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ABSTRACT

A general procedure to identify instability regions which lead to multifragmentation events is presented. The method covers all possible sources of dynamical instabilities. Informations on the instability point, like the time when the nuclear system enters the critical region, the most unstable modes and the time constant of the exponential growing of the relative variances, are deduced without any numerical bias. Important memory effects in the fragmentation pattern are revealed. Some hints towards a fully dynamical picture of fragmentation processes are finally suggested.

1. Introduction

Heavy ion collisions at intermediate energy have been widely described through some kinetic approaches (VUU, BUU, BNV) which are able to correctly reproduce the mean trajectory of the system in phase space. However, when one is interested in unstable processes such as, for instance, multifragmentation events, these methods are not valid anymore since they do not contain fluctuations, which are very important in a not equilibrated system.

Nevertheless we will show that the use of such kinetic equations, solved within the test particle approach, can still be very useful to extract quite precise informations on physical properties of the instability regions.

Actually, since always some numerical noise is present when the systems enters an instability region, we see in the solution of these transport equations a fast growing of fluctuations, symmetry breakings and cluster formation. A lot of excitement and discussion has been recently raised around the preferred spacial geometries of fragmentation events deduced from different transport codes[1]. Indeed these geometries depend sensitively on the configuration of the system at the time of instability, on the leading unstable modes and on the time scale for cluster formation. In this paper we will show how such quantities are related to the used phase space discretization procedure (*i.e.* number of test particles per nucleon in the numerical algorithm) and we will suggest the way to extract unbiased physical informations.

Moreover we will try to go further and to follow the fragmentation pattern studying the memory effects versus the dynamical evolution of the system in different situations (*i.e.* changing the beam energy or the nuclear matter compressibility).

2. Instabilities in nuclear dynamics

As already stressed, the kinetic equation used to study heavy ion collisions describes only the mean phase space trajectory of the one-body distribution function, including two-body and Pauli correlations but averaging over the related fluctuations. It can be used only to extract mean values of one-body observables and not variances, as already known from the Time Dependent Hartree-Fock (TDHF) theory used at low energies. Since the inclusion of fluctuations is essential to form fragments in critical regions, the BNV equation seems to be useless for a fully dynamical picture of fragmentation.

However the test particle method also naturally introduces some numerical fluctuations in the initial conditions and in the Pauli blocking. These can be reduced just increasing the number of test particles and therefore they can be easily eliminated in regions of dynamical stability. In critical regime the fluctuations are growing exponentially and this numerical noise can substantially modify the relative dynamics.

The instability point, for a particular degree of freedom of the system, is reached when the corresponding RPA mode attains an imaginary frequency[2–4]. This is equivalent to say that small amplitude fluctuations in that degree of freedom will show an exponential growth. Therefore we expect that important informations on the unstable modes and on the instability point could be extracted from the study of the mean phase space trajectory if the evolution of the mean field is suitably described and, of course, if some fluctuations, from any type of source, are present. At variance, the time scale for cluster formation is very much related to the absolute value of the amount of fluctuations present in some explosive degree of freedom. We see fragments when the variances become quite large and the corresponding time evolution starts to deviate from the exponential growth, the system having reached a new region of stability. All that has been recently confirmed by some model calculation for nuclear matter[4,5]. The conclusion is that we expect any noise, in a good mean trajectory dynamics, suitable to extract informations on the instability point (most unstable modes and instability time). On the contrary, in order to get the right fragment formation, we should be able to reproduce the correct physical fluctuations at the instability point. We have checked these ideas performing a series of numerical analyses solving BNV-type transport equations for two dimensional collisions of nuclear matter slabs.

3. Dynamics and fragmentation

We have performed calculations using a simplified model. We consider a two-dimensional collision between two slabs of nuclear matter in a box of lengths 63 fm and 21 fm in order to reduce the computational time, the mean field has been averaged on the y direction. The x radius of the two nuclei is equal to 4 fm (the

