bandwidths and central frequencies was applied to the reflectometric signals. In Fig. 10 it is presented the raw data signal (a) and the filtered signal (b) of the K-band reflectometer (ASDEX \# 27294).

The parameters of the F.I.R. filters have to be carefully selected; the filter bandwidth, for example, must be large enough to preserve the relevant phase information but has to be sufficiently narrow to provide an effective filtering of the signal perturbations due to the plasma fluctuations.

In situations where the phase shift derivative due to the plasma exhibits large variations, as it is the case in H-mode plasmas with flat bulk profiles, it is rather difficult to meet the requirements needed for the correct filtering, and a method based on the phase filtering is more adequate.
Phase filtering

The phase shift characteristic is evaluated by integration of the distorted beat frequency trajectory and a polynomial curve or a spline is fitted into that characteristic. The fitting polynomial is then differentiated providing a first estimate, $\tilde{f}_B(t)$, of the correct beat frequency. The "filtered" frequency curve is finally obtained as, $f_B(t) = \tilde{f}_B(t) + g(e(t))e(t)$, where $e(t) = f_B(t) - \tilde{f}_B(t)$ and $g$ is a suitable even-type function. If a Gaussian function is used, $g(x) = \exp(-0.5x^2/\sigma^2)$, the variance parameter $\sigma^2$ controls the degree of smoothing: for $\sigma^2 = 0$ the maximum smoothing is obtained, $\tilde{f}_B(t) = f_B(t)$; for $\sigma^2 = \infty$ no smoothing is performed and $\tilde{f}_B(t) = f_B(t)$. Other even-type functions, namely the rectangle function, can also be used.

This method was applied to the evaluation of plasma profiles in a wide range of plasma regimes. However, it cannot be used when the level of fluctuations is very high and a good beat frequency fitting cannot be obtained.

In this situation the signal frequency spectra broadens preventing also the application of bandpass filtering. A technique, based on the detection of the signal zero crossings, was implemented allowing the disturbed part of the reflectometric signal to be recovered. This technique was complemented with the nonlinear filtering approach to determine the beat frequency characteristic.

Stochastic model-based approach

The application of stochastic nonlinear filtering techniques allows to overcome, to a certain degree, the errors due to plasma density perturbations, [2]. This approach uses all the available data points providing a beat frequency characteristic with an increased definition. The $f_B(t)$ characteristic is estimated by applying a stochastic nonlinear filter where
... *a-priori model* is assumed for the $f_B(t)$ trajectory. A Brownian motion is considered whenever some model based on physical evidence is not available.

![Fig. 11](images/fig11.png)

**Fig. 11**

Figs. 11(a), (b) and 12(a) show three steps in the estimation of the beat frequency for ASDEX shot #29285, $t = 1580$ msec, Ka-band. The raw data of Fig. 11(a) are bandpass filtered and full-wave rectified. An envelope is estimated by fitting a cubic spline into the local maxima of the rectified signal. A new signal, with approximately constant amplitude, is then obtained, (Fig. 11(b)). This signal is processed by a discrete stochastic nonlinear filter to produce the estimated frequency trajectory of Fig. 12(a). Fig. 12(b) shows the result of applying the detection of minima to the same signal. The result is a less detailed frequency characteristic.

By modifying the parameters of the nonlinear filter different degrees of smoothing for the $f_B(t)$ trajectory can be obtained without further computations.

Profiles are evaluated by Abel integration of the estimated phase derivative using the usual numerical integration techniques. However, as the number of points of the beat frequency characteristic is very high (when compared to the detection of minima) a *fast Abel integration technique* was developed allowing significant computation savings, [2].

The results obtained with the stochastic model-based approach revealed its great potentialities for studying in detail the phase derivative characteristic. In particular, it can be used to analyse the temporal evolution of magnetic islands during locking and until the disruption. It should be noted that during mode locking Mirnov coils cannot provide any mode measurements.
VII - Concluding remarks

Broadband Microwave Reflectometry on ASDEX enabled to obtain density profile measurements with good spatial and temporal resolution in a wide range of plasma regimes (see for example [3] in this Workshop). The measurements do not require any information from other diagnostics. Plasma fluctuations and modes can cause significant errors in the evaluated profiles and either improved hardware (fast sweeping time), or software techniques have to be used. In ASDEX sweeping time was limited to 2 ms and therefore data processing including filtering techniques were developed to analyse the ASDEX data. These techniques will be the basis for the automatic evaluation of density profiles on the ASDEX-Upgrade system, presently under development.

VIII - References


B: THEORY AND SIMULATIONS
Abstract

The refractive indices and locations of cutoffs are investigated for cold, hot and relativistic plasma models. Significant relativistic modifications of refractive indices and locations of cutoffs are found in regimes relevant for reflectometry in large Tokamaks. It is demonstrated that these effects may shift the location of the reflecting layer by a significant fraction of the minor radius and that the cold model may lead to considerable underestimations of the density profile when X-mode is used. Relativistic effects predicted for O-mode reflectometry are smaller than for X-mode, but not negligible. An algorithm for reconstruction of density profiles which allows a relativistic plasma model to be used is presented.

1 Introduction

Electron temperatures found in large tokamaks (5-15 keV) remain small relative to the rest mass energy of the electrons (511 keV). It may therefore appear reasonable to assume that the relativistic modification of the plasma dielectric tensor is small in absolute terms and hence only noticeable in phenomena which vanish in a non-relativistic theory (e.g. off resonance cyclotron emission perpendicular to the magnetic field) or where a dielectric property is particularly sensitive to modifications in the plasma response.

Recent investigations have revealed significant relativistic shifts in the locations of cutoffs and modifications of the refractive index in the regions leading up to these cutoffs. This is caused partly by the fact that the refractive index in the vicinity of a cutoff is quite sensitive to the plasma response. It is, however, also found that the relativistic modifications of the Hermitian part of the dielectric tensor are larger than would be expected from a straightforward comparison of the electron temperature with the electron rest mass energy.

The relativistic modifications of refractive indices and cutoffs have practical consequences for the analysis of reflectometry data.

2 Relativistic plasma model

The cold plasma model is well known. In this paper the hot plasma model refers to a magnetized plasma with an isotropic Maxwellian velocity distribution for the electrons. In the hot model the dielectric tensor, \( \varepsilon \), is derived from Maxwells equations and the non-relativistic Vlasov equation. Like the cold model the hot model can be found in many text books [1]. In the relativistic model the constitutive equations are

\[
\begin{align*}
J &= \sum_j n_j q_j \int v_j f_j(p) \, dp, \\
\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} + q(E + \mathbf{v} \times B) \cdot \frac{\partial f}{\partial \mathbf{p}} &= 0
\end{align*}
\]

(1)

(2)
where \( \mathbf{v} = \mathbf{p}/\gamma m, \gamma = \sqrt{1 + (\mathbf{p}/m c)^2}. \) The unperturbed velocity distribution corresponding to thermodynamic equilibrium is

\[
f^{\text{a}} = \frac{\alpha \exp(-\alpha \gamma)}{4\pi (mc)^2 K_{2}(\alpha)}, \quad \alpha = \frac{m_e e^2}{T_e}.
\]

\( K_n \) is the modified Bessel function of the second kind and order \( n. \)

The relativistic dielectric tensor is derived from Maxwell's equations with the plasma current response described by equations (1), (2) and (3). Various expressions for the fully relativistic dielectric tensor have been derived [2]. One result, due to Trubnikov [3], is given here:

\[
\epsilon = \mathbf{I} + \frac{i\omega^2 \alpha^2}{\omega \omega_c K_{2}(\alpha)} \int_0^\infty dy \left\{ \frac{K_2(x)}{x^2} \mathbf{T}^{(1)} - \frac{c^2 K_3(x)}{\omega^2 x^3} \mathbf{T}^{(2)} \right\}.
\]

The relativistic dielectric tensor is derived from Maxwell's equations with the plasma current response described by equations (1), (2) and (3). Various expressions for the fully relativistic dielectric tensor have been derived [2]. One result, due to Trubnikov [3], is given here:

\[
\mathbf{I} = \begin{cases} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{cases}, \quad \mathbf{T}^{(1)} = \begin{cases} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{cases},
\]

\[
\mathbf{T}^{(2)} = \begin{cases} k_{\perp}^2 \sin^2 y & -k_{\perp}^2 \sin y (1 - \cos y) & k_{\perp} k_{\parallel} y \sin y \\ -k_{\perp}^2 (1 - \cos y)^2 & k_{\perp} k_{\parallel} (1 - \cos y) & k_{\parallel}^2 y \\ k_{\perp} k_{\parallel} y (1 - \cos y) & k_{\parallel} y & k_{\parallel}^2 y \end{cases}
\]

\[
x = \sqrt{(\alpha - i \frac{\omega}{\omega_c})^2 + 2 \left( \frac{k_{\perp} c}{\omega_c} \right)^2 (1 - \cos y) + \left( \frac{k_{\parallel} c y}{\omega_c} \right)^2}.
\]

Fully relativistic expressions for \( \epsilon \) including the one given in equation (4), are difficult to handle numerically. A much more tractable form known as the weakly relativistic dielectric tensor, derived from Trubnikov's result (4) by Shkarofsky [4], is given below. It is valid for \( \alpha \gg 1 \) and \( \lambda < 1 \) and is a very good approximation at the temperatures found in large Tokamaks.

\[
\epsilon_{ij} = \mathbf{I} - \frac{\alpha \omega^2}{\omega^2} \sum_{N=-\infty}^{\infty} \sum_{p=0}^{\infty} a_{pn} \lambda^{p+n-1} M_{ij}, \quad \lambda = \left( \frac{k_{\perp} v_t}{\omega_c} \right)^2, \quad v_t = \sqrt{\frac{T_e}{m_e}},
\]

\[
M = \begin{cases} n^2 F_{p+n+3/2} & -iN(p + n) F_{p+n+3/2} \\ -M_{12} & (p + n)^2 - \frac{(p+2n)}{2n+2p-1} F_{p+n+3/2} \\ M_{13} & M_{13} \end{cases}
\]

\[
a_{pn} = \frac{(-1)^p(n + p - 1/2)!}{p!(n + p/2)!(n + (p - 1/2))2^n}, \quad n = \lfloor N \rfloor,
\]

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\[ F_q = -i \exp(-\psi^2) \int_0^\infty (1 - it)^{-\frac{\mu}{2}} \exp[-i\phi^2 t + \psi^2/(1 - it)] dt. \]

\[ F^m = \frac{\partial^m F}{\partial(\phi^2)^m} \quad \psi = \frac{k_v e^2}{\sqrt{2} \omega v_t} \quad \phi^2 = \psi^2 - \alpha \left( \frac{\omega - \nu \omega_c}{\omega} \right). \]

The required Shkarofsky functions, \( F^m(\phi, \psi = 0) \), are readily evaluated by the use of recursion relations and expressions relating the lower order functions to the Fried and Conte dispersion function [4].

3 Refractive index

Electromagnetic waves in a homogeneous source-free plasma satisfy the homogeneous wave equation

\[ \nabla E = 0, \quad \nabla^2 E = \epsilon(k, \omega) + \mu^2 (k^2 - I), \]

where \( \mu \) is the refractive index, \( \omega \) the angular frequency, \( k \) the wave vector, \( k_v \) the unit wave vector and \( I \) the identity tensor. Non-trivial solutions to (6) only exist if

\[ \Lambda = |\Lambda| = 0. \]

This is the dispersion equation. In a cold plasma an explicit expression for \( \mu \) can be derived by solving for \( \mu \) in (7). In hot and relativistic plasmas, where \( \epsilon \) is a function of \( \mu \), (7) is a transcendental equation in \( \mu \). In this case the refractive indices of the modes of the plasma are found numerically by searching for the roots of \( \Lambda \) in the complex \( \mu \) plane. Whereas in the cold plasma at most 3 modes exist, in the hot and relativistic plasmas there are, due to the transcendental nature of (7), in general an infinity of modes, though most of them are heavily damped. A typical situation at frequencies above the R--cutoff is illustrated in figure 1.

In reflectometry using O--mode the frequency, \( f \), of the probing beam is below the cyclotron frequency, \( f_c \). Cold, hot and weakly relativistic calculations of the O--mode refractive index as a function of density at \( f < f_c \) and \( T_e = 10 \) keV are presented in figures 2 (a) and (b). The cold and the hot curves are indistinguishable.

X--mode reflectometry makes use of waves with frequencies between the 1st and 2nd harmonics of the electron cyclotron frequency and above the R--cutoff frequency. Calculations of the refractive index in this region are presented in figures 2 (c)–(f). Cold, hot and weakly relativistic calculations for \( T_e = 10 \) keV and \( f = 100, 110 \) and 120 GHz are compared in figures 2 (c)–(e). While the hot model produces only a small change relative to the cold model and tends towards the cold model at the R--cutoff, the weakly relativistic model predicts a substantial change in the refractive index and the density of the R--cutoff is increased considerably. In figure 2 (f) the refractive index is calculated relativistically for four different temperatures to display the temperature dependence of the relativistic effects. The \( T_e = 0.05 \) keV curve is indistinguishable from the cold curve.
Figure 1: Contour plots of the logarithm of the hot plasma dispersion function, \( \log(|\lambda|) \), in the complex \( \mu \) plane. The weakly relativistic modes appear as singularities in this plot. The cold O- and X-mode refractive indices are also indicated. Parameters: \( T_e = 15 \, \text{keV}, n_e = 6.0 \times 10^{19} \, \text{m}^{-3} \), \( B = 3.0 \, \text{T}, \angle(B, k) = 90^\circ, f = 124 \, \text{GHz} \). (\( f_s \approx 84 \, \text{GHz}, f_p \approx 70 \, \text{GHz}, f_R \approx 123 \, \text{GHz} \))

Figure 2: Refractive index, \( \mu \), as a function of electron density. Parameters: \( B = 3.4 \, \text{T}, \angle(B, k) = 90^\circ \). (a)-(e) cold, hot and weakly relativistic, \( T_e = 10 \, \text{keV} \),
(a) O-mode, \( f = 30 \, \text{GHz} \),
(b) O-mode, \( f = 60 \, \text{GHz} \),
(c) X-mode, \( f = 100 \, \text{GHz} \),
(d) X-mode, \( f = 110 \, \text{GHz} \),
(e) X-mode, \( f = 120 \, \text{GHz} \).
(f) weakly relativistic,
\( T_e = 0.05, 5, 10, 15 \, \text{keV} \),
X-mode, \( f = 120 \, \text{GHz} \).
It is noteworthy that the relativistic effects are more important than the effects found with the hot model in all the regimes explored here. The following analysis will therefore only be concerned with the cold and relativistic models.

4 Cut-offs

The locations of the X-mode R cut-off and the O-mode cut-off predicted by fully relativistic model are given by [5,6]

\[
\left( \frac{\omega_p^2}{\omega^2} \right)_{R \text{ cut-off}} = \frac{3K_2(\alpha)}{\alpha^2 \int_0^\infty \left( 1 + \frac{\omega_c e/\omega}{\gamma} \right) \frac{p^4 e^{-\alpha \gamma}}{p^2 + 1 - (\omega_c e/\omega)^2} \, dp}
\]

\[
\left( \frac{\omega_p^2}{\omega^2} \right)_{O \text{ cut-off}} = \frac{3K_2(\alpha)}{\alpha^2 \int_0^\infty (p^4/\gamma^2) e^{-\alpha \gamma} \, dp}
\]

where \( K_2 \) is the modified Bessel function of second order.

The locations of cut-offs found with the weakly relativistic code agree accurately with the results found with the fully relativistic expressions (8) and (9). In figure 3 the locations of the R cut-off and O-mode cut-off predicted by the fully relativistic model (equations (8) and (9)) are plotted in an \( \omega_c/\omega \) versus \( \omega_p^2/\omega^2 \) diagram (CMA diagram) for \( T_e = 0.1, 5, 10, 15, 20 \) keV.

![Figure 3: Locations of the R cut-off and O-mode cut-off predicted by the fully relativistic model, plotted in an \( \omega_c/\omega \) versus \( \omega_p^2/\omega^2 \) diagram (CMA diagram). \( T_e = 0.1, 5, 10, 15, 20 \) keV. The \( T_e = 0.1 \) keV curves are almost identical to the cold curves.](image)

As may be seen in figures 2 and 3, the largest relative shift occurs for the R cut-off near the cyclotron frequency. Relativistic effects in reflectometry should therefore be most noticeable in this region.
To explore the consequences of the relativistic modifications for reflectometry, plasmas with the following electron density, $n_e$, electron temperature, $T_e$, and magnetic field, $B$, profiles are assumed:

\begin{align*}
n_e &= n_{e0} \left(1 - \left(\frac{r}{a}\right)^2\right)^{p_n} + n_{el} \\
T_e &= T_{e0} \left(1 - \left(\frac{r}{a}\right)^2\right)^{p_T} + T_{el} \\
B &= \left(\frac{B_0 R_0}{R_0 + r}\right) \\
R_0 &= 3 \text{ m}, \ a = 1 \text{ m}, \ B_0 = 3.5 \text{ Tesla} \\
n_{e0} &= 6 \cdot 10^{19} \text{ m}^{-3}, \ p_n = 0.5, \ n_{el} = 1 \cdot 10^{17} \text{ m}^{-3} \\
T_{e0} &= 0, 5, 10, 15 \text{ keV}, \ p_T = 1, \ T_{el} = 100 \text{ eV}.
\end{align*}

Figure 4: Cut-off frequency as a function of major radius. Plasmas are defined in equations (10). (a) $O$-mode, (b) $X$-mode, $B_0 = 3$ T, (c) $X$-mode, $B_0 = 5$ T.
The cut-off frequency as a function of major radius is given in figures 4 (a)-(c) for a range of central temperatures, \( T_e \), and in the case of X-mode, for \( B_0 = 3 \) and 5 Tesla. It is clear that in both modes the cutoff point may, in the central region, be shifted by a significant fraction of the minor radius.

5 General algorithm for density profile reconstruction

It is evident that the relativistic modification of the refractive index and in particular the shift of the cut-off density will change the relation between the density profile and the phase change which a probing wave undergoes in the plasma. An algorithm for reconstruction of the density profile from reflectometric data which is valid for both O-mode and X-mode in the relativistic model is therefore required. One such algorithm is derived here. It is of course also valid for the cold model.

It is assumed that the phase shift which the probing wave undergoes in the plasma is \( 2(\omega/c)\Psi(\omega) - \pi/2 \) where \( \omega \) is the frequency of the probing wave, \( c \) is the speed of light and \( \Psi(\omega) \) is the optical distance from the plasma edge to the cut-off layer [7],

\[
\Psi(\omega) = \int_{r_{\text{cut-off}}}^{r} \mu \, dr. \tag{11}
\]

\( e \) is analytical in \( \mu^2 \) at \( \mu = 0 \) (cf. equation (5)). From this it follows that \( \Lambda \) is analytical in \( \mu^2 \) at \( \mu = 0 \) and hence

\[
\frac{\partial \Lambda}{\partial \mu} = 2\mu \frac{\partial \Lambda}{\partial \mu^2} = 0 \text{ at } \mu = 0, \tag{12}
\]

while in general

\[
\frac{\partial \Lambda}{\partial \mu^2} \neq 0 \text{ at } \mu = 0. \tag{13}
\]

Since \( \Lambda \) is analytic in \( X \), where \( X \) equals \( B \), \( n_e \) or \( T_e \) (cf. equation (5)), it follows from (13) that

\[
\frac{\partial \mu^2}{\partial X} = \frac{-\partial \Lambda}{\partial X} \frac{\partial \Lambda}{\partial \mu^2} \tag{14}
\]

tends to a finite limit as \( \mu \to 0 \), while

\[
\frac{\partial \mu}{\partial X} = \frac{-\partial \Lambda}{\partial X} \frac{\partial \Lambda}{\partial \mu} \tag{15}
\]

does not.

Expanding \( \mu^2 \) around the cut-off we get to lowest order in \( x \)

\[
\mu^2(x) = \left( \frac{\partial \mu^2}{\partial B} \frac{\partial B}{\partial x} + \frac{\partial \mu^2}{\partial n_e} \frac{\partial n_e}{\partial x} + \frac{\partial \mu^2}{\partial T_e} \frac{\partial T_e}{\partial x} \right) x \tag{16}
\]

where \( \mu(x) \) is the refractive index at the distance \( x \) from the cutoff. Let \( \delta \Psi \) be the integral of \( \mu \) from the cutoff out to a distance \( \Delta \)

\[
\delta \Psi = \int_0^\Delta \mu(x) \, dx = \sqrt{\frac{\partial \mu^2}{\partial B} \frac{\partial B}{\partial x} + \frac{\partial \mu^2}{\partial n_e} \frac{\partial n_e}{\partial x} + \frac{\partial \mu^2}{\partial T_e} \frac{\partial T_e}{\partial x} \frac{2 \Delta^{3/2}}{3}}. \tag{17}
\]
The second equality is valid to lowest order in \( \Delta \). From equations (16) and (17) we get an expression for \( \frac{\partial n_e}{\partial x} \) in the vicinity of the cut-off which together with the expression for \( \Delta \), implicit in (16), forms the basis for the inversion algorithm:

\[
\frac{\partial n_e}{\partial r} = \left( \frac{2\mu^3}{3\delta \Psi} - \frac{\partial \mu^2}{\partial B} \frac{\partial B}{\partial r} - \frac{\partial \mu^2}{\partial T_e} \frac{\partial T_e}{\partial r} \right) \frac{\partial \mu^2}{\partial n_e}
\]

(18)

\[
\Delta = \frac{3\delta \Psi}{2\mu}.
\]

(19)

In equations (18) and (19) \( \mu \) is the refractive index at the distance \( \Delta \) from the cut-off while the gradients, which vary slowly, can be evaluated anywhere in the vicinity of the cut-off. The assumptions about analyticity made in the derivation clearly also hold for the cold model.

From equations (18) and (19) the following algorithm is readily derived.

\[
\begin{align*}
or & = a \\
\tilde{n}_0 & = n_{e,\text{cut-off}}(\omega_0, B_0, T_0); \quad B_{i+1} = B(\tilde{r}_{i+1}) \quad ; \quad T_{i+1} = T_e(\tilde{r}_{i+1}) \\
\omega_i & = \omega_0 + i\delta \omega \\
(\mu^2)_i & = \mu^2(\omega_i, \tilde{n}_{i-1}, B_{i-1}, T_{i-1}) \\
\left( \frac{\partial \mu^2}{\partial X} \right)_i & = \left( \frac{\partial \mu^2}{\partial X} \right)(\omega_i, \tilde{n}_{i-1}, B_{i-1}, T_{i-1}); \quad X = n_e, B, T_e \\
\tilde{\Psi}_i & = \int_{r_{i-1}}^{r_i} \tilde{\mu}(\omega_i, r) \, dr \quad ; \quad \tilde{\mu}(\omega_i, r_i) = \mu(\omega_i, \tilde{n}_j, B_j, T_j) \\
\delta \Psi_i & = \Psi_i - \tilde{\Psi}_i \\
\Psi_i & = \int_{r_{\text{cut-off}}(\omega_i)}^{r_i} \mu_{\text{rel}}(\omega_i, r) \, dr
\end{align*}
\]

(\partial n) \left. \frac{\partial}{\partial r} \right|_i = \left( \frac{2\mu^3}{3\delta \Psi_i} - \left( \frac{\partial \mu^2}{\partial B} \right)_i \frac{\partial B(\tilde{r}_{i-1})}{\partial r} - \left( \frac{\partial \mu^2}{\partial T_e} \right)_i \frac{\partial T(\tilde{r}_{i-1})}{\partial r} \right) \left( \frac{\partial \mu^2}{\partial n_e} \right)_i

\[
\Delta_i = \frac{3\delta \Psi_i}{2\mu_i}
\]

\[
r_i = \tilde{r}_{i-1} - \Delta_i
\]

\[
\tilde{n}_i = \tilde{n}_{i-1} - \left( \frac{\partial \tilde{n}}{\partial r} \right)_i \Delta_i
\]

\( \tilde{n}_i \) is the reconstructed electron density at the minor radius \( r_i \) (\( r_i \) is negative on the inside of the plasma).

6 Reconstruction of density profiles from simulated data

To simulate reflectometric data the phase function \( \Psi(\omega) \), as given in equation (11), was calculated relativistically for a range of plasmas given by equations (10). Density profiles
were then reconstructed from $\Psi(\omega)$ with the algorithm given above using the cold and the relativistic plasma model.

The reconstructed density profiles are identical to the actual density profiles when the reconstruction is based on the relativistic plasma model. This demonstrates that the above algorithm is numerically stable and accurate, and that reconstruction based on a relativistic plasma model is feasible.

Figure 5: Actual electron density profile and reconstructed density profiles derived using the cold plasma model for analysing phase functions, $\Psi(\omega)$, obtained with the relativistic model. The plasmas are defined in equations (10). (a) O-mode, $B_0 = 5$ T (b) X-mode, $B_0 = 3$ T (c) X-mode, $B_0 = 5$ T. For X-mode the maximum probing frequency and hence the maximum depth to which the density profiles could be reconstructed was limited by absorption in the outer region of the plasma at the second harmonic the cyclotron frequency.
When the reconstruction is based on the cold plasma model the reconstructed density profiles underestimate the actual density profiles, by a considerable amount in X-mode and by a smaller, though still significant, amount in O-mode.

Examples of density profiles reconstructed with the cold model are given in figures 5 (a)-(c).

7 Conclusions

The dielectric properties of plasmas have been investigated for propagation perpendicular to the magnetic field at frequencies in the range of the electron cyclotron frequency and the plasma frequency. Calculations of the Hermitian part of the dielectric tensor, refractive indices and locations of cut-offs based on cold, hot and relativistic plasma models were compared. While only small differences were found between the cold and the hot models, substantial differences in all three quantities were found between the cold and hot predictions on the one hand and the relativistic on the other. The differences are larger than a simple comparison of $T_e$ with $m_e c^2$ might suggest.

Cold and relativistic predictions for reflectometry were compared. It was found that relativistic effects are of practical importance for X-mode reflectometry in large Tokamaks, because (a) cold analysis leads to a considerable underestimation of the electron density profile and (b) the location of the cutoff may be shifted by a significant fraction of the minor radius. (a) implies that for density profile measurements using X-mode reflectometry (an attractive option for ITER) the data must be analysed with a relativistic plasma model. (b) has consequences for the determination of where fluctuations observed with correlation reflectometry are situated in the plasma.

While the relativistic modifications found in O-mode are smaller than in X-mode they may still have to be taken into account, except at the plasma edge.

A code for relativistic reconstruction of the electron density profile from the phase shift function has been written and tested with simulated reflectometric data.

References

A Numerical Study of Correlation Reflectometry

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Abstract

In this report some basic features of the performance of a correlation reflectometer used as a diagnostic for random plasma fluctuations are studied. Using a realistic and tractable model for the plasma fluctuations we derived some analytical results for correlation and crosscorrelation functions for the temporally varying phase of the reflected signals. Numerical simulations were performed to illustrate the practical applicability of the basic ideas of the reflectometer.

1 Introduction

Recently the principles of a new technique for diagnosing microturbulence called correlation reflectometry was presented (Costley and Cripwell, 1989, and Cripwell and Costley 1991). Two microwave beams with a small difference in frequency are launched against the density profile. The two phases can be measured versus time and since these phases are functions of plasma cut-off layer positions, it is possible by a crosscorrelation technique to detect the relative motion of plasma perturbations between two different positions. By using a variety of frequency differences between the two microwave beams it has been possible (Costley and Cripwell, 1989) to obtain a full dispersion curve for the plasma waves giving essential information about the plasma turbulence.

We carry out a performance study of a model of a two-frequency reflectometer. A level of random plasma density fluctuations is modelled in plane geometry by superimposing moving density pulses on a given density profile. By the proper choice of the shapes of these pulses, we are in principle able to model any spectrum for disturbances propagating in the direction along the density gradient. With the speed of propagation known in the numerical experiment, we are able to determine the accuracy of the predictions of the characteristic velocity deduced from the crosscorrelation of the fluctuating phase signals of the reflectometer. Studies are carried out for statistically distributed disturbance velocities and for varying levels of a superimposed small-scale random noise component. The analysis uses a fullwave solution and results are compared with WKB-solutions. More details on this work are given by Michelsen and Pécseli (1991).

Section 2 gives the wave equation and derives the appropriate boundary conditions, and presents some numerical results. In Sec. 3. a discussion of the plasma model and the correlation analysis can be found. Finally, discussions and conclusions are given in Sec. 4.
2 The full wave solution

We consider electromagnetic waves propagating in the $z$-direction in a plasma with an inhomogeneous density $n(x)$ in a constant magnetic field $B_0$. The wave equation can be written as:

$$\frac{d^2 E}{dx^2} = -\varepsilon(x) E,$$

where $\varepsilon = \varepsilon_0$ with $k_0$ the wavenumber in free space. We have for the O-mode that $E = E_z$ and $\varepsilon = \varepsilon_{xx}$ and for the X-mode that $E = E_y$ and $\varepsilon = \varepsilon_{xx}^2/\varepsilon_{yy}$. Here the components of the dielectric tensor are

$$\varepsilon_{xx} = 1 - \frac{X(1 + iZ)}{(1 + iZ)^2 - Y^2}, \quad \varepsilon_{xy} = i\frac{XY}{(1 + iZ)^2 - Y^2}, \quad \varepsilon_{zz} = 1 - \frac{X}{1 + iZ} \tag{2}$$

The normalized density $X$, the normalized magnetic field $Y$, and the normalized collision frequency $Z$ are defined by

$$X = \frac{\omega_p^2}{\omega^2}, \quad Y = \frac{\omega_{ce}}{\omega}, \quad Z = \frac{\nu}{\omega}, \tag{3}$$

where $\omega_p(\xi) = (n(\xi)e^2/\varepsilon_0m)^{1/2}$ and $\omega_{ce}(\xi) = eB(\xi)/m$ are the electron plasma frequency and the gyro-frequency, respectively.

To solve the equation (1) we have to specify the necessary boundary conditions. Assume the inhomogeneous part of the plasma is surrounded by a homogeneous plasma i.e.: $n = n_0$ for $\xi < 0$ and $n = n_1$ for $\xi > a$. In the homogeneous plasma ranges the wave solution is the solution to eq.(1) with constant $\varepsilon$:

$$E(\xi) = c_1 \exp(iN_x\xi) + c_2 \exp(-iN_x\xi) \tag{4}$$

where $N_x$ is the x-component of the refractive index. The matching condition at the border between the homogeneous and the inhomogeneous plasma is determined by that the E-field and the derivative of the E-field ($\alpha B$-field) must be continuous across the boundary. At the left boundary we let the incoming wave have the amplitude $c_1 = 1$. If the E-field at the boundary is $E_0$ the boundary condition can be written as:

$$iN_xE_0 + E'_0 = 2iN_x0 \tag{5}$$

where $E'_0$ is the derivative of $E(\xi)$ at $\xi = 0$ and $c_2 = (E_0 - 1)$. At the right boundary, $\xi = a$ there will be no left-going wave, i.e.: $c_2 = 0$, which gives the boundary condition:

$$iN_xE_a - E'_a = 0 \tag{6}$$

where $E'_a$ is the derivative of $E(\xi)$ at $\xi = a$ and $c_1 = E_a \exp(-iNxa)$. This means that the wave solution in the homogeneous regions is:

$$E = \exp(iN_x0\xi) + (E_0 - 1)\exp(-iN_x0\xi), \quad \text{for} \quad \xi \leq 0$$

$$E = E_a \exp[iN_x1(\xi - a)], \quad \text{for} \quad \xi \geq a \tag{7}$$

where $E_0$ and $E_a$ are the E-fields at the left and right boundary, respectively.

The full wave equation with the given boundary conditions was solved by use of the numerical code COLSYS by Ascher, Christiansen and Russel (1979). This code can solve boundary-value problems for mixed-order systems of ordinary differential equations. The solution method is based on spline collocation, and the code automatically finds an appropriate distribution of mesh points in order to keep the local error within certain limits specified by the user. A similar but more general system of equations taking into account oblique propagation with respect to the magnetic field solved by COLSYS was treated by Hansen et al. (1988a), in order to investigate wave conversion.
In Fig. 1a the wave solution for an ordinary wave propagating against a steep density gradient is shown. The density is zero at the left boundary and it is equal to twice the critical density at the right boundary. In Fig. 1b corresponding curves for a wave in a density distribution with a smooth gradient are shown. A WKB solution will in this case give nearly the same solution.

In order to see how a small narrow pulse will influence on the reflected wave Fig. 2 shows a model with a pulse moving along a smooth density gradient. The pulse half-width is in this case equal to five wavelengths and has an amplitude equal to 0.2 times the critical density. In Fig. 3a the phase of the reflected wave at the left boundary is shown as a function of time, calculated by COLSYS (solid line) and by a standard WKB approximation (Ginzburg 1964). Similar curves are shown in Fig. 3b with a more narrow pulse with a half-width equal to one wavelength and of the same amplitude as in Fig. 2. It is seen here that large differences in the wave-field will appear when a density pulse with a width comparable to the wavelength moves in a standing wave.
Figure 3: Phase of reflected wave versus time when a pulse is moving along the density gradient. Solid line is full wave solution, and dashed line is WKB solution. a) The half-width of the pulse is five wavelength, b) The half-width of the pulse is one wavelength.

3 Reflectometry modelling

With the full-wave calculation we only have to introduce two assumptions concerning the plasma motion and density profile. First we assume that plasma motion is slow compared to the speed of the electromagnetic modes, and therefore, we can calculate the wave pattern and the phase of the reflected wave for a stationary plasma profile neglecting plasma motion. The other assumption requires that the one-dimensional plasma is surrounded by regions of constant density in order to fulfill the boundary conditions.

