WAVELENGTH SELECTION IN TRAVELING-WAVE CONVECTION IN A FLUID MIXTURE

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ABSTRACT

The mechanisms by which a one-dimensional pattern of traveling waves changes wavelength (i.e. the Eckhaus instability) is studied in a binary fluid mixture. Propagating wavelength modulations develop when the Rayleigh number of the system is decreased below a wavelength-dependent threshold, commonly referred to as the Eckhaus boundary. These wavelength modulations increase in amplitude and narrow in spatial extent until they trigger the creation or annihilation of convection roll pairs and thereby change the average wavelength of the system. We find qualitatively different dynamics for wavelength-increasing events and wavelength-decreasing events; these differences are due to the strong wavelength dependence of the group velocity.

INTRODUCTION

Nonequilibrium systems form an important part of our physical world. An important class of nonequilibrium systems is those in which traveling waves play a central role. The work discussed here has relevance to systems such as lasers, ocean waves, atmospheric flows, and certain kinds of crystal growth. A fundamental question in pattern selection and dynamics concerns the mechanism by which a periodic pattern changes wavelength in response to changes in control parameter. In the Eckhaus instability, a wavelength instability results when the control parameter is lowered below a wavelength-dependent threshold. The case in which the underlying pattern is stationary has received considerable experimental and theoretical attention [1,2,3]. In contrast, wavelength instabilities in pattern forming systems in which the underlying state consists of traveling waves have only recently been studied. The first experimental studies of the Eckhaus instability for traveling waves were done by Janiaud et al [4]. They studied wavelength instabilities on wave trains which resulted from the oscillatory instability in a convecting gas. While their results were interesting, experimental constraints limited their work to a small aspect-ratio system.

Furthermore, the fact that the wavelength modulation in their experiment is a tertiary instability made detailed theoretical analysis of their system difficult. Recently the Eckhaus instability for traveling waves has been investigated in the binary fluid system by both our group and by complementary experiments elsewhere [5,6].

Binary fluid mixtures of ethanol and water provide a model system in which to study nonequilibrium pattern-forming systems, since the underlying fluid equations are well understood and the control parameters of the system can be precisely controlled. The binary fluid system has been the subject of extensive study in recent years and consequently much is known about the nonequilibrium behavior of this system.

Binary fluid convection is closely related to convection in a pure fluid. In pure fluid convection, by heating from below, a temperature difference is imposed across a horizontal fluid layer, causing the fluid to expand and rise under the buoyant force. The Rayleigh number, R, is proportional to the imposed temperature difference and is the main control parameter in convection experiments. Due to the stabilizing effects of heat diffusion and viscosity, the onset of convection occurs only after a minimum temperature difference is imposed across the fluid layer at a critical Rayleigh number, R_c. (In this paper we use the reduced Rayleigh number $r \equiv R/R_c$.) In binary fluid mixtures, two additional parameters are required to describe the system. The Lewis number, $L \equiv D/k$, is the ratio of diffusivity of concentration to the thermal diffusivity and thus characterizes the relative time scales on which concentration and heat diffuse. The "separation ratio", Ψ, is a measure of the concentration-driven density changes due to the Soret effect which couples the temperature gradient to concentration gradients [7]. For $\Psi < 0$, the lighter component diffuses toward the colder region and a linear concentration gradient develops which stabilizes the fluid layer against convection. Consequently, the Rayleigh number at which the onset of convection occurs in the mixture is greater than that in a pure fluid with the same fluid properties.

When the Rayleigh number is increased above threshold, the linear concentration gradient is destroyed by the onset of convection; the fluid is well-mixed in the interior of the rolls and the concentration gradients occur only in boundary layers at the top and bottom of the fluid layer [8,9,10]. Concentration from these horizontal boundary layers is fed asymmetrically into the vertical upflow and downflow regions at the roll boundaries, producing a lateral modulation in concentration. In ethanol-water mixtures $L << 1$; and for $\Psi < 0$, the convection patterns are generally found to propagate laterally as traveling waves. It is the small phase shift between the lateral concentration and temperature fields which is predicted to cause the translation of the convection rolls.