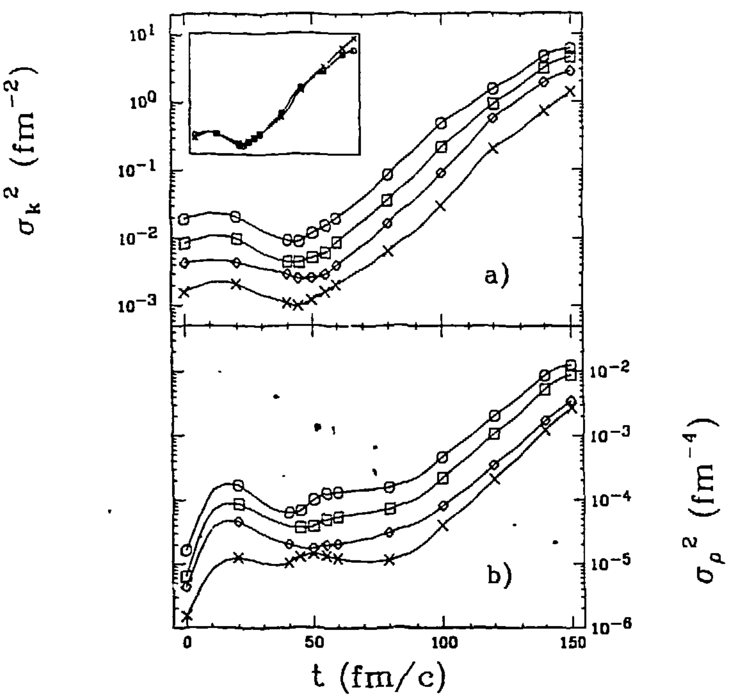

Fig. 1. Time evolution of variances for the two slab collision described in the text. a): $\sigma^2(t)$ of the most unstable modes; b) $\sigma^2(t)$ of the monopole mode. Circles: $N_{TP} = 20$; squares: $N_{TP} = 40$; diamonds: $N_{TP} = 80$; crosses: $N_{TP} = 200$.

radius of a ^{40}Ca nucleus) while the total number of nucleons is fixed requiring that the density of the two slabs is equal to the normal density, which is 0.55 fm^{-2} in two dimensions. We have simulated a central collision at a beam energy $E_{lab}/A = 40\ MeV/u$. As the time increases we see the formation of a nearly uniform region at half normal density and at a temperature around $T = 8\ MeV$. This system lies in the spinodal instability region[5]. In such a simplified situation it is possible to demonstrate that, for infinite nuclear matter, the eigenmodes of the density are plane waves, characterized by a wave number k. Even if, in our case, the system is finite, it is interesting to perform a Fourier analysis of the density fluctuation and therefore to define the following quantity:

$$\sigma_k^2 = \int_0^{L_x} dx \int_0^{L_x} dx' e^{-ik(x-x')} < \delta\rho(x)\delta\rho(x') > . \qquad (1)$$

The quantity σ_k^2 represents the spectral correlator.

In fig. 1a) is shown the time behaviour of σ_k^2 for the most unstable mode ($k = 5 \div 6$), calculated using 20, 40, 80 and 200 test particles per nucleon, 50 trajectories are considered. We see that all the variances reach the minimum point at the same time $t_{inst} = 45 fm/c$; this point does not depend on the noise and it corresponds to the time when the system enters the instability region. The scaling law is preserved also in the next evolution of σ_k^2; it means that the time τ, characteristic of the exponential growing, does not depend on the number of test particles. The scaling disappears when the system enters a saturation regime, corresponding to a new equilibrium reached by cluster formation. This can be seen in the box inside fig.1a where we report two curves of this figure ($N_{TP} = 20, 200$) suitably rescaled. As expected the saturation appears earlier for the trajectories with less test-particles.

The fragmentation time, *i.e.* the time when we observe fragment formation, is therefore fully dominated, in this representation, by the value of the fluctuation σ_k^2 at $t = t_{inst}$. In this sense it would be extremely important to know the right physical amount of fluctuations that are present at that point.

We have also studied the time dependence of the density variance of the trajectories in the a circle of radius 4 fm in the overlapping zone:

$$\sigma_\rho^2 = \frac{1}{N_{events}} \sum_{i=1}^{N_{events}} (\rho_i(t) - \bar{\rho}(t))^2 \tag{2}$$

which is related to the fluctuations of the distribution function, after some phase space averaging. In the stability region the variance follows the density as it should be in a Poisson-type distribution of test particles. Indeed if the variance on the number of test particles in the circle follows a Poisson law $\sigma_n^2 = \bar{n}$ we get for the variance on density the relation:

$$\sigma_\rho^2(t) = \frac{\bar{\rho}(t)}{S\ N_{TP}} \tag{3}$$

where S is the surface of the circle. The presence of this classical behaviour of fluctuations is not surprising since we are looking at properties of phase space averaged observables and then we expect quantum fluctuations to be washed out.

When the system enters the instability region we observe a sharp transition to an exponential behaviour:

$$\sigma_\rho^2(t) = \sigma_0^2 e^{2t/\tau} \tag{4}$$

which clearly indicates the presence of an unstable mode with associated imaginary energy $E = i\hbar/\tau$ [3,4]. σ_0^2 represents the amount of fluctuations present at the time of instability and clearly scales with the number of test particles per nucleon N_{TP}. But it is very important to remark that the time of instability, *i.e.* the time of the transition to the exponential, and the time τ, characteristic of the exponential growing of fluctuations, do not depend on the number of test particles we are using, *i.e.* on the noise introduced in the calculation. They are related just to the mean evolution and to the mean properties (density and temperature) of the system which, as we explained before, is always well described until the instability spinodal region is reached. Starting from this point, the amount of fluctuations present in the system becomes of crucial importance since the dynamics is now dominated by the exponential amplification of the initial noise and consequently calculations using a different number of test particles will lead to different results.