The plasma density profile is separated into three parts. The stationary background density is chosen to: \( n(\xi) = n_0 \xi^2 (3 - 2\xi) \) which is the lowest order polynomial with zero slope at \( \xi = 0 \) and at \( \xi = 1 \), and which is equal to 0 at \( \xi = 0 \) and equal to \( n_0 \) at \( \xi = 1 \). On top of the stationary density profile a noisy background is superimposed. The noise is generated by superimposing a large number of bell-shaped pulses with random positive or negative amplitudes within a certain range, with given width, and with a random velocity direction. Finally the fluctuations in plasma density are described as composed of a linear superposition of pulses having constant shapes and propagating with constant velocity. The pulses may have \( M \) different shapes labelled by the index \( \ell \). One such pulse gives rise to a certain phase variation \( \phi_\ell(t - t_{j,\ell}) \) of the electromagnetic wave as detected at the receiving antenna outside the plasma. We let \( t_{j,\ell} \) denote the time where the peak value of the density pulse (with label \( \ell \)) passes through the cut-off position in the unperturbed plasma profile. The individual pulses are assumed to be integrable and to vanish for \( |t| \to \infty \) but otherwise they can be chosen arbitrarily. In the following we assume the density perturbations to be pulse-like, but any other form (such as a wave-packet) can be chosen depending on the actual model for the fluctuations. With the density pulses injected randomly into the plasma and uniformly distributed in time we may write the temporally varying response in the phase signal as

\[
\tilde{\phi}(t) = \sum_{\ell} \sum_{j} N_{\ell} \phi_\ell(t - t_{j,\ell})
\]

where the number of pulse responses \( N_{\ell} \) originating from shapes of type \( \ell \) is itself a quantity which varies over the ensemble. With \( t_{j,\ell} \) being uniformly distributed in a time record much longer than the duration of an individual response, we readily obtain (Rice, 1944) the autocorrelation
function for the fluctuating phase signal

\[ R(\tau) \equiv \langle \phi(t) \phi(t + \tau) \rangle = \sum_{\ell} \nu_\ell \int_{-\infty}^{\infty} \phi_\ell(t) \phi_\ell(t + \tau) dt + \left[ \sum_{\ell} \nu_\ell \int_{-\infty}^{\infty} \phi_\ell(t) dt \right]^2, \]  

(9)

where \( \nu_\ell \) is the average number of structures of type \( \ell \) passing the cut-off layer per unit time. Note that there is in general no logical reason for the last term to be vanishing. A power spectrum \( S(\omega) \) for the phase fluctuations is defined as the Fourier transform of \( R(\tau) \), and it is important to note that an arbitrary prescribed spectrum can be realized by the model (8), actually in indefinitely many ways, by the appropriate choice of \( \phi_\ell \) for \( \ell = 1, 2, \ldots, M \).

The result (9) refers to a one-frequency reflectometer, but it is easily generalized to its two-frequency counterpart. When the density pulse passes the cut-off layer corresponding to the frequency of the second reflectometer it gives rise to a phase variation

\[ \psi(t) = \sum_{\ell} \sum_{j} \psi_\ell(t - t_{j, \ell} - D/u_j) \]  

(10)

where \( u_j \neq 0 \) is the velocity of the \( j \)-th pulse and \( D \) is the distance between the two cut-off layers. The time \( t_{j, \ell} \) is still referring to the crossing of the first cut-off layer as in (8). Note that we use different notations for the two phase signals, i.e. \( \phi_\ell \) and \( \psi_\ell \) will probably look rather similar, but because of the difference in local plasma density (and possibly density gradients) at the two positions they will not be identical. The autocorrelation function for \( \psi(t) \) is obtained in a form quite similar to (9) while the more interesting crosscorrelation takes the form

\[ R_c(\tau) = \langle \phi(t) \psi(t + \tau) \rangle = \sum_{\ell} \nu_\ell \int_{-\infty}^{\infty} \phi_\ell(t) \psi_\ell(t + \tau - D/u) P(u) du dt + \left[ \sum_{\ell} \nu_\ell \int_{-\infty}^{\infty} \phi_\ell(t) dt \right] \left[ \sum_{\ell} \nu_\ell \int_{-\infty}^{\infty} \psi_\ell(t) dt \right] \]  

(11)

where \( P(u) \) is the probability density of velocities \( u \), which is here taken to be independent of pulse shape. For cases of interest here both polarities of density pulses are equally probable and the last, constant term in (9) and (11) is vanishing.

As working hypothesis we first assume that \( \phi(t) \approx \psi(t) \), which can actually be a good approximation when the two cut-off layers are close. With the previous definition of \( S(\omega) \) we obtain the Fourier transform \( S_c(\omega) \) of (11) as

\[ S_c(\omega) = S(\omega) \int_{-\infty}^{\infty} e^{-i\omega D/u} P(u) du. \]  

(12)

For the case where all density pulses have the same velocity, i.e. \( P(u) = \delta(u - u_0) \) we have the particularly simple case

\[ S_c(\omega) = S(\omega) e^{-i\omega D/u_0}. \]  

(13)

showing that all frequency components undergo a phase change proportional to \( \omega \) with a coefficient \( D/u_0 \) which can be used to determine \( u_0 \). In this particular case the crosscorrelation (11) is just a shifted copy of the autocorrelation (9) and a characteristic velocity is obtained unambiguously. Here \( S(\omega) \) coincides with the crossspectrum with the present assumptions. However, in the case where the difference between \( \phi(t) \) and \( \psi(t) \) is nontrivial, it is no longer possible to write \( S_c(\omega) \) as a real spectrum with the phase variation given in the form as in (13), and a velocity of propagation is no longer uniquely defined.

In the case where the pulse velocities are statistically distributed, we have to solve (12) with the actual probability density \( P(u) \) even when the approximation \( \phi(t) \approx \psi(t) \) remains applicable.

We considered two examples: first a box-like distribution

\[ P(u) = \begin{cases} \frac{1}{b-a} & \text{for } 0 < a < u < b \\ 0 & \text{elsewhere} \end{cases} \]  

(14)
The function $\theta(\Omega)$ calculated for a box-like pulse velocity distribution for two values of $\alpha = a/b$, see (14) and (15). The dotted line shows the average pulse velocity. a) $\alpha = 0.67$ and b) $\alpha = 0.33$.

The integral in (12) can then be solved analytically with the result

$$\int_a^b e^{-i\omega D/u} du = -\frac{1}{a\Omega} e^{-i(\alpha-1)\Omega} - i\Omega E_1(\alpha\Omega) + \frac{1}{\Omega} + i\Omega E_1(\Omega)$$

(15)

where $\alpha \equiv a/b$ and $\Omega = \omega D/a$ while $E_1(x) \equiv \int_x^\infty \frac{1}{t} e^{-t} dt$. Using (15) we can rewrite (12) in the form $F(\Omega)e^{-i\theta(\Omega)}$ with $\theta(\Omega)$ shown in Fig. 4 for two values of $\alpha$. Evidently, for $\alpha = 1$ we recover the representation (13), i.e. $\theta(\Omega) = \Omega$. For $\alpha < 1$ we find that $\theta(\Omega)$ is no longer a straight line, but has a curvature, which increases with decreasing $\alpha$. Another observation is that the curve deviates from the slope corresponding to that given by the average of pulse velocities, i.e. $(a + b)/2$ in the present model for $P(u)$. This line is dotted on Fig. 4.

In another model we assumed

$$P(u) = \frac{1}{\sigma \sqrt{\pi}} e^{-[u-u_0/\sigma]^2}.$$  

(16)

The integral in (12) was evaluated numerically and the curve $\theta(\Omega)$ with $\Omega \equiv \omega/\sigma$ was calculated. Again, in the limit $\sigma \to 0$ we recover (13). For $\sigma > 0$ we again find that $\varphi(\Omega)$ is no longer a straight line and that its average slope can deviate from the one determined by $u_0$ in (16). The model (16) is used only in cases where it is safe to assume that $P(u \leq 0) \approx 0$.

We may conclude that the slope of the phase function $\theta(\Omega)$ gives a quite acceptable approximation to the average pulse velocity for narrow pulse velocity distributions $P(u)$ in (12). This approximation deteriorates for increasing scatter of pulse velocities, and eventually the results can depend critically on the actual choice of $P(u)$.

Finally, we model plasma waves also by pulses of random positive or negative amplitudes, random widths, and moving with a given average velocity. The pulses each have a constant velocity which is the sum of the average velocity plus a certain random velocity specified to some limited interval. Initially the pulses are distributed randomly in space. During a run pulses disappear when they move out of the plasma, but new pulses are injected at the other boundary at random times with a given average injection rate, specified by the average number of pulses. In Fig. 5a an example of a density profile with noise pulses included is shown, and in Fig. 5b both noise pulses and wave pulses are included.

The phase of the reflected wave is calculated for each time step. Phase curves for two different reflection points i.e. two frequencies are produced for correlation analysis. An example of the
Figure 5: Density profile versus time. a) Only noise pulses are included, b) Both wave and noise pulses are included.

Figure 6: An example of the phase as function of time for 8192 time steps.

Phase variation with time is shown in Fig. 6. The phase variation is restricted to the interval \([-\pi; \pi]\). In practice it need not be evident how this restriction is achieved and problems may occur, which are not accounted for in the constructions (8) or (10).

In the present study we used density perturbations generated by a superposition of puls-like individual perturbations. Evidently any form can be used for \(\Phi\) and \(\Psi\) in (8) and (10), also long wave packets and similar. It is possible to perform a quite detailed simulation of any actually observed spectrum of perturbations.

From our model calculation we get the two phase curves as discussed above. In a real experiment the amplitude of the reflected wave will be smaller than the injected wave due to geometrical expansion of the beam, the scattering due to plasma turbulence and to plasma damping. However, these processes should not have any effect of importance on the phase of the reflected wave. Since the phase shift depends on the variation of the plasma density between the antenna and the reflection point it is not possible to know a priori which information can be extracted from the phase measurements. By considering a small perturbation moving along \(x\) we find a large contribution when the perturbation passes the cut-off point which decreases when the perturbation moves to lower densities. This decrease is monotonic when the perturbation
length is larger than the wavelength, but oscillating when the perturbation is short compared to the wavelength (See Fig. 3). The phase change in the reflected signal will depend on the size and shape of the perturbation, and the density function in front of the cut-off layer, especially the slope of the density near the cut-off layer. If the reflection point of the two reflectometers are close, the two phase change signals should be similar and since the large contribution comes at the time when the perturbation passes the cut-off layer it should be possible by correlation techniques to determine the time of flight of the passing perturbations. If the distance between the two reflection points is determined by single reflectometry the perturbation velocity may subsequently be calculated. To investigate how well this can be done if there is a spread in perturbation velocity and there is other kind of turbulence in the plasma is the objective for the following investigation. If the phase signal exactly gives the position of the cut-off layer the problem is easy. However since this is not the case the solution to the problem is not obvious.

Let the two phase-signal be \( \phi_1(t) \) and \( \phi_2(t) \). Then we can calculate the correlation function and the Fourier transform of the correlation function:

\[
R(\tau) = \frac{1}{T} \int_0^T \phi_1(t)\phi_2(t - \tau)dt, \quad F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(\tau)e^{i\omega\tau}d\tau
\]

(17)

From a numerical point of view the Fourier transform of the correlation function can, however, be found in a more easily wave as:

\[
F(\omega) = G(\phi_1(\omega))H^*(\phi_2(\omega))
\]

(18)

where \( G \) is the Fourier transform of \( \phi_1 \) and \( H^* \) is the complex conjugate of the Fourier transform of \( \phi_2 \). In this way the calculation can utilize the fast Fourier transform.

If the phase responses from a perturbation passing the cut-off layers corresponding to the two reflectometer frequencies are similar we should expect the correlation function \( R(\tau) \) to be peaked, and the time shift of the peak should be the time of flight of the perturbation. From the Fourier transform \( F(\omega) \) we can obtain the distribution of perturbation amplitudes and the phase shift versus frequency, and thereby the perturbation velocity.

In Fig. 7a the correlation function is shown in a case with wave pulses propagating in a plasma without any noise. All the pulses move with the same velocity. The corresponding cross-amplitude spectrum is shown in Fig. 7b, and in Fig. 7c is shown the phase shift of the various Fourier components. The peak of the correlation function is, of course, shifted corresponding to the pulse velocity. However, the shift is of the order of 20 time steps and, therefore, not easily recognizable on Fig. 7a.

The shown correlation functions have been calculated from phase curves consisting of 8192 points (time steps). Some improvements in the correlation function can be obtained by dividing the phase curve into two or more equal parts, calculating the correlation function for each and taking the average of the results. This will reduce the uncertainty of the individual Fourier components on the expense of a reduced resolution of the spectrum caused by the reduction in individual record lengths.

From other calculations (see Michelsen and Pécseli 1991) it is evident that the estimate on \( \theta(\omega) \) becomes increasingly uncertain when the velocity-spread of the structures or pulses is increased. The addition of small scale noise, has a similar effect. The estimated value of \( \theta(\omega) \) becomes particularly uncertain at frequencies where the spectral amplitude is small, since the phase is here obtained as the ratio of two small quantities.
4 Discussion and Conclusions

The present study have assumed the density perturbations as a priori given, without discussing the actual nature of these fluctuations. In the report of Costley and Cripwell (1989) an interpretation in terms of drift waves was advocated. However, this particular wavetype propagates predominantly in the direction perpendicular to both the local magnetic field $\mathbf{B}$, and the density gradient $\nabla n_0(r)$, with small modifications induced by magnetic shear. A significant radial propagation velocity is most likely to be found for acoustic type fluctuations where our model is directly applicable or cyclotron waves, which however will have a significant dispersion of individual pulses. Since Alfven waves are incompressible, it might be expected that they should not be observable by reflectometer techniques. However, if these waves are propagating in a plasma density gradient, they may still give rise to local fluctuations in density, when the local plasma velocity associated with the wave moves plasma in and out along the gradient.

Due to limitations in the COLSYS code, the present studies were carried out in one spatial dimension. In the limit where the WKB approximation is applicable it is actually possible to carry out the numerical simulations in a fully three dimensional toroidal model using codes applied for different problems by Hansen et al. (1988b, 1988c) or Bindslev and Hansen (1991).
In summary we may state that our results indicate that a two-frequency reflectometer can in a number of cases prove to be a most versatile method for diagnosing local density fluctuations in fusion related plasma experiments. When the density perturbation have a velocity component in the direction defined by the probing electromagnetic wave beams, then this velocity component can be determined relatively accurately by a crosscorrelation of the modulated phase of the reflected waves, where the modulation is caused mainly by density perturbations propagating through the reflection point (i.e. cut-off layer) for the two waves. The studies by Costley and Cripwell (1989) were concerned primarily with density perturbation propagating in the radial direction of the plasma i.e. they used normally incident probing waves, although also other angles of incidence could be used.

References

A full wave, one dimensional, model of the phase response of reflectometers to density fluctuations is presented. The model is based on obtaining numerical solutions to the electromagnetic wave equation in a turbulent dielectric medium using a differencing method. In order to test the accuracy of the model, its results are compared with analytic solutions of the wave equation.

The model is used to determine the phase response of a single channel reflectometer and a two channel correlation reflectometer to density fluctuations. For the single channel device, it is concluded that it is only possible to obtain qualitative information on the density fluctuations. However for the two channel correlation reflectometer it is shown that it is possible to determine, under certain conditions, the radial dispersion relation of the fluctuations and the radial correlation lengths.

1: Introduction

Reflectometry has been used to study density fluctuations on a number of different devices, /1,2/. However a clear relationship between the density fluctuations in the plasmas and the fluctuating reflectometer signal has not been established and makes interpretation of the reflectometer signals difficult. Recently there have been some attempts to model the effects of density fluctuations using the Born approximation, /3/, which indicate that the reflectometer response is localised close to the reflecting layer in certain conditions. In this paper, a one dimensional numerical solution to the wave equation in a turbulent inhomogeneous dielectric medium is presented. No approximations are made restricting either the size of the perturbations (WKB approximation) or the number of wave-perturbation interactions (Born approximation). This is used to investigate the effects of density fluctuations on the reflectometer signals and to model the response of both the single channel reflectometer and the radial correlation reflectometer to density fluctuations in the plasma.
The electric field structure of an electromagnetic wave in a plasma is described by the one dimensional wave equation:

\[ \frac{\partial^2 E}{\partial x^2} - k^2 \cdot \varepsilon(x) \cdot E = 0 \]  \hspace{1cm} (1)

where \( E \) is the electric field, \( k \) is the free space wavenumber and \( \varepsilon \) is the dielectric constant of the plasma. The solution to this equation is a standing wave electric field which is oscillatory for \( \varepsilon > 0 \) and which is damped to zero for values of \( \varepsilon \) less than zero. The reflection point is defined as the position where the dielectric constant is zero.

The equation is solved numerically using a shooting code approach. In this the phase of the \( E \) field at the edge of the plasma is varied and the equation for the \( E \) field is solved through the plasma. The correct edge phase is the one that predicts an evanescent electric field after the reflecting layer.

In practice, this method generates an approximation to the edge phase of the standing wave rather than an exact solution. However it is an iterative method and therefore can determine the correct edge phase for the standing wave field to an accuracy of \( \pi/2^{n-1} \cdot 4/n \), where \( n \) is the number of iterations on the value of the edge phase. This numerical solution may be compared with analytic solutions of the wave equation to determine the accuracy of the technique. For a linear dielectric constant profile, the wave equation has a solution in terms of the Airy function, \( \text{Ai}(\xi) \). As may be seen in figure 1, the numerical solutions are very good approximations to the analytic solution up to and beyond the reflection point.

\[ \begin{array}{c}
\text{Figure 1: A comparison between the analytic solution, (solid line) and the numerical solutions to the wave equation for a linear dielectric constant profile.}
\end{array} \]
3: Description of Model Parameters

To determine the dielectric constant profile, $\varepsilon(x)$, for the plasma the equilibrium electron density and magnetic field profiles are required. We take the electron density profile to be quasi-parabolic with an exponential decay at the edge of the plasma:

$$n_e(r) = (n_0 - n_{\text{lim}}) \left[ 1 - \frac{r^2}{a^2} \right]^{-\alpha} + n_{\text{lim}} \ldots 0 \leq r \leq a$$  

$$n_e(r) = n_{\text{lim}} \cdot e^{\frac{(a-r)}{\beta}} \ldots r \geq a$$  

$n_{\text{lim}}$ is the density at the last closed flux surface and $\alpha$ and $\beta$ are constants.

The magnetic field profile is assumed to be of the standard form:

$$B(R) = \frac{B_0 R_0}{R}$$  

where $B_0$ and $R_0$ are the value of $B$ and $R$ at the centre of the plasma.

4: Numerical Results

The results of the model calculations are presented for two reflectometer configurations: single channel reflectometry and two channel correlation reflectometry. For the single channel case, the response of a reflectometer to a single gaussian density pulse is determined and this is used to illustrate the localisation properties of the reflectometer phase measurement (case 1). The phase response of the reflectometer to a set of coherent and localised broad band density fluctuations is then determined (case 2).

The response of the two channel correlation reflectometer to a single gaussian pulse is modelled to illustrate the potential of the technique (case 3). The response of the correlation reflectometer to a set of localised, coherent broad band fluctuations is then determined, concentrating on the effect on the crossphase spectrum of fluctuations with different wavenumber spectra (case 4). Finally, the response to incoherent and non-localised density fluctuations is considered (case 5).

4.1: Single Channel Reflectometry

Case 1: The phase response of the reflectometer is determined by adding a small gaussian density pulse to the equilibrium density profile and by calculating the resultant difference in the phase of the standing wave.
The density pulse is located at different radial positions and the response of the reflectometer is calculated (figure 2).

![Graph showing the variation of the reflectometer phase response to a gaussian perturbation as the central position of the perturbation is moved through the reflecting layer.](image)

**Figure 2:** The variation of the reflectometer phase response to a gaussian perturbation as the central position of the perturbation is moved through the reflecting layer.

**Case 2** In practice, however, the experimentally observed fluctuations are not consistent with single gaussian density pulses, but suggest that fluctuations exist over a wide frequency range. To model these, we assume density perturbations of the form:

\[
\Delta n(r) = \sum_{m=1}^{M} A_m \cdot \sin(\omega_m t - k_m r - \phi_m) \cdot \exp \left( -\frac{(r-r_c)^2}{l_{cm}^2} \right) 
\]

where \( A_m \) is the amplitude of the mode at each frequency, \( k_m \) is the wavenumber at each frequency, \( \phi_m \) is the phase offset at each frequency (random), \( r_c \) is the central position of the gaussian envelope (constant as a function of time) and \( l_{cm} \) is the radial correlation length at each frequency. This set of fluctuations is specified and then added to the equilibrium density profile about a radial position \( r_c \). The system is allowed to evolve with time and the edge phase of the standing wave is recalculated for each time point \( t \). In this way the variation of the phase as a function of time to a broad band set of density fluctuations is determined.

A record of the local density fluctuations may be determined from the variation of the calculated density at a single radial position as a function of time. From these time signals the autopower spectra of the reflectometer phase and the density fluctuations may be determined [Bendat 1980] (figure 3). As may be seen there is good agreement between the shape of the two autopower spectra, indicating that the reflectometer gives a reasonable indication of the autopower spectrum of the fluctuations.
The sensitivity of the reflectometer to the different parameters that specify the density fluctuations may be investigated by varying each parameter while keeping the others constant. As seen earlier, the phase response function of the reflectometer varies as a function of radius. By changing $r_c$ the effect of radial position on the response of the reflectometer to a broad band set of density fluctuations may be investigated. Four different radial locations are shown in figure 4a and the autopower spectra of the phase responses of the reflectometer are shown in figure 4b. As can be seen, variations in the power on the order of 20 dB can be observed by changing the position of the fluctuations by less than 50 mm. The shape of the autopower spectrum, however, is not significantly dependent on the position of the fluctuations.

Varying the correlation length, $l_{cm}$, of the density fluctuations can also have a significant effect on the amplitude of the phase response of the reflectometer, figure 5a. However, this does not have a significant effect on the shape of the phase autopower spectrum.

Varying the wavelength of the fluctuations can effect the autopower spectrum of the reflectometer phase (figure 5b). There is some change in the
amplitude of the phase response but the shape is essentially constant and gives a good indication of the shape of the autopower spectrum of the density fluctuations, curve 4, figure 5.

\[ k_m, \lambda \]

\[ \lambda = \frac{\lambda}{\lambda} \]

\[ 100, 300, 500 \]

Frequency (Hz)

Figure 5a: Change of the autopower spectrum as \( l_c \) is varied.
1) \( l_c = 0.06 \text{ m} - 0.005 \text{ m} \).
2) \( l_c = 0.006 \text{ m} - 0.0005 \text{ m} \).
3) \( l_c = 0.3 \text{ m} - 0.025 \text{ m} \).

Curve 4 is the autopower spectrum of the density fluctuations.

\[ 100, 300, 500 \]

Frequency (Hz)

Figure 5b: Change of the autopower spectrum as \( k_m \) is varied.
1) \( k_m = 62.83 \text{ m}^{-1} - 523.6 \text{ m}^{-1} \).
2) \( k_m = 125.6 \text{ m}^{-1} - 1047.2 \text{ m}^{-1} \).
3) \( k_m = 251.0 \text{ m}^{-1} - 2094.4 \text{ m}^{-1} \).

Curve 4 is the autopower spectrum of the density fluctuations.

If the amplitude of the density fluctuations increases, the amplitude of the phase response also increases, (figure 6). As the amplitude of the density fluctuations becomes very large, however, the autopower spectrum of the phase tends to flatten at high frequencies due to the generation of harmonics from lower frequencies. In general, the rise in the amplitude of the reflectometer phase response will reflect any increase in the amplitude of the density fluctuations. This must be contrasted with the very high sensitivity of the reflectometer phase response to the location of the density fluctuations.

\[ 100, 300, 500 \]

Frequency (Hz)

Figure 6: As the amplitude of the density fluctuations is decreased (lower 1,2,3 curves) the amplitude of the reflectometer response also decreases (upper 1,2,3 curves).
4.2: Two Channel Correlation Reflectometry

Case 3 We now model the response of a two channel correlation reflectometer to a single radially propagating gaussian density pulse. We assume that the width of the pulse is taken to be greater than the free space wavelength of the probing radiation.

Case 4 The reflecting layers of the reflectometers are at $x_{c1}$ and $x_{c2}$ respectively, figure 7a. The gaussian pulse is assumed to be propagating radially outwards with a velocity $v$. At $t = t_1$ the leading edge of the pulse is at $x_{c1}$. At $t = t_2$ the peak of the pulse is at $x_{c1}$ and at $t = t_3$ the peak is at $x_{c2}$. The phase response of the reflectometers to this pulse is shown in figure 7b. The passage of the peak through the reflecting layers can easily be identified and the time difference between the peaks on the two reflectometer phase responses, $\Delta t = t_3 - t_2$, can easily be measured. If the interlayer distance, $\Delta x$, between the two reflecting layers is known, the radial velocity of the pulse can easily be determined from the relation $v = \Delta x / \Delta t$.

Figure 7: The phase response of the reflectometers reflects the propagation of the pulse through the two reflecting layers. The time difference between the two peaks in the phase response may be used to determine the radial velocity of the perturbation.

Case 4 For broad band fluctuations, the radial velocity of the fluctuations may be derived from the dispersion relation $\omega = \omega(k)$. The correlation reflectometer gives a measurement of the phase difference between the signals on the two reflectometers $2\theta(\omega)$ from the crossphase spectrum. This can be used to determine the wavenumber spectrum of the fluctuations from the relation

$$k(\omega) \cdot \Delta x = \Delta \theta(\omega)$$  (5)
The accuracy of the crossphase spectrum as a measurement of the fluctuation wavenumbers is tested using the full wave model. For this comparison the interlayer distance $\Delta x$ is less than the correlation lengths, $l_{CM}$, of the density fluctuations. This implies that the coherence fraction $\gamma_{12}(\omega) > \gamma_r$ across the whole spectrum. Here $\gamma_r$ is the level of coherence corresponding to two totally random signals.

For density fluctuations propagating radially outward, the crossphase spectrum gives an accurate determination of the wavenumbers of the density fluctuations (figure 8). The crossphase spectrum is given by the circles and the line is calculated from the specifications of the fluctuations from the relationship $2\pi \Delta x / \lambda_m(\omega) = \Delta \Theta_m(\omega)$. The crossphase spectrum can also be used to determine the direction of propagation of the density fluctuations (figure 9). In this case the fluctuations are propagating radially inwards ($\lambda_m < 0$). This is reflected in the crossphase by the negative slope of the spectrum.

For density fluctuations propagating transverse to the direction of the probing beam ($\lambda_m >> 1$, $k_m = 0$), the crossphase spectrum is flat with a value of zero across the whole band (figure 10) because the fluctuations perturb both reflecting layers in phase. A zero value of crossphase therefore indicates that the observed density fluctuations are propagating transverse to the direction of the probing beam.

**Figure 8:** Comparison between the model crossphase spectrum and the density fluctuations with negative phase difference calculated using eqn 5.

**Figure 9:** Crossphase spectrum for fluctuations with wavenumbers $\neq 0$.

**Figure 10:** Crossphase spectrum for fluctuations with wavenumbers $= 0$.
Changing the wavelength of the density fluctuations changes the slope of the crossphase spectrum (figure 11). In this case the two sets of fluctuations have identical parameters except that the wavelengths used to determine (a) are twice those used to determine (b). This is seen in the doubling in the slope of the crossphase spectrum.

![Figure 11](image)

**Figure 11**: A decrease in the wavelength of the fluctuations is observed as an increase in the slope of the crossphase.

In principle, it is possible to determine the correlation length of the density fluctuations from the decrease in the level of coherence between two reflectometers with increasing interlayer distance. However, as demonstrated in a previous section the phase response is not localised to the reflecting layer and therefore there will be an overlap of the response functions of the two reflectometers. This implies that there will be a significant level of coherence between two reflectometers over some distance (over) even if the radial correlation length of the fluctuations is zero.

**Case 5** To estimate the effect of the overlap of the response functions the phase response of the correlation reflectometer to a totally random set of density fluctuations is modelled. We take the density fluctuations to be described by

\[ n_e(r,t) = n_e^{\text{equl}}(r,t) \cdot [1 + D(r,t)\Delta n] \]  

(5)

where \( D(r,t) \) is a random number (-1 < D < 1), generated at each radius at each time point, \( \Delta n \) determines the fluctuation amplitude.

We find that autopower spectra of both the density and the reflectometer phase response is flat across the whole spectrum as would be expected for random noise (figure 12). The uncertainty in the level of coherence increases as the level of coherence approaches the random level, \( \gamma_r \). Therefore in the following the average coherence across the whole band is used for presentation purposes.
Figure 12: The autopower spectra of the density fluctuations and the reflectometer response produced using random fluctuations is flat.

Figure 13: As the layers are moved apart the coherence drops to the random value at a distance of 30 mm.

When the reflecting layers are close to the edge, the overlap length is of the order of 10 - 20 mm (figure 13). For this case, the probing radiation is ~ 75 GHz, polarised in the extraordinary mode and the reflecting layers are located at $\rho \sim 0.91$. Thus the overlap length is on the order of 2.5 - 5 free space wavelengths. If the frequency of the probing waves is increased, so that the position of the reflecting layers is changed, the overlap distance is increased to greater than 60 mm for a radial position of $\rho \sim 0.5$ (figures 14 and 15). It is not clear, however, if this increase is due to an increase in the propagation distance or a decrease in the probing wavelength. This may be investigated by decreasing the value of the central magnetic field $B_0$, thereby changing the reflecting layer position for a given probing frequency. As can be seen at $\rho = 0.5$, $l_{\text{overlap}} \sim 60$ mm which corresponds to $> 15$ free space wavelengths for a 75 GHz probing beam. This indicates that the increase in $l_{\text{overlap}}$ is dominated by the increase in path length. This is as expected because a decrease in probing wavelength should not have a large effect on the phase response to density fluctuations with a zero correlation length.
Figure 14: As the frequency of the probing radiation is increased, the overlap distance increases to 60 mm. (The circles correspond to $\rho \sim 0.9$, the squares to $\rho \sim 0.75$ and the crosses to $\rho \sim 0.5$).

Figure 15: The probing frequency is kept constant but the reflecting layer position is moved to the centre of the plasma. This illustrates that the increase in the overlap length is a function of the propagation path length.

Conclusions

A one-dimensional, full-wave, model has been used to predict the ability of both a single channel reflectometer and a two channel correlation reflectometer to diagnose density fluctuations in a turbulent plasma. Results on the single channel reflectometer show that the autopower spectrum of the reflectometer may be used to give qualitative information on the density fluctuations and, in particular, on the spectral shape. It cannot be used to give information on the amplitude of the density fluctuations due to the sensitivity of the phase response to parameters such as the radial position and correlation length of the fluctuations.

For totally random fluctuations, there is significant coherence between the two channels of a correlation reflectometer for interlayer distance in the range 10 - 50 mm, depending on the length of the propagation region. This implies that the correlation reflectometer cannot be used to give a reliable estimate for the correlation lengths of density fluctuations that are less than 10 - 50 mm. The crossphase spectrum, however, gives an accurate estimate of the wavenumbers of the observed density fluctuations if the correlation length of the fluctuations is greater than the interlayer distance.

References

Abstract

A general analysis is presented of the sensitivity of reflectometry to perturbations of the plasma profile, using a full-wave description in one-dimension. Correlation reflectometry is investigated. It is found that the phase correlation is substantial, regardless of the correlation length of the fluctuations, unless either the wave attenuation is substantial or else the fluctuation correlation function is non-monotonic, corresponding to narrow-band turbulence. This behaviour can be understood in terms of the relative importance of forward and backward scattering.

1. Introduction

Reflectometry has gained considerable attention as a diagnostic in large tokamaks (e.g. Simony et al., 1985. Hubbard et al., 1987), especially for fluctuations, but the interpretation has been based mostly on the WKBJ approximation.

Recently, experiments have been conducted forming correlations between signals from reflectometers operating at adjacent frequencies (Cripwell et al., 1989, Hanson et al., 1990) and simulation experiments and code development have been begun (Baang, et al., 1990) in an attempt to try to understand these types of experiments.

The purpose of the present work is to present a systematic full-wave analysis of the sensitivity of reflectometry to changes in the density, including, therefore, fluctuations. The analysis is based on what amounts to the first Born approximation (Pitteway, 1959). The present approach parallels more recent calculations by Mazzucato and Nazikian (1991), Garcia et al. (1989) and Zou et al. (1991) but goes further in presenting more systematic numerical results, specifically concerning correlations. Also, the inclusion of an imaginary part to $k^2$, modelling beam attenuation or divergence, has been studied.

It may be that intrinsically multi-dimensional effects, such as Bragg reflection from rippled surfaces, are predominant in the experiments (Irby, 1990). Nevertheless, it seems essential to conduct this more thorough analysis of the one-dimensional problem so as to understand what can and cannot be explained on the basis of a one-dimensional approach.
2. One-dimensional full-wave reflectometry

We consider a plasma slab in which all gradients are perpendicular to the magnetic field, \( \mathbf{B} \), which is in the z-direction. We suppose the wave under analysis to propagate in the direction of the gradients, which we take as the x-direction. In the context of a full-wave analysis, this means that the only non-zero derivatives in the problem are \( \partial / \partial x \). This is thus a one-dimensional analysis.

