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CONTRIBUTION AU XXX METING D'HIVER DE BORMIO

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Abstract

A microscopically derived Langevin equation is applied to thermally induced nuclear fission. An important memory effect is pointed out. A strong friction coefficient, calculated from microscopic quantities, tends to decrease the stationary limit of the fission rate and to increase the transient time.

There are mainly two competiting ways of deexcitation for hot nuclei, neutron evaporation and fission. Here, we shall just consider fission. In 1940, Kramers, in a cornerstone paper [1], proposed to describe fission as a diffusion over a barrier of a collective variable and to use a Langevin Equation (LE) to study the phenomenon. The slow collective motion with its high mass is the brownian particle, and the fast nucleonic degrees of freedom are the heat bath. He performed a study of the stationary flow over the saddle point with a Fokker-Planck Equation (FPE), equivalent to the LE, and gave a formula for the stationary fission rate (or reaction rate for the chemistry applications). More recently, a complete study of the fission process was perfomed numerically with both FPE and LE [2,3]. A long transient time, that could allow more pre-scission neutrons to evaporate, was pointed out by both groups. These calculations were completely phenomelogical; all the parameters of the equation, such as the friction coefficient, were fitted to reproduce the experimental data. Since then, a LE has been derived microscopically from the Boltzmann-Langevin Equation (BLE) and can be applied to this problem. In this case, all the parameters are explicitely given from canonical microscopic values [4].

The derivation of this new LE is recalled in a first part and the influence of the memory dependance and of the large friction coefficient on the fission rate is shown in a second part.

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Former approaches. Langevin equation and Fokker-Planck equation

The markovian LE used in the phenomenological studies is, for the collective variable q

$$M\ddot{q} = - \frac{\partial V}{\partial q} - \beta M q + \sqrt{M\beta T}\, w(t) \quad (1)$$

where w(t) is a white noise stochastic force:

$$<w(t)> = 0 \text{ and } <w(t)w(t')> = 2\, \delta(t-t') \quad (2)$$

In equation (1), V(q) is the potential and β is the friction coefficient. This equation is equivalent to the FPE if the noise is gaussian:

$$\left(\frac{\partial}{\partial t} + \frac{p}{M}\frac{\partial}{\partial q} - \frac{\partial V}{\partial q}\frac{\partial}{\partial p}\right) P = \frac{\partial}{\partial p}\left(- \beta p + D \frac{\partial}{\partial p}\right) P \quad (3)$$

This equation determines the time evolution of the probability distribution $P(q,p,t)$ in terms of the diffusion coefficient $D=M\beta T$ and the friction.

We have chosen to use a LE rather than a FPE because the first one is a simple differential equation with a stochastic term and therefore more tractable numerically than a second order partial differential equation. In addition the LE is more general because it can more easily be extended to non-markovian processes, multidimensional problems and non-gaussian noises.

Derivation of a Langevin equation

To derive microscopically a LE, the authors of the reference [4] started from the BLE:

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{p}}{m} \nabla\mathbf{r} + \nabla\mathbf{r}Uf\, \nabla\mathbf{p}\right) f(\mathbf{r},\mathbf{p},t) = K(f) + \delta K(f) \quad (4)$$

which is an extended version of the Boltzmann equation which takes into account dynamical fluctuations. In equation (4), K(f) is the Uehling-Uhlenbeck collision term:

$$K(f_1) = \frac{1}{h^3} \int d\mathbf{p}_2\, d\mathbf{p}_3\, d\mathbf{p}_4\, w(12,34)\, (f_3 f_4 \bar{f}_1 \bar{f}_2 - f_1 f_2 \bar{f}_3 \bar{f}_4) \quad (5)$$

with $f_i = f(\mathbf{r}_i,\mathbf{p}_i,t)$ and $\bar{f} = 1 - f$. The mean spin-isospin averaged transition rates are given by the cross section:

$$w(12,34) = \frac{4}{m}\frac{d\sigma}{d\Omega}\,\delta(\mathbf{p_1}+\mathbf{p_2}-\mathbf{p_3}-\mathbf{p_4})\,\delta(\varepsilon_1+\varepsilon_2-\varepsilon_3-\varepsilon_4) \quad (6)$$