In fig.1b) is displayed the density variance. The scaling behaviour here is less evident because in σ_ρ^2 we have the superposition of different modes corresponding to different values of k. This superposition seems to create an uncertainty on the time when the system enters the instability region, since, using 200 test particles, the exponential growing starts later. Actually this time does not change and it is still $t = 45\ fm/c$ for all curves. When the amplitude of fluctuations is not large enough (as it happens using 200 test particles per nucleon), the system expands before exploding and therefore the growing time τ reduces, according to the reduced new density, until the explosion becomes the dominant one. In this case it happens at $t = 80\ fm/c$.

4. Memory effects in fragmentation

We focus now our attention on memory effects in the fragmentation mechanism versus the dynamical evolution of the system.

In order to study the sensitivity of fragment formation to the dynamics of the reaction, we have performed two-dimensional calculations changing the beam energy and the equation of state. The e.o.s. can be easily modified using a different value of the parameter σ in the Skyrme-like form of the mean field potential $U = A(\rho/\rho_0) + B(\rho/\rho_0)^\sigma$.

Requiring a binding energy of 16 MeV at $\rho = \rho_0$, *i.e.*

$$E/A(\rho = \rho_0) = \frac{A}{2}(\rho/\rho_0) + \frac{B}{\sigma+1}(\rho/\rho_0)^\sigma + \frac{1}{4m}\hbar^2\pi\rho = -16 \qquad (5)$$

(in order to simulate three dimensional nuclear matter) and the minimum condition for this point ($\partial(E/A)/\partial\rho = 0$), we obtain the following sets of values for the parameters A and B :

$$\sigma = 2 \qquad A = -100.3 \qquad B = 48 \qquad (I)$$

$$\sigma = 3 \qquad A = -84 \qquad B = 32 \qquad (II)$$

In the following we will refer to the parameterization (I) as "soft" equation of state, while the parameterization (II) will give the "stiff" equation of state.

Let us discuss first the effects that can be seen in the fragment formation if we change the beam energy. Increasing the beam energy, the system is allowed to perform larger oscillations in density (see fig. 2a). Because of these oscillations, a volume instability region is reached and then the formation of fragments becomes the most likely mechanism of de-excitation. Corresponding to the different initial conditions, the value of the density when the system starts to explode may be different, since the temperature in the participant zone will change. Consequently, the

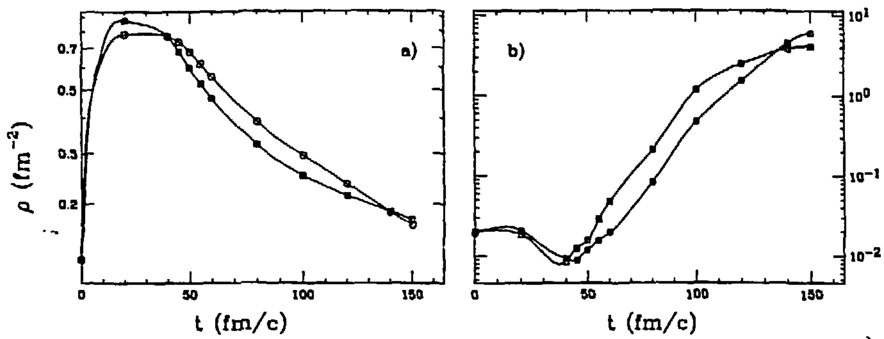

Fig. 2. Time evolution, for the two slab collision, of the mean density in the participant zone (a) and of the variance of the most unstable mode (b) for $E/A = 40\ MeV/u$ (circles) and $E/A = 70\ MeV/u$ (squares).

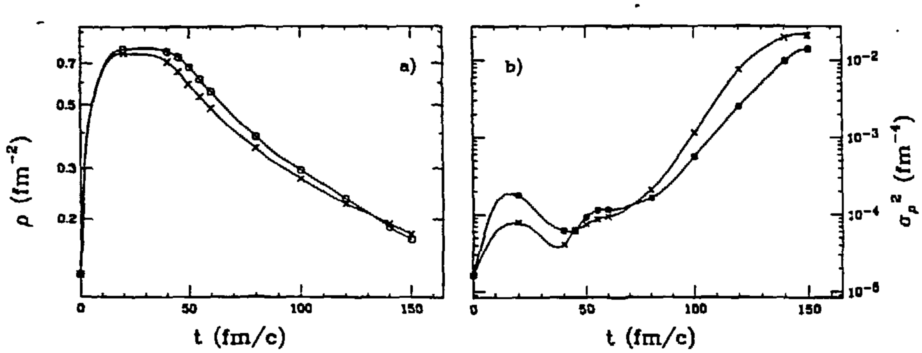

Fig. 3. Time evolution. for the two slab collision, of the mean density in the participant zone (a) and of the variance of the monopole mode (b) for a "soft" e.o.s. (circles) and a "stiff" e.o.s. (crosses).

pattern followed by the fragmentation and, in particular, the number of fragments that we obtain in the final state may also change. (Indeed this number depends both on density and temperature at the critical point).