In the cold plasma approximation, and indeed under some more general assumptions, it may be shown that the wave equation can be written

\[
\frac{d^2 E}{dx^2} + k^2 E = 0,
\]

i.e. of the Helmholtz type but with the parameter \( k^2 \) a function of \( x \). For the ordinary wave

\[
k^2 = (\omega^2 - \omega_p^2)/c^2 = k_v^2(1 - n/n_c),
\]

where \( k_v = \omega/c \) and \( n_c \) is the critical density, \( \omega^2 \epsilon_0 m_e / \epsilon_2 \). While for the extraordinary wave

\[
k^2 = k_v^2 \left[ 1 - \frac{\omega_p^2(\omega^2 - \omega_p^2)}{\omega^2(\omega^2 - \Omega^2 - \omega_p^2)} \right].
\]

The general situation in reflectometry is illustrated in Fig. 1(a). The wave number, \( k^2 \), has a functional form such that for large positive \( x \), deep inside the plasma, \( k^2 \) becomes large and negative, the wave is cut off. For increasingly negative \( x \), there is some value \( x_c \), equivalent to the plasma edge, beyond which \( k^2 \) is constant and positive. In the intervening region \( k^2 \) varies continuously.

The second order linear differential equation (1) has two independent solutions, which we will denote \( \psi_1 \) and \( \psi_2 \). The physically significant solution is, of course, the one that tends to zero for large \( x \). Let us suppose that \( \psi_1 \) is this solution. It is shown in Fig. 1(b) for our illustrative profile.

We wish to understand the sensitivity of the reflectometer signal to influence from different positions. To discuss this we consider a small perturbation to the \( k^2 \) profile, arising, for example, from a density perturbation.

Proceeding in a manner that amounts to the Born approximation, we seek a solution to the perturbed equation

\[
\frac{d^2 E}{dx^2} + [k^2(x) + \widetilde{k}^2(x)]E = 0,
\]

by assuming that \( \widetilde{k}^2 \) is small compared to \( k^2 \) so that the solution may be obtained approximately in the form of an expansion \( E_0 + \widetilde{E} \) with \( E_0 \) the solution of the original equation (1) and

\[
\frac{d^2 \widetilde{E}}{dx^2} + k^2 \widetilde{E} = -\widetilde{k}^2 E_0.
\]
(Details of this derivation have been given elsewhere by Hutchinson, 1991). This first approximation will be good provided $E_0 \ll E$ at all $x$. Putting $E_0 = \psi_1$ as the zeroth order solution, get find that in the vacuum region

$$
\bar{E}(x) = -[\psi_2(x) - \frac{U(\psi_2)}{U(\psi_1)} \psi_1(x)] \int k^2(\xi) \frac{[\psi_1(\xi)]^2}{W} d\xi.
$$

where $W$ is the Wronksian, and $U'(\psi) = 0$ is the vacuum boundary condition.

In the vacuum region, where $k_1$ is constant, the unperturbed solution, $\psi_1$, can be decomposed into forward and backward propagating waves with amplitudes $A_1$ and $B_1$. The second solution, $\psi_2$, can also be described in terms of such amplitudes, $A_2$ and $B_2$, which we are free to choose as we like, so long as the result is linearly independent of $\psi_1$. By making a convenient choice of these, such that $W = 4k_1A_1B_1$, it may be shown (Hutchinson, 1991) that if the launched (forward) wave amplitude is fixed, the perturbation gives rise to backward amplitude

$$
\bar{B} = 2iB_1 \int \frac{k^2 \psi_2^2}{W} d\xi.
$$

For one-dimensional reflectometry, the key quantity is usually the phase difference of the reflected wave. This may be deduced immediately from Eq(7). The total reflected wave is $B_1 + \bar{B}$, whose phase angle relative to $B_1$ is

$$
\bar{\phi} = -2\Re \left( \int \frac{k^2 \psi_1^2}{W} d\xi \right),
$$

provided $\bar{\phi}$ is small. [$\Re(f)$ denotes real part of $f$.]

Thus, for perturbations of the system resulting in a phase perturbation that is everywhere small, the phase perturbation outside the plasma is given by a simple integral, over

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{An example of (a) the squared wavenumber, $k^2$, (b) the resulting solution, $\psi_1$, and (c) the weighting function, $\psi_1^2/W$ compared with the corresponding WKBJ approximation $1/2k$.}
\end{figure}
the whole plasma, of the perturbation to \( k^2 \) times a weighting function that is the square of the solution of the unperturbed system, divided by the (constant) Wronskian, \( W \). This weighting function is illustrated in Fig. 1(c).

For comparison, it may be noted that the usual WKBJ approximation, \( \sigma = -2 \int k \, d\xi + \pi/2 \), gives rise to a linearized phase perturbation expression.

\[
\bar{\sigma} = -2 \int \tilde{k}^2 \frac{1}{2k} \, d\xi,
\]

which is also plotted in Fig. 1(c). Our full-wave result shows that this WKBJ estimate of the phase shift is incorrect in not accounting for the modulation of the reflectometer sensitivity proportional to \( \psi_1^2 \). What is more, our result shows how properly to account for the region where \( k^2 \) passes through zero. The full-wave treatment avoids the unphysical divergence in the sensitivity that occurs in the WKBJ approximation.

This linearized response also allows us to calculate immediately the group delay, \( d\sigma/d\omega \). This is the quantity that is measured by an experiment that uses amplitude or frequency modulation so as to avoid the ambiguities of the phase delay, \( \sigma \) (e.g. Doane, 1981). The group delay gives the phase shift of the modulation, and is of course equivalent to a measurement of the round-trip time travelling at the group velocity, \( d\omega/dk \).

One can also allow \( k^2 \) to have an imaginary part, to model attenuation or beam divergence. Details of the effects on the results are given elsewhere (Hutchinson, 1991).

Although the above is completely general, we will from now on give examples based in linear profiles of \( k^2 \), which therefore give unperturbed solution that is the Airy integral function:

\[
\psi_1 = \frac{(\pi k_v)^{1/2}}{|k'^2|^{1/4}} \text{Ai} \left(|k'^2|^{1/3}[x - x_c]\right)
\]

We are writing \( k'^2 \) for \( \partial k^2/\partial x \), which should be understood as being evaluated at the reflection point, \( x_c \).

The scale factor, \( |k'^2|^{1/3} \), relating physical position to Airy function argument, is particularly simple for the ordinary mode, described by Eq(2). We find

\[
|k'^2|^{1/3} = |k_v^2 n'/n_c|^{1/3} = \left(k_v^2/L_n\right)^{1/3},
\]

where \( L_n \) is the density scale length. Thus, for example, the full-width-half-maximum in physical space of the last lobe of \( \psi_1^2 \) is \( 1.63 L_n/(k_v L_n)^{1/3} \), intermediate between the density scale length and the inverse vacuum wavenumber \( k_v^{-1} \).
3. Correlation Reflectometry

We shall assume that the output of each reflectometer channel is proportional to the phase shift, $\phi$. A simple homodyne reflectometer, however, gives a signal that is proportional not to $\phi$ but to $\sin(\phi)\dot{\phi}$. That is, its sensitivity is modulated according to the zeroth order phase shift. This effect alone could be responsible for observation of low correlation between reflectometry channels at different frequencies; since variations in $\phi_0$, different in different channels, due to slow changes in the average density profile, could rapidly average the correlation to a small value.

We shall consider, then, two reflectometer signals, $\tilde{\phi}_a$ and $\tilde{\phi}_b$ obtained from the same plasma path but with different frequencies, $\omega_a$ and $\omega_b$. We shall drop the subscript 1 from the solution of the unperturbed equation and use subscripts $a$ and $b$ to refer to the two channels. The product of the two reflectometer signals is then

$$\tilde{\phi}_a \tilde{\phi}_b = 2 \int k_a^2(\xi_a) \Re \left( \frac{\psi^2(\xi_a, \omega_a)}{W(\omega_a)} \right) d\xi_a 2 \int k_b^2(\xi_b) \Re \left( \frac{\psi^2(\xi_b, \omega_b)}{W(\omega_b)} \right) d\xi_b$$

$$= 4 \int k_a^2(\xi_a) k_b^2(\xi_b) \Re \left( \frac{\psi^2(\xi_a, \omega_a)}{W(\omega_a)} \right) \Re \left( \frac{\psi^2(\xi_b, \omega_b)}{W(\omega_b)} \right) d\xi_a d\xi_b. \quad (12)$$

When we take the ensemble average of this equation so as to obtain the quantity $\langle \tilde{\phi}_a \tilde{\phi}_b \rangle$ we obtain an identical double integral of the quantity $\langle k_a^2(\xi_a) k_b^2(\xi_b) \rangle$. The general expression for the correlation coefficient is

$$\rho_{ab}(\omega_a, \omega_b) \equiv \frac{\langle \tilde{\phi}(\omega_a) \tilde{\phi}(\omega_b) \rangle}{\left[ \langle \tilde{\phi}^2(\omega_a) \rangle \langle \tilde{\phi}^2(\omega_b) \rangle \right]^{1/2}}, \quad (13)$$

where, in our case, the correlation function is

$$\langle \tilde{\phi}(\omega_a) \tilde{\phi}(\omega_b) \rangle = 4 \int k^2(\xi_a) k^2(\xi_b) \Re \left( \frac{\psi^2(\xi_a, \omega_a)}{W(\omega_a)} \right) \Re \left( \frac{\psi^2(\xi_b, \omega_b)}{W(\omega_b)} \right) d\xi_a d\xi_b. \quad (14)$$

4. Turbulent Fluctuations

We consider a Gaussian shaped correlation function in the scaled dimension, $\xi$, with width $\sigma$:

$$\langle k^2(\xi_a) k^2(\xi_b) \rangle = C_k^2(\xi_a - \xi_b) = \frac{1}{(2\pi)^{1/2}2\sigma^2} \exp \left( -\frac{(\xi_a - \xi_b)^2}{2\sigma^2} \right). \quad (15)$$

This corresponds to a Gaussian fluctuation power spectrum of unit height, and width $1/\sigma$. The resulting phase correlation functions and coefficients can be obtained by performing the integrals in Eq(14).

The extent of the integrations in Eq(14) is, by implication, only up to the edge of the plasma, $x_e$. We may choose to measure distances from the cut-off position of one of the waves and to regard the correlation function and the correlation coefficient as functions of the distance of the plasma edge from the cut-off position in scaled units. The results are then universal for these 'stationary' turbulence profiles.
Fig. 2  The correlation function for plasma edge a distance $x_e$ from the cut-off position of the higher frequency wave. Curves are labelled with the value of the shift of the lower-frequency cut-off. Two correlation lengths are shown: (a) $\sigma = 0$ (b) $\sigma = 1$. The spatial coordinate is in scaled units. Physical position is given by $x_e/\left|k^2\right|^{\frac{1}{2}}$.

Figs. 2 and 3 show some examples. For various fixed values of the $x$-shift distance, $x_{ca} - x_{cb}$, between the cut-off positions of the two frequencies, we plot, as a function of $x_e$, the integrals from position $x_e$ to infinity, where $x_e$ is measured relative to the cut-off position $x_{ca}$. We thus obtain universal curves that allow us not only to obtain the observable correlation functions for a range of different plasmas, but also to deduce where in space the contribution to the correlation function comes from. This second, and very important, factor is given by realizing that the contribution arising from plasma between any two positions $x_1$ and $x_2$ to the correlation function for plasma whose edge, $x_e$, is further from the cut-off position, $x_{ca}(= 0)$, than $x_1$ and $x_2$ is simply the difference between the values of the correlation function evaluated at $x_e = x_1$ and $x_e = x_2$.

**Fig. 3**  The correlation coefficient, $\rho_0$ corresponding to the case of Fig. 2(b).
As a summary, we also show in Fig. 4 plots of the correlation coefficient evaluated at \( x = 32.5 \) (the left hand edge) versus the \( x \)-shift, \( x_{ca} - x_{cb} \), for various values of the width, \( \sigma \), which is the correlation length of the fluctuations.

\[
\text{Correlation Coefficient}
\]

\[
x\text{-shift}
\]

Fig. 4 The correlation coefficient, \( \rho_\phi \), versus \( x \)-shift of the lower frequency cut-off position (solid line). The labels indicate the width, \( \sigma \), of the corresponding fluctuation correlation coefficient, and the dashed lines plot its form.

As shown in Fig. 4, the correlation coefficient of the reflectometer signals as a function of \( x \)-shift bears only a distant relationship to that of the \( k^2 \) fluctuations. In particular, the reflectometer correlation coefficient remains large regardless of how short the fluctuation correlation length is: for example roughly 0.5 at an \( x \)-shift of 10 for \( \sigma s \) in the range 0 to 3. Thus the observation of substantial correlation at large shifts cannot safely be interpreted as indicating fluctuations with comparable correlation lengths.

When a substantial imaginary part to \( k^2 \) is assumed, one finds that similar calculations can give a small correlation length for \( \rho_\phi \) for very small \( \sigma \). However, for \( \sigma \) greater than about 0.3, \( \rho_\phi \) again looks as in Fig. 4.

5. Coherent Fluctuations
The coherent case can generically be described by a correlation coefficient

\[
C_{k^2}(\xi) = \exp(-\xi^2/2\sigma^2) \cos(k_f \xi) .
\]

Thus \( \sigma \) is the overall width, but is the envelope of a wave with dominant wave-number \( k_f \). Such a correlation arises, of course, from a fluctuation wave-number power spectrum of Gaussian shape, width \( 1/\sigma \), centered at \( k_f \).

The reflectometry correlation functions and coefficients that arise from such fluctuations have been evaluated. Examples are shown in Fig. 5. The dominant contribution to
the correlation function arises from the position at which the oscillations of the fluctuation match those of the weighting function $\Re(\psi^2/W)$. This occurs where $2k = k_f$, which is naturally the Bragg condition for backscattering.

Fig. 5 Correlation function versus edge plasma position for coherent waves. Curves are labelled with the x-shift of the reflectometer channels. Total width, $\sigma$, is 3.0. Wave numbers, $k_f$, are (a) 1, (b) 5.

A remarkable transformation of the reflectometry correlation coefficient takes place as we increase the product $k_f\sigma$, which is basically the number of oscillations in the total width of the fluctuations' correlation, $C_k(z)$. For $k_f\sigma \gtrsim 2$, the reflectometry correlation plotted versus x-shift, as shown in Fig. 6, rapidly assumes a form that is identical to the form of $C_k(x)$. This proves to be the case regardless of whether the dominant contribution is coming from near the cut-off or not.

A way of understanding this behaviour is to think of the process in scattering terms. One obtains contributions either from forward scattering or from backward scattering. The selection rules for these processes are $k_n = 0$ and $k = 2k$ respectively, where $k_n$ is the fluctuation-spectrum wave-number. The relative magnitude of the fluctuation spectrum at $k_n = 0$ for the correlation function of Eq(16) is $\exp(-k_f^2\sigma^2/2)$. Thus the magnitude of $k_f\sigma$ determines whether the forward scattering, $k_n = 0$, is significant or not. When it is not, there is close agreement between the phase correlation coefficient and the fluctuation correlation coefficient. It is clear that this backward scattering component cannot be described using the smooth WKBJ estimate but requires a full-wave analysis such as this.
Fig. 6 Reflectometry correlation coefficient versus x-shift for coherent waves. The width, \( \sigma \) is 3.0, and curves are labelled with the value of the fluctuation wavenumber, \( k_f \). The dashed lines are the correlation coefficients of the fluctuations.

6. Summary

The full-wave weighting function, giving the contribution of \( k^2 \) perturbation to phase perturbation, has an average in the oscillatory region that is equal to the WKBJ result, but avoids the singularity at the cut-off point. The width of the last lobe of the weighting function is approximately \( 1.63/|k''|^{\frac{1}{3}} \).

Correlation reflectometry must make use of quadrature information, otherwise the variation of fluctuation-sensitivity proportional to the sine of the unperturbed phase will render the signal correlations meaningless. Even with a proper phase signal, turbulent fluctuations with monotonic correlation coefficients give reflectometry correlation coefficients that only distantly reflect the correlation coefficients of the fluctuations. In particular, the correlation observed should be substantial even for shifts much greater than the correlation length, unless the wave attenuation effects are important. In this latter case, low correlation can occur, but mostly because of short-wavelength contributions well away from the cut-off position (Hutchinson, 1991).

Coherent waves, represented by non-monotonic correlation coefficients, contribute most from a position where their wavelength is equal to half the wavelength of the unperturbed solution. The reflectometry correlation coefficient in this case can closely resemble the correlation coefficient of the fluctuations.

The distinction between these two cases is fundamentally whether forward or back-
ward scattering contributions dominate. The coherent wave case corresponds to negligible forward scattering and can not be represented using the WKBJ analysis.

The localization of the reflectometry contribution, whether of coherent or incoherent fluctuations, is never to a region narrower than about one (local) wavelength of the unperturbed wave function.

Finally, one should emphasize that these conclusions are based on a one-dimensional analysis. Inherently multi-dimensional effects might predominate in actual experiments. Therefore great caution should be exercised in inferring the nature of the fluctuations from reflectometry measurements.

References


To calculate the propagation of electromagnetic waves in an inhomogeneous medium in one and two dimensions a spatial network is used, which represents the equivalent circuit for the propagation of electromagnetic waves in a plasma. This method has been used to describe the one dimensional propagation of radio waves in the ionosphere, including absorption due to collisions. The method gives the full wave solution for arbitrary density profiles.

As a reminder of the relationship of experiments in the ionosphere with reflectometry in fusion plasmas we reproduce the corresponding equivalent circuit from a textbook on high-frequency engineering where the derivation of the circuit elements for the 1-dimensional case can be found /1/.

**Fig. 1a:** Circuit elements for the 1-dimensional case with absorption due to collisions.

**Fig. 1b:** Equivalent circuit for the lossless ionosphere.
Since the main emphasis of our work is on 2-dimensional effects only one example, namely the phase change due to a localised density perturbation is shown, which for the case of a linear density profile has been checked against the analytic solution /2/.

Fig. 2: 1-dimensional simulation of a reflectometry experiment with a parabolic electron density profile showing the phase change from a localised density disturbance \( \hat{N}_n(x) \).

The circuit elements for the 2-dimensional network are depicted in Fig. 3 where the correspondence between the field quantities \( E, H \) and the network quantities \( U, I \) is listed.

\[
\begin{align*}
\frac{\partial V_y}{\partial z} &= -L \frac{\partial I_z}{\partial t} ; \\
\frac{\partial V_y}{\partial z} &= -\frac{\partial I_z}{\partial t} ; \\
\frac{\partial I_z}{\partial z} + \frac{\partial I_z}{\partial x} &= -2C \frac{\partial V_y}{\partial t} ; \\
\frac{\partial H_x}{\partial z} - \frac{\partial H_x}{\partial x} &= \epsilon_n \frac{\partial E_y}{\partial t}
\end{align*}
\]

Fig. 3: Network element for the 2-dimensional case.
If desired the network can be terminated by suitable impedances \( Z(b) \), \( Z(e) \) as indicated schematically in Fig. 4a. The properties of this network are represented by an impedance matrix whose size is determined by the number of input ports. The equivalent circuit for the antenna matched to the free space impedance \( Z(0) \) in front of the antenna is shown in Fig. 4b. The field distribution in the aperture plane is determined by the source voltages \( U(a) \). By choosing appropriate amplitude and phase distributions for \( U(a) \) the radiation pattern of an antenna can be realized, i.e. the transmitted power can be focussed at a given distance.

![Fig. 4: a) Schematic representation of the network grid representing the plasma region with absorbing boundaries. b) Schematic representation of the network representing the antenna matched to the impedance \( Z(0) \) in front of the antenna.](image)

In the following first computational results will be given to illustrate the capabilities of the 2-dimensional model. The incident wave frequency is kept constant:
1.) Divergence of the beam due to the density profile in the propagation direction:

The divergence of the incident and reflected beam in a stratified medium is usually determined by ray tracing methods. The effect depends on the electron density profile and leads to a reduction in reflected power seen by the receiver antenna. Figs. 5a,b show the amplitude distribution across the antenna for the transmitted and reflected signal. In Fig. 5a the electron density in front of the cutoff position is far below the cutoff density (i.e. the refractive index is close to the vacuum case) and far above cutoff for larger distances resulting in a plane reflecting mirror (Profile A). The antenna is focused onto this mirror to match the reflected field to the antenna (which serves both as transmitter and receiver). For comparison Fig. 5b shows the results for density profile which increases linearly up to and beyond the cutoff layer (Profile B). The resulting strong divergence of the reflected beam can be clearly observed.

![Fig. 5a,b: Distribution of the transmitted and reflected field in the aperture plane of the antenna for two different density profiles A and B. Transverse width = 10*λ_{vacuum}.](image)
2: Changes in the phase of the reflectometry signal due to fluctuations propagating transversely:

There is an upper limit to the wavenumber of transversely propagating fluctuations that can be detected in a reflectometry experiment due to the following effects:

- For fluctuation wavenumbers $k_{\text{fluct}}>0$ radiation will be scattered back at scattering angles $<180^\circ$ reducing the modulated signal at the receiver antenna. This is equivalent to a lowpass filter in $k_{\text{fluct}}$-space. The characteristics of this lowpass filter are determined by the beam profile along the distance travelled and thus by the electron density profile. Fig. 6a shows the magnitude of the fluctuating field measured by the receiver antenna for profile A and Fig. 6b corresponds to profile B. In the numerical calculations the change in refractive index due to the transversely propagating electron density fluctuations is simulated in a layer with thickness $\lambda/10$ directly in front of the cutoff.

Fig. 6a,b: Magnitude of phase change due to a transversely propagating fluctuation. The normalised wavenumber of the fluctuation is $k=k_{\text{fluct}}/k_{\text{vacuum}}$ for profiles A and B.
- The results of a transversely propagating periodic density fluctuation can only be observed, if the time-dependent part of the electromagnetic wave is not evanescent, i.e. if $\lambda_{\text{fluct}} > \lambda_{\text{incident}}$.

2. Detection of radiation scattered at angles $< 180^\circ$ from transversely propagating fluctuations:

If one observes the scattered signal at angles $< 180^\circ$ it should in principle be possible to determine the wavevector of the transversely propagating density fluctuations. However there are two complications:
- Only non-evanescent waves can be observed in the far-field corresponding to a cutoff for high wavenumbers of fluctuations.
- The propagation direction of the waves scattered at angles $< 180^\circ$ changes while propagating through the electron density gradient.

Figs. 7a,b: Magnitude of the "scattered" signal amplitude for profiles A and B.
Fig. 7 shows the magnitude of a second receiver antenna positioned at a transverse distance of $5\lambda_{\text{vacuum}}$ from the transmitter antenna and a distance $3\lambda_{\text{vacuum}}$ from the cutoff layer (i.e., in the near field) for the electron density profiles A and B. One can see that scattered field depends sensitively on the density profile chosen.

Fig. 8 shows the transverse distribution of the magnitude of the electromagnetic field component due to transversely propagating fluctuations at a distance of $3\lambda_{\text{vacuum}}$ from the cutoff layer for the density profile B. The spatial variation of the density fluctuation is indicated in the bottom of the diagrams.

Fig. 8:  
(a) Magnitude of the "scattered" field for $k=0$.  
(b) Magnitude of the "scattered" field for $k=.4$.  

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Although the antenna aperture is still situated in the near field, for $k=0.4$ it can be seen that the two sidebands begin to separate.

Conclusions: Further detailed studies of 2-dimensional effects will have to be performed to fully assess the potential of reflectometry for fluctuation measurements in fusion experiments.

References:

In reflectometry experiments with stationary density profiles the desired information is contained in the phase of the reflected signal, which is a measure of the line integral
\[ \int n(x) \, dx \]
along the refractive index \( n(x) \) of the plasma. It can be interpreted as "interferometry term" and shall be considered stationary on the time scale of the fluctuations to be measured.

If fluctuations are present, the reflected signal contains three types of contributions:

**Term 1:**
A phase shift which contributes to the line integral and thus to the "interferometry term". It arises if the integral
\[ \int \hat{n}(x) \, dx \]
along the interaction region is non-zero, where \( \hat{n}(x) \) is the fluctuating part of the plasma refractive index.

**Term 2:**
A backscattering term, which is not contained in the WKB approximation. The signal which reaches the receiver is composed of the following contributions:

a) Part of the incident beam is directly backscattered from the fluctuation ("direct" backscattering).

b) The transmitted beam is reflected from the cutoff region, part of this beam is backscattered from the fluctuation, travels towards the cutoff layer, is reflected and reaches the receiver ("indirect" backscattering).

**Term 3:**
Forward scattering term due to transversely propagating density fluctuations.

a) Part of the incident beam scattered forward and then reflected back from the cutoff layer back to the receiver.

b) Part of the reflected beam again undergoes forward scattering.
The wavenumber acceptance of the antenna generally limits this effect to small values of the fluctuations wavenumber $k$. Therefore a reflectometry experiment acts like a lowpass filter for the wavenumbers of transversely propagating density fluctuations. Examples from 2-dimensional numerical simulations are given in /3/.

One-dimensional model:

All the transmitted power will reach the receiver (absorption due to collisions can be neglected in fusion type plasmas). For simplicity we will assume that the scattered power is small compared to the power transmitted into the plasma.

The total field $E_{\text{ant}}$ at the receiver antenna is

$$E_{\text{ant}} = E_\text{s} \sin(\omega t + \phi_1) \quad \text{direct backscattering}$$
$$+ E_\text{s} \sin(\omega t + \phi_2) \quad \text{long way backscattering}$$
$$+ E_{\text{ref}} \sin(\omega t) \quad \text{reflection from cutoff layer}$$

with:

$\phi_1, \phi_2$ the phase with respect to the reflected signal $E_{\text{ref}}$.

In the one dimensional case $E_\text{s} = E_\text{s}$.

The interference of the scattering terms yields the well known node-anti-node structure which characterises one-dimensional reflectometry experiments /1/. The total scattered field is

$$E_s = 2E_s \sin(\omega t) \cos(\phi_1 - \phi_2)$$

which contains a modulation of the amplitude through the cos-term although the fluctuation amplitude is constant. Depending in the relative phase (i.e. spatial localisation) a density fluctuation can be seen or not.

Two-dimensional model:

In this context two cases have been discussed during the meeting:

- A laboratory experiment with well diagnosed density fluctuations /2/.
- A two-dimensional numerical simulation /3/.
Part of the transmitted microwave power will not reach the receiver antenna due to the following effects:

- Due to beam divergence in the macroscopic 1-dimensional density profile.
- Due to scattering from transversely propagating fluctuations. This effect depends on the position of the fluctuations along the microwave beam.
- Resonance absorption, refraction, and/or interference effects /2/.

Obviously the resulting attenuation for term 1, term 2 and term 3 defined above can differ due to the different path lengths they have to travel.

**Measurement of the received signal:**

A.) Information about term 1 (i.e. the "interferometry term") is obtained by measuring the reflected signal phase.

B.) We look at the field $E$ at the antenna for the case of backscattering:

The term resulting from backscattering is given by $E_s \sin(\omega t + \varphi)$.  

$$E = E_s \sin(\omega t + \varphi) + E_{ref} \sin(\omega t)$$

where:

- $E_s$ is the attenuated backscattered field
- $E_{ref}$ is the attenuated reflected transmitter field.

The fluctuating part in the phase of the antenna signal $\alpha$ is

$$\alpha = \tan^{-1}\left(\frac{\sin(\varphi)}{\cos(\varphi) + r}\right)$$

where:

- $r = \frac{E_{ref}}{E_s}$ is the reflected field normalised to the backscattered field.

It can be seen that the phase fluctuation at the receiver due to backscattering depends crucially on the strength of the reflected signal normalised to the backscattered signal at the receiver. The two limiting cases are:

**Case 1:** $r >> 1$ leads to following approximation:

$$\alpha(t) \approx \frac{\sin(\varphi(t))}{r},$$

i.e. the measured phase increases with decreasing reflected signal.
Case 2: \( r << 1 \) leads to the following approximation:

\[
\alpha(t) \approx \varphi(t)
\]

i.e. in the case of backscattering the measured phase is independent of the scattered amplitude as expected.

For the case where the fluctuations are dominated by backscattering one should subtract the signal \( E_{\text{ref}} \sin(\omega t) \) (where \( E_{\text{ref}} \) is assumed to be constant on the time scale of the fluctuations) from the total field \( E \) at the receiver antenna to obtain the back-scattered term \( E \sin(\omega t + \varphi) \). A possible method to obtain the reference signal \( E_{\text{ref}} \) is by extracting it using phase-locked loop with an appropriate time constant or the numerical equivalent. In a heterodyne detection system, which is generally used in scattering experiments, one will observe a downshifted (corresponding to indirect backscattering) and upshifted (corresponding to direct backscattering) sideband. In contrast the analysis of the "interferometry term" yields two sidebands with equal amplitude characteristic of a time-dependent phase modulation.

A discussion of the question, if and to what degree it is possible to determine the spatial distribution density fluctuations by reflectometry under realistic conditions is beyond the scope of this note.

References:


/2/ T.L. Rhodes et al. " Fundamental Investigation of Reflectometry as a Density Fluctuation Diagnostic ", presented at this meeting.

/3/ E. Holzhauer et al. " Numerical Simulation of Reflectometry Experiments in One or Two Dimensions ", presented at this meeting.
C: MEASUREMENT OF DENSITY TRANSIENTS AND FLUCTUATIONS
Reflectometry as a Diagnostic for Density Fluctuations: A Comparison of Laboratory and Computer Modeling Results


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ABSTRACT. Reflectometry is currently used to diagnose density fluctuations and turbulent correlation lengths in fusion plasmas. Various models have been used to both interpret the experimental data and to determine the regimes of validity of the reflectometer fluctuation measurements. However, these models have, heretofore, not been validated by direct comparison with experiment. In this paper the first comparison between a controlled laboratory experiment and a one-dimensional numerical model is presented. It is found that the model is unable to predict the observed high degree of spatial localization and dependence on perturbation wavenumber. The implications of these disagreements are discussed, together with suggestions for their resolution.
INTRODUCTION

Reflectometry is currently used for the determination of density profiles [1-6] and as a density fluctuation diagnostic [7-15] on fusion plasma devices. Its use as a density profile diagnostic is considered to be well understood [1-6]. However, a fundamental understanding of the reflectometer signal due to density fluctuations and its interpretation does not exist at this time. Questions include the diagnostic sensitivity to different fluctuation wavelengths, density scale length, and fluctuation amplitude as well as the spatial localization of the signal. The UCLA P. sma Diagnostic Group has initiated an investigation into these basic physics questions and has reported some initial results [16]. There it was reported that in a laboratory plasma, the reflectometer response to grid launched ion acoustic wave packets was highly localized, showing the same number of cycles and same frequency as the launched wave. In addition, a one dimensional numerical model was developed and showed promising initial results [17]. This code models the plasma-microwave interaction by solving the full wave equation with specified plasma density profile, density perturbation waveform and wavenumber, and microwave frequency.

This paper describes the first comparison of laboratory results to these numerical calculations. The code used here is equivalent [18-22] to or similar in result [23] to codes and models used elsewhere to interpret reflectometer data. These codes are currently being used to calculate absolute density fluctuation levels in fusion plasmas [24] from experimental reflectometer data. In addition, it has been concluded [19,21] on the basis of model predictions that correlation reflectometers cannot measure correlation lengths as short as have been reported [10,12-15]. However, it
is important to realize that no direct experimental confirmation of the models utilized in these codes has heretofore been performed. The results presented here indicate that one-dimensional and semi-two-dimensional codes do not adequately describe the physics present in the laboratory plasma-reflectometer interaction. In particular they omit 2-D effects such as refraction, diffraction, interference, and resonance absorption [25]. It is concluded that reliance on these unconfirmed models to interpret experimental data is at best questionable and can result in false and misleading conclusions. It should be noted that these comments apply as well to the experimentally unconfirmed WKB method of calculating absolute values of $\bar{n}$ from reflectometer data [8,9].

The paper is organized as follows: in Sections 2 and 3 the laboratory experiment and computer code are described; the results from each are described and compared in Sec. 4 with conclusions and discussion presented in Sec. 5.

**DESCRIPTION OF EXPERIMENT**

The laboratory plasma is produced in a cylindrical vacuum chamber 80 cm long and 60 cm in diameter. A diagram of the device is shown in Fig. 1. A plasma discharge is produced by repetitively pulsing (~ 7 Hz) a set of filaments at one end of the chamber producing a plasma of approximately 2 msec duration (Fig. 2). Argon gas is employed, with electron and ion temperatures and densities in the range $T_e=1-2$ eV, $T_i=0.1-0.2$ eV, $n_i=n_e=1-10\times10^{11}$ cm$^{-3}$. During the flat top portion of the discharge, a density gradient is formed with high density in the region of the discharge filament (top of chamber), which decreases as the microwave antenna is approached (bottom of chamber). Approximately 1500 small
permanent magnets are used on the chamber walls to improve the plasma confinement while leaving the bulk plasma magnetic-field free. The plasma is normally quiescent with density fluctuations $\bar{n}/n < 10^{-3}$. In order to study the reflectometer response to density fluctuations, ion acoustic waves are launched by applying a bias signal of appropriate waveform to a movable grid placed within the plasma. The grid positions are indicated in Fig. 1 and consist of wire mesh with ~20 wires/cm with grid dimensions 12.8×8.8 cm. Two launch directions are currently available, one down the density gradient and one transverse to it. The density perturbation due to the ion acoustic wave is normally small ($\bar{n}/n < 1\%$) with frequencies in the range 15-200 kHz ($\omega_{\text{acous}}/\omega_{\parallel} < 2 \times 10^{-3}$). A microwave antenna at one end of the vessel launches radiation up the density gradient where interaction with the ion acoustic waves can occur. The microwave frequency is kept fixed during the plasma discharge but can be varied between 3 and 8 GHz between shots. Quadrature detection of the reflected signal is used to obtain both phase and amplitude information. The microwave circuit is also shown in Fig. 1.