As this term only describes the average effects of the two-body collisions, a fluctuating term which contains all the other correlations, $\delta K(f)$, is added (eq. (4)). It is caraterized by its first two moments:

$$\begin{aligned} &\langle \delta K(\mathbf{r},\mathbf{p},t)\rangle = 0 \\ &\langle \delta K(\mathbf{r},\mathbf{p},t)\,\delta K(\mathbf{r}',\mathbf{p}',t')\rangle = C(\mathbf{p},\mathbf{p}')\,\delta(\mathbf{r}-\mathbf{r}')\,\delta(t-t') \end{aligned} \quad (7)$$

To use BLE to study a large amplitude collective motion q(t), an irrotational-incompressible velocity field is chosen:

$$\mathbf{v}(\mathbf{r},t) = \dot{q}(t)\,\nabla\phi(\mathbf{r}) \quad (8)$$

The BLE is projected onto q using a fluid dynamic reduction in the diabatic and quasistatic approximation.

One eventually obtains a memory dependant LE. usually called a Generalized Langevin Equation (GLE)

$$M\ddot{q} + \frac{1}{2}\frac{\partial M}{\partial q}\dot{q}^2 + \frac{\partial V}{\partial q} = -\int_{-\infty}^{t} dt'\,\gamma(t-t')\,M\dot{q}(t') + \delta F(t) \quad (9)$$

where the friction kernel is given by

$$\gamma(t-t') = \frac{\Gamma}{M}\exp\left(-\frac{t-t'}{\tau}\right) \quad (10)$$

and the correlation function of the random force δF is

$$\langle \delta F(t)\,\delta F(t')\rangle = TM\,\gamma(t-t') \quad (11)$$

All the coefficient are explicitely given from microscopic values (using the notations of reference [4])

$$M(q) = m\int d\mathbf{r}\,\rho_0(\mathbf{r},q)\,\nabla\phi\nabla\phi \quad (12)$$

$$\Gamma = \frac{24}{5}A\,\varepsilon_F\,T\,\xi \quad (13)$$

$$\frac{1}{\tau} = 8\,\sigma\,v_F\,\rho\left(\frac{T}{\varepsilon_F}\right)^2 \quad (14)$$

where ξ is the mean value over density of $\sum(\partial_i\partial_j\phi(\mathbf{r}))(\partial_i\partial_j\phi(\mathbf{r}))/6$ and ε_F and v_F are respectively the Fermi energy and velocity. Here, τ is given for a quadrupolar mode [5].

One of the main differences with the phenomenological LE is the memory dependence of this equation. If τ vanishes, which is the case when T increases, $\gamma(t-t')\approx\beta\delta(t-t')$ and one gets back the markovian LE with $\beta=\tau\Gamma/M$. To see how this effect can affect the final results, we study single trajectories. Without the Langevin force one solves analytically equation (9) and one gets

$$q(t) = A\exp(-\frac{\pm\omega_0^2 t}{\beta\pm\omega_0^2\tau}) + B\exp(-\frac{t}{2\tau})\cos(\omega t+\Psi) \quad (15)$$

where

$$\frac{1}{M}\frac{\partial V}{\partial q} = \pm\,\omega_0^2 q \text{ and } \omega = \sqrt{(\frac{\beta}{\tau}\pm\omega_0^2)-\frac{1}{4\tau^2}} \approx \Omega \;\; (\Omega >> \omega_0) \quad (16)$$

In eq. (15), A, B and Ψ depend on the initial conditions, and we have assumed that the friction is large as compared to the other fre encies. In addition to a drift term, some damped oscillations appear, the stochastic force tends to reactivate these oscillations, as can be seen in figure 1. We can easily understand that the memory effect can change the time of evolution of an event and then change the fission rate.

Application to fission

To apply the GLE to symetric fission, we choose a quadrupolar velocity field,

$$\phi(\mathbf{r}) = (2z^2-x^2-y^2)/2 \quad (17)$$

and the quadrupolar moment for q, $q=\alpha_2/2$, where the radius of the nucleus is parametrized by

$$R = R_0\,(1 + \alpha_2\,P_2(\cos\theta)) \quad (18)$$

We solve numerically

$$\begin{aligned} \dot{q} &= \frac{p}{M} \\ \dot{p} &= -\frac{\partial V}{\partial q} - \frac{p^2}{2M}\frac{\partial M}{\partial q} - \beta p + \sqrt{M\beta T}\,w(t) - \tau\dot{F} \\ \dot{F} &= \frac{1}{\tau}\left(-\beta p - F + \sqrt{M\beta T}\,w(t)\right) \end{aligned} \quad (19)$$

to evaluate the fission rate

$$r(t) = -\frac{1}{Pr(t)}\frac{dPr(t)}{dt} \quad (20)$$

where Pr(t) is the probability that the nucleus is compound, i.e. the number of nuclei with $q<q_s$ (q at the saddle point) over the total number of nuclei. All the parameters are

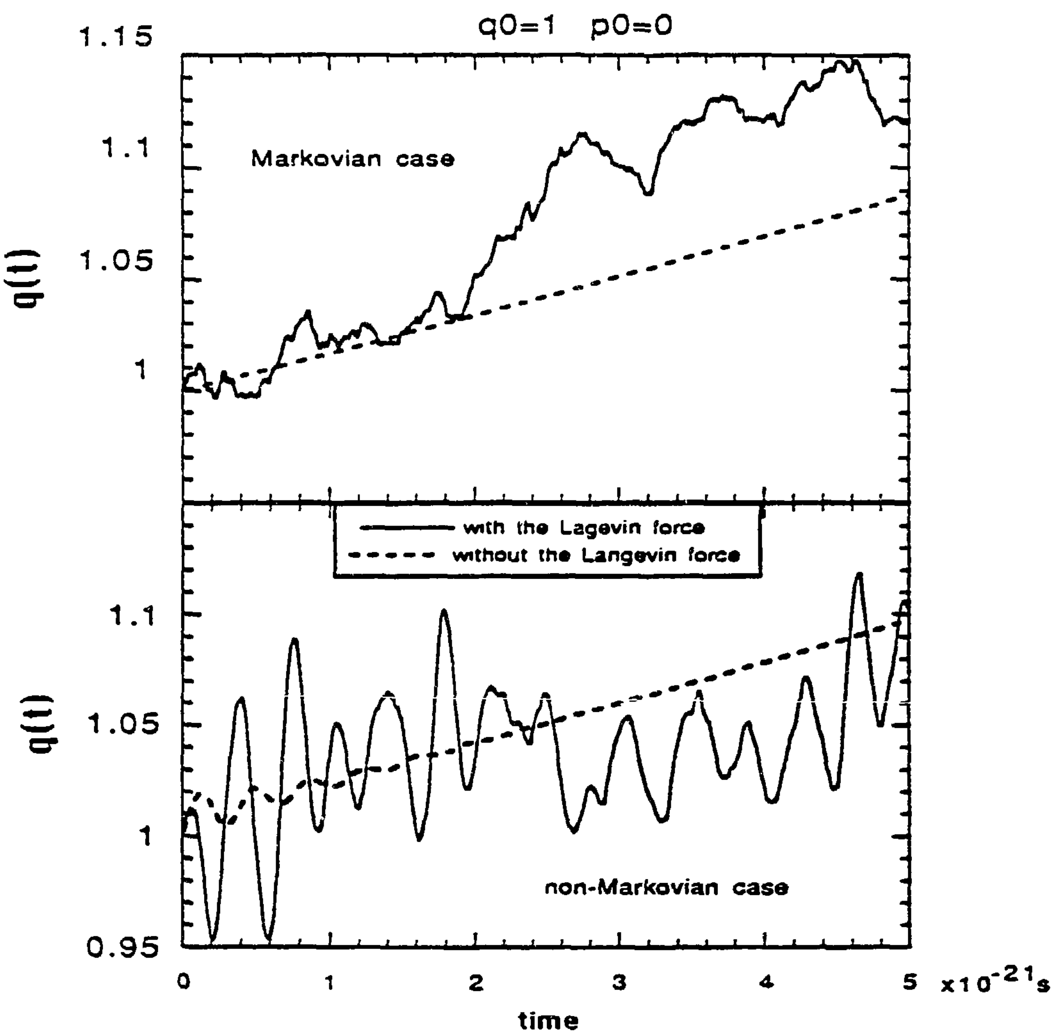

figure 1: Effect of the Langevin force on a single trajectory, in the Markovian case and the non-Markovian case. The initial condition is far beyond the saddle point. The oscillations are a signature of the memory effect.

set, $\xi=1$, $\beta=\tau\Omega^2$, where Ω is frequency of the Giant Quadrupolar Resonance. From the Liquid Drop Model, we know the position of the saddle point, we can build a potential with two parabolas

$$V(q) = \begin{cases} 135.5\ q^2 & \text{for } q<0.122 \\ -135.5\ (q-0.24)^2 + 4 & \text{for } q>0.122 \end{cases} \quad (21)$$

The width of the barrier is arbitrarily chosen. The initial condition for q and p is randomly picked following the distribution

$$d(q,p) = \exp - \frac{1}{2T_0}\left(\frac{p^2}{M} + 135.5\ q^2\right) \quad (22)$$

which is a cold gaussian, T_0=0.3MeV, and F_0=0.

Results

We calculated the fission rate of the ^{248}Cf at T=4.88MeV. Note that as we do not have the same density parameter as the one of the reference [2], this temperature correspond to 4MeV in the case of reference [2]. The other relevant parameters read β=0.1c/fm=30 $10^{21}s^{-1}$, τ=33fm/c=1.1 10^{-22}s. In that case, τ is small enough to do the markovian approximation. As the friction coefficient is large, 6 times larger than the largest one of the reference [2], we did the calculation in the overdamped limit too:

$$\dot{q} = -\frac{1}{M\beta}\frac{\partial V}{\partial q} + \sqrt{\frac{T}{M\beta}}\,w(t) \quad (23)$$

which means just keeping the drift term of the equation (15). The results are shown on the figure 2. Due to this larger friction the stationary fission rate is smaller and the transient time larger, but the orders of magnitude are the same as the ones of reference [2]. The results of the calculation in the overdamped limit are close to the other ones.

We did the calculation at T=3MeV too. Then one has β=0.27c/fm=80 $10^{21}s^{-1}$, τ=88fm/c=3 10^{-22}s. One cannot do the markovian approximation anymore, so we used the GLE to evaluate the fission rate. The results are shown on figure 3. They are not accurate due to a small statistics, but one can extract the bulk behavior. The stationary fission rate is about 10 times smaller and the transient time 10 times larger than the ones obtained in the phenomenological case. This means a life time of a nucleus much longer. These differences are due to the fact that our friction coefficient depends on temperature, $\beta \propto 1/T^2$. Our friction coefficient may be a bit too large by a factor 2, for the range of temperatures we consider, because the BLE is markovian. If the memory effects are taken into consideration in the Boltzmann equation, then

$$\frac{1}{\tau} = 8\,\sigma\, v_F\, \rho\, \left(\left(\frac{T}{\varepsilon_F}\right)^2 + \frac{3}{4}\left(\frac{\hbar\Omega}{\pi\varepsilon_F}\right)^2\right) \tag{24}$$

for a quadrupolar oscillation [5]. Of course taking into account the memory effect in BLE would change the GLE, but the expression (24) gives us a rough indication.

Conclusion

We have derived from a microscopic model a GLE wich allows us to calculate the fission rate. All the parameters of the equation, and especially, the friction coefficient, are given explicitely from microscopic values of the nucleus. The friction coefficient is much larger than the one used in the phenomenological approach even if a strong memory effect tends to decrease its effects. This means a longer life time for the nuclei which allows more pre-scission neutrons to evaporate. The conclusion that nuclear matter may be more viscous than we thought was reached by another group with another method [7]. But the friction value we used may be too large because we have neglected the important memory effects in the BLE.