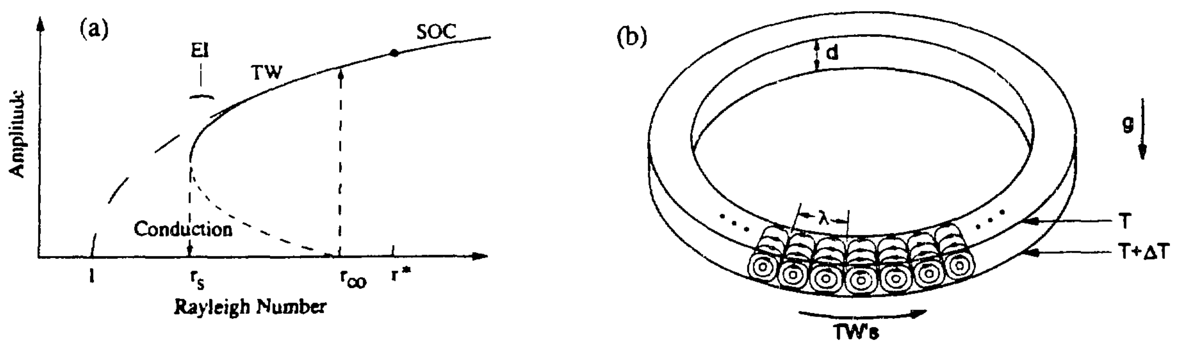


Figure 1. Shown in (a) is a schematic of the bifurcation diagram for pure and binary fluid convection: pure fluid (long dashed line), binary fluid mixture at $\Psi = -0.25$ (solid line), and unstable branch in the binary fluid (short dashed line). The bifurcation is supercritical in the pure fluid and subcritical in the mixture. The Eckhaus instability is encountered in the small range of Rayleigh numbers indicated as "EI". Shown in (b) is a schematic of the experimental geometry.

In this paper, we report results for a fluid mixture at $\Psi = -0.25$. The bifurcation diagram for this value of Ψ is shown schematically in Fig. 1(a). For this value of Ψ, the bifurcation is subcritical. Convection begins at $r = r_{co}$ as a Hopf bifurcation. The convective amplitude grows via a long transient to a state of slow-moving traveling waves (TW) with a period of approximately 1/30 the period of the Hopf bifurcation. As r is decreased, this nonlinear state remains stable until the Eckhaus instability (EI) is reached. The instability is encountered in a small region of Rayleigh numbers near the saddle-node bifurcation. If the initial wave number corresponds to the minimum of the Eckhaus boundary, the saddle-node bifurcation point is encountered at $r = r_s$, where the amplitude of the pattern becomes unstable. As the Rayleigh number is decreased, the convective amplitude decreases, and the phase velocity increases monotonically. If r is increased, the phase velocity decreases and goes to zero at a point denoted by r^*, resulting in a state of stationary overturning convection (SOC)[11] .

DESCRIPTION OF THE EXPERIMENT

The experiments are conducted in a large aspect ratio annular channel of rectangular cross-section which is depicted schematically in Fig. 1(b). The cell is constructed of plastic sidewalls sandwiched between a mirror-polished rhodium-plated copper bottom plate and a sapphire top plate which provides optical access from above. The height of the cell is $d = 0.309 \pm 0.002$ cm and the width and mean circumference are, in units of d, 1.288 and 67.09 respectively. The convection rolls align radially and propagate azimuthally, thus the annular geometry provides periodic boundary conditions in the direction of roll (TW) propagation. The top plate temperature is maintained at 25.000 ± 0.001°C, and the bottom plate temperature is varied from 29.6°C to 32.0°C with similar regulation. The working fluid is a water-ethanol mixture which is 8% ethanol by weight. At the onset of convection, the mean fluid temperature is 27.53°C and the fluid parameters are $\Psi = -0.25$, $Pr = 9.16$, and $L = 0.008$ [12]. The fluid is visualized from above using the shadowgraph technique. The image of the convection pattern is recorded by a 720 element annular CCD array.

The Eckhaus boundary is experimentally determined by starting the system in a state of uniform wavelength and slowly decreasing the Rayleigh number until the wavelength of the pattern becomes unstable. By starting with different initial numbers of rolls in the cell, we can determine the points at which each possible wavelength becomes unstable and thus map the Eckhaus boundary which is shown in Fig. 2(a) . This stability boundary is approximately parabolic and has a minimum at a wave number which we denote by k_o. For experimentally realizable initial wave numbers, k_i, the range of Rayleigh numbers at which the Eckhaus instability is encountered is in a small region near the saddle node bifurcation, as is shown in Figs. 1(a) and 2(a). One of the most striking features of our results is that the dynamics of wave number changing events are qualitatively dependent on the value of the local wave number relative to k_o.

The technique of complex demodulation [13] is used to follow the dynamics of