A typical Langmuir probe trace showing the time history of the plasma density is shown in Fig. 2. Note the small fluctuation near 1.5 msec which is due to the ion acoustic wave launch and is a combination of electromagnetic pickup and an actual density perturbation. Also shown is the reflectometer response to the plasma evolution. For plasma densities less than the critical density $n_{e,\text{crit}} = (4\pi^2 f_{\text{mw}}^2 e_0 m_e)/(e^2)$ the reflectometer acts as an interferometer and produces the observed fringes (Fig. 2). Here, $f_{\text{mw}}$ is the microwave frequency, $m_e$ and $e$ are the electron mass and charge, and $e_0$ is the permittivity of free space. Approximately 0.75 msec after the discharge initiation, critical density is reached within the plasma.
and the reflectometer signal is cutoff becoming roughly constant with time until the end of the discharge.

Figure 2b shows both the reflectometer and Langmuir probe response to the ion acoustic wave using an expanded time scale. The Langmuir probe has been positioned within the plasma so that the time delay is coincident with respect to the reflectometer. Note the pickup on the Langmuir probe indicating the launch time. Little pickup is seen on the reflectometer. The time delay is an indication of the distance that the wave travels before interacting with the probe, while the position of the probe (since the time delay is coincident with the microwave signal) determines the interaction region of the ion acoustic wave and the reflectometer. The frequency and number of cycles seen on both signals are very similar, indicating that the reflectometer signal is well localized in space. Note that the Langmuir probe is more sensitive than the reflectometer to high frequency density fluctuations (Fig. 2b) as evidenced by the more sharply pointed peaks. This is due to the wavenumber sensitivity of the reflectometer and will be discussed more fully later in the paper.

Measurements of the absolute electron density were made using movable Langmuir probes which had been absolutely calibrated using a microwave interferometer. The location of the microwave/ion acoustic wave interaction region was determined by locating a Langmuir probe within the plasma so that the time delay between wave launch and detection (as detected by the Langmuir probe) was the same as that observed on the reflectometer. Since the reflectometer measures the same number of cycles and frequency as the Langmuir probe, the position of the Langmuir probe is then the location of the microwave/ion acoustic wave interaction region.
These combined measurements indicated that, within the measurement accuracy, the reflectometer detects the ion acoustic wave near the calculated cut-off position (i.e., at n=n\textsubscript{crit}) for any given microwave frequency.

**DESCRIPTION OF NUMERICAL MODEL**

In order to complement the laboratory investigations, a one dimensional model has been developed. The plasma-microwave interaction is modeled using the time independent full wave equation:

\[
\frac{d^2E}{dx^2} + N^2 k_0^2 E = 0 \quad (1).
\]

Here, \(N(x) = (1 - f_{pe}(x)^2/f_{mw}^2)^{1/2}\) is the plasma refractive index for the ordinary mode and contains the density profile information \(n_e(x)\). \(f_{pe}(x)^2 = n_e(x)e^2/(4\pi^2\epsilon_0 m_e)\) is the local electron plasma frequency, \(E = E(x)\) is the local value of the microwave electric field, and \(k_0\) is the microwave vacuum wavenumber. Note that this allows arbitrary microwave frequencies and density profiles to be used and can thus model a wide range of plasmas varying from a laboratory plasma to fusion tokamak plasmas. It represents a first order approximation to the laboratory plasma which neglects possibly important effects such as resonance absorption, refraction, mode conversion, etc. Equation 1 is then solved numerically for the electric field. An analytic solution of Eq. 1 is known for several density profiles, specifically for a linear density profile and a density profile of the form \(n(x) = (\epsilon_0 4\pi^2 f_{mw}^2 m_e/e^2)(1 - \alpha^2/x^2)\), where \(\alpha\) is a constant [1]. Comparison of the code results to these solutions showed that the code correctly solves the full wave equation. More detailed information on the numerical code can be found in [26].
The ion acoustic wave is modeled as a density perturbation superimposed upon the density profile. This perturbation is then propagated down the density gradient to the edge of the plasma where its amplitude is decreased to zero. The code calculates the phase fluctuation that the reflectometer signal would undergo as a result of the density fluctuation. To accomplish this, the wave equation is first solved without a density perturbation. This produces an unperturbed electric field $E_0$. The perturbation is subsequently introduced and the new electric field $E_1$ calculated.

The phase shift $\Delta \phi$ induced by the density perturbation is then calculated from $\Delta \phi = k_0 \Delta x$. Here $k_0$ is the vacuum microwave wavenumber and $\Delta x$ is the distance between the first zero crossings (measured in the vacuum region) of the unperturbed and perturbed electric fields. The density perturbation is propagated down the density gradient in small time steps $\Delta t$ with a new electric field $E_1$ and phase perturbation $\Delta \phi$ recalculated at each time step. These values are stored and can be displayed as a phase fluctuation versus time. Note that the model assumes that the plasma-microwave interaction can be modeled as a time independent problem, i.e., that the microwaves propagate on a much faster time scale than the ion acoustic waves. This is in general well satisfied since $c_{\text{acous}} = 2 \times 10^5$ cm/s and $f_{\text{ion}}/f_{\text{mw}} < 10^{-4}$. Currently the code does not include damping, refraction, or divergence of the microwave beam; however, these effects will soon be added.
Laboratory measurements show that the ion acoustic wave damps significantly as it propagates down the density gradient, with a typical e-folding length of \(-12\) cm, approximately independent of frequency. The code models this effect by using a spatially dependent ion acoustic wave amplitude with an e-folding length \(\Delta=12\) cm. The wave is tapered to zero amplitude at the plasma edge-vacuum boundary in order to avoid negative densities.

**COMPARISON OF NUMERICAL MODEL AND EXPERIMENTAL RESULTS**

Figure 3 shows the code and laboratory results for ion acoustic waves with \(k_n/k_0\) ranging between 0.7 and 1.4 and with \(\bar{n}/n_{crit}=0.5\%\). The code inputs were the density profile from the laboratory plasma, the density perturbation form and amplitude, and the microwave frequency (3.3 GHz). Both the measured and calculated responses show approximately the same frequency and number of cycles as the actual density perturbation.

Note that as the ion acoustic wave frequency is increased the code predicts an increase in the time delay. This result is in agreement with a scattering model of interaction where an incident wave \(k_{inc}\) is scattered through an angle \(\theta\) due to the interaction with a plasma wave \(k_n\). This leads to the familiar wavenumber matching condition \(k_n=2k_{inc}\sin(\theta/2)\), which for \(\theta=\pi\) results in the condition for 180° backscattering \(k_n=2k_{inc}\). It was found that the dominant response predicted by code is consistent with this backscattering wavenumber matching condition. There are indications of a generally much smaller signal originating near the cutoff layer which will be discussed later.
As $k_n$ was increased, the calculated location of matching moved progressively closer to the vacuum boundary and hence the time delay increased. This latter point is due to the fact that the wavenumber of the microwave radiation $k_{inc}$ decreases from its vacuum value $k_0$, at the boundary, towards zero at the cutoff point as $k_{inc}=k_0N$, where $N=(1-n_e/n_{e,\text{crit}})^{1/2}$ is the local index of refraction. This result indicates that the predicted reflectometer response is non-local in the sense that different wavenumbers interact at different positions within the plasma. Thus, a perturbation containing many wavenumbers would appear spread out in time while in reality it is quite localized within the plasma. These predictions are the basis of the conclusion [19,21] mentioned in the introduction regarding the correlation reflectometer results.

From Fig. 3b one sees that there is no observable change in the experimental time delays as $k_n/k_0$ is varied. This lack of variation in the delay time was found to hold as the perturbation wavenumber was varied over the range $k_n/k_0=0.5\rightarrow5.0$ - a factor of 10. Thus, the experimental time delay or location of the signal does not vary substantially with the wavenumber of the density perturbation. A pulse with a wide wavenumber range would then be observed as very localized within the plasma, so that experimentally, the reflectometer signal is much more localized than predicted by the 1D code. This is a clear, quantitative departure of experiment from the model and may indicate why correlation reflectometer measure lengths much shorter than these models predict.

Using quadrature detection the time evolution of the phase $\phi(t)$ and amplitude $A(t)$ of the reflectometer signal can be experimentally determined. Fluctuations in these quantities, $\tilde{\phi}$ and $\tilde{A}$ respectively, due to
the ion acoustic wave can then be compared to the code predictions. The code predicts that $\phi$ varies linearly with the density fluctuation level $\bar{n}$: $\phi_{\text{model}} \sim \bar{n}$ ($\phi$ also depends on perturbation wavenumber to be discussed below). Note that, since the current code is lossless, $A_{\text{model}} = 0$. Experimentally, both amplitude and phase fluctuations are observed, both of which vary linearly with $\bar{n}$: $\phi_{\text{exp}} \sim \bar{n}$ and $A_{\text{exp}} \sim \bar{n}$. The magnitude of the experimentally measured phase fluctuation is a factor of 20 or more greater than predicted by the model. Since the code is lossless and can contain no amplitude fluctuations, this is perhaps the first indication of the reason behind the observed differences between model and experiment. Physically, $A$ can be related to losses in the returned signal due to refraction, diffraction, interference, resonance absorption, etc. These effects will be discussed more fully later.

It should be noted that the scattered response should be present in the experiment. However, due to the magnitude of the experimentally measured values it is quite possible that it (the scattered component) is in the current experimental noise level. It is planned to improve the signal-to-noise ratio and look for the wavematched signal.

The reflectometer response to perturbation wavenumber was investigated by varying $k_\bar{n}$ while the density profile, microwave frequency, $\bar{n}$, etc. were held constant (Fig. 4). Two curves are shown in Fig. 4, one experimental and one numerical. The numerical model predicts a large increase in response below $k_\bar{n}/k_0 \approx 0.3$, due perhaps to a mirror like movement of the whole profile (since the wavelength is comparable to the scale length of the plasma). This is currently under investigation. This is followed by a region of relatively flat response and then, as $k_\bar{n}/k_0$ is
increased above 2, the code predicts a response that rapidly decreases (Fig. 4). Some, but not all, of the decrease occurring before \( k_{\tilde{\alpha}}/k_o = 2 \) is due to the damping of the ion acoustic wave near the vacuum boundary (imposed to prevent negative densities). Further study of this effect is underway. If backscattering were the only effect contained in the code there would be no response for \( k_{\tilde{\alpha}}/k_o > 2 \), since there are no incident wavenumbers with that value. There is some preliminary evidence that the predicted response above \( k_{\tilde{\alpha}}/k_o > 2 \) is due to small, mirror-like movements of the cutoff layer as the pulse passes through.

The laboratory experiment was conducted by varying the ion acoustic wavenumber \( k_{\tilde{\alpha}} \) while monitoring the level of the density fluctuation with a Langmuir probe. The probe was placed in the microwave/ion acoustic wave interaction region to ensure that \( \bar{n} \) due to the ion acoustic wave in the detection region was constant, independent of the wavenumber and of damping. Experimentally, it was observed that the response decreased with increasing \( k_{\tilde{\alpha}} \), qualitatively similar to the code predictions (Fig. 4). However, quantitatively, the reflectometer response decreases more rapidly than the model predicts for \( k_{\tilde{\alpha}}/k_o < 1 \), and then much less rapidly for \( k_{\tilde{\alpha}}/k_o > 1 \) (Fig. 4). The response extends to \( k_{\tilde{\alpha}}/k_o \approx 5 \), well beyond that predicted by the code.

**DISCUSSION AND CONCLUSIONS**

The results of the experiment/model comparison are summarized in Table I. From the discussion above and from Table I it is clear that the numerical model does not predict the experimental measurements. The question naturally arises as to why not? Is the model incorrect? As mentioned earlier, it is expected that the scattered signal should be
experimentally present, although evidently much below the magnitude of the currently measured signals. One of the future goals of the experiment is to attempt to observe this scattered signal, by a combination of improving the signal-to-noise ratio, increasing the injected power, etc. Thus, the model is not believed to be incorrect per se as it does predict a one-dimensional lossless reflectometer/plasma interaction. However, in practice no plasma, and in particular no fusion plasma, is completely lossless and 1D. Other physical effects need to be included in order to successfully model reflectometry.

Some physical effects not contained in the current model include resonance absorption of the microwave beam, refraction, and interference effects. They are presented and discussed below not as explanations, but rather as potential lines of research, meant to stimulate further investigations.

The first effect, resonance absorption, occurs when the RF electric field has some vector component parallel to a plasma density gradient [25]. Conversion of the electromagnetic energy of the incident microwave beam to electrostatic fluctuations, in the form of plasma-oscillations, occurs at the layer $n_e=n_{e,\text{crit}}$ [25]. The cutoff layer itself then becomes a function of the angle $\Theta$ between the incident RF wavevector and the density gradient, and occurs at a lower density than the case of $\Theta=0$. It is known that the absorption layer is relatively narrow and fixed in space [25], thus meeting the requirements of spatial localization near the cutoff layer. Modulation of the absorption by the density fluctuation $\tilde{n}$ results in a modulation of the amplitude and phase of the returned signal [27]. An investigation of this effect as applied to the reflectometer is currently underway. This effect is
intrinsically two dimensional and can not occur in 1D or semi-2D models. It should be noted that while this effect can in principle work for O-mode (E parallel to the magnetic field B) reflectometers, it is believed that for X-mode (E⊥B) the upper hybrid resonance is generally (although not always) too far from the right hand cutoff for any significant amount of energy to tunnel through. If strong changes in refraction of the signal, at or near the cutoff layer, are induced by \( \tilde{n} \) then an effect similar to resonance absorption may be generated. It is planned to investigate these effects using a combination of 2D full wave and particle codes.

Another 2D effect not contained in the present model is time varying interference or speckle due to what might be called a moving crinkled mirror. If only a portion of such an object is observed, large amplitude and phase fluctuations in the detected signal can occur. For example, depending on the size of the detected area, amplitude fluctuations of as much as 100% can have a finite probability of occurrence. It is planned to experimentally test a variation of this concept in the laboratory plasma by using larger and smaller ion acoustic wave launching grids. Effects of changing the 'wrinkling of the mirror' can then be investigated using the natural diffraction of the ion acoustic wave as it is launched from varying grid sizes and angles. Two dimensional interference effects will also be investigated using numerical models. It should be pointed out here that some initial work has already begun using a fully two dimensional model of the plasma-microwave interaction [28].

In conclusion, models and codes are frequently needed to physically interpret experimental data. However, it is important that these models and codes contain the dominant physics effects present in the system under
consideration. In particular, these codes need to be validated and should be able to explain, to some level, the observed data. The laboratory reflectometer response to a controlled launch of ion acoustic waves was found to be highly localized. This localization was independent of the perturbation wavenumber and the position was close to the cutoff layer. A comparison between these observations and a simple one dimensional model has shown that the model is unable to account for these results. Additionally, reported correlation reflectometer results [10,12-15] indicate very short correlation lengths compared to model predictions. The laboratory results tend to support these correlation length measurements, indicating that the reflectometer spatial resolution is significantly better than expected on the basis of simple models. Thus, observations indicate the presence of some type of physical effect (2D, resonance absorption, interference, refraction) in the experiment that is not contained in the current models. Future work is directed towards a fundamental understanding of reflectometry as a fluctuation diagnostic to allow the full potential of this diagnostic to be realized.

ACKNOWLEDGEMENTS

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Figure Captions

Figure 1. Diagram of laboratory plasma and microwave circuit.

Figure 2. (a) Langmuir probe electron saturation current (upper trace) and reflectometer signal (lower trace). (b) Reflectometer and Langmuir probe responses to the ion acoustic wave.

Figure 3. Variation of time delay as perturbation wavenumber \( k_n \) is varied. (a) Code predictions and (b) experimental results.

Figure 4. Dependence of magnitude of reflectometer response on perturbation wavenumber \( k_n \) for code and experiment.
<table>
<thead>
<tr>
<th>Spatially localized?</th>
<th>Phase and Amplitude Fluctuations</th>
<th>$k_{fl}/k_o$ response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical Model</td>
<td>$\tilde{\phi}$ linear with $\tilde{n}$.</td>
<td>$\tilde{A}=0$ (-10dB for $k_{fl} \geq 2k_o$).</td>
</tr>
<tr>
<td></td>
<td>No amplitude fluctuation: $\tilde{A}=0$</td>
<td></td>
</tr>
<tr>
<td>Experiment</td>
<td>$\tilde{\phi}$ linear with $\tilde{n}$; $\phi_{exp} &gt;&gt; \phi_{model}$.</td>
<td>Extends well above $k_{fl}=2k_o$.</td>
</tr>
<tr>
<td></td>
<td>Amplitude fluctuations present: $\tilde{A} \neq 0$, depends linearly on $\tilde{n}$.</td>
<td></td>
</tr>
</tbody>
</table>
LONGITUDINAL GRID

MESH ANODE

FILAMENTS

TRANSVERSE GRID

WAVE GUIDE INPUT

FN

HORN

Interferometer

RF Sweeper
3-6 GHz

RF Coupler

Phase Shifter

Power Meter

Quadrature Mixer

to data acquisition
Fig. 2.
Figure 3

**Code**

- $k_n/k_o = 0.54$
- $k_n/k_o = 1.0$
- $k_n/k_o = 1.2$
- $k_n/k_o = 1.4$
- $k_n/k_o = 1.6$
- $k_n/k_o = 1.8$

**Experiment**

- $f_n = 3.29 \text{ GHz}, \quad k_0 = 0.68 \text{ cm}^{-1}$

Reflectometer Signal (a.u.)

$t$ (msec)
MEASUREMENTS OF MHD ACTIVITY WITH REFLECTOMETRY ON ASDEX

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ABSTRACT

Density perturbations induced by MHD activity were studied with broadband and fixed frequency O-mode microwave reflectometry on ASDEX. The system operated in the frequency range 18-60 GHz, allowing to probe densities $n_e \sim 0.4 - 4.5 \times 10^{13} \text{cm}^{-3}$. By obtaining from the broadband raw data the average phase shift radial pattern corresponding to the (indisturbed) plasma profile and the component due to fluctuations, the radial distribution of the fluctuations could be determined. With fixed frequency measurements the detailed temporal evolution of MHD modes at specific density layers could be estimated from the power spectra of the detected signals. Here we report on results obtained in Lower Hybrid Current Drive (LHCD) scenarios, and in H-mode discharges.

I - MHD ACTIVITY DURING LHCD

1. Influence of MHD modes in density profiles

(i) Experimental results

Resistive MHD tearing modes can develop magnetic islands near the rational surfaces, where $q = m/n$. The magnetic structure of rotating modes in a tokamak plasma is well known from the measurements of the modulations of the poloidal magnetic field. Large amplitude modes have been directly localized in ASDEX by reflectometry. The width of the magnetic islands was estimated in situations where locking of the $m = 2, n = 1$ mode occurred /1/.

Fig. 1(a) shows data referring to #31070. A distribution of the frequency power spectrum (as contour plots) is shown in Fig. 1(b), for a constant density layer, probed with fixed frequency. The slowing down of the $m = 2$ mode towards locking is clearly resolved, preceding the disruption at $t \sim 1.94 s$. The $m = 2$ mode frequency obtained from reflectometry agrees with the corresponding data given by Mirnov coils (Fig. 1(c)).
In fixed frequency measurements only a limited number of plasma layers can be probed. While with broadband swept measurements the plasma is continuously probed. Results are shown for \#29285 (where the \( m = 2 \) mode was locked during 1577-1585 ms). Referring to: (i) magnetic island rotation at \( t = 1565 \) ms and (ii) mode locking at \( t = 1580 \) ms. Before locking, the strong effect of the magnetic islands on the plasma profile is clearly observed in the phase shift, \( \Delta \phi / \Delta F \), of the reflected microwaves (Fig. 2a). The shape of the disturbed beat frequency corresponds to outward and inward movements of the plasma layers during the measuring sweep time, \( \Delta t = 2 \) ms\(( \Delta t = 0.5 \text{kHz} )\). The corresponding deformations can be seen on the evaluated profile (Fig. 3a).

When the mode locked, the phase shift \( \Delta \phi / \Delta F (18 - 40 \text{ GHz}) \) presents two abrupt jumps in a narrow density region. (see Fig. 2b), revealing the flattening of the plasma in those density ranges. The evaluated density profile (Fig. 3b), presents two density plateaus, respectively for (1) \( r = 29 - 32.5 \) cm, and \( r = 39 - 40.5 \) cm.

From the observed deformations the width (W) of the magnetic island can be estimated. Its apparent width while rotating, as observed in a fixed position, is \( W \times (1 + \cos(\theta \phi + \omega \tau + \psi))/2 \)^{1/2}. When the \( m = 2 \) mode is locked, the toroidal angular distance of the antennae (in the mid-plane, \( \theta = 0^\circ \)) to viewing the "O" point is \( \psi \sim 150^\circ \): so the measured plateau should roughly correspond to only 30% of the island width, yielding a width of about 10 cm, centered at \( r \sim 30 - 31 \) cm. This is in agreement with the results derived from the magnetic data for the \( m = 2 \) mode. The second plateau must likely be located near the rational surface \( q = 3 \).

(ii) Numerical modelling

A numerical study was performed where the effects on the density due to the tearing mode with \( m = 2, n = 1 \) were simulated, considering a parabolic shaped profile. The observed density plateau was modelled around \( r = 30 \) cm\(( q = 2 \) ), whose width can vary accordingly to the rotating island. When the mode is locked, \( \omega \tau = 0 \) kHz, \( \psi = 150^\circ \), and \( W (1580 \text{ms}) = 10 \) cm as concluded before; a 40% increase in the width from 1565 ms to 1580 ms was estimated (which is also revealed from magnetic measurements of the poloidal field), so \( W (1565 \text{ms}) = 7 \) cm for \( \omega \tau = 1.4 \) kHz.

Fig. 4 shows profiles as seen by the probing waves during the measuring time of 2 ms. for (a) \( \omega \tau = 0 \) kHz and (b) \( \omega \tau = 1.4 \) kHz. Numerical simulations were also performed where discrete data was considered (with \( F_j - F_{j-1} \) such that \( \Delta \phi = 2\pi \)); the localization of each density layer \( x_j (F_j, t_j) \) is obtained by the integration of the phase shift for lower frequencies (launched at earlier times when the profile was different), so the reconstructed profiles differ from the profiles seen by the launched waves namely when the island is rotating (Fig. 5). The shape of the measured profile is therefore depending on the rate.
between the measuring time interval and the periodicity of the perturbation, as it is clear from the example of Fig. 6 (sweep time 200μs < T = 1/ f_{rot} ~ 700μs. for f_{rot} = 1.4kHz).

The numerical modelling clearly confirm the interpretation of the experimental results. For example from the measured deformation of the profile at t = 1565 ms it can also be concluded that a density plateau within the island structure should already exist before mode locking occurs, rotating with the magnetic perturbation.

2. Changes in the q-profile during LHCD

From the radial distribution of the density perturbations due to the destabilization of MHD modes at rational surfaces the temporal evolution of the q-profile was estimated from single shot measurements /2/.

Two plasmas with different safety factors (q ~ 3.3 and 4.8) were studied. In both cases locked modes (m=2, n=1) precede the disruption.

From the evaluated phase shift characteristics, phase jumps are observed (Fig. 7) due to the flattening of the plasma profile, caused by the locked modes at q ~ 2. Perturbations are also seen at the expected location of other rational surfaces. The electron density profiles obtained from the averaged phase shift data of Fig. 7 are shown in Fig. 8. The locations of density perturbations for the two plasmas are presented in Fig. 9: the q profiles for ohmic plasmas as derived from the T_e(r) profile, assuming neoclassical conductivity, are also shown for comparison. Reflectometry measurements agree rather well with the expected q profiles for the two plasmas.

LHCD discharges with different compound launched spectra were studied, where significant changes of the current density profile j(r) are expected to occur. Fig. 10 refers to # 29273, with Δφ = 90°/90° + 180° and P_{LH} = 0.75MW/0.75 + 0.3MW. Fig 10(a) shows a slow decrease in (β_{equ} − β_p − l_i)/2, for the second LH plateau, attributed to a decrease of the internal inductance l_i and therefore a broadening of j(r). The overshoot in the decrease of the U_{Loop} is an independent confirmation of this because it is caused by a decrease of the poloidal magnetic field energy due to the drop in l_i.

The local modification of j(r) can be inferred from the temporal evolution of the radial location of MHD modes as described above. The results obtained from several reflectometric samples during the plasma discharges (for 20 < r < 40cm) are presented in Fig. 10(b). From the OH phase to the steady state phase of the first LH plateau (with Δφ = 90°), an outward radial shift of the radial position of the observed modes is found. For t > 1.4s (second LH plateau, 90°+180°) a clear outward shift of the innermost detected surface (at q ~ 3/2) is seen; surfaces with higher q remain roughly at their position during this stage. The observed shifts are in agreement with Li beam measurements which give the local j(r) profile (see Fig. 11).
The range of variation of the radial position of each mode estimated from reflectometry is shown in Fig. 11(a), where the q-profile for the ohmic phase (from $T_e(r)$) is plotted for comparison. For another set of discharges with the same plasma parameters but different LH spectra ($\Delta \varphi = 90^\circ + 150^\circ$), the modification of $j(r)$ was also detected. The surfaces $q = 3, 5/2$ and $2$ were identified and an inward radial shift was measured for $q = 2$, corresponding to a flattening of the $j(r)$ profile near the $q=2$ surface (Fig. 11(b)).

II - MHD ACTIVITY IN H-MODE PLASMAS

Combined broadband and fixed frequency measurements, enabled the detailed study of the density profile development and of the temporal evolution of density fluctuations from the Ohmic to the H phase /3/.

Plasma density profiles were obtained from the scrape off layer until close to the plasma center ($0.4 - 4.5 \times 10^{13} \text{cm}^{-3}$, $43 \geq r \geq 8 \text{cm}$) and fluctuations were localized in the measured profiles. Fig. 12(a) shows a typical L-mode profile and the development of the plasma density during the H mode; Fig. 12(b) shows the corresponding fluctuation levels in the L and H-phases estimated from the perturbations induced in the radial phase shift characteristic $[(\Delta \phi/\Delta F(r))]$ of the broadband signals.

In the ohmic and L phases, fluctuations are present both in the edge plasma (between 35 and 40 cm) and in the central region (close to and within the $q = 1$ rational surface). In the intermediate zone ($16 < r < 35 \text{cm}$), MHD modes with low level of density perturbation could be detected close to the expected locations of $q=3/2$ and $q=2$.

$\text{L-H transition:}$ At the onset of the H phase, the level of edge fluctuations decreases as fast as the $H_\alpha$ drops within the divertor chamber, to levels well below those observed in the OH phase. The edge density gradient steepens; density shoulders were detected 0.4 ms after the L-H transition.

In Fig. 12(a) a density shoulder is shown 1.5 ms after the transition. At the edge, a region with reduced fluctuations is formed that extends radially beyond the steep gradient zone. After the rapid formation of the edge shoulder at the L-H transition, the interior plasma profile ($r < 35 \text{cm}$) flattens along a much slower time scale, while fluctuations in that region increase to the levels observed in the OH phase. In H regimes with an ELMy phase the flattening of the interior plasma is initiated after the last ELM; in regimes without ELMs or with a single ELM the flattening occurs some 30-40 ms after the L-H transition. The observation of a fast time scale for the density rise at the edge and a slow one for changes in the bulk plasma, suggests that the improvement of the bulk confinement is rather a consequence of the changes in the edge conditions, whose effects slowly propagate inward.

$\text{ELMy Phase:}$ Coherent MHD modes (namely around $q=2$ and $q=3/2$) that had been
supressed after the L-M transition often reappear. The broadband structure of ELMs. (as observed by the magnetic diagnostics), was detected at the edge plasma and seems also to affect the inner plasma layers.

ELMs- Fig. 13 shows the frequency distribution of density fluctuations (for $n_{ec} = 0.5 \times 10^{13} \text{cm}^{-3}$) at the end of the quiescent H-phase. and the Hα trace at the divertor, for #33143. A precursor appears $\sim 800\mu s$ before the first ELM, with frequency $f \sim 100kHz$. During the ELM, broadband turbulence is seen up to the highest measured frequency ($\sim 350kHz$); however, the background level of fluctuations is clearly below the one in L-phase suggesting that ELMs should not be considered as a H-L back transition phenomenon.

H Quiescent Phase: A large increase of density occurs during the quiescent phase: contrary to expectation, however, strong fluctuations can occur inside the edge density shoulder. A strong narrowband mode with decreasing frequency was found, of electrostatic nature. MHD activity is detected at the edge plasma, coupled with the destabilized central $m=1$ mode.

$m=1$ satellite mode activity- In ASDEX, with strong NBI ($P \geq 1MW$), a satellite mode of the central $m=1$. $n=1$ mode is often observed. Two types of discharges were analysed (with hydrogen and with deuterium co-injection) where the time evolution of the satellite mode frequency was measured at the plasma edge with (fixed frequency) reflectometry.

In the discharges with D injection the torques applied to the plasma are higher and the L-H transition occurs earlier ($\geq 35ms$ after injection, instead of $\sim 200ms$ for hydrogen) and the plasma toroidal rotation speed might still increase during the H-phase. The example of Fig. 14 refers to a discharge where $P_{N1}(D) = 2.3 \text{ MW}$ was applied at $t=1.2 \text{ ms}$. Fig. 14(a) shows the frequency distribution of density fluctuations at the edge, $n_{ec} = 0.5 \times 10^{13} \text{cm}^{-3}$. A $m=1$ satellite mode is detected, with frequency increasing from $\sim 14kHz$ in the L-phase to $24kHz$ in the H-phase; identical frequencies are observed by Mirnov coils (Fig. 14(c)). The $m=1$ activity and the coupled mode supressed after the first sawtooth observed in the H phase and are destabilized shortly before the second sawtooth: the frequencies then decrease as a corollary to the decrease of the toroidal plasma rotation.

The measurements of the local mode frequency, $f$, provided an indirect way to estimate the central plasma toroidal velocity: $v \sim 2\pi R(f - f_d)$, where $R$ is the plasma major radius and $f_d$ is the diamagnetic drift frequency at the $q=1$ surface. For the case of Fig. 14, $v$ is expected to increase from $\sim 2.1 \times 10^5 \text{ms}^{-1}$ at $1.24s$ (when $f_d \sim 4.1kHz$) up to $v \sim 2.7 \times 10^5 \text{ms}^{-1}$, at $1.3s(f_d \sim 2.3kHz)$

The radial evolution of the fluctuation levels induced by the satellite mode, $\Delta n/n$, (estimated from the relative amplitude of the frequency harmonics in the power spectra) suggests that the modes should be localized in regions where $n_e < 0.5 \times 10^{13} \text{cm}^{-3}$, seem-
ingly close to but outside the separatrix /4/. Magnetic measurements, taken within the divertor chamber, indicate a mode location within the SOL.

**H-L transition:** Prior to the back transition, fluctuations increase and recover the pattern of the L-phase. A further increase of both incoherent fluctuations and MHD activity observed after the H-L transition.

### 4. Concluding remarks

During LH regimes the detailed deformation of the plasma profile due to large magnetic islands was measured by reflectometry. The evolution of the \(m=2\) magnetic islands structure approaching locking was investigated, and locked modes could be directly localized for the first time. In cases of LHCD discharges with different compound spectra, the radial shift of the locations of the MHD modes was detected, indicating changes of the current profile: the study showed the potential of this technique for the estimation of the temporal evolution of the q-profile from single shot measurements.

Steep edge density gradients were measured after the L-H transition (\(\leq 400\mu s\)), as well as the flattening of the interior plasma which occurs along a slower time scale (\(\sim 30\) ms). Turbulent fluctuations above 30 - 40 kHz are drastically reduced at the L-H transition. Several magnetic modes were analysed during the H phase. Localized frequency spectra of ELMs and their precursors were obtained, and the influence of ELMs on the central plasma was discussed. The so-called satellite of the central \(m = 1, n = 1\) mode was observed at the reflectometer channels probing the plasma edge. The measurement of the local mode frequency provided an indirect way of estimating the central toroidal velocity of the plasma.

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Fig. 1
BROADBAND HETERODYNE REFLECTOMETRY:
APPLICATION TO THE W7-AS STELLARATOR

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ABSTRACT

A broadband heterodyne reflectometer, operating in the frequency range 75-110 GHz, has been installed and operated at the W7-AS Stellarator, for the study of density fluctuations and fast density profile determination.

One of the main aims of the system is to allow for broadband operation, without the limitations of phase-locked sources. Both LO and RF oscillators can be swept over the whole frequency range. After a first IF stage in the range 5-8 GHz, several downconversion steps lead to a last IF (60 MHz), which is provided by a quartz oscillator and carries the phase delay from the plasma as a phase modulation. The phase information is detected by sin/cos detection around the 60 MHz carrier. Due to the balanced detection scheme, all drifts, as well as the broadband noise of the BWO (RF and LO) oscillators, are cancelled at the different downconversion stages.

The system offers broadband operation together with the possibility to decouple phase and amplitude oscillations of the reflected beam. In addition, the detection system shows a very high dynamic range (up to 80 dB) which is able to overcome the large amplitude oscillations of the reflected beam, phenomenon which causes very often loss of fringes in reflectometry systems.

The reflectometer described offers a good basis for further developments on large devices, which will require a very high sensitivity due to the long waveguide runs, together with the advantage of the operation in a continuous frequency range and fast sweeping capability.