Another effect of the increased energy is an earlier fragment formation due to the faster dynamics.

In our case we get 6 fragments for $E/A = 40\ MeV/u$ and 7 fragments for $E/A = 70\ MeV/u$. Moreover, as we can see from fig. 2b, where we plot σ_k^2 for $K = 6$ as a function of time, the instability time (*i.e.* the minimum point of σ_k^2) changes from $t \approx 50 fm/c$ to $t \approx 40 fm/c$ and correspondingly the saturation is observed before, when we pass from $E/A = 40$ to $E/A = 70\ MeV/u$. It is interesting to remark that the fragment formation time, *i.e.* the time interval between instability and saturation, seems to not depend on the beam energy. This effect has been recently observed in velocity correlation measurements[6].

Let us consider now the effects of changing the e.o.s. keeping constant the beam energy (40 MeV/u).

Smaller oscillations in the density are observed using a "stiff" equation of state (see fig. 3a). The number of fragments reduces therefore to 5.

Moreover, the expansion velocity increases, giving an effect similar to that one obtained using a larger beam energy; in fact the instability time and the saturation point are reached before (see fig. 3b). We stress here that all calculations have been done using the same number of test particles (20 per nucleon). It is very important to remind that the fluctuations present in our approach are strictly related to the used number of test particles. Thus if we increase this number (*i.e.* we correspondingly reduce the amplitude of fluctuations), the system may expand before going to fragmentation. It means that there is an interplay between the dynamical expansion and the explosion of instabilities, due to the presence of fluctuations, until the new reduced density is low enough to form fragments.

Of course, in this case, the number of fragments and the structure of the fragmentation process will depend on this new situation and consequently on the number of test particles we are using. We mean that, because of the dynamical effects, no prediction can be made about the fragmentation pattern, since it strongly depends on the amplitude of fluctuations and, in our case, on the number of test particles.

Moreover we observe that these dynamical effects are more and more important going to higher beam energies or considering a "stiff" equation of state, since in both cases we have a higher velocity of expansion.

Memory effects of the observed number of fragments are therefore lost when fluctuations are small, the fragment formation time is long and spherical symmetric dynamical expansion is dominant. In this sense, in order to correctly describe the behaviour of the system, the right physical amount of fluctuations that the system contains must be known. However, from our calculations, since the variances at the instability time should be rescaled, we predict quite large fluctuations at the critical point and therefore a corresponding short fragmentation time, of the order of 100 fm/c, is expected. This seems to be in agreement with some first experimental evaluations[6,7]. Therefore we believe that there is a good chance of observing such collective dynamical effects in the pattern of fragmentation events.

5. Conclusions

We have shown that, when one is interested in the dynamics of critical situations, it is possible to extract very important informations just solving the BNV equation within the test particle approach. In fact, if the number of test particles is large enough to avoid spurious effects, the dynamical evolution is correctly described until the system becomes unstable. Therefore the time when the spinodal region is reached and all the observables related to the mean properties (density, temperature, etc...) at that point may be calculated without any ambiguity. In particular we can extract the growing time τ, characteristic of the exponential increasing of fluctuations, which will lead to fragmentation.

We have also clearly shown the presence of memory effects in the fragmentation process. The leading unstable modes will completely fix the final fragmentation pattern (number and size of fragments, space and momentum structure and so on). This is a warning for the use of methods for fully chaotic systems, like percolation, to study nuclear fragmentation. The dynamics is playing a fundamental role. The paradox is that properties of the average phase space trajectory in the stable region will univocally determine the mean features of fragment production.

On the other hand we have also proved that time and pattern of this fragment formation depends, in a crucial way, on the noise present in the system, *i.e.* on the number of test particles. Therefore the right physical value of fluctuations is needed in order to correctly describe the dynamics. Concerning this subject, work is presently in progress.

It is finally evident that the method presented here is not limited to the study of spinodal (volume) instabilities. The recipe is to look at the time dependence of the variances σ_i^2 of various collective degrees of freedom, that can be done within the test particle approach. In this way all possible sources of instabilities (surface, Coulomb and so on) will be revealed and suitably studied.

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