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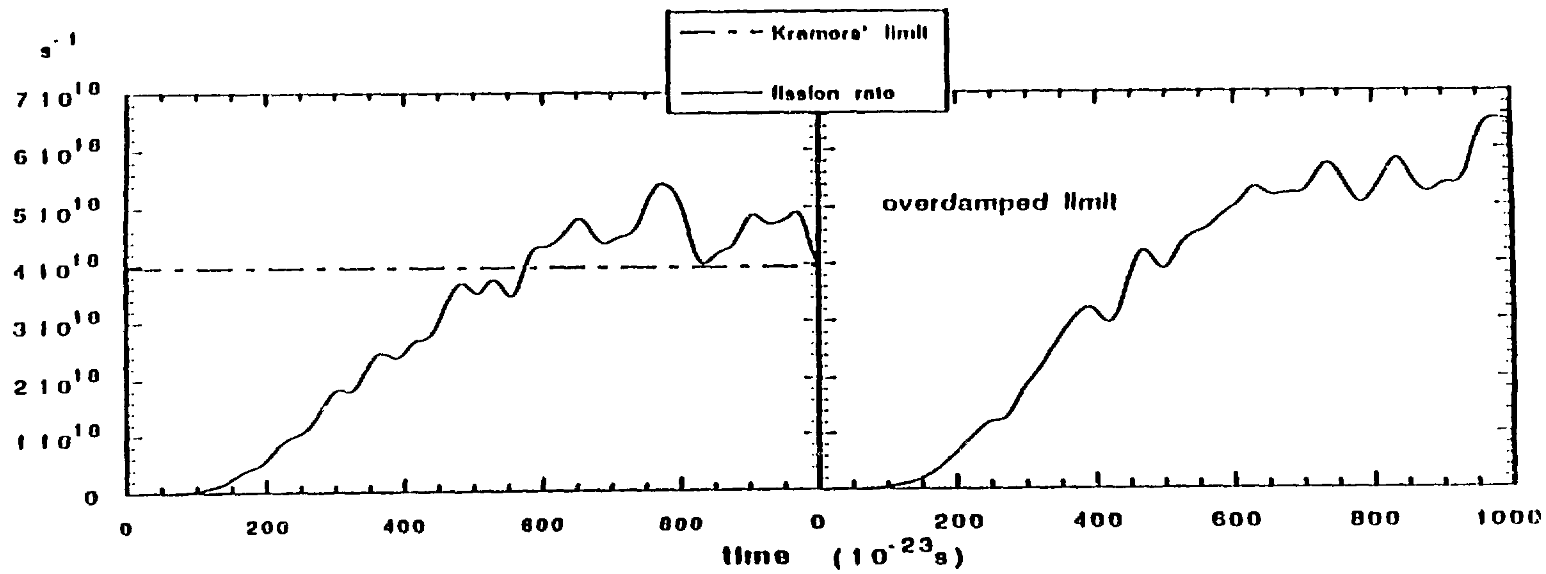


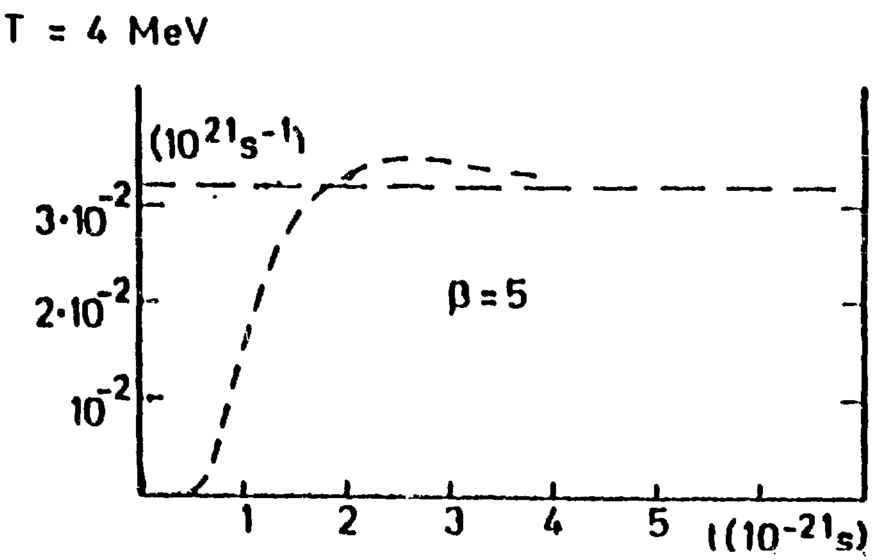
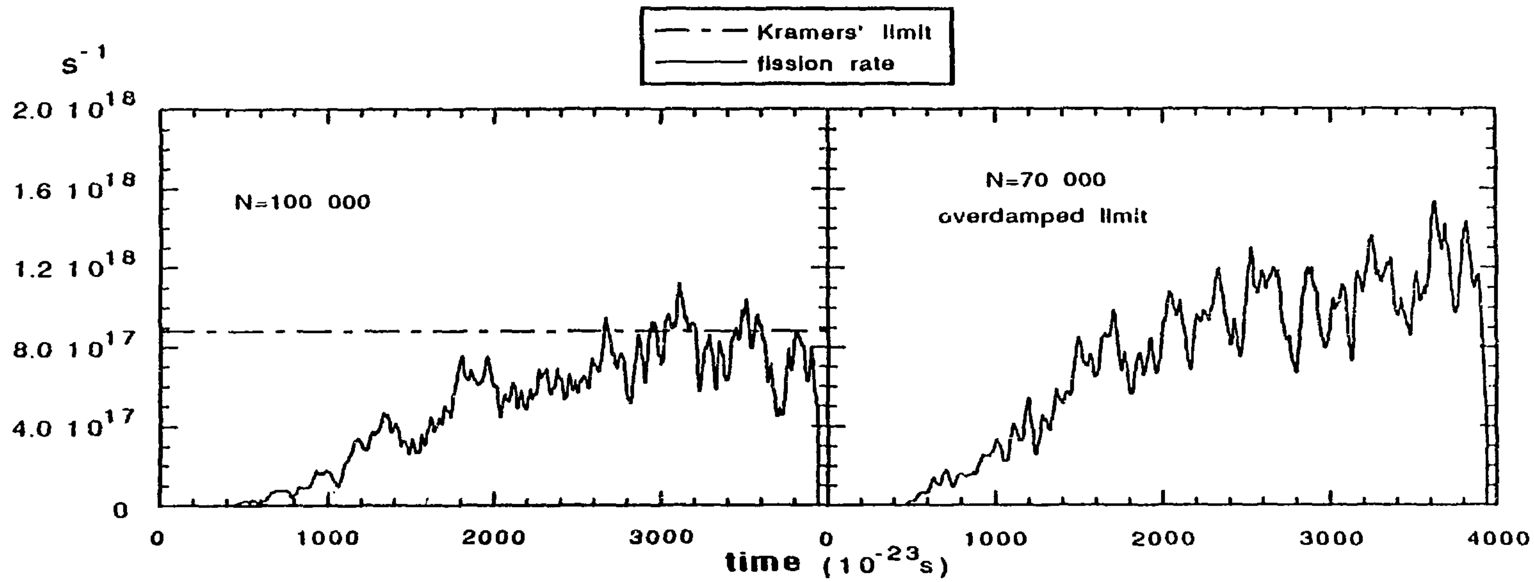