After installation on W7-AS, first results reproduced the original findings of the former homodyne system (1): location of rational surfaces and coherent modes and a correlation between fluctuations and loss of optimum confinement.
New results on the radial distribution of density fluctuations have been obtained. The fluctuation levels and spectra behavior are compared for the different plasma conditions and heating methods (ECRH and NBI).

A phase drift due to asymmetries in the spectrum around the last IF has been observed, this effect is strongly dependent on the plasma conditions and can be caused by plasma rotation.

INTRODUCTION

Most of the reflectometers use homodyne detection techniques. This method has the major advantage of simple broadband operation but has an ambiguity in the determination of the phase delay, \( \phi \), between the reference and the reflecting layers, because \( \cos(\phi) \) and not \( \phi \) is measured. This leads to interpretation problems due to the coupling between phase and amplitude and to the nonlinearity of the cosine function. For these homodyne systems, broadband capability and swept frequency operation are needed to overcome this ambiguity (2).

Heterodyne systems permit unambiguous phase determination but are in general narrowband. Due to the very stable IF required, feedback synchronization of the two high frequency oscillators is needed to avoid a large drift of the two oscillators, which would require a large IF bandwidth.

The heterodyne reflectometer described herein uses several downconversion steps which allow for a large first IF bandwidth with a very stable last IF. The reflectometer diagram is shown in figures 1 (front end) and 2 (IF section). Two free running BWO oscillators, working in the frequency range 75-110 GHz, are used as RF and LO. The first IF is allowed to drift between 5.6 and 7.7 GHz due to the two additional downconversion steps used: 4 GHz and 60 MHz. The last IF stability is provided by a quartz oscillator and its low frequency allows for a narrower bandwidth, leading to a higher s/n ratio. The dynamic range of the system is very high (>80 dB) allowing plasma losses =40 dB. Thus, the measurement is not affected by the large amplitude oscillations, permitting a continuous tracking of the phase signal with a minimum amount of lost fringes.
Figure 1: Experimental device: FRONT END

Figure 2: Experimental device: IF SECTION
A phase drift due to asymmetries in the spectrum around the last IF has been observed, its magnitude and also its sign depend strongly on plasma conditions and radial position. This makes difficult the low frequency (< 5 kHz) fluctuation measurements and the profile determination. In figure 3 we show the spectra for two different radial positions under the same plasma conditions (ECRH, B=2.5 T, \( <n> = 0.8 \times 10^{19} \text{ m}^{-3} \)), the phase drift is negative in the first case (\( \approx 150 \text{ rad/ms} \)) and positive in the other one (\( \approx 120 \text{ rad/ms} \)).

![Figure 3: Asymmetric spectra of the phase fluctuations for two different radial positions.](image)

The radial dependence of the asymmetry is shown in figure 4, in which the deviation from the symmetric case (mean frequency equals zero) is represented.

![Figure 4: Radial dependence of the asymmetry and density profile (measured by Thomson scattering system).](image)

A change in the sign of the asymmetry appears close to the limiter position, where the velocity shear layer is located. This change could be due: a) **Symmetric perturbation**: in this case an antenna misalignment is required to interpret the experimental results, and the sign changes because the poloidal rotation direction reverses. b) **Asymmetric perturbation**: an antenna misalignment is not required and the sign changes either because the poloidal rotation direction reverses or because the asymmetry reverses.
RESULTS AND DISCUSSION

In order to interpret the measured phase fluctuations as density fluctuations near the cutoff layer location the WKB approximation must be valid along the propagation path, otherwise the full wave equation must be solved to find the phase change due to a given density fluctuation. The condition for the validity of the WKB approximation is given by (3):

$$\Lambda = \frac{\lambda_0}{2\pi} \left| \frac{1}{\eta^2} \frac{d\eta}{dr} \right| << 1$$

where $\lambda_0$ is the vacuum wavelength and $\eta$ is the refraction index of the plasma. We have checked whether this condition is satisfied when density fluctuations are present. Considering the result from the saturation models ($k\rho_s<1$), the estimated mean wavelength, $\Lambda$, of the fluctuations in W7-AS is $\Lambda > 1.5$ cm at the plasma edge and $\Lambda > 3$ cm at $r=14$ cm
(r/a=0.7). In figure 6 we show the results obtained when the cutoff layer is located at r=14 cm (fig. 6.a) and r=18 cm (fig. 6.b), they correspond to the most unfavorable cases.

![Figure 6: WKB approximation validity (A<<1) for two different cutoff positions: r=14 cm (a) and r=18 cm (b)](image)

The linear approximation at the cutoff layer is valid when (3):

\[\left|\frac{de}{dr}\right| \gg \left|\frac{d^2e}{dr^2}\right|\Delta r\]

where \(e = \eta^2\) and \(\Delta r\) is the width of the reflecting layer. For W7-AS parameters this width is \(\approx 2\) mm, which means that the condition is fulfilled for perturbations with \(\Lambda = 1.5 - 3\) cm.

In order to obtain the density fluctuations, \(\delta n\), from the phase fluctuations, \(\delta \phi\), the expression:

\[\delta n = \frac{\lambda}{4\pi} \nabla n \delta \phi\]

is used. This is valid either for O-mode or for X-mode with constant magnetic field along the minor radius, as it is the case for the W7-AS reflectometer line of sight. The probing beam wavelength in the plasma, \(\lambda\), is usually larger than \(\lambda_0\), but is not in general very well defined. The value of \(\lambda\) for the typical \(\Lambda\) values (1.5 - 3 cm) lies between \(\lambda_0\) and 2\(\lambda_0\). A numerical search for W7-AS parameters at B=2.5 T leads to:

\[\lambda = \lambda_0 \left[1 + 3\left(\frac{f - f_{sc}}{\nabla n}\right)\right]\]

where \(f\) is given in GHz and the density gradient in \(10^{19}\) m\(^{-4}\).
The radial distribution of the density fluctuations has been measured under different plasma conditions. The relative density fluctuation level shows a continuous increase from the plasma bulk (=1% in r/a=0.5) to the edge (=15% in r/a=1), (see figure 7), while the mean frequency and the spectral width show a small change along plasma radius, in both, ECRH and NBI-heated discharges (see figure 8).

Figure 7: Density fluctuation profiles (together with density profiles measured by Thomson scattering system) during ECRH and NBI methods.

Figure 8: Radial dependence of the mean frequency and the spectral width.

Mixing length theory predicts a density fluctuation level given by:

$$\frac{\delta n}{n} = g \frac{1}{L_n}$$

where the function "g" shows different behavior according to the different theories. In figure 9 the value of the "g" parameter is represented for the previous discharges. Its values are very similar for the different plasma conditions and there is almost no variation along plasma radius.
As a consequence of the asymmetry in the spectra we can observe the velocity shear layer location (where the asymmetry, described as the value of the center of gravity of the frequency spectra, sign changes) and its influence on density fluctuations. In figure 10 the radial dependence of the density fluctuation level is represented together with the asymmetry.

Due to the fact that W7-AS has almost no magnetic shear, the confinement depends strongly on the magnetic configuration. Reduced confinement is associated with low order rational values of the rotational transform at the boundary: 1/3, 1/2, and optimum confinement is found close to these values. During the iota scan experiments, the phase fluctuations were measured for a fixed incident frequency (radial position = constant), obtaining a clear correlation between fluctuation level and loss of optimum confinement. In figure 11 the phase fluctuation level and the plasma diamagnetic energy are represented for the different values of iota at the boundary, for NBI-heated discharges at B=1.25 T. Similar results were obtained for different plasma conditions (B=2.5 T and ECRH).
SUMMARY AND FUTURE DEVELOPMENTS

The broadband heterodyne reflectometer works in a reliable way:
A complete description of the radial distribution of turbulence has been stored for several Thomson profile series:

\[ \delta n, \delta n/n, \langle \omega \rangle, \sigma_\omega, \delta n/\text{grad}(n), \ldots \]

\( \delta n/n \) can be evaluated for \( f > 5 \text{ kHz} \) within \( \pm 25\% \) (provided 1-Dim. WKB remains valid). The obtained values are: 15% at the plasma edge and 1% at \( r/a=0.5 \).

A phase "run away" appears possibly due to rotating structures: Shear layer is observed close to the plasma edge.

A correlation is observed between fluctuation level and loss of optimum confinement during iota scan experiments.

In the next experiments the sweeping frequency method will permit to determine \( \delta n(r) \) in a single shot.

Time delay measurements by A. M. reflectometry will be used for density profile determination.

With a few modifications the system should be able to measure the radial correlation length of the density fluctuations.

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MEASUREMENT OF DENSITY TRANSIENTS
USING THE MULTICHANNEL REFLECTOMETER AT JET

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Abstract
In this paper we discuss the use of the multichannel reflectometer at JET for measuring density transients. We present data on the following topics: density pulse propagation and density fluctuations during the sawtooth crash, the change in the electron density profile and the fluctuations during the L to H transition, and the spatial localisation of MHD modes.

1. Introduction
At JET the multichannel reflectometer(1,2) is used for measurements of density transients. The reflectometer consists of twelve channels which probe electron densities between 0.4 and 8.0x10^{19} m^{-3}. The probed densities and their probing frequencies are given in table 1. The channels are equipped with phase detectors and homodyne detectors. The phase detectors measure the phase between the reference arm and the plasma arm with a resolution of 1/256 fringe and a time resolution of 0.3 ms. They are mainly used for measuring density profiles. The homodyne detectors, also called the coherent detectors, are used to measure fast density fluctuations. With the present system density fluctuations up to 100 kHz can be recorded. The coherent detectors are sensitive to both amplitude and phase fluctuations in the signal reflected from the plasma.

It is possible to operate the reflectometer in two different ways: a fixed frequency mode and a narrow-band sweep mode. The narrow-band sweep mode is used for measuring density profiles whereas the fixed frequency mode is well suited for measuring fast fluctuations. In the paper by A.C.C. Sips et al.(3) the technique used to obtain density profiles from the phase measurements is presented. In this paper results from this analysis are combined with the density fluctuation data.
2. Particle transport studies

The sawtooth crash has been used extensively to study (perturbative) heat transport in thermonuclear plasmas (4-6). During the sawtooth crash the temperature profile in the centre is flattened in less than a micro second up to the inversion radius (see fig. 1) and the heat from the centre is deposited between the inversion radius and the mixing radius. Outside the mixing radius the plasma is not affected by the crash. After the crash the excess of heat between the mixing radius and the inversion radius is transported to the edge of the plasma by a diffusive process. Experimental data for such a heat pulse is shown in the left panel of fig. 2. The crash affects both the temperature profile and the density profile. A small fraction of the central electron density is pushed outward to the region between the mixing radius and the inversion radius. This fraction is sometimes visible as an outward going density pulse on the phase data of the reflectometer. A second much bigger density pulse is often observed following the arrival of the heat pulse at the limiter which releases some gas from the limiter into the plasma. A typical example of such a density pulse is shown in the right panel of fig. 2. This density pulse is measured with the phase detectors. Note that this density pulse is propagating inward. Both the temperature and density pulses can be described by a single (linearised) diffusive matrix equation (6):

\[ \frac{\partial z}{\partial t} = A \nabla^2 z + B \nabla z + C z + S \]  

with

\[ z = \left( \frac{n_i}{n_0} \right) \quad \quad \quad \quad S = \left( \frac{S_T}{Q} \right) \]

the perturbation of the temperature and density profile. In this equation the matrix, A, contains the diffusion coefficients, \( \chi^{hp} \) and \( D^{dp} \), together with the coupling coefficients between the heat and density pulses:

\[
A = \begin{bmatrix}
D^{dp} & \frac{\partial D}{\partial T} \frac{T_0}{n_0} \nabla n_0 \\
\frac{2}{3} D^{dp} + \frac{2}{3} \chi^{hp} \frac{n_0}{T_0} \nabla T_0 & \frac{2}{3} \chi^{hp} + \frac{2}{3} \frac{\partial D}{\partial T} \frac{T_0}{n_0} \nabla n_0
\end{bmatrix}
\]
The matrices B and C represent the convection and damping terms, while the sources and sinks are represented by the matrix S. This equation is now solved and the diffusion coefficients are adjusted in such a way that a good agreement with the data is obtained. In fig. 2 the dashed curves are the result of such a fit. At JET we have studied a large number of heat and density pulses this way. For $\chi^{hp}$ we have obtained values between 2 and 8 m$^2$/s and for $D^{dp}$ 0.3 and 1.2 m$^2$/s. In ref. 4 a more extensive report is given on these results.

3. Fluctuations during the sawtooth crash

The heat conductivity as obtained from perturbative methods, such as the sawtooth crash and ECRH modulation experiments, is in general much larger than the values obtained from a power–balance analysis. An increase of high frequency density fluctuations may be associated with this discrepancy. At JET we studied the change in the level of fluctuations during and after the sawtooth crash. For this study we have been using monster sawtooth crashes. The diffusive region for these sawteeth is typically between 3.8 and 4.2 m minor radius. In these pulses the reflectometer was sampling this region as can be seen from fig. 3. In fig. 4 the density fluctuation spectra for the channel at $2.5 \times 10^{19}$ m$^{-3}$ are shown before and after the sawtooth crash. It can be seen from this figure that the fluctuations between 60 and 100 kHz are enhanced. The time development of these fluctuations is shown in fig. 5, where we have plotted the average fluctuation level between 60 and 100 kHz normalized to the fluctuation level just before the sawtooth. These measurements suggest that the increased heat conductivity following a sawtooth crash is associated with an increase in the level of electron density fluctuations.

4. L to H transition

During the L to H transition an efficient transport barrier is formed in the edge of the plasma. In order to investigate the behaviour of the electron density fluctuations during the L to H transition we have used the reflectometer for both measuring density fluctuations and electron density profiles. In fig. 6 the development of the electron density profile is shown. From this figure it can be seen that the edge gradient steepens considerably when the transition occurs. In fig. 7 we show the density fluctuations before and after the L to H transition for two reflectometer
channels. In the lower part of fig. 7 a channel at the plasma edge is displayed, whereas in the upper part a channel deeper inside the plasma is shown. From this figure it can be seen that there is a significant drop in the fluctuation level at the edge, whereas inside the plasma no reduction is observable. This observation we confirm that, so far as the electron density fluctuations are concerned, the L to H mode transition is an edge localized phenomenon. Deeper inside the plasma keeps its L mode character as can be seen from the upper part of fig. 7.

5. Localisation of MHD modes

The combination of slow phase detectors and fast homodyne detectors is well suited to determine the radial position of MHD modes in the plasma. From the phase detectors, electron density profiles are obtained in the usual way on a 10 to 50 ms time-scale. The homodyne detectors are used to sample the density fluctuations, in our case up to 100 kHz. In fig. 8 the density profile as measured directly before the fast fluctuation window is shown. In this case it is a measurement on one of the two high performance deuterium—tritium plasmas in the L—mode phase 80 ms after the H—mode phase was terminated. In fig. 9 the density—fluctuation spectra are shown. In these spectra coherent modes can be recognized on a continuous background, together with a broad peaked structure starting at 70 kHz and extending beyond 100 kHz, the experimental cut-off frequency. Some of the discrete frequencies are only visible on the edge channels, whereas other frequencies are only found on more central channels. From this data we can determine the spatial extent of the discrete modes. To study the MHD activity in more detail we would like to make a quantitative comparison between measured amplitudes in the different channels. Therefore, it is necessary to calibrate the relative response between the channels and we do this by assuming that the broadband fluctuation level is the same in each channel. We can thus calibrate the observed discrete peaks to the background fluctuation level by taking the ratio of the peak height and the background. The result of this calculation is shown in fig. 10. With this assumption we can draw continuous curves through the different channels showing the rise and fall of the different MHD modes. At the inner channels, a low frequency mode at 4.5 kHz is visible with some of its harmonics. There is a mode at 29.3 kHz present only in the edge (3.89 m) where the density gradient is large (see fig. 8). Possibly the most
interesting feature, however, is the broad structure between 80 to 100 kHz. This has a maximum at the edge channels and a second maximum at 3.73 m. From this example we conclude that it is possible to obtain information about the localisation of MHD activity in the plasma. Under the assumption of a homogeneous background of broadband fluctuations we can obtain the relative amplitudes of the MHD modes.

6. Conclusions
In this paper we have reported on various transients in JET plasmas observed with the multichannel reflectometer. With this apparatus electron density profiles as well as electron density fluctuation spectra can be obtained on the same pulse. From measurements of the heat and density pulses, generated by the sawtooth crash, perturbative heat and electron diffusion coefficients can be obtained. From a coupled analysis, it was found that there is some coupling between the heat and density pulses. However, this coupling, as expressed as the off–diagonal components in the diffusion matrix, is small compared to the diagonal components. In an attempt to find the cause of the anomalous heat diffusivity, we have found that the electron density fluctuation level is increased during the passage of the heat pulse. Further experimental evidence that electron density fluctuations are associated with the anomalous heat diffusivity comes from the observation of the decreased fluctuation level at the plasma edge during H mode phases, where the transport barrier is formed. Deeper inside, the L–mode fluctuation level is maintained. Finally, it is shown that reflectometry can be used to spatially resolve MHD activity if the relatively slow electron–density profile measurements are performed in combination with a homodyne detection of the reflected signal on a fast time scale.

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TABLE 1. The probing frequencies and critical densities used in the JET multichannel reflectometer.

FIG. 1. The flattening of the temperature profile during the sawtooth crash. Right of the profile the time development of the recovery of the temperature is shown in the three different regions.
FIG. 2. The temperature and density traces as measured with the ECE polychromator and the reflectometer, respectively. The solid curve represents the data. The dashed curve is the best-fit solution of equation 1. The dotted curve is a spline to the data.

FIG. 3. The temperature and density profiles as measured with the ECE polychromator and the reflectometer, respectively, for a monster sawtooth crash.
FIG. 4. The density fluctuation spectra between 10 and 100 kHz for the reflectometer channel probing the density at $2.5 \times 10^{19}$ m$^{-3}$. The solid dotted curve is obtained just before and the solid curve just after the sawtooth crash.
FIG. 5. The time dependence of the average density fluctuations between 50 and 100 kHz during a sawtooth crash normalized to the fluctuation level just before the sawtooth crash. The position of the channels is shown in fig. 3.
FIG. 6. The electron density profile development before and after the L to H transition.
FIG. 7. The density fluctuation spectra before (dotted curve) and after (solid curve) the L to H transition. In the upper part a channel well inside the bulk of the plasma \((n_e = 3.1 \times 10^{19} \text{ m}^{-3})\) is shown whereas in the bottom part a channel at the edge \((n_e = 0.7 \times 10^{19} \text{ m}^{-3})\) is shown.
FIG. 8. The electron density profile $n_e$ obtained from the swept frequency data immediately before the frequencies of the transmitters in the reflectometer were kept fixed to measure the fluctuation spectra as shown in fig. 9.
FIG. 9. Power spectra of the density fluctuations as measured just after the profile in fig. 8 was measured. The individual curves have been displaced vertically for clarity.
FIG. 10. The amplitude of some of the discrete peaks as shown in fig. 9 normalized to the broadband fluctuation level.
MEASUREMENTS OF ELMs AND ASSOCIATED FLUCTUATIONS WITH THE MULTICHANNEL REFLECTOMETER SYSTEM.

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Abstract

The potential and limitations of JET's Multichannel Reflectometer System for studying ELMs have been evaluated. Two new methods for calibrating fixed frequency data have been developed to cope with the problems caused by ELMs. Results on both the temporal and radial evolution of the ELMs and associated fluctuations are presented, and the significance of the relative proportion of phase and amplitude fluctuations is discussed.

Introduction

Edge Localised Modes (ELMs) are characterised by bursts of D₃ emission during the H-mode and the L-H transition. The ELMs are generally associated with a temporary decrease in confinement, which results in increased particle and energy transport across the last closed flux surface (lcsf), and consequently increased D₃ emission. As the ELMs thus reduce both the plasma and impurity densities in the edge region, they have potential for controlling the density and prolonging the H-mode.

The plasma region of interest is the edge region spanning 20 cm inwards from the lcsf, i.e. at relative minor radius \( p = 0.8 - 1.0 \). The ELMs occur on a timescale of a few ms, and they are associated with fluctuations over a wide frequency range, up to at least 100 kHz. The interval between ELM's varies from 0.5 ms for a series of small ELMs occurring during the L-H transition, to seconds for large singular ELM's later in the H-mode.

The Multichannel Reflectometer System at JET.

The multichannel reflectometer system probes the plasma along the horizontal midplane with microwaves polarized in the ordinary mode. It employs separate launch and reception antennas, oversized wave guides for transmitting the microwave power to and from the antennas, Gunn oscillators as sources and a heterodyne detector system [2]. There are twelve discrete probing frequencies in the range 18 - 80 GHz corresponding to critical electron densities from \( 4 \times 10^{18} \) m\(^{-3} \) to \( 8 \times 10^{18} \) m\(^{-3} \).

During an H-mode the density profile is relatively constant across most of the plasma, decreasing steeply in the edge region. Thus a spatial resolution of around 5 cm or less is needed to obtain quantitative measurements of the edge density profile. The reflectometer has the advantage that it measures the radial positions of layers of fixed density. Most of the reflection points are found where the density gradient is steepest, resulting in a good spatial resolution in the edge region.

The reflectometer has two independent arrays of detectors:

1) The coherent detectors measure a combination of changes in phase caused by movements of the reflecting layer and changes in the amplitude of the reflected wave,
equivalent to homodyne detection. Frequency spectra of the density related fluctuations can be computed from the data; the maximum sampling rate is 500 kHz.

2) The fringe counters measure the phase part of the signal, \( \phi(t) \), in units of fringes (one fringe is equivalent to a phase change of \( 2\pi \) radians). The resolution is 1.128 fringe corresponding to movements of the reflecting layers of \( \approx 0.2 \) mm. In order to eliminate the effects of high frequency density fluctuations the fringe counter data is filtered at a bandwidth of 3 kHz. Apparent 'jumps' in the measured phase still occur occasionally, mostly caused by momentary loss of signal from the plasma arm. These phase jumps are eliminated by taking the time derivative of the signals and removing the spikes in \( d\phi/dt \) corresponding to the jumps.

The reflectometer has two modes of operation: fixed frequency, for monitoring the relative movements of the critical density layers; and narrow band swept frequency, for generating density profiles. In the latter case, the frequency of each source is swept over a narrow band (typically 100 MHz). The resultant change of phase is measured, at a rate of around 25 samples sweep. In order to eliminate phase changes caused by movements of the reflecting layers during the sweep the reflectometer employs a sweep dwell technique. A number of fixed frequency samples is taken in the dwell periods between each frequency sweep [2]. These measurements are used to reconstruct a baseline corresponding to the movements of the reflecting layers. This can then be separated from the phase changes caused by the frequency sweeps. Alternatively the baseline can be used as fixed frequency data.

**Calculation of Density Profiles.**

The phase delay \( \phi \) of an ordinarily polarised wave of frequency \( F_c \) reflected from the cut-off layer is \([1]\):

\[
\phi_c = \frac{4\pi F_c}{c} \int_{R_e}^{R_{out}} \mu(n(R)) \, dR - \frac{\pi}{2}, \quad \mu = \left(1 - \frac{n(R,t)}{n_c}\right)^{1/2}
\]

where \( n(R) \) is the electron density at radial position \( R \), \( n_c = F_c^2 / 4\pi \varepsilon_0 m_e c^2 \) is the critical density, \( R_c \) is the position of the reflecting layer, \( R_{out} \) is a reference position outside the plasma edge, and \( \mu \) is the refractive index.

From (1) it can be seen that changes in the phase delay can be caused by changes in both the density profile and the reflectometer frequency. Self-calibrated density profiles can be calculated from measurements of the change in phase with frequency, \( d\phi_c/dF_c \), using an Abel inversion technique [2].

Measurements of \( \phi_c(t) \) at fixed frequency give the relative movements of the critical density layers. Several techniques can be employed to convert these into density profiles, all of which require a reference density profile for calibration.

**Linear Method:**

The density profile is approximated by a linear model with a constant gradient, \( n_e/\Delta R \), from the plasma edge \( R_{edge} \) to a position \( R_{edge} - \Delta R \) in the plasma, and a constant value \( n_e \) throughout the rest of the plasma. Inserting this model in (1) gives the density gradient and the edge position:

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The multichannel reflectometer provides simultaneous measurements of $\phi_c$ at several critical densities $n_c$. With these data the density gradient and plasma edge position can be calculated from (2) employing linear regression.

In order to calculate the phase constants $\psi_{\psi_0}$, a set of values for the density gradient and edge position is obtained from a reference density profile. These are then inserted in (2) to give $\psi_{\psi_0}$ for each $n_c$.

**Piecewise Linear Method:**

The density profile is assumed to be linear between each critical density $n_c$ at radial position $R_c$, with gradient $\Delta n_c / \Delta R_c$ between $n_c$ and $n_{c-1}$. Assuming that the plasma edge position $R_{edge}$ is known, the position of each subsequent critical density $n_c$ is given by:

$$R_c = R_{c-1} - \frac{n_c}{\Delta n_c} \left[ \frac{3}{2} (\phi_{\psi_0} - R_{out} - R_{edge}) + n_c \sum_{i=1}^{c-1} \frac{\Delta R_i}{\Delta n_i} (\mu_i^1 - \mu_{i-1}^1) \right]$$ (3)

Here, $\Delta R_c = R_c - R_{c-1}$, $\Delta n_c = n_c - n_{c-1}$, $\phi_{\psi_0} = \frac{c}{4\pi} (\phi_c - \phi_{\psi_0})$, $R_0 = R_{edge}$ and $n_0 = 0$.

In order to calculate the phase constants $\psi_{\psi_0}$ a reference density profile is used. The positions of the critical densities are obtained from the reference profile. Inserting these in (3) then gives $\psi_{\psi_0}$ at each critical density.

**Fringe Counter Phase Measurements during ELMs.**

The narrow band sweep profiles are generated at a rate of one per sweep dwell period i.e. with a temporal resolution of 15-30 ms, and it is only possible to generate profiles when the positions of the reflecting layers do not change abruptly during the sweep. The density evolution during ELMs is thus too fast to be studied using narrow band sweep profiles.

The temporal resolution of the fixed frequency phase data is $\approx 0.3$ ms, fast enough to resolve the ELMs. The fixed frequency phase data have the following limitations:

- 3 kHz filters limit the movement of the reflecting layers that can be observed; the maximum observable radial velocity is $\approx 10$ m/s.
- All methods of generating profiles from fixed frequency data require a reference density profile for calibration. However it is usually possible to use a narrow band swept profile from the reflectometer, independently of other diagnostic systems.
- Since the fringe counters measure the total phase change, a fixed frequency profile will be corrupted by any phase ‘jumps’ in the period between the time of the reference profile and the time of interest. Thus a new reference profile must be used in each period of good data between phase jumps.

The linear method depends on linear regression of data from several channels. This results in a fairly good tolerance of phase jumps on any single channel. Furthermore the error on the linear fit at any given time can be calculated. This is due to phase jumps and to changes in the profile shape, in the period since the reference profile time. As the
change in profile shape during ELMs is small, the error gives an estimate of the data
corruption caused by phase jumps during the ELMs. Comparisons show that the linear
approximation to the density profile is usually good in the edge region, particularly
during the H-mode [3].

The piecewise linear method was developed to overcome the limited radial
resolution of the linear method. It allows the movement of each critical density layer to
be calculated individually. Thus the radial propagation of a density pulse can be
resolved. The main disadvantage is the lower tolerance for phase jumps, as the
calculation of each critical position is corrupted by errors in the phase data at any lower
density. Furthermore it is also necessary to obtain the plasma edge position from
another diagnostic.

**Edge Density Gradient**

The linear method has been used to calculate the average density gradient in the
edge region. In fig.1 the top traces show the density gradient during one medium and
two small ELMs. The calculation is based on a reference profile in the beginning of the
period. The uncertainty on the gradient increases during each ELM. The reason for this
is most likely that the reflecting layers move faster than can be observed by the fringe
counters due to the 3 kHz filtering. The maximum velocity that can be resolved by the
filters is $\approx 10 \text{ m s}^{-1}$, whereas a propagation velocity of $\approx 1 \text{ m s}^{-1}$ is observed by comparison
of several channels.

The middle trace in fig.1 shows the electron temperature gradient, which decreases
sharply during the ELM. The relative decrease is around 10 %, of the same order as the
observed decrease in density gradient. The timescale of the decrease is however much
shorter. This is partly instrumental as the density gradient is calculated as an average
over the whole edge region, disguising the radial evolution. Part of the difference may
also be physical as the heat transport is faster than the particle transport [2].

**Radial Density Evolution**

In fig.2 a contour plot of the critical densities against radius and time generated
by the piecewise linear method is shown, during the three ELMs of fig.1. The larger ELM
originates between $n = 1.4 \times 10^{19} - 2 \times 10^{19} \text{m}^{-3}$, causing an outward propagating density
pulse. The propagation velocity is found by calculating the time delay between the pulse
on different density contours, thus overcoming the limited time response of each channel
due to the 3 kHz filters. The propagation velocity is 13 m/s, that is $\approx 25 \%$ larger than
the local diffusion velocity [3]. In the edge region inside $n = 1.7 \times 10^{19} \text{m}^{-3}$ a density
decrease is observed spreading inwards.

The low amplitude ELMs start further inwards, at $n = 2 \times 10^{19} - 3 \times 10^{19} \text{m}^{-3}$. They
also generate outward propagating density pulses, but these appear to die away before
they reach the plasma edge and cause a D$_e$ burst.

**Coherent Detector Fluctuation Data**

The coherent detectors measure fluctuations in both amplitude, $A$, and phase, $\phi$,
of the reflected signals. The reflected signal is given by:

$$S(t) = A(t) \cos(\phi(t))$$ (4)

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The fluctuation level of the signal depends on the value of \( \phi \); if \( \phi \approx \pm \pi/2 \) the fluctuations in phase dominate the signal, and if \( \phi \approx 0 \) the fluctuations in amplitude dominate the signal. It is not usually possible to establish whether the coherent detectors are measuring phase or amplitude fluctuations at any given time. When the data is analysed it is necessary to be aware that changes in the fluctuation signature could be due to a change from amplitude fluctuations to phase fluctuations or vice versa rather than a physical effect.

The fringe counter data during ELMs show that movements of the reflecting layers correspond to a phase change of \( \approx \pi \). The coherent detector data will thus contain periods with both mainly phase and mainly amplitude fluctuations during an ELM.

The absolute phase, \( \phi \), will be randomly distributed over the different channels of the reflectometer, and also at the start of separate events on each channel. Thus any observations that are consistent for several channels and several events must be physical phenomena, independent of the absolute phase. The fluctuation characteristics of the ELMs including the outwards movement of the precursor oscillation have been observed in enough cases to be proved physical rather than instrumental.

### Density Related Fluctuations during Medium ELMs

Fig. 3 shows the fluctuations at several densities in the edge region, during one medium and one small ELM. 4-5 ms prior to the larger ELM a high frequency oscillation starts at around \( n = 2.5 \times 10^{19} \text{m}^{-3} \), and gradually spreads outwards to the rest of the edge region. A similar oscillation also starts 2-3 ms prior to the low amplitude ELM.

Fig. 4 shows the autopower spectrum of the density fluctuations vs. time and frequency at four radial positions. A quasi-coherent oscillation at 40-70 kHz (referred to as a precursor) is observed 2-6 ms prior to the \( D_s \) burst at both ELMs. The precursor to the larger ELM decreases gradually in frequency towards the ELM. It peaks earlier on the inner channels, suggesting an outwards movement of the oscillation. The precursor to the low amplitude ELM does not decrease significantly in frequency, and there is no clear outwards movement. At the onset of the main \( D_s \) burst a period of enhanced broad-band turbulence is observed for both ELMs. The duration of this coincides with the duration of the \( D_s \) burst.

### Density Evolution and Fluctuations during Small ELMs

The fluctuation measurements at several densities are shown in fig. 5, during a series of small ELMs at the L-H transition. On the top three traces, at \( n = 1.4 \times 10^{19} - 2.5 \times 10^{19} \text{m}^{-3} \) or \( \rho = 0.8 - 0.9 \), the density gradient is increasing steadily. The reflecting density layers are moving outwards causing a phase change of several times \( 2\pi \). This results in a sinusoidal evolution of the reflectometer signals. At densities below \( n = 1.1 \times 10^{19} \text{m}^{-3} \) outside \( \rho = 0.9 \) the underlying density profile remains unchanged.

The ELMs are seen as fast changes and oscillations superposed on the base signal, and they are localised to the edge region outside \( n = 1.9 \times 10^{19} \text{m}^{-3} \). On the top three traces, at densities above \( n = 1.4 \times 10^{19} \text{m}^{-3} \) the absolute phase at each ELM can be determined from the sinusoidal curve. The perturbation caused by each ELM seems fairly independent of the absolute phase, i.e. whether it is mostly a phase or amplitude perturbation. Furthermore the direction of movement of the reflecting layers during an
ELM can be determined; at densities above $n = 1.4 \times 10^{19} m^{-3}$ the ELMs cause a sharp decrease in density followed by slightly slower recovery. At densities below $n = 1.1 \times 10^{20} m^{-3}$ the ELMs result in a sharp increase in density. Here the direction of movement is obtained from fringe counter measurements. The overall effect is a decrease of the density gradient during each ELM.

At the onset of each ELM a burst of oscillations is seen on the traces at $n \leq 1.1 \times 10^{20} m^{-3}$. On one trace only, at $n = 1.1 \times 10^{20} m^{-3}$, one or more bursts of high frequency oscillations are observed prior to each ELM. These precursor oscillations are thus localised to a narrow radial region of $< 5$ cm. On the traces outside this, an outwards movement of the reflecting density layers is observed during each oscillation burst. This could be due to fluctuation enhanced transport. Another interpretation is also possible; if the precursor before each ELM is continuous but localised to a very narrow region, the observed density layer may be moving in and out in relation to the oscillation, causing it to appear as several separate bursts.

Fig.6 shows the autopower spectrum against time and frequency for $n = 1.1 \times 10^{20} m^{-3}$, one or more quasi-coherent precursor oscillations are seen prior to each ELM. Each successive precursor before the ELM has a lower peak frequency. Just prior to and coinciding with the D, burst, a period of broad-band turbulence is observed. Unlike the precursors this is also seen in the rest of the region affected by ELMs.

Density and Magnetic Fluctuations during Large ELMs.

Fig.7 shows the autopower spectrum of the density fluctuations at two radial positions in the edge region, and of the fluctuations in $B_{pol}$ at the outer midplane, during a large ELM event. At $n = 0.72 \times 10^{20} m^{-3}$ the density fluctuations show a quasi-coherent precursor oscillation at 80-100 kHz, starting $\approx 20$ ms prior to the ELM. Broad-band turbulence is observed just prior to and coinciding with the D, burst, though at a lower level than the precursor. In the outer edge region the precursor oscillation peaks 2-10 ms prior to the ELM. Further inwards, at $n = 2.5 \times 10^{19} m^{-3}$, the precursor peaks earlier and dies away before the ELM, suggesting an outwards movement of the oscillation. A slight decrease in frequency towards the ELM is also observed, and it is possible that the precursor starts even earlier, but at a higher frequency than the instrumental cut-off at 100 kHz.

The precursor oscillation is also observed on the magnetic field, confirming the reflectometer results. The magnetic precursor seem more continuous. This could be because the magnetic field measurements are not as localised radially as each of the reflectometer channels.

Summary

- The evolution in density gradient and profile in the edge region has been investigated during ELMs using two new techniques for calibrating fixed frequency phase data.
- A significant decrease in density gradient and a density pulse propagating outwards have been observed during the ELM.
- Measurements of density and magnetic fluctuation spectra show a quasi-coherent precursor oscillation prior to the ELM, and a period of broad-band turbulence coinciding with the main D, burst.
• For larger ELMs the precursor oscillation originates in the inner edge region and spreads outwards. For high frequency, low amplitude ELMs the precursors are localised to one channel only, at \( n = 1.1 \times 10^9 \text{m}^{-1} \).

• The problem of whether the coherent detectors at JET are measuring phase or amplitude fluctuations in the reflected signal, depending on the absolute phase, has been addressed. The density related fluctuation signature of the ELMs is shown to be fairly independent of the absolute phase.

**Aknowledgements**


**References**


![Figure 1: Density and temperature gradient during medium ELMs.](image-url)
Figure 2: Density contours during medium ELMs.

Figure 3: Density fluctuation measurements during two medium ELMs; high frequency oscillations are seen before each ELM.
Figure 4  Autopower spectrum of the density fluctuations in the inner, middle and outer edge region; the precursor to the large ELM peaks 4-5 ms before the $D_r$ burst at the inner position and 0-2 ms before at the outer position.
Figure 5: Density fluctuation measurements during a series of small ELMs at the L-H transition.

Figure 6: Autopower spectrum of the fluctuations at $n = 1.1 \times 10^{19} \text{m}^{-3}$. 
Figure 7: Autopower spectrum of the density related fluctuations at two densities in the edge region, and of the fluctuations in $B_{pol}$ at the outer midplane during a large ELM.
Abstract

Two correlation reflectometers are used on JET: a four channel radial correlation reflectometer and a two channel toroidal correlation reflectometer. Measurements are made for a wide range of plasma conditions - ohmic, H-mode and L-mode. The measurements suggest that fine scale density structures exist in the plasma. The clearest results are obtained under L-mode conditions when it appears that the structures grow radially under the action of additional heating and rotate toroidally with the plasma. Under ohmic conditions, no significant evidence of structures is observed. Under H-mode conditions, it appears that the structures have a relatively small radial extent and a significant poloidal component to their motion. An additional poloidal correlation reflectometer is required to determine this motion.

1: Introduction

It has been known for many years that reflectometers are sensitive to density perturbations /1,2/. However, in general, it has not been possible to obtain quantitative information on the perturbations from an analysis of the reflectometer signals. Correlation reflectometry is a new technique that can provide, under certain conditions, information on the wavenumbers and correlation lengths of the density perturbations /3,4/. In this paper we describe our work at JET with two correlation reflectometers: a four channel radial correlation reflectometer and a two channel toroidal correlation reflectometer.

2: Principle

In radial correlation reflectometry, two independent reflectometers operating with microwave frequencies f₁, f₂ with the same polarisation, probe the
plasma along the same line of sight. The reflecting layers are at \( R_1 \) and \( R_2 \) separated by a distance \( \Delta x \). The fluctuating reflectometer signals are recorded with a wide bandwidth (typically 100 kHz) during a time of interest in a plasma pulse.

The data are analysed using standard spectral analysis techniques, /5/. The autopower spectra, \( G_i(\omega), i = 1,2 \), crosspower spectrum, \( G_{12}(\omega) \), and the crossphase spectrum, \( \Theta_{12}(\omega) \) are calculated. The phase difference between the two signals is assumed to be due to the propagation of a density perturbation between \( R_1 \) and \( R_2 \), and so it may be related to the wavenumber of the perturbation, \( k_r \), in the line of sight by:

\[
\Theta_{12}(\omega) = k_r(\omega) \cdot \Delta x \tag{1}
\]

\( \Delta x \) may be effectively increased by changing one of the probing frequencies. As \( \Delta x \) approaches the coherence length \( l_c(\omega) \) of the waves, the power common to both signals decreases. This effect may be quantified in the coherence function \( \gamma_{12}(\omega) \) which is defined as:

\[
\gamma_{12}^2(\omega) = \frac{|G_{12}(\omega)|^2}{G_1(\omega) \cdot G_2(\omega)} \tag{2}
\]

where \( |G_{12}(\omega)| \) is the magnitude of the crosspower spectrum. By analysing the variation of \( \gamma_{12}(\omega) \) as a function of \( \Delta x \) it should be possible to determine \( l_c(\omega) \). There is a lower level of coherence \( \gamma_r \), which is the level of coherence that would be found between two totally random signals. A level of coherence at or below this value implies that the two signals are totally uncorrelated.

There are a number of effects that limit the information that may be obtained by this technique and these have been detailed elsewhere /4,6/.

In toroidal correlation reflectometry two reflectometers operate at the same frequency and polarisation and probe the plasma along different lines of sight, separated toroidally. As in the radial case, \( G_i(\omega), i = 1,2 \), \( G_{12}(\omega) \) and \( \Theta_{12}(\omega) \), are calculated but in this case the crossphase spectrum is related to the wavenumber of the waves in the toroidal direction:

\[
\Theta_{12}(\omega) = k_T(\omega) \cdot \Delta x \tag{3}
\]

From this the toroidal group velocity of the wave, \( v_{T-g} = \frac{d\omega}{dk} \) may be determined from the slope of the crossphase spectrum:
3: Radial Correlation Reflectometry

A four channel radial correlation reflectometer has been constructed at JET. It operates on the plasma midplane with all the channels operating along the same line of sight. The radiation is polarised in the extraordinary mode and reflection is off the upper cutoff layer. In this way most of the plasma electron density profile may be investigated and the effects of changes in the electron density gradient on the channel location and interlayer distance are minimised.

The probing radiation is from high stability Gunn oscillators with frequencies at 75.5 GHz, 75.6 GHz, 75.75 GHz and 76.2 GHz respectively. The sources are isolated from each other (> 20 dB) to prevent tracking of one diode by another. The radiation is combined and then transmitted to the plasma along oversized waveguide. The radiation is launched and received by a single antenna on the plasma midplane. The reference signals arise from reflection along the transmission line and from reflection from the fused quartz window at the vacuum break in the tokamak vessel.

The reflected signal is frequency separated and detected using a heterodyne detection system that was originally designed for electron cyclotron emission (ECE) electron temperature measurements, figure 1. The heterodyne system gives a high signal to noise ratio (> 30 dB) and a high channel isolation (~ 30 dB).

![Figure 1: Schematic of the JET Radial Correlation Reflectometer](image)

**Plasma Results**

Measurements were made under three different plasma conditions (ohmic, L-mode and H-mode) and for different radial positions of the reflecting layers ($0.5 < \rho < 0.95$, where, $\rho = r/a$). To display the results we plot the coherence averaged over all frequencies (0 - 62.5 kHz) against the interlayer distance.
In ohmic plasmas significant coherence is found only between channels with an interlayer distance \( \leq 5 \text{ mm} \), figure 2a. Generally there is no significant variation of the coherence with interlayer distance as the radial positions of the reflecting layers are varied. In all cases where there is significant coherence the crossphase spectrum is approximately constant across the whole frequency band under consideration, figure 2b. The large amount of scatter in the data is consistent with the low level of coherence between the signals associated with this crossphase spectrum.

![Figure 2a: Change of \( \gamma \) with \( \Delta x \) for Figure 2b: Crossphase spectrum for two different values of \( \rho \). Squares, \( \rho = 0.3 \); signals with a coherence = 0.35. Crosses, \( \rho = 0.55 \); circles, \( \rho = 0.9 \).](image)

In H-mode plasmas significant coherence occurs only with an interlayer distance < 5 mm. There is no significant variation with radial position of the reflecting layers, figure 3a. In all cases where there is significant coherence, the crossphase spectrum has a value of approximately zero across the whole frequency band, figure 3b.

![Figure 3a: Change of \( \gamma \) with \( \Delta x \) for Figure 3b: Crossphase spectrum for two different values of \( \rho \). Squares, \( \rho = 0.6 \); signals with a coherence = 0.48. Crosses, \( \rho = 0.8 \); circles, \( \rho = 0.9 \).](image)
In L-mode plasmas a significant level of coherence is observed between the different reflectometer channels for interlayer distances of greater than 10 mm, figure 4a. When the coherence is significant the crossphase spectrum has a value approximately constant and of order zero, figure 4b. As the additional power level is increased, the level of coherence increases so that for additional power levels of 15 MW, significant coherence is observed for interlayer distances of greater than 30 mm, figure 4a.

![Graph](image)

**Figure 4a:** Change of $\gamma$ with additional power levels, $p = 0.75 - 0.9$. **Figure 4b:** Crossphase spectrum for two signals with a coherence = 0.8.

There are a number of implications from the radial results. The ohmic and H-mode results imply that the reflectometer response is localised to at least 5 mm. This is in contradiction to what is expected from a one dimensional model, /7/. The zero crossphase observed in L-mode plasmas implies that the observed density perturbations do not propagate radially but that they are propagating perpendicularly to the line of sight, i.e. in a toroidal and/or poloidal direction. The radial scale length of the perturbations is in the range 10 to > 30 mm. To investigate the possibility of toroidal propagation of the density perturbations, a toroidal correlation reflectometer was constructed and operated on JET.

### 4: Toroidal Correlation Reflectometry

A two channel toroidal correlation reflectometer which probes the plasma on the mid plane has been constructed. The two antennas are separated toroidally by 155 mm, figure 5. The probing radiation is polarised in the ordinary mode and so reflection is off the plasma frequency layer. The radiation from each reflectometer has the same frequency so that the two reflecting layers have the same radial position. There are two possibilities for the probing frequencies, 29 or 34 GHz, corresponding to electron densities of $1 \times 10^{19}$ m$^{-3}$ and $1.4 \times 10^{19}$ m$^{-3}$.
respectively. The detection systems in both cases are of the homodyne type and the signals are digitised at 200 kHz giving a Nyquist frequency of 100 kHz.

![Figure 5: Schematic of the JET toroidal correlation reflectometer.](image)

**Plasma Results**

In ohmic plasmas the level of coherence observed between the two toroidally separated channels is generally low or at the random level $\gamma_r$. The crossphase spectrum shows wide scatter which is consistent with the random coherence level, figure 6.

![Figure 6: Coherence and crossphase spectra obtained under ohmic conditions.](image)

In L-mode plasmas the level of coherence is significant across the whole frequency band under observation (0 - 100 kHz), figure 7. The crossphase spectrum increases linearly with frequency. This implies that the perturbations are propagating toroidally in a clockwise direction ie. in the same direction as the rotation of the plasma due to the action of the neutral beams. The toroidal group velocity of the observed density perturbations may be determined from the slope of the crossphase spectrum (equation 4). For the case shown in figure 7, the toroidal velocity of the observed perturbations is 19 (±4) km.s$^{-1}$. The uncertainty arises from the error in the measured slope of the crossphase spectrum.
The radial dependence of the toroidal velocity of the plasma is measured on JET with the charge exchange (CX) diagnostic /8/. A comparison can therefore be made between the toroidal velocity of the observed density perturbations and the toroidal velocity of the plasma. In the comparison an uncertainty arises from the uncertainty in the position of the reflecting layers in addition to the intrinsic uncertainties in both measurements. When the comparison is made the two velocities agree over a wide range, figure 8. The results suggest that the observed density perturbations are rotating with the plasma.

If we assume that the lifetime of the perturbations is long relative to the time that they take to pass in front of the antenna, then we can estimate the toroidal extent of the perturbations from the product of the autocorrelation time ($T_c$), of one of the reflectometer signals /5/ and the toroidal velocity of the perturbations. A typical autocorrelation function is shown in figure 10, and we
define $T_c$ as the time for the autocorrelation function to decay from one to zero. In this case $T_c = 80 \, \mu s$, $v_T = 19 \, \text{km.s}^{-1}$ and so the toroidal extent of the perturbation is $1.5 \, \text{m}$. Applying the same analysis to our data set gives toroidal extents in the range $1 - 5 \, \text{m}$. On the other hand, $T_c$ may be an estimate of the lifetime of the perturbations. From our measurements, we find $T_c$ in the range $20 - 200 \, \mu s$. In order to select between these two possibilities, measurements with a third reflectometer at a different toroidal location are required.

![Autocorrelation function](image)

**Figure 9:** The autocorrelation function for reflectometer data taken under L-mode conditions. The toroidal velocity of the perturbations is $19 \, \text{km.s}^{-1}$.

In H-mode plasmas significant coherence is usually observed between the channels of the toroidal correlation reflectometer (figure 10). However, the crossphase is independent of frequency. This suggests that the observed perturbations are either moving toroidally at very high velocity or, more plausibly, poloidally. The CX diagnostic indicates that under these conditions there is usually a poloidal component to the plasma motion of similar magnitude to the toroidal component. It is most likely, therefore, that under these conditions the reflectometer measurements are dominated by the poloidal motion. In order to determine the poloidal motion of the structures a poloidal correlation reflectometer is required but such a device has not been installed on JET.

![Coherence and crossphase spectra](image)

**Figure 10:** Coherence and crossphase spectra obtained under H-mode conditions.
5: Conclusions

Measurements with a radial correlation reflectometer on JET show that the response of the reflectometer is highly localized (\(\leq 5\) mm) to the reflecting layer. This is contrary to predictions based on a one-dimensional model, /7/.

Measurements with radial and toroidal correlation reflectometer suggest that fine-scale density structures exist in the plasma. Under ohmic and H-mode conditions, the structures have a radial extent \(\leq 5\) mm. Under L-mode conditions, they have a radial extent which depends on the level of additional heating power and for high power levels (15 MW) is \(\geq 30\) mm. The measurements show that the structures do not propagate radially but rather toroidally and/or poloidally. Under L-mode conditions, comparison with CX diagnostic measurements shows that the structures rotate toroidally with the plasma. On the other hand, under H-mode conditions they appear to rotate poloidally. In order to determine the poloidal motion, an additional poloidal correlation reflectometer is required.

References

/7/ P. Cripwell. This conference.
DETERMINATION OF THE RADIAL CORRELATION LENGTH OF THE DENSITY FLUCTUATIONS BY SWEPT HOMODYNE REFLECTOMETRY IN THE TJ-I TOKAMAK

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ABSTRACT

A method for determination of radial correlation lengths of the electrostatic turbulence has been developed and applied to the TJ-I Tokamak. The method uses homodyne detection and is based on the slow sweeping of one oscillator while the other remains at fixed frequency. The decay of the coherence can be observed describing maxima and minima as the distance between the two probing points increases. The maxima correspond to the relevant coherence values. The correlation length for the overall turbulence as well as the correlation length for the different frequency intervals can be directly determined, providing a higher accuracy than the standard two-point measurement.

By the operation in the Q-band (33-50 GHz) extraordinary mode, the system reaches most of the plasma radius. Correlation lengths obtained at the edge are compared with those measured at deeper positions in the plasma.

A critical issue for this comparison is the detailed knowledge of the local density gradient. The proposed method can provide the density profile information, avoiding the effect of lost fringes, even in the presence of turbulence. This fact can be used to determine the local density gradient which is needed to evaluate the separation between both reflecting points. Initial experimental results and a discussion on its degree of accuracy is presented.

INTRODUCTION

A number of attempts have been made trying to evaluate the turbulence levels dn/n in toroidal devices. Although the spatial location of reflectometry is suitable for that kind of studies, the relationship between the turbulence level dn and the reflectometer signal must be carefully analyzed on each experiment.

The determination of dn/n from the reflectometer signal is not a straightforward procedure. The phase oscillation of the reflected beam must be known and then the movements of the position of the reflecting layer must be determined from the phase changes. Here only rough approximations can be taken, the most usual one being the use of the vacuum wavelength of the incoming beam instead of the actual one. This
approximation provides a lower limit for the density fluctuation level.

In the other hand radial coherence measurements offer the possibility to determine the k-spectrum and the correlation length of the density fluctuations. The determination of this radial structure requires the use of multipoint (at least two) measurements. Reflectometry allows the simultaneous probing of two radial positions by the use of two oscillators at different frequencies which are feed to the same antenna system. The access to different separations between the reflecting points is performed by changing the frequency of the oscillators.

The determination of the coherence by reflectometry is much more direct than the measurement of the turbulence level: the coherence between the density fluctuation values at two neighboring points is very similar to that between the phase delays obtained by both reflectometers reflecting at those positions.

$$\Gamma (n_1, n_2) = \Gamma (\phi_1, \phi_2)$$

The main reason for that is that the ration of the density fluctuation to the phase fluctuation is very similar for neighboring reflecting points.

Taking advantage of this fact, coherence measurements have been performed (1-2). However, an additional difficulty arises: most of reflectometers use homodyne detection, this is useful for broadband operation, which is necessary for changes in the radial position of the probing point, but has the drawback of the nonlinear relationship between the reflectometer signal and the phase delay to the reflecting point. This problem can be solved either by heterodyne or by dual homodyne (sin/cos) detection, in both cases the system is complicated and broadband operation becomes difficult.

A third way to solve the problem was proposed recently (3): the use of two single homodyne reflectometers with slow frequency sweeping in one of them. In the initial paper the idea of the method and some promising simulations were presented. In this paper, experimental evidence for that behavior is presented, with its application to the determination of radial correlation lengths in the TJ-I tokamak.

COHERENCE BETWEEN HOMODYNE SIGNALS

The method most widely employed for the measurement of correlation lengths (Lc) and wavenumbers (k) of the density turbulence is the so-called two point correlation
technique. It is based on the calculation of the coherence \( g \) between the fluctuating density values at two points in the plasma, \( L_c \) is obtained from the value of \( \Gamma \) and the wave-number distribution can be derived from the phase of the cross-spectrum.

In order to measure the correlation length of the density fluctuations, two microwave beams with frequencies \( f_1 \) and \( f_2 \) are launched and reflected at neighboring points in the plasma. A description of the experimental setup is shown on fig.1.

The signals at the detectors of the two reflectometers are:

\[
v_1 = a \cos (\phi_1(t) + \phi_1) \quad \text{and} \quad v_2 = b \cos (\phi_2(t) + \phi_2).
\]

Where \( f_1(t) \) and \( f_2(t) \) are the oscillating phase delays for both reflectometers. The phase offsets \( \phi_1 \) and \( \phi_2 \) play a very important role in the coherence determination.

The coherence between \( v_1 \) and \( v_2 \) is only equivalent to the coherence between \( \phi_1 \) and \( \phi_2 \), if: \( \delta = \phi_1 - \phi_2 = n\pi \), in the other extreme case, for \( \delta = (2n+1)\pi/2 \), the apparent coherence \( \Gamma(v_1,v_2) \) is clearly lower than \( \Gamma(\phi_1,\phi_2) \). This result is due to the nonlinear behavior of the \( \cos \) function, and depends also on the amplitude of the oscillations of \( \phi_1(t) \) and \( \phi_2(t) \). This phenomenon can cause large errors in the determination of the correlation lengths by homodyne reflectometry.
SLOW SWEEPING METHOD

In order to overcome the aforementioned problem keeping the simplicity of homodyne detection, a method has been proposed. It is based on the slow frequency sweeping of one of the oscillators, while the other one remains at fixed frequency. If we divide the sweeping period in finite intervals assigning for each of them a given position of the moving reflecting point, we can evaluate the dynamic coherence between the signals in both reflectometers for each interval. The phase difference $\delta$ increases monotonically as the frequency is swept and the coherence shows maxima and minima. The maxima, corresponding to $\delta=n\pi$, can be taken as the true coherence values. Simulations have been made with a simple model in order to evaluate whether this assumption makes sense when the coherence is measured in such a dynamic way. The results of the simulation are shown on fig. 2, the 'true coherence' of the density fluctuation and that measured with the reflectometer (assuming WKB propagation) are compared: the pattern with the maxima and minima is observed, the maxima being the good approach to the true values.

EXPERIMENTAL RESULTS

A very good agreement with the expectations was obtained in the experiment. As fig. 3 shows, the reflectometer coherence describes a clear oscillation with decreasing

![Fig.2 Comparison of the 'true' and reflectometer coherence, during the frequency sweeping (simulation)](image-url)
Fig. 3 TJ-I experiment: reflectometer coherence obtained for the different channel separation, during the frequency sweeping.

amplitude as the distance between reflecting points increases. For this experiment, the fixed frequency oscillator (Gunn diode) was placed at 32 GHz whereas the movable one was swept between 32 and 34 GHz in xx ms. The reflection position, was placed around \(r/a=0.8\), for a standard TJ-I discharge with \(n_e(0)= 1.5 \times 10^{13}\) cm\(^{-3}\), according with the typical Thomson Scattering profiles.

The observed coherence length is relatively short: 0.2 cm. and can be directly obtained from the coherence decay, this procedure gives a much higher accuracy that the standard determination by the two point approximation.

Fig 3. is showing the coherence for the whole turbulence spectrum, it can be also determined, using the same data, for the different spectral intervals, producing the correlation lengths for the different frequencies. Fig 4. shows the spectral dependence of the correlation length \(L_c(\omega)\) for this experiment, as it is expected, the longer correlation lengths appear at the lower frequencies. Taking the assumption of a Gaussian-like shape for the \(S(k,\omega)\) spectral density of the fluctuations, the width of the k-spectrum \(\sigma_k(\omega)\) can also be known, since it is inverse proportional to the correlation length \(L_c(\omega)\).

The correlation length for deeper positions in the plasma increases, as it can be seen on fig. 5, values closer to 1 cm are obtained.

The pattern presented on Fig. 3 includes information not only on the turbulence characteristics but also on the electron density profile. The maxima and minima on that pattern, obtained as the frequency is swept, correspond to increments of \(\pi/2\) in the phase.
offset of the reflectometer: the distribution of $d\phi/df$ can be determined and the density profile inverted if this information is known along the whole profile. This effect can be the basis of a method to recover the density profile information in systems where the poor access and complicated waveguides producing parasitic references or the strong turbulence destroy the ‘profile fringe’ pattern. As the plasma reflection is the only one showing turbulent behavior, the correlation technique is a powerful tool to extract the relevant information.

An important test to be performed was to observe the sensitivity of the fringe pattern to changes in the phase offset of the fixed frequency reflectometer. First of all a number of identical discharges was done with the sweeping method in order to check the reproducibility of the positions of maxima and minima, which happened to be very good. Then, identical discharges were produced changing the phase offset in the fixed frequency system by means of the variable short circuit. As fig. 6 shows, the maxima and minima become accordingly shifted in the coherence signal. This is a clear indica-

![Fig. 4 Radial correlation length for the different turbulence frequencies](image)

![Fig. 5 Coherence decay vs distance for internal positions in the plasma (r/a=.54)](image)
tion that the result is not a consequence of the signal pattern in the swept arm, and even more, for the fact that the phase and not the amplitude of the reflecting beam is dominating the coherence in the homodyne signal: the amplitude is not sensitive to the phase offset of the fixed frequency arm.

DISCUSSION

The swept frequency method provides the information on correlation length, width of the k-radial spectrum and local $d\Phi/df$ in a single shot. The determination of the correlation length is more accurate than that obtained from two single point approximation, since the coherence shows the decay, leading to the direct determination of the correlation length.

The correlation lengths obtained are short even compared with the radial resolution of the reflectometer, this phenomenon, observed also in devices like JET, DIII-D and in some cases TFTR is presently under study by the different groups.

The swept method might not be valid when the phase oscillations due to the density fluctuations are large ($>2\pi$), in that case the pattern with maxima and minima in the coherence becomes smoothed and the correlation length become shorter than the real one. This effect would also be present in any homodyne or dual homodyne system which does not use the recovered phase signal as the input for the coherence calculation.

The determination of $d\Phi/df$ can be used to obtain the density profile in systems with parasitic reflections and turbulent signals.

REFERENCES

USING OF CROSS-DETECTION REFLECTOMETER FOR THE TURBULENCE STUDY IN THE TOKAMAK PLASMA

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In recent years microwave reflectometry has been more and more used as a convenient method for the plasma turbulence study in tokamak. However some problems have been encountered in experimental data interpretation connected with the coupling of the phase and amplitude oscillations in the output reflectometer signals. The decoupling of the oscillations may be achieved with the cross-detection and quadruple reflectometers. The cross-detection reflectometer technique is discussed and the half of year experimental run in Tuman-3 tokamak is given in the report.

The initial investigations of the plasma edge turbulence in the Tuman-3 tokamak were carried out with the simple homodyne reflectometer \( ^{1} \). Some features of the received signal spectra have been similar to those of the DIII-D tokamak \( ^{2} \). Namely the high frequency spectral components were drastically decreased just after the L-H transition. The typical output signal of the homodyne detector is shown in Fig 1. This alternating signal component may be written as follows

\[
U_1(t) = 2 \sqrt{P_{LO} P_R} \cos(\phi(t) + \Delta)
\]

Here \( P_{LO} \) and \( P_R \) are the reference and the received signal powers correspondingly and \( \phi(t) \) is a phase delay in plasma. This signal \( U_1(t) \) may be represented as a projection of the phase trajectory (Fig 2). The cut-off layer displacement leads to the vector rotation on the phase diagram and therefore the fringe oscillations in the output signal \( U_1(t) \). The additional oscillations are induced by the amplitude modulation effect. So one can see that the temporal variation of the output signal \( U_1(t) \) is influenced by phase and amplitude oscillations simultaneously. For the fringe component has not been evidently observed in the output signal the strong amplitude modulation is assumed to be existing near the Tuman-3 tokamak periphery.
The cross-detection reflectometer has been used for the decoupling of the phase and amplitude oscillations. The principle of such detection is based on simultaneous measurements of the two phase trajectory projections: \( U_1(t) \) and
\[
U_2(t) = \sqrt{P_0} \sin(\phi(t)) \sin(\Delta)
\]
It's easy to see that the received signal module can be calculated and actual phase delay can be reconstructed as well. So the cross-detection sine-cosine reflectometer allows to decouple the phase and amplitude fluctuations.

The simple single antenna scheme has been designed for this purpose (Fig 3). There are two similar homodyne channels in which the reference signal phases have been tuned to obtain the additional phase shift \( \pi/2 \). This shift is achieved using the movable reflectors in the directional couplers. The shift \( \pi/2 \) could be measured and tuned while observing the 2D phase diagram on the oscilloscope. The sine and cosine output signals are amplified and memorized in the digital form with the sampling step of 1 or 0.5 mks. The corresponding algorithm has been used for the step by step phase calculation provided the phase jump is no more than \( \pi \) to exclude the phase ambiguities.

The cross-detection reflectometer was installed on the Tuman-3 tokamak. The main tokamak and reflectometer parameters as follows:

- \( R = 55 \text{ cm} \quad a = 24 \text{ cm} \)
- \( B_T = 0.4 - 0.6 \text{ T} \quad I = 100 \text{ KA} \)
- \( F = 16 - 26 \text{ GHz} \) ( O or X mode )
- \( n_c = (0.3 - 0.8) \times 10^{13} \text{ cm}^{-3} \) ( O-mode )
- \( n_c = (0.1 - 0.5) \times 10^{13} \text{ cm}^{-3} \) ( X-mode )

Here: \( F \) - is a microwave probing frequency, \( n_c \) - cut-off layer electron density. The notation of other parameters are commonly used. The experiments were performed in the various tokamak discharge condition: with the ohmic H-mode initiated by means of gas puffing or radial electric field switching on, in the fast current ramp down and during the pellet injection.

The circular antenna of 6 cm in diameter placed at the low magnetic field side was taken to probe the plasma. A few experiments were made with low gain antenna with the 2 cm horn diameter. The antenna was taken for plasma probing along the central vertical chord.
While using the cross-detection reflectometer the amplitude and phase oscillations could be treated separately. The Fig 4 shows the typical calculated phase and amplitude spectra. The spectra have been obtained with FFT procedure in the 512 mks time window. The amplitude spectra are commonly broader than the phase ones. It is worth to compare the total reflectometer signal spectrum with the amplitude one. As an example both spectra are shown in Fig 5. As one can see the amplitude spectrum falls down in the frequency band below 50 -100 KHz. It may be connected with the fact that the fringe oscillations to be withdrawn due to the phase calculating procedure are essential in this band. Both amplitude and output signals were found to be the similar in the high frequency band. So that the falling down of the high frequency spectral component which was observed previously during L-H transition seems to be connected with the amplitude fluctuation dumping. The additional results have been obtained while studying the fluctuation spectra. The plasma probing with the different microwave frequencies shows that the high frequency fluctuation dumping exist during the L-H transition only in localized region near the plasma edge. The result has been obtained which contradicts with the DIII-D data. Namely the bursts of the high frequency component are happened to be observed during the L-H transition. This bursts are strongly correlated with the ones of the soft and hard X-rays and D emission signals and may be associated with the saw-teeth oscillations. The step by step reconstruction procedure allows to obtain the phase evolution over the all period of the discharge. As an example the phase temporal variation during the tokamak discharge with a disruption is shown in Fig 6 together with the D emission signal. The phase signal rump down in the time frame from 58 to 61 ms reflects the plasma decay after the discharge break down. During this quiet plasma period the phase curve corresponds to the actual cut-off displacement. The phase trajectory in this time window is well defined (Fig 7). The both reflectometer signals are presented here as well. These data prove the correct action of the overall cross-detection scheme. During the main tokamak discharge the rate of the phase changing is observed to be anomalously large (Fig 8). The 100 Μ per division is here. So this phase variation is equivalent to
cut-off displacement which happened to be large than tokamak minor diameter. It is clear that the long time periods of phase ramp up or down are not due to the cut-off movement and are of another origin. The same phase runaway effect was recently observed in the stellarator plasma 3. It is important to note that the runaway periods are strongly correlated with the discharge condition changing. Fig 5 illustrates the event of the ohmic L-H transition where the phase changing is observed just after the L-H transition. The same effect occurs after the positive potential application to the biased electrode (Fig 9). The two fold fast current ramp down is another example (Fig 10). One can see the phase runaway is slowing off in a few ms after the ramp down.

The phase runaway phenomena may be induced by different reasons. According to simulation experiments the fault of the \( \pi/2 \) phase shift can not be responsible for the anomalous phase changing. The low level secondary reflection between antenna horn and plasma was stated to be unimportant as well. The phase distortion could be happened if the high frequency range of spectrum is virtually lost. In such a case the chaotic phase jumps may occur as it's often observed in plasma microwave interferometer. This effect may be enhanced with a noise. However the experiments with two different sampling steps (1 and 0.5 \( \mu s \)) have shown the same phase behavior. This fact is the evidence that the spectra are reconstructed correctly in a whole signal frequency band.

Our view concerning the phase evolution behavior is as follows. Let us consider the grating-like cut off layer which rotates in the poloidal (or toroidal) direction (Fig.12). The rotation leads to continuous shift of the cutoff layer in the antenna direction with the speed of \( V_c = V_p \delta / \lambda_p \) (Here: \( \delta \) is the depth of grating groove modulation, \( \lambda_p \) - the poloidal wave length of the fluctuation). The microwaves are reflected from this rippling layer and interfere into the antenna directivity diagram. As a result the Doppler frequency shift have to be appeared (\( \Omega t \)). The frequency shift \( \Omega \) leads to the linear phase uprise. Such effect is known to be commonly used in the laser interferometry for the frequency shift via the rotating of the reflection grating. With this view the
experiments with the low gain antenna are worth to be mentioned. Because of the wide directivity diagram the wide range of the reflection angles is accessible and so the profile of the plasma grating and its displacement could be determined. This displacement was observed just after the L-H transition by the well defined vector rotation on the 2D phase diagram (Fig. 13). Each point on the diagram corresponds to a single sampling step. A full phase trajectory pattern is drawn for the restricted time period of the discharge from 0 to 50 ms. The observed phase trajectory loop has somewhat diffusive structure because the phase and amplitude oscillations. It have to be noted that the phase runaway effect would occur due to the strong back (or forward) scattering process. The scattering has to induce the frequency shift signal and therefore the linear phase changing provided the scattering plasma wave fronts slightly tilted towards the minor radius (see Fig.12). The Tuman-3 experiments and the W7AS stellarator investigations [2] show the strong influence of the two dimensional fluctuations on the reflectometer phase signals therefore the 2D solutions in reflectometry data processing would be appreciated.