figure 2: Fission rate as a function of time for the ^{248}Cf at T=4.88 MeV. The calculation was made with a LE in the Markovian approximation and in the overdamped limit. The initial condition distribution is "cold", T_0=0.3MeV. The number of events calculated for both graphes is N=250 000. The results of reference [2] with T=4MeV and $\beta=5\times10^{21}$s are indicated. The stationary limit is calculated analytically [6].



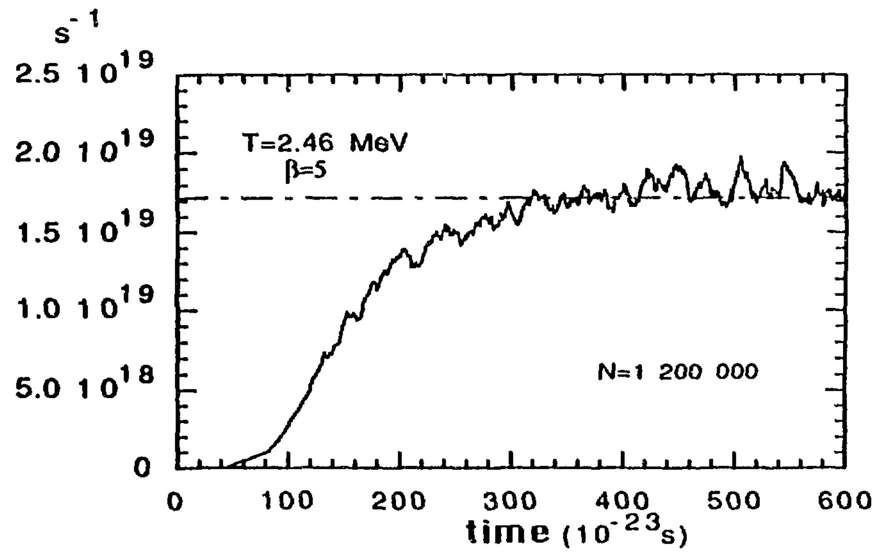


figure 3: Fission rate as a function of time for the ^{248}Cf at T=3 MeV. The calculation was made with the GLE and in the overdamped limit. The result of a calculation made in the phenomenological case of the reference [2] for $\beta=5\times10^{21}$s and at T=2.46MeV, but at the same excitation energy, is also shown. The initial condition distribution is "cold", T_0=0.3MeV. The stationary limit is calculated analytically [6]. N is the number of events calculated.