The assumption on the grating like plasma fluctuations is in accordance with the theoretical outlook on the tokamak turbulence [4]. According to this theory the high poloidal wave number islands which are turned over their magnetic axes form the grating-like space structure (Fig. 12). The experimental study of this turbulence pattern is a rather difficult problem. These plasma perturbations perhaps could be observed via the cross-detection reflectometry technique.

In conclusion the authors express the gratitude to the whole of the Tuman-3 Group for their assistance in performing the experiments.

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Fig. 1 Temporal dependence of the homodyne reflectometer output signal.

Fig. 2. Cross detection phase trajectory.
Fig. 3. Cross detection reflectometer layout on Tuman-3 tokamak. HGA- high gain antenna.

Fig. 4. Phase (1) and amplitude (2) spectra, 3- reconstructed phase.
Fig. 5. Output signal (1) and module (2) spectra obtained at the same time period (in arb. unit).

Fig 6 Phase signal in tokamak discharge with a disruption. -RMS value of the sine output signal in the frequency band above 250 KHz (in arb. unit).
Fig 7 Temporal signal dependence during plasma decay. 1- phase trajectory, 2- reconstructed phase, 3- sine output signal, 4- cosine one, 5- amplitude oscillation.

Fig 8
Fig. 9. Discharge with L-H transition induced by positive potential on the biased electrode.

Fig. 10. Discharge with current ramp down.
Fig. 11. a- Sketch of magnetic island pattern in minor cross section of tokamak /4/. b- grating-like structure of cutoff layer. c- back-scattering front, diagram.

Fig. 12. Phase trajectory pattern.
D: NEW APPROACHES
A multichannel reflectometer is under construction for the RTP tokamak (RTP = Rijnhuizen Tokamak Project). The system will be heterodyne and is based on the experience gathered with the JET reflectometer system\(^1\)). However, on RTP a different approach will be used to generate and detect the multiple frequencies. The main reason for redesigning a working system is the fact that the total costs of the system are about a factor of three less than that of the corresponding JET reflectometer. The system is especially suited for use on small tokamaks.

**Introduction**

The RTP tokamak\(^2\) is a medium-sized tokamak (see Table 1), especially dedicated to the study of transport processes. It is equipped with a number of fast multichannel diagnostics,\(^3\) two ECRH gyrotrons and a pellet injector. Each gyrotron has a power of 200 kW at 60 GHz. The power of both gyrotrons can be modulated with a changeable frequency, modulation depth and duty cycle. One gyrotron is connected to a launcher at the low field side, which is mostly operated in the O-mode, but which can be switched also to the X-mode. The other gyrotron is equipped with a high field side launcher in the X-mode, but with a launching angle that can be changed. The pellet injector can fire eight pellets of different mass and speed during a plasma shot. The two gyrotrons and the pellet injector can be operated simultaneously.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>R</td>
<td>0.72 m</td>
</tr>
<tr>
<td>a</td>
<td>0.16 - 0.185 m</td>
</tr>
<tr>
<td>(B_{\text{max}})</td>
<td>&lt; 2.5 T</td>
</tr>
<tr>
<td>(I_{\text{max}})</td>
<td>&lt; 200 kA</td>
</tr>
<tr>
<td>(n_e(0))</td>
<td>(1.10^{18} - 1.10^{20} \text{ m}^{-3})</td>
</tr>
</tbody>
</table>

Table 1: RTP parameters
The electron density profile in RTP is accurately measured by a fast 19-channel interferometer and by a Thomson-scattering system. The latter diagnostic measures the density profile in a single laser shot (only once per plasma discharge). Since the density profile measurement is well covered, the reflectometer could be designed as a fixed frequency heterodyne system. The reflectometer will have eight channels: five in the Ka-band (26.5 - 40 GHz), and three in the U-band (40 - 60 GHz). It will be equipped with antennae at both the high and the low field side of the tokamak. All the channels will probe the plasma in the O-mode. The bandwidth of the system will be 100 kHz and the phase resolution 7.2°.

**Design of the reflectometer**

The principle of the RTP reflectometer is derived from the conventional heterodyne system (see Fig. 1). In such a system, two Gunn oscillators, locked to a frequency difference of $\Delta f$ by a phase locked loop (PLL), are used for each frequency channel. This $\Delta f$ is taken as the reference signal. The signal of the probing oscillator is sent to the plasma where it is reflected at the critical density layer, received by a second antenna and mixed with the signal coming from the local oscillator to give a frequency difference of $\Delta f + \phi$, with $\phi$ the phase shift induced by the plasma. The two IF frequencies ($\Delta f$ and $\Delta f + \phi$) are fed into a phase comparator. To probe simultaneously different density layers in the plasma one has to use a multi-frequency system, each frequency having its own channel similar to the one indicated in Fig. 1.
The five-channel Ka-band system for the RTP reflectometer is presented in Fig. 2 as far as the microwave components are concerned. Only the low field side system is given here. Measurements from the high field side can be performed at the expense of one additional harmonic mixer. It can be seen from Fig. 2 that, although the system looks more complicated than a conventional reflectometer, one needs fewer expensive microwave components.

![Fig. 2: Microwave components for the RTP reflectometer. This is the total five-channel system for low field side measurements. For performing high field side measurements as well, only one extra harmonic mixer is needed.](image)

In this system, the principle of AM-modulation is used to create upper and lower side bands according to the simple formula:

\[
\sin(\alpha) \sin(\beta) = \frac{1}{2} \cos(\alpha - \beta) - \frac{1}{2} \sin(\alpha + \beta).
\]

The modulation is done using an up-converter (type: Hughes 47471H-2230). This is a planar balanced mixer to which two signals are applied (IF and LO). The LO-frequency is in the same band as the RF output frequency. The bandwidth of the LO is ±5% of the central frequency. The IF frequency can be swept from DC to 18 GHz. The maximum input power is +17 dBm for both the IF and the LO, while the maximum DSB output
power is +8 dBm integrated over all the frequency components. The typical output spectrum of an up-converter when it is driven by an LO of 34 GHz and an IF of 100 MHz is shown in Fig. 3.

![Fig. 3: Typical output spectrum of an up-converter. LO = 34 GHz, +16 dBm, IF = 100 MHz, +15 dBm. Total output power is +8 dBm. The real spectral lines are indicated with an arrow.](image)

Detection is done by harmonic mixers (type: Millitech MHP-28-1) driven with a local oscillator at 3010 MHz. These mixers have a conversion loss of 17 - 45 dB depending on the harmonic number used (see Fig. 4). In our case harmonic numbers between 9 and 13 are used corresponding to a conversion loss between 35 and 38 dB. In Table 2, the transmitted frequencies, the mixing frequencies, being n times 3010 MHz, and the corresponding IF frequencies are presented.
Fig. 4: Conversion loss as a function of harmonic number for the harmonic mixers employed in the RTP reflectometer (Millitech: MHP-28-1).

<table>
<thead>
<tr>
<th>transmitted frequency</th>
<th>harmonic of LO</th>
<th>IF frequency</th>
<th>Cut-off density $\times 10^{19}$ m$^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>27150</td>
<td>27090</td>
<td>60</td>
<td>0.92</td>
</tr>
<tr>
<td>30150</td>
<td>30100</td>
<td>50</td>
<td>1.13</td>
</tr>
<tr>
<td>33150</td>
<td>33110</td>
<td>40</td>
<td>1.37</td>
</tr>
<tr>
<td>36150</td>
<td>36120</td>
<td>30</td>
<td>1.62</td>
</tr>
<tr>
<td>39150</td>
<td>39130</td>
<td>20</td>
<td>1.90</td>
</tr>
</tbody>
</table>

Table 2: The different channels in the system as represented by their frequencies. All frequencies are in MHz.

In Fig. 5, the IF components are shown. In a conventional reflectometer all the IF-frequencies are the same. For the RTP-reflectometer, however, a somewhat more complicated system has to be used to convert all the IF signals to the same frequencies. Since the low frequency components needed for this are relatively cheap, it will not have a large effect on the total price of the system.
Fig. 5: The IF components of the reflectometer. To convert all the channels to the same IF frequency, some extra components are needed.

Accuracy of the system

In the design of the reflectometer a minimum phase accuracy ($\Delta\phi$) of 7.2° was requested. As the phase comparator and all the other IF components are designed within an accuracy of 2°, the limiting factor for the accuracy will be the signal-to-noise ratio (SNR). It can be proven that for phase measurements the minimum SNR is given by:

$$
\text{SNR} > \frac{1}{2\pi^2} \left( \frac{360}{\Delta\phi} \right)^2.
$$

For the given phase accuracy the minimum needed SNR is 126 (≈21 dB). The receiver noise is estimated to be around -118 dBm, being the thermal noise taken at room temperature with a bandwidth of 100 kHz. The noise from the plasma is estimated to be ≈ -72 dBm at an average electron temperature of 1 keV. So the dominant noise will come from the plasma. In Fig. 6, a measurement of the reflected power by the plasma is shown at a frequency of 34 GHz and a bandwidth of 200 kHz. Before the discharge starts the losses between the two antennae, including vacuum breaks and two times one meter of fundamental waveguide, is measured to be -30 dB. During the discharge an extra loss of 7 dB is measured, so the total losses do not exceed -37 dB. In Table 3, a power balance has been made using the measurement presented in Fig. 6.
Fig. 6: Attenuation measured between the two antennae during a RTP discharge. Before the discharge the attenuation is -30 dB. During presence of the plasma the attenuation becomes maximally 7 dB larger.

<table>
<thead>
<tr>
<th>noise contributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>plasma noise</td>
</tr>
<tr>
<td>conversion loss of mixer</td>
</tr>
<tr>
<td>noise after detection at one harmonic</td>
</tr>
<tr>
<td>contribution of 5 harmonics</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>signal contributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>source power per frequency component</td>
</tr>
<tr>
<td>conversion loss of mixer</td>
</tr>
<tr>
<td>loss in plasma</td>
</tr>
<tr>
<td>signal after detection</td>
</tr>
<tr>
<td>signal-to-noise ratio</td>
</tr>
</tbody>
</table>

maximal achievable accuracy | 3.22°

Table 3: Power balance for the RTP reflectometer. It can be concluded that the design accuracy can be met.
Future plans

After the system has been put in operation at RTP, an upgrade to larger bandwidth of 400 kHz is envisaged, which is the maximum set by the data acquisition system. Furthermore efforts will be undertaken to perform correlation reflectometry. The simplest way to do this is to modulate the up-converter at 10 and 20 MHz. Then sidebands are created at frequencies between 33.13 and 33.17 GHz. These frequencies are mixed with the eleventh harmonic of 3010 MHz to get again IF frequencies of 20, 30, 40, 50, and 60 MHz. In this way the frequency change between the two outermost channels is only 0.12% so the density changes is only 0.24%. To perform correlation reflectometry at a larger separation between the channels or at other frequencies will take a larger effort and still has to be studied.

Costs of the system

In a conventional system a total of ten Gunn oscillators and 15 mixers is needed to measure with five channels from both the high and the low field side. For the RTP system one needs for the same number of channels only one Gunn oscillator, four harmonic mixers, one up-converter and one master oscillator. The total costs for both systems are estimated in Table 4. The difference in price is obvious.

<table>
<thead>
<tr>
<th></th>
<th>conventional reflectometer</th>
<th>RTP reflectometer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gunn oscillator</td>
<td>$4.450</td>
<td>$4.450</td>
</tr>
<tr>
<td>mixer</td>
<td>$3.000</td>
<td>$2400</td>
</tr>
<tr>
<td>waveguides</td>
<td>$500</td>
<td>$10.000</td>
</tr>
<tr>
<td>total 1 channel</td>
<td>$18.400</td>
<td>$3.550</td>
</tr>
<tr>
<td>5 channels</td>
<td>$92.000</td>
<td>$500</td>
</tr>
<tr>
<td>total for 5 channels</td>
<td>$28.100</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Comparison of costs for the two different systems.

Conclusion.

The advantage of the system here presented is that only a few expensive microwave components are needed making the system relatively cheap. This makes it very attractive for groups with low budgets.
The disadvantages is that the system has a low signal power and a high conversion loss, which makes it impossible to use at large tokamaks where long waveguide runs are needed.

Acknowledgement

This work was performed as part of the research program of the association agreement of Euratom and the 'Stichting voor Fundamenteel Onderzoek der Materie' (FOM) with financial support from the 'Nederlandse Organisatie voor Wetenschappelijk Onderzoek' (NWO) and Euratom.

Literature:
A new radar system is presented. It can make very accurate measurements of times of flight of waves at different frequencies which are reflected by the plasma, thus allowing an evaluation of the electron density profile by the well known Abel inversion.

1 Theory of FM-CW radar:

The resolution of a radar is the ability to separate two echoes. In all the cases, the resolution $\Delta R$ is inversely proportional to the frequency bandwidth $\Delta F$:

$$\Delta R = \frac{C}{2\Delta F} = 15 \text{ cm GHz}$$

In the case of reflectometry on plasma, resolution is very interesting to separate the reflection from the plasma from spurious reflections like on vacuum windows, metal pieces in access port, VSWRs (one antenna set-up), coupling between antennae (two antennae set-up). In some cases, the reflection at the plasma edge may be mixed with the wave reflected at the cut-off layer[1].

The accuracy of distance measurement in FM radar can be much better than the resolution. The theoretical limit is given by the famous Woodward formula[2]. The variance of the error of the time of flight measurement is:

$$\sigma_t = \frac{1}{2\pi \beta \sqrt{\frac{S}{N}}}$$

$$2\pi \beta = 1.8 \Delta F$$

$S/N$ represents the signal to noise ratio (assuming that $N$ is a gaussian noise). Knowing that $\tau = 2R/c$, one finds if $\sigma_R$ is the variance of the error on the distance measurement:

$$\frac{3\sigma_R}{\Delta R} \approx \frac{3}{2\sqrt{\frac{S}{N}}}$$

At $3\sigma_R$, the measurement can be considered as certain so $3\sigma_R$ can be taken as the accuracy (or spatial resolution). For example, $3\sigma_R = 3\text{mm}$ for $\Delta F = 1 \text{ GHz}$ and
This theory is exact for reflection on a fixed reflector in vacuum, but we shall see in this paper an application to plasma.

2 FM radar with a synthetic "chirp":

![Diagram of chirp]

The proposed solution for frequency modulation is to use a synthetic "chirp" (see drawing) which consists of a set of N discrete frequencies sent one by one. Let $\delta F$ be the frequency step, the total frequency bandwidth $\Delta F$ is equal to $N\delta F$. This chirp is equivalent in theory to a short pulse of duration $1/\Delta F$. For instance a 2 GHz chirp would replace a 500 ps pulse. Nevertheless, it is easier to achieve very large bandwidths than ultra short pulses. A continuous wave has more energy than a pulse, so the signal to noise ratio of a chirp radar is improved compared to a pulsed radar by a ratio called the compression factor, which is equal to $\Delta t \Delta F$.

The desired frequencies are obtained by direct synthesis, so compared to other sweeping systems, the advantage is that there is neither phase noise neither amplitude modulation noise (the synthetiser is automatically power leveled). Furthermore, the sweep is very linear in frequency compared to analogic sweepers like YIG, VCO, BWOs ... The commutation time from one frequency to any in a range of 20 GHz is less than 1μs standard, but 100 ns of commutation time seems possible in a near future. A simple acquisition system 1 MHz (or 10 MHz) only is needed.

The measurement of time of flight is as follows. On each frequency step $F=F_0+i\delta f$, one measures in amplitude and phase the wave reflected by the
plasma. The amplitude phase detector delivers the two projections 
\( A_i \cos \Phi_i, A_i \sin \Phi_i \). The group delay \( \tau = \Phi'(F)/2\pi \) is obtained by inverse Fourier transform of the set of \( \{ A_i e^{-i \Phi_i} \} \). \( \tau \) is derived from the maximum of the spectrum in an expected time window (spurious signals have different times of flight). It is assumed to be the time of flight of the average frequency \( F_0 + \Delta F/2 \). Note that only the phase is needed to compute the time of flight, but at this point one cannot divide by the amplitude because it is the amplitude of the whole signal (useful signal mixed with spurious signals). The Fourier transform approach is interesting because it can admit small random phase changes in frequency (that is in position of the reflective layer). The system may be insensitive to small density fluctuations at the contrary of fringe counting systems that sum the errors instead of averaging.

As the frequency is quantified by \( \delta F \), the spectrum in time domain is repeated every \( 1/\delta F \) so one has an unambiguous distance \( \delta R \) (or ambiguity) equals to only \( C/2\delta F = N\Delta R \). One must take care to have an ambiguity distance superior to the distance range to be measured.

An example of radar could be:
- frequency bandwidth \( \Delta F = 2 \) GHz
- resolution \( \Delta R = 7.5 \) cm (about 10 cm with apodisation)
- number of frequencies \( N = 32 \)
- ambiguity in vacuum \( \delta R = 2.4 \) m (1.2 m in plasma : mean refractive index of .5)
- signal processing: 1024 points IFFT with zero padding

The amount of data (here 32 points x number of frequencies = about 320 points) is much smaller than for systems involving fringe counting.

An interesting point is the complete changeability of the chirp configuration. For example, one could try a 32 ms measurement by the two alternatives 32x1\( \mu \)s or 250x128ns (or else). The differences in the results can give some informations about density fluctuations.

\section{Radar calibration:}

Because there is generally a long transmission line to carry the waves to the plasma, one needs to take into account the frequency dependance of the transmission line. The best way to do this is to put a reflector in the vacuum chamber like a trihedral corner cube in order to make an amplitude versus frequency calibration. In the case we use such a strong reflector, parasitic reflections are assumed to be at a much lower level. Another solution is to
couple the emission waveguide and the receiving one with a variable attenuator.

It is also necessary to calibrate the time of flight of the transmission line. We have:

\[ \tau_{\text{plasma}} = \tau_{\text{measured}} - \tau_{\text{vacuum}} - \tau_{\text{guide}} \]

For a waveguide, one can define an index (like in O-mode plasma)

\[ \mu = \sqrt[2]{\frac{F_c^2}{F^2 - \frac{c}{v}}} \]

where \( F_c \) is the cut-off frequency

The group index is

\[ \mu_g = \frac{\partial}{\partial F}(\mu F) = \frac{1}{\mu} \]

\[ \tau_{\text{guide}} = 2 \mu_g D/c, \text{ so the line can be considered as a vacuum length of } D/\mu. \]

The second point is the ambiguity calibration. Obviously, it is interesting to have only a few number of frequencies in the chirp to make the fastest measurements. So the ambiguity distance must be chosen as short as possible, in particular much shorter than the transmission line equivalent length. We measure \( \tau \) modulo \( 1/\delta f \) (\( \tau_k = \tau_0 + k/\delta f \) with \( k \) unknown). The use of a known reflector (at a known position) removes the ambiguity. In the case of changes due to some dilatation of the line for instance, spurious reflections or the direct coupling between emitter and receiver in a two antennae scheme could be used as position references.

Note that there is no possible phase versus frequency correction. This is why mismatches that disturb the phase must be avoided at maximum in the set-up and the transmission line. A problem may also appear in case of mode trapping in the transmission line. If this appears, any correction could be difficult to make.

4 Simulations:

First simulations have been made for realistic plasmas. In the case of reflection on plasma, the spectrum is broader than in the case of reflection on a mirror. Furthermore, the shape is more complex and the maximum is not exactly at the middle. So there is a compromise between taking large \( \Delta F \) for resolution and low \( \Delta F \) for localisation of the reflection layer. Simulations have been made for \( \Delta F \) from 500 MHz to 4 GHz. We present here a simulation for a linear density profile with:

- central electron density \( 5 \times 10^{19} \) m\(^{-3} \)
- plasma radius 1.2 m
- distance antenna plasma .3 m

chirp data:
- frequency bandwith 1 GHz
- 32 frequencies
- 1024 points FFT

Figure 1 shows the results of FFT (spectrum modulus in dB normalised to the maximum) for different wave frequencies varying from 10 to 60 GHz. One clearly sees the differences of time of flight. For each average frequency \( F = F_0 + \Delta F/2 \), \( \tau \) is given by the peak value or by the average of the two values corresponding to -3 dB (half peak). The second process gives slightly better accuracy.

<table>
<thead>
<tr>
<th>cut-off freq</th>
<th>theor. radius</th>
<th>obtained radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 GHz</td>
<td>1.167 m</td>
<td>1.168 m</td>
</tr>
<tr>
<td>20 GHz</td>
<td>1.075 m</td>
<td>1.077 m</td>
</tr>
<tr>
<td>30 GHz</td>
<td>.923 m</td>
<td>.925 m</td>
</tr>
<tr>
<td>40 GHz</td>
<td>.711 m</td>
<td>.712 m</td>
</tr>
<tr>
<td>50 GHz</td>
<td>.439 m</td>
<td>.440 m</td>
</tr>
<tr>
<td>60 GHz</td>
<td>.108 m</td>
<td>.109 m</td>
</tr>
</tbody>
</table>

This table shows a comparison between exact radii of the reflecting layers and radii deduced from obtained times of flight. The maximum difference is 2 mm. Other plasma shapes are being tested. A problem is expected with very flat profiles where \( \tau \) increases very much. Note also that the spatial resolution obtained here is much less than the error caused by the WKB approximation in ordinary mode.

5 Description of possible microwave set-up:

We have described the use of a "chirp" with frequencies sent one by one. One could imagine to improve time resolution by sending the frequencies at the same time. There will be no problem at emission but a filter bank should be present at the detection. This system is more complex because of the number of acquisition
channels.

The radar needs two antennae (one at emission, one at reception). In cases of lack of accessibility, a one antenna monostatic arrangement with a directivity coupler can also be used.

The basic component of chirp radar is a fast switching synthetiser. Such a device is now available in centimetric waves (DC-20 GHz). Phase locked millimetric oscillators can up-convert waves to millimetric ranges. Note that the performances do not decrease when going toward high frequencies in contrary of other sources like carcinotrons.

There are two alternatives for the chirp frequency bandwidth: moderate bandwidths (.5-2 GHz) or large bandwidths (10-20 GHz). The first choice is similar to a multichannel reflectometer like in JET. The second one would replace broadband reflectometers like in Tore Supra or ASDEX. In this case, signal processing is similar to sliding FFT [3]. On large tokamaks, because of long transmission line, vacuum windows ... a moderate bandwidth seems easier to achieve.

A set of millimeter oscillators phase locked by the synthetiser reference are used. The signal is down-converted to lower frequencies, amplified by a low noise microwave amplifier for phase detection (figure 2). Eventually, a direct coupling between emitter and receiver will be added for reference positionning. This homodyne system is very sensitive but the phase detection is less accurate than in heterodyne detection. (Nevertheless, the phase error versus frequency can be corrected). The heterodyne solution uses a second fast switching synthetiser with a small frequency difference (one step). Phase detection is made at this low frequency and is very accurate.

6 Physics studies:

i) One can measure directly the density gradient with large bandwidths ΔF. This direct measurement could be very useful for transport studies for instance in divertor experiments[4]. The density gradient should be deduced from the spectrum width.

Let us assume a linear density profile where \[ \tau = 4 \frac{(x-x_0)}{c} \]

\[ x-x_0 = \frac{n_e(x)}{n'_e(x)} \]

in differentiating relatively to \( F = F_{pe} \alpha \sqrt{n_e(x)} \), one finds that the gradient is inversely proportional to Δτ.

\[ \frac{dn_e}{dx} \propto \frac{AF}{\Delta F/\Delta \tau} \quad \text{with} \quad F_{av} = F_0 + \Delta F/2 \]
Preliminary studies show that ΔF must be in excess than 5 GHz.

ii) As the radar system makes an efficient phase detection at every frequency and has a frequency agility of about 20 GHz/μs, it is a good Doppler radar to monitor plasma turbulence [5]. It could be used to measure electron density fluctuation profiles with large frequency increments δF.

iii) When the plasma presents MHD activity, there is a profile flattening inside the magnetic islands so τ may increase suddenly with F increasing then decrease after. An improved data processing like wavelet-transform or other time frequency transforms could give some information about islands widths. A map of islands could be made by sending a chirp every 100 μs.

7 Conclusion:

We have described a synthetic chirp millimeter wave radar that emits a reduced set of pure frequencies. It contains a highly sensitive phase detection that eliminates amplitude modulation. The use of Fast Fourier Transform gives an efficient signal processing. We have seen that we can separate spurious reflections from the useful reflection and that an accurate calibration is possible. This radar delivers an electron density profile in some tens μs perhaps some μs. It works with a moderate bandwidth of about 1 GHz that enables a careful matching study of the set-up and the transmission line. It can be in some cases easily extended to full waveguide band operation. It is useful for fluctuation studies and other physics studies.

References:

The author is grateful to Michel Paume from CEA/DRFC
HIGH RESOLUTION MILLIMETER-WAVE SYSTEM
(HOMODYNE)

Figure 1

Figure 2
ARBITRARY DENSITY PROFILE AND SHAFRANOV SHIFT RECONSTRUCTION IN LARGE TOKAMAKS VIA DUAL-POLARIZATION REFLECTOMETRY AND MAGNETIC DIAGNOSTICS

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Institute of Physics & Technology, Kharkov 310108, Ukraine


1. One of the most important tasks of plasma diagnostics in fusion devices with magnetic confinement is the measurement of the electron density profile $n_e(r)$. For this purpose the ordinary (o-) wave one-chord multifrequency reflectometry has been discussed and recognized promising for ITER [1,2]. This method was proposed in the early sixties. By now it is developed in great detail and is successfully employed in tokamaks. However this way enables one to reconstruct the density profile only to the first maximum and can not be used in the cases of flattened or hollow density profiles.

Last years extraordinary (x-) wave reflectometry was proposed and employed for plasma density profile measurements in a number of devices (e.g., see [3-5]). But the methods developed in [3-5] don't take into account the influence of the poloidal magnetic field on the x-wave phase delay. Implicitly they supposed that Shafranov shift was equaled to zero. So this methods are invalid for the devices with large toroidal current (ITER, Tore Supra, Asdex Upgrade etc.).

Here we offer the method for reconstructing the plasma density profiles of an arbitrary shape and Shafranov shift determination. It is based on the use of DPR phase delay data and information of magnetic diagnostics about the position of the separatrix (two points in the equatorial plane) and values of poloidal magnetic field in these points. The reconstruction of the poloidal field profile in the outer part of the plasma is also possible.

2. The method proposed is based on measuring the phase delay of the o-wave and the x-wave which is reflected at the upper cut-off point $R_c$ where
Here $\omega$ is the probing wave frequency, $\omega_c(R_c) = eB(R_c)/m_e c$, $\omega_p(R_c) = (4\pi n_e(R_c)e^2/m_e)^{1/2}$, $3(R_c) = (B_t^2 + B_p^2)^{1/2}$ is the modulus of the confining field, $B_t$ and $B_p$ being the toroidal and poloidal magnetic field respectively, $R$ is the coordinate along the major radius of the torus in the equatorial plane. We consider the waves with the wave vector component along the magnetic field $k_B = 0$.

The necessary condition for the applicability of the method is the frequency (1) growth as $R$ decreases. In the ITER this condition is well fulfilled in a wide range of plasma density (Fig. 1).

The x-wave phase delay is given by

$$\tau(\omega) = (2c/\omega) \int_{R_b}^{R_c} \left[ \frac{\omega^2 \omega^2(R) - (\omega^2(R) - \omega^2)^2}{\omega_c^2(R) + \omega_p^2(R) - \omega^2} \right] dR + \pi/2$$

where $R_b$ is the plasma boundary position at the outer side of the torus. The plasma density and the magnetic field $B$ at the $R = R_b$ are assumed to be known.

3. The general way of solving the problem of profile reconstruction is as follows.

i) Based on the measured o-wave phase delay we reconstruct $n_e(R)$ to the point of the first maximum $R_m$.

ii) Solving eq.(2) we find the dependence of the cut-off frequency (1) on the radius $\omega(R)$ and the $\omega_c(R)$ in the ($R_b + R_m$) region.

iii) We assume that magnetic axis is shifted from the geometrical axis of the torus $R_0$ by the $A(0) = A_0$ distance and the magnetic surfaces are shifted by the distance

$$\Delta(a) = \Delta_0(\exp(\xi) - \exp(\xi a^2/b^2))/(\exp(\xi) - 1)$$

where $\xi$ is the parameter, $a$ is the radial coordinate - label of the magnetic surface, $b$ is the boundary surface coordinate. We assume that in the region from $R_0 + \Delta_0$ to $R_m$ the poloidal field $B_p(a)$ changes by the linear law from 0 to the $B_p(R_c)$ value determined in (ii). This assumption can be adopted because the region mentioned above lies in the plasma interior where $B_p \ll B_t$. 

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iv) Solving eq. (2) and using the results of the x-wave phase delay measurements we find \( \omega(R) \) and \( n_e(R) \) in the region from \( R_0 + \Delta \) to \( R_m \). The original algorithm [6] was created for solving eq. (2) and applied in ii) - iv). As compared to already known [3,5] it allows us to get \( R(\omega) \) more accurately.

v) Regarding the plasma density to be constant on the magnetic surface we extend the obtained function \( n_e(R) \) to the region between \( R_0 + \Delta \) and the inner boundary of the torus. Then we use the data of the x-wave phase shift measurements for the frequency band where the cut-off point is on the inner part of plasma column. Taking these we minimize the functional

\[
\Psi(\Phi_0, \xi) = \sum_k \left[ \Phi(\omega_k) - \Phi_s(\omega_k, \Delta_0, \xi) \right]^2
\]

(4)

over the parameters \( \Delta_0 \) and \( \xi \). Here \( \Phi \) is the measured phase delay and \( \Phi_s \) is that reconstructed by the use of assigned \( \Delta_0, \xi, n_e(R) \). The contours of the \( \Psi \) are shown in Fig. 2. The most reliable way of the minimum \( \Psi \) finding is based on the using of the data of the magnetic diagnostics. These data allows us to connect the parameters \( \xi \) and \( \Delta_0 \) through the \( B_{\text{pout}} / B_{\text{pin}} \):

\[
\Delta_0 = - \frac{B_{\text{pout}} R_{\text{out}} - B_{\text{pin}} R_{\text{in}}}{B_{\text{pout}} R_{\text{out}} + B_{\text{pin}} R_{\text{in}}} \frac{b}{2\xi} (1 - \exp(-\xi))
\]

(5)

Then the functional (5) has a pronounced minimum over the variable \( \Delta_0 \) (see Fig. 2).

The reconstructed profiles of the plasma density and the poloidal magnetic field in ITER are shown in Fig. 3 (the magnetic configuration at the \( t=70 \) s of discharge) and in Fig. 4 (\( t=90 \) s). Dashed lines - arbitrary prescribed profiles, solid - found profiles. The first, second and final step of iterations are shown.

4. To summarize we may state that an efficient method is offered to reconstruct plasma density profiles of an arbitrary shape and to determine Shafranov shift via measurements of phase delays of \( \alpha \)- and \( \times \)-waves and magnetic diagnostics data. One needs the data on the phase delay of the \( \times \)-wave at approximately 20 frequencies for ITER. We studied the stability of this method with respect to the errors of the phase delay measurements (Fig. 5).
results are rather optimistic and allow us to confirm that we can get the reliable information about plasma density profile, Shafranov shift, and outer part of poloidal field profile within a acceptable range of errors.

References.
Fig. 1
Fig. 2
Fig. 4.
Fig. 5

---
- o - wave phase delay error = 0
- o - wave phase delay error = $e$ - wave phase delay error
AMPLITUDE MODULATION REFLECTOMETRY FOR DENSITY PROFILE
AND FLUCTUATIONS STUDIES

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Abstract

One of the main problems which arises in density profile measurements by reflectometry is the need for continuous tracking of the phase delay: fast density fluctuations and strong modulation on the amplitude of the reflected signal usually bring to 'fringe jumps' in the phase signal.

Amplitude Modulation Reflectometry performs a time delay measurement by the determination of the phase delay of the modulating envelope of a millimeter wave reflected by the plasma. The phase delays involved are small, the measurement is not affected by the fluctuations and can be directly performed without complicated fringe counters: the method provides a promising possibility for real time determination of the plasma position and density profile.

In the present paper the principles of the method are presented as well as the analysis on the effects of phase and amplitude fluctuations, dispersive effects and accuracy requirements. The application to the future JET Divertor plasmas will be also presented together with an initial design of the system.

Introduction

The relevant information used in reflectometry as the input set of data for density profile studies is the differential phase shift $\partial\phi/\partial\omega$ which the beam undergoes for the different reflecting frequencies ($\omega$). This information is usually obtained by sweeping the frequency of the launched beam, either in narrow or broadband.

Due to the short vacuum wavelengths imposed by the relevant critical frequencies in present day plasmas, most of swept reflectometers have to deal with multiradian phase changes, induced during the sweep or by the broadband density fluctuations present in the plasma. This problem will become more severe for the next step devices, operating at high densities and magnetic fields. The large phase changes involved, lead to a number of problems. The main of
them is the need for a continuous tracking of the phase 'history' during the sweep, to keep memory of the number of fringes. Any temporary loss of information, normally caused by fast transients or strong decreasing in the amplitude of the reflected beam, will make impossible further tracking of the phase value (lost fringes). In addition, spurious phase contributions can be introduced by phase-runaway effects during the sweep, caused by geometrical effects of the reflection on rotating structures. Finally, complicated filtering and huge data recording are needed to produce the final set of $\partial \phi / \partial \omega$ data.

A.M. Reflectometry

Amplitude Modulated (A.M.) reflectometry is to some extension an intermediate solution between the classical phase delay reflectometry, so far applied to small distances, and the time domain reflectometry, used for ionospheric studies and recently also proposed for fusion plasmas (1). Proof of principle experiments with AM signals reflected at the plasma have been performed in the T-10 Tokamak (2).

If a wave with periodical amplitude modulation is launched to the plasma, the group delay time $\tau$ of the returning reflected beam can be easily evaluated from the phase delay $\Delta \phi$ of the modulation signal, which leads directly to the differential phase delay:

$$\tau = \partial \phi / \partial \omega = \Delta \phi / \Omega \quad [1]$$

$\Omega$ being the angular frequency of the amplitude modulation.

With a typical modulation frequency of 50 MHz, the 'vacuum wavelength' of the modulation signal will be 6 m, then we will expect moderate phase shifts ($\leq 2\pi$) for the usual dimensions and density gradients in plasma devices (even JET-ITER). In the other hand the carrier can have the suitable frequency (10-200 GHz) for reflection in the plasma. The modulation frequency can be adjusted to the particular plasma dimensions.

Experimental system

A generic diagram of an A.M. reflectometer is shown on fig.1. The mm wave signal is modulated at approx.: 50 MHz and launched into the plasma.
PIN diodes can be used as modulators for the complete range of applications. The reflected signal is received by a second antenna and carried to the detector. The detector diode provides the modulating 50 MHz envelope which is downconverted to a lower IF (1-5 MHz), amplified and phase compared with the original one at the phase meter. Direct detection can be used for the smaller devices with low losses on the waveguides, but an additional heterodyne front end will be required for large devices like JET, where long waveguide runs are needed, producing high attenuation of the beam. The proposed scheme can be applied to most of present

Fig. 1.- Generic experimental system for AM reflectometry: O1: mm wave oscillator (=100 GHz), O2: modulation generator (= 50 MHz), O3: local oscillator, phase locked to O2 (=45 MHz). Heterodyne front end can be included for higher sensitivity.
day reflectometers without significant interference on the normal phase measurements. As it happens in all reflectometers, calibration of the waveguide paths by standard techniques is necessary.

Results simulation

Fig. 2 shows the behavior of the time delay for the different launching frequencies reflecting along a typical exponential density profile, similar to those expected for the JET Divertor plasmas. For frequencies higher than the highest cutoff (63.5 GHz for \( n_e(0) = 5 \times 10^{19} \) m\(^{-3} \), O-mode) interferometer-like operation, with reflection at the opposed wall, is assumed. The time delay is expressed as phase delay for the 50 MHz selected modulation frequency (1 rad = 3.2 ns), as it can be seen, moderate phase shifts (≤ 0.75 rad) are involved. The magnitude of the time delay depends on both the distance to the reflecting point and the density gradient along the propagation path.

Discussion on the advantages and limitations of the method

In this section we are going to take into consideration the effect of the different perturbations which affect the AM measurement: fast density fluctua-

![Fig. 2.- Typical exponential-like density profile for the JET Divertor (a) and corresponding phase delay for a 50 MHz modulation signal (b).](image-url)
tions, pulse deformation and parasitic reflections. In addition a few considerations will be done on the degree of accuracy needed for a given spatial resolution.

The density fluctuations affect the reflected beam either through phase and amplitude oscillations.

Phase fluctuations with amplitude of the order of $2\pi$ are usually affecting the millimeter wave beam producing strong frequency broadening at the phase detectors of classical swept reflectometers. Being the AM signal sensitive only to the differential effects, the phase oscillations of the modulating envelope become typically 1000 times smaller and do not affect the measurement, furthermore the linearity is kept for those small fluctuations and the time average of the phase delay can be used to obtain the 'average' density profile.

Amplitude fluctuations: the phase delay measurement is not affected provided the modulation frequency is clearly larger (factor 100 for a final phase accuracy in the range of .5 degree) than the frequencies of the amplitude oscillations of the reflected beam (typically 100 kHz versus 50-100 MHz for the modulation frequency). In the other hand the amplitude oscillations can cause temporary loss of the phase readout (signal goes down to the noise level). This can be overcome either by using a receiver with high dynamic range, such as that used on the W7AS stellarator (3) or by restricting the bandwidth of the time delay measurement. An additional possibility is the use of an amplitude detector for providing a 'low signal' warning which can be used to avoid the average of significant and non-significant signals.

'Pulse' deformation: The AM signal, as happens in pulse radar systems, is detected via a square law detector. Different phenomena in the plasma lead to pulse deformation and produce indetermination in the time delay to be measured. This is one of the main drawbacks of time delay methods and the effect becomes more severe for pulses with broad spectrum. The main causes for pulse deformation are:

- higher order derivatives of the phase delay versus frequency and spectra deformation by dispersive effects on waveguides and plasma.

The effect of higher order derivatives can be present in AM reflectometry only through the second derivative, since the AM wave is formed by only three spectral components. It appears when the phase shift between the carrier and each of the
two sidebands is significantly different. The effect might be completely sup-
pressed by removing the carrier or one of the sidebands in the AM spectrum, in
general this is not needed for moderate modulating frequencies (50-100 MHz)
since the effect is very small:

If the launched wave with carrier frequency \( \omega \) and AM frequency \( \Omega \) is:

\[
E_{in} = a \sin (\omega t) \left(1 + b \cos \Omega t\right)
\]

The signal from the square law detector, proportional to the square of the ampli-
tude of the reflected beam is:

\[
V_{out} = c_1 + c_2 \cos \alpha \sin \Omega (t + \tau) - c_3 \cos 2\Omega (t + \tau)
\]

with:

\[
\alpha = \Omega^2 \cdot \frac{\partial^2 \Phi}{\partial \omega^2}
\]

The effect of the finite second derivative is a decrease in the amplitude of the first
harmonic of the modulation signal after the detector. The typical values of \( \alpha \) lie in
the range of \( 10^{-3} \) rad for \( \Omega/2\pi = 50\text{-}100 \text{ MHz} \) and the final effect is negligible.

Spectrum deformation by dispersion in the plasma and waveguides, which can severely affect pulse reflectometry, does not affect at all AM measure-
ments, due to the simplicity of the spectrum and the separation of the different
harmonics:

Let the amplitudes of the spectral components of the AM signal, after the refllec-
tion and dispersive effects, be: \( a, b, c \) for the carrier and both sidebands. The de-
tector output, proportional to the square of the signal amplitude, can be written as:

\[
V = a^2 + \frac{b^2 + c^2}{2} + a(b + c)\sin \Omega (t + \tau) - \frac{b \cdot c}{2} \cos 2\Omega (t + \tau)
\]

The time delay \( \tau \) of the first harmonic is not affected by the values of \( a,b,c \). The
signal even survives if one of the sidebands is completely suppressed. The pulse,
originally sinusoidal, has been deformed by the detector, but we can easily sup-
press higher order harmonics and recover the time delay.

Parasitic reflections, produced in the waveguides, vacuum win-
dows..., are one of the main problems for AM reflectometry: the error in the di-
rect determination of the phase delay is of the order of \( a/b \) rad, for \( a \gg b \), being a
and b the amplitudes of the main and parasitic signal respectively. Ideally a number n of parasitic references can be distinguished if we operate with n+1 harmonics of the fundamental frequency in the modulating signal but, in the experiment the situation becomes extremely complicated and parasitic references must be avoided by using separated waveguides and antennae for the launched and reflected beam. This is not a severe limitation, since the AM system does not need strongly oversized waveguides or horns due to its immunity to pulse deformation.

Accuracy requirements: As the phase delays involved are relatively small, high accuracy is required in the phase determination: This implies linear phase detectors and a good signal to noise ratio. For AM frequency 100 MHz, a vacuum spatial resolution of 5mm implies 1.2 degree phase accuracy, which is easily achievable with a s/n ratio of 17dB in amplitude in front of the phase meter. For specific applications requiring high accuracy, simultaneous operation with higher modulation frequencies can be used.

Summary

Amplitude Modulation offers the possibility of a direct readout of the differential phase delay, which is the relevant parameter for density profile studies in reflectometry. The typical filtering and fringe counting procedures are not necessary and the associated errors (fringe losses) are avoided. In the other hand pulse deformation and density fluctuations do not affect the measurement. This makes of AM reflectometry a very attractive alternative, not only for standard density profile measurements, but also for fast monitoring of density profile and plasma position in large fusion devices.

References

Technical aspects of a multichannel pulsed radar reflectometer

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1. Introduction

Pulsed radar reflectometry has been proposed as a viable option for measuring electron density profile in thermonuclear plasmas [1, 2, 3 and 4]. Pulsed radar is based on the measurement of the time-of-flight between launching and receiving a short (< 1 ns) microwave pulse which is injected into the plasma and reflected by the cut-off layer. The flight time is directly related to the position of the cut-off density. Density profiles can be deduced by measuring the flight times of pulses with various frequencies. Additional information about the local density gradient can be gained from the pulse shape deformation during the reflection process.

2. Motivation

The effort on the development of a multichannel pulsed radar reflectometer is motivated by the need for reliable density profile measurements in highly turbulent plasmas, e.g. in the scrape-off layer of the JET New Phase Diverter. Reflectometer measurements are often plagued with disappearances of the reflected signal, probably caused by density fluctuations which reflect the signal in a direction outside the antenna pattern of the receiving horn. The phase measurement of the received signal needs to be continuous over the lifetime of the plasma, otherwise the phase integration process fails and the position of the cut-off layer becomes indeterminate. The principle of time-of-flight measurement can offer a great advantage in this situation. Although it can be expected that some of the reflected pulses will not be detected, this will not cause problems since every pulse received back carries all information needed to deduce the position of the cut-off layer.

Other interesting properties of pulsed radar include the negligible plasma movement during the flight time of the pulses through the plasma, and the simple elimination of spurious reflections, e.g. from the vacuum window or components in the waveguide runs, because the corresponding echoes will fall outside the time window which is of interest for the density measurement. Therefore, a system with a single
antenna can be employed, which can be of importance if the access to the tokamak is limited.

The principle of different echoes from a single pulse in different time windows can also be used for the simultaneous measurement of the positions of the X- and O-mode cut-off layers, if both cut-off layers can be probed with the same frequency. This offers the possibility for magnetic field measurements. Such a measurement can be performed by injecting the E-vector of the probing wave with a 45° angle with respect to the magnetic field. Since both layers will not have the same position, the result will be two echoes with a different time-of-flight. Moreover, mode conversion due to fluctuations can be expected in high poloidal field devices like ITER. This will not hamper the pulsed radar measurements, in contrast to conventional reflectometry.

3. Pulsed radar measurements at RTP

Measurements with pulsed radar at a frequency of 34 GHz, corresponding to a cut-off density of $1.4 \times 10^{19} \text{ m}^{-3}$, have been performed at the RTP tokamak [2,3]. Typical echoes, measured above (straight line) and below cut-off (dashed line), are depicted in Fig. 1.

![Figure 1: Measured reflections above (straight line) and below cut-off (dashed line)](image)

The reflected signals were recorded with a sampling scope (type: Tektronic CSA 803) with a sampling frequency of 200 kHz in sequential equivalent time. The
launched pulses had a gaussian shaped envelope with a FWHM of 1.5 ns. Both curves clearly show a reflection at 4 ns (A), which originates from the vacuum window. The reflection (C) from the back wall of the vacuum vessel can be observed if the density is below cut-off, while a large echo (B) appears when the electron density exceeds the cut-off density. The flight time of echo B is clearly smaller than the flight time of echo C. The measured time delays are in good agreement with values derived from the density profile measured by a 19-channel interferometer.

4. Multichannel pulsed radar.

Microwave pulses with various carrier frequencies are needed to deduce the electron density profile by measuring the individual flight times. This will lead to a multichannel system. The construction of such a multichannel pulsed radar reflectometer is planned before the end of 1992 at RTP.

Two four-channel systems in the Ka-band are proposed. Both systems use a heterodyne detection scheme to maximize the dynamic range. The system given in Fig. 2(a) uses four microwave sources with four different frequencies (29, 32, 35 and 38 GHz), which are multiplexed sequentially into a waveguide. The waves are modulated with a fast pin-switch (type: Hughes 47971H-2000) into a pulse shape with a FWHM of 500 ps to 1 ns. The received pulses are mixed with a local oscillator with a frequency of 11 GHz. The IF pulses are fed to an envelope detector, after which the flight time of the pulses is measured. In this set-up, the reflection from the vacuum window is used to start the time-of-flight measurement. This has the advantage that the time interval which has to be measured very accurately is minimized and the repetition rate of the measurement is maximized. The multiplexing process, the modulation of the microwaves and the time-of-flight measurement have to be well controlled. Disadvantages of this set-up are the number of relatively expensive microwave sources which are needed, and the different IF frequencies of the mixer corresponding to the different RF frequencies. This makes signal processing after the mixer very difficult or even impossible due to broadband spectral requirements.

Both problems are reduced in the system given in Fig. 2(b). Here, different microwave frequencies are generated by switching the IF input of an up-converter (type: Hughes 47471H-2230). The IF output of the mixer is always equal to 26 GHz because the IF input of the up-converter is identical to the LO input of the mixer. This scheme has the disadvantage that the output power of the up-converter is limited to approximately 8 dBm.

It is expected that there will be no need for a PLL to stabilize the frequency of the microwave sources, because a small frequency shift will lead to a minor change of
the value of the cut-off density only. This will have a negligible effect on the measured flight time.

Figure 2: Multichannel pulsed radar systems (numbers indicate frequencies in GHz)

- **a)** Multiplexing of RF frequencies;
- **b)** Multiplexing of UP-converter IF frequencies.

5. Accuracy

Many effects which will influence the accuracy of the pulsed radar measurements have been identified:

1. noise, e.g. due to plasma emission;
2. the effect of reflected power fluctuations;
3. the finite bandwidth of the pulses;
4. the density dependency of the propagation velocity of the pulse.

A basic radar detection scheme [5] consists of a bandpass filter, an envelope detector, followed by a threshold circuit. The bandpass filter needs to be matched to the
input waveform to maximize the signal-to-noise ratio. The threshold circuit has to
distinguish the reflected pulses from the background noise. The signal-to-noise
requirement at the input of the threshold stage is firmly related to the probability of
detection, the probability of false alarm and the achieved accuracy of the time-of-flight
measurement.

A lower bound on delay measurement errors in high signal-to-noise ratio cases,
close to the Cramer-Rao bound for delay errors relevant to radar applications, can be
found in [5]. Assuming a carrier signal with a gaussian shaped envelope:

\[ s(t) = g(t) \cos(\omega_c t + \phi(t)) \]  

with:

\[ g(t) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(\frac{-(t-T)^2}{2\sigma^2}\right) \]  

and a matched filter with an impulse response \( h(t) \):

\[ h(t) = K s(t_m-t) \]  

one can calculate the output signal of the matched filter:

\[ s_o(t) = \int_{-\infty}^{\infty} s(\tau) h(t-\tau) d\tau = \frac{K}{4\sigma \sqrt{\pi}} \exp\left(\frac{-(t-t_m)^2}{4\sigma^2}\right) \cos(\omega_c(t-t_m)) \]  

This signal is fed to an envelope detector, so the input of the threshold stage is the
envelope of \( s_o(t) \). Assuming additive white noise with a two-sided power spectral
density of \( N_0/2 \), the signal to noise ratio at the threshold input can be written as:

\[ \text{SNR} = \frac{|s_o(t_m)|^2}{n_o^2(t)} = \frac{1}{4\sigma N_0 \sqrt{\pi}} \]  

Referring to Fig. 3, we note that the signal plus noise crosses the threshold \( \Delta t \) time
units before the signal alone would have crossed it. This is the delay error due to the
noise \( n(t) \). Because \( n(t) \) could be considered a constant around the threshold crossing,
we can write:

\[ \frac{n(t)}{\Delta t} = \frac{d}{dt} |s_o(t)| \]
Using Eqs (4), (5) and (6) one can calculate the RMS value of the delay error, with $t = t_m - \sigma \sqrt{2}$:

$$\sigma_t = \sqrt{(\Delta t)^2} = \sigma \sqrt{2} \exp\left(\frac{1}{2}\right) \sqrt{\frac{1}{\text{SNR}}}.$$  \hspace{1cm} (7)

Eq. (7) shows that the RMS value of the delay error will increase if the SNR decreases. From Eq. (7) the lower bound of the required SNR can be calculated if $\sigma_t$ is given. For example: if $\sigma_t < 100$ ps and $\sigma = 500$ ps, then the SNR must be at least $136 = 21$ dB.

\[\text{Figure 3 : The effect of noise on the measurement of the flight time for a pulse with a gaussian shaped envelope}\]

It is clear that fluctuations of the reflected power can cause large measurement errors. These errors can be reduced significantly by using the timing information of the rising edge of the reflected pulse as well as the information of the falling edge. This method assumes that the reflected pulses are symmetric. A feedback system can be employed to set the optimum threshold level as a function of pulse power.

Due to the finite bandwidth of the radar pulses, the emitted pulse shape will be deformed after the reflection because the different frequency components of the pulse will reflect from different cut-off layers. This effect will depend strongly on the local density gradient. In principle, the local density gradient can be deduced from the pulse shape deformation, as is shown in [2, 3]. Dispersion in the long oversized waveguide run, anticipated for large fusion devices, can be neglected compared to pulse deformation due to the plasma reflection process.

The propagation velocity of the microwave pulses will depend on the electron density. This effect must be taken into account by employing the information gained from the reflection at low cut-off densities to correct the measured flight time of pulses reflected at higher cut-off densities.
6. Time-of-flight measurements

For studying the feasibility of a pulsed radar reflectometer for future fusion devices, a high resolution time-of-flight measurement set-up with a high repetition rate is needed. A temporal resolution of 70 ps, corresponding to approximately 1 cm spatial resolution when reflecting from a metal mirror, is required. This resolution, in combination with a repetition rate of $> 2 \times 10^6$ measurements per second, cannot be obtained by means of commercially available time-interval counters.

![Diagram of Time-of-flight measurement with parallel averaging.](image)

Figure 4: Time-of-flight measurement with parallel averaging.

The time-interval counter, which will be developed for pulsed radar reflectometry at RTP, is based on the conventional principle of counting gated clock pulses (see Fig. 4). The resolution of this method is equal to one clock period, due to the ± 1 count ambiguity. The proposed system will employ several parallel counters instead of one single counter. Each counter is driven by the same clock, but phase shifted. The gates are opened simultaneously during the time-interval between the start and the stop pulse. The measured time-of-flight of the pulses will be equal to the average of the output of the counters. This will improve the resolution by a factor equal to the number of counters. The amount of data is reduced by adding the output of the counters. After the addition, data is stored into a memory. For a multichannel radar system, data has to be demultiplexed and synchronization with the multiplexing of the microwave sources has to be included. Other control features are also needed, e.g. if no
plasma reflection is detected, the gates should be closed and the counters should be reset before the arrival of the next start pulse.

7. Conclusion

Research on the use of pulsed radar techniques for reflectometry is in progress. The hardware of a multichannel pulsed radar reflectometer for RTP will be constructed during 1992 and is planned to become available in the beginning of 1993. Many effects which will influence the accuracy of the measurements have been identified. Although most of these effects can be canceled or reduced significantly, the accuracy of pulsed radar reflectometry will be less than the accuracy of conventional reflectometry. This potential weakness of the proposed system is compensated in many applications by several unique properties of pulsed radar reflectometry, e.g. the possibility to measure density profiles in highly turbulent plasmas; the simple elimination of spurious reflections and the possibility to perform magnetic field measurements by measuring simultaneously the position of the X- and O-mode cut-off layer.

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NEW APPROACH TO MICROWAVE REFLECTOMETRY:
CORRELATION REFLECTOMETRY VIA STOCHASTIC NOISE SIGNALS.

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1. Introduction

Main problems of microwave reflectometry using measurement of
phase delay of reflected wave- density fluctuations and trapped modes
in the oversized waveguide transmission systems - still need solution.
An alternative approach which is discussed now - to use pulse radar
reflectometry and time of flight measurements [1]. The difficulty of
this approach - the need of ultra short (<1ns) microwave pulses - was
successfully overcome [2].

In this report we discuss a new approach to reflectometry technic- use of generators of stationary wide-band noise microwave
signals and correlation technic for measurements of plasma cut-off
layer position. The key components of this scheme are the microwave
oscillator producing wide-band stochastic noise signal (F= 35 GHz, F≤1
GHz, P=1W) [3,4] and controlled PIN diode delay line.

In the first part of this report we discuss a problem of
propagation in a plasma microwave signal with amplitude described by a
random function having a limited frequency band. Auto- and cross-
correlation were calculated for launched and reflected in a plasma
stochastic microwave signal. The calculations were done for the upper
X-mode reflectometry for two devices: "Uragan-2M" torsatron (Kharkov
Institute of Physics and Technology) and ITER.

In the second part of report the principal layout and parameters
of the noise correlation reflectometer (NCR) which is being developed
for U-2M torsatron in the Institute of Radiophysics and Electronics of
the Academy of Science of Ukraine (Kharkov, IRE) are described.
2. Auto-and cross-correlation of stochastic microwave signals reflected in a plasma.

Stochastic microwave signal propagation in a plasma was modeled by means of a numerical code NOISIM (Fig.1). The input block of code was generating a stationary random function describing a time behavior of stochastic microwave signal amplitude. Second block was producing a random function with the frequency band corresponding to that of the NOISE BWO developed in IRE and described a time behavior of probing microwave signal amplitude. Then for each Fourier component of microwave signal with a frequency corresponding to that of the upper X-mode the phase delay was calculated and reflected signal time behavior was obtained by means of inverse FFT. Finally auto- and cross-correlation of input and output signal were calculated.

Plasma parameters necessary for microwave propagation studies (electron density and magnetic field) were calculated by means of numerical codes describing 2D plasma equilibrium for tokamak with a double- null divertor (EQUT) and stellarator (EQUS). The calculations were done for ITER Standard Case (B=4.85 T, J=22kA, n(0)=1·10^{14} cm^{-3}, β_p=0.6, l_i=0.65 ) and U-2M Standard Case (B=2.4 T, n(0)=1·10^{13} cm^{-3}, l(0)=0.24, τ(a)=0.47 ). Fig.3 shows the radial profiles of 0- and UX-mode cut-offs for both devices.

The typical auto-correlations for input (R_{11}) and output (R_{12}) signals and crosscorrelation for input and output signals (R_{12}) for "U-2M" UX-mode reflectometry are shown in Fig.4. One can see that auto- correlations of input and output signals are similar and their half width is equal to 2.5 ns (inverse of frequency band width). The cross correlation demonstrates clearly the time lag of the noise signal in a plasma. The time lag for signals which are reflected at the same distance from the plasma edge (40 cm) for two devices with a different plasma radius (ITER- 2.15 cm, U-2M- 0.3m) is shown on Fig.5. The time lag versus the cut-off layer distance for both experiments which was obtained by the probing frequency variation are shown on Fig.6. These data can be used for the noise correlometer delay line parameter specification. They show a strong diminution of average velocity $\bar{V}_n$ of noise signal propagation in a plasma($\bar{V}_n(\text{ITER})\approx 0.1 \cdot c$, $\bar{V}_n(\text{U-2M})\approx 0.03 \cdot c$) and give grounds for nontrivial and encouraging conclusion about comparatively larger slowing-down of noise microwave signal in devices of lesser size. Reason of this phenomenon is a considerable decrease of a wave group velocity only in a region.
adjacent to cut-off layer. This is illustrated by Fig.7 where schedules of stochastic micro-wave signal propagation in U-2M and ITER plasmas are shown.

Numeric calculations using code NOISIM allowed to evaluate the integration time which is necessary for obtaining of acceptable signal-to-noise ratio. Fig. 8 shows a sequence of reflected signal cross-correlations calculated for noise signals of different time duration. From such calculations one can make a conclusion that for noise oscillator with a noise frequency band $\Delta F \approx 0.5$ GHz the acceptable integration time is of order of 10 mksec. Of course the correlation technic can be used in a pulse radar reflectometry. Unquestionable advantage of the noise oscillator is much larger (factor of 30-50) output detector signal at equal radar oscillator power and correlometer detector integration time. (fig.9).

Noise signals allow to use another technique for design of the distance-measuring system. The technique is based on the phenomenon of the power spectrum modulation of the wide-band noise signal, which appear when reflected signal is added to transmitting one under the condition that reflector is placed at the distances, exceeding the signal coherence length [3,5]. The frequency period $T_f$ of the modulation is determined by phase velocity $v$ and distance $L$ to the reflecting layer $T_f = L/2 \cdot v$. The principal layout of experiment is shown on Fig.10. The experiment, carried out with non-dispersive reflector, shown the high efficiency of the technique when amplitudes of the signals are equal. The power spectrums of the output signal is shown on Fig 11. Period of the frequency modulation proved to be $T_f = \frac{v}{2L}$, where $v$ is the velocity of the waveguide wave. The measurement sensibility and accuracy ($\approx 3$ sm) is observed at a few number (3-5) of modulation minimums in the power spectrum of the sum signal.

3. Noise correlation reflectometer for "Uragan-2M" torsatron

Fig.10 shows a principal layout of a noise correlation reflectometer which is being developed now in IRE for UX-mode reflectometry on the "Uragan-2M" torsatron. Basically this layout repeats the homodyne reflectometer layout; main differences are the noise oscillator (NO) and controlled delay line (2). Delay line (1) will be used for a time lag compensation in waveguide between oscillator and antenna. The controlled delay line have to provide a variable time lag corresponding to that in "Uragan-2M" plasma.
The noise oscillator (NO) is a source of stationary random signals of middle power (2-10 W) in the K_a band. Operating principle of oscillator is based on the effect of dynamic stochastisation of O-tye backward wave tube oscillations due to nonlinear interaction between electron beam and longitudinal electric field of an intensive surface EM wave field of slow-wave structure. Stable generation and electron retuning of carrying frequency of stochastic oscillation spectrum is provided by use of wide band weakly resonant oscillatory system. A typical example of stochastic oscillation spectrum for 35 GHz noise oscillator is shown in Fig.2. Such generators allow to obtain stochastic signals in the frequency range of 30-100 GHz with the noise band 0.3-1.5 GHz and power of 10-0.5 W. Now the noise generators in the frequency band of 34-64 GHz with the noise band of 0.3-1 GHz and power of 5-1 W are manufactured in the IRE.

The microwave controlled delay line consists of a set of short single-mode waveguides connected to each other via PIN-diode switches providing step-wise change of time lag during a time less then 10 ns. In the delay line which is being developed now the time lag step is chosen to be equal of 0.1 ns which corresponds for U-2M reflectometry the cut-off distance shift of order of 1 mm.

4. Conclusion.

We have shown that the stochastic microwave signals can be used in reflectometry for fusion plasma. Estimates of time lag ranges for UX-mode reflectometry for the "Uragan-2M" torsatron and experimental tokamak- reactor ITER are made. It is shown that the "noise" radar and pulse radar can be used for measurement of cut-off layer position even in a magnetic traps with plasma radius of 20-30 cm. The accuracy of cut-off layer position measurements is increasing in a large tokamaks. We hope that the proof-of principle experiment which is planned now for the U-2M torsatron will reveal a possible difficulties which are not seen usually for a new approach.

References

Numerical code for simulation of noise/pulse propagation in a plasma

\[ \tilde{A}(t) \longrightarrow \tilde{A}_i(t) \longrightarrow S_{A_i}(\omega) \longrightarrow \Delta \varphi(\omega + \Delta \omega_0) \]

\[ \tilde{A}_o(t) \]

\[ F(\omega) \]

\[ R_{id}(\tau) \]

\[ R_{od}(\tau) \]

Fig. 1
Typical output signal spectrum of $K_A$ band noise generator

Noise generator side view

Fig. 2
O- and UX- mode cut-off position for ITER and U-2M

U-2M Reflectometry
B(0)=2.4 T, n(0)=1E13 cm^-3

Cut-off frequency, GHz

Normalized plasma radius, r/a

REFLECTOMETRY in ITER
Upper X- and O-mode

Cut-off frequency, GHz

Normalized plasma radius, r/a

Fig. 3
U-2M 70GHz noise propagation

**R(T)**

**R11**

**R(T)**

**R22**

**R(T)**

**R12**

*Fig. 4*
ITER
F=140GHz, Rc=39.5cm

$R(T)$

0.14
0.13
0.12
0.11
0.1
0.09

0 10 20 30 40 50 60 70
Tlag(ns)

U-2M
F=80GHz, Rc=41cm

$R(T)$

0.034
0.032
0.03
0.028
0.026
0.024

0 10 20 30 40 50 60 70
Tlag(ns)

Fig. 5
Noise time lag
versus cut-off layer distance
for ITER and U-2M

Fig. 6
Microwave pulse propagation

**Figure 7**
Influence of integration time on crosscorrelation of input/output noise signals
ITER, $F=140\text{GHz}$

$dT=0.25\text{mks}$

$dT=0.5\text{mks}$

$dT=2\text{mks}$

$dT=8\text{mks}$

$T_{\text{lag}}$ [ns]

$R_{12}$

Fig. 8
Pulse and noise propagation for ITER

\[ R_{12}(T) \]

\( F = 180 \text{GHz} \)

- Pulse radar
- Noise radar

Fig. 9
Fig. 10
Spectral approach to the cut-off layer distance measurement via noise generator

\[ \Delta F = \frac{c}{2L} \]
A 'COMB' REFLECTOMETER: A SIMPLE DEVICE FOR DETERMINING THE PEAK DENSITY IN DIFFICULT EXPERIMENTAL SITUATIONS

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Abstract

A simple 'comb' reflectometer which can provide some information on the electron density in difficult experimental situations is described. Results obtained with a four channel device which probes the X point region on JET are presented. Possible further developments are briefly discussed.

1. Introduction

In some experimental situations it is not possible to apply the existing reflectometry techniques for density profile measurements, i.e. broad band swept or multiple frequency narrow band swept techniques. Examples are the X point region of JET and the pumped divertor region of the planned new phase of JET. In both cases, very complicated, long, oversized waveguide runs containing many bends cannot be avoided. Mode conversion and reflections in such waveguide systems generate spurious signals when the probing frequency is swept and can make accurate phase measurements impossible.

Observations with a multiple fixed frequency reflectometer on JET [1,2], have shown that it is possible to determine the maximum frequency in reflection, and therefore to estimate the peak density in the line-of-sight, simply from the fluctuation level of the reflected signal. Fixed frequency reflectometers can operate with very complicated waveguide runs and so this observation suggests the design of a simple device for determining the peak electron density even in difficult experimental situations. Since an array of frequencies is involved, we have termed the device a 'comb' reflectometer. In this paper, we describe a four channel version and summarize our experience in using the device to probe the X-point region of some JET plasmas.

2. Principle

In a comb reflectometer, a number of beams of different fixed frequencies probe the plasma along the same line-of-sight. Fluctuations in the electron density generate broad band 'noise', considerably in excess of the detector noise.
on those channels in reflection (figure 1). Hence the maximum frequency in reflection, and therefore the density band corresponding to the maximum density in the line-of-sight, is obtained. Since accurate phase measurements are not involved, the device can be used with complicated waveguide runs.

3. Implementation on JET

A prototype comb reflectometer has been constructed at JET. The outputs from four Gunn oscillators operating at 27, 40, 60, and 90 GHz are combined into a single, oversized (WG10), waveguide run (figure 2). The run is about 40 m long, contains 14 mitre bands (both E plane and H plane) and crosses the torus vacuum through a quartz window interface. The microwave radiation irradiates the X-point region in the ordinary mode and the reflected radiation is transmitted with the same waveguide run to two heterodyne detection systems. Each one has a wide bandwidth mixer/amplifier followed by narrow band (250 MHz) filters tuned to pass the intermediate frequencies corresponding to the Gunn oscillators. The signals are detected and recorded with a bandwidth of 300 Hz.

4. Plasma Measurements

Measurements have been made for a range of plasmas with X-points including H-mode plasmas. A typical result is shown in figure 3. In this case, the X-point is formed at 4 s and the plasma is in the H-mode between 7.4 s and 8.3 s. The times when the different channels are believed to be in reflection are marked on the figure, and the time dependence of the peak density in the line-of-sight is estimated.
5. Problems and Possible Improvements

The main problem experienced in operating the device was that it was not always obvious whether a particular channel was in reflection or in transmission. Changes in the plasma, for example L to H-mode transitions, or movements of the reflecting layers, could introduce significant changes in the fluctuation level which could be confused with a change between reflection and transmission. Considerable care has to be exercised, therefore, when interpreting the measurements obtained with the present device.

Several possible improvements can be envisaged.

(i) The amplitude of the probing beam could be modulated at high frequency and a narrow band pass filter centred at the modulation frequency used in the detection system. The frequency would be chosen to be above that at which the signal is fluctuating significantly, typically ≥ 100 kHz. The transition between transmission and reflection would then be determined by the change in amplitude of the reflectometer signal rather than by the change in the fluctuation level.
Figure 3: A typical result obtained during an X-point plasma with an H-mode phase.
(ii) Measurement of both the reflected and transmitted beams should give a clearer distinction between reflection and transmission although, of course, requires additional waveguides in the vacuum vessel.

(iii) Spectrally analysing the signals in real-time may give a clearer distinction and could be implemented in either hardware or software.

6. Conclusions

A simple 'comb' reflectometer can give an estimate of the peak electron density in difficult experimental situations. Difficulties can arise due to changes in the plasma and/or movements of the reflecting layers which confuse the distinction between those channels in reflection and those in transmission. Several possibilities exist for improvements which should give clearer results.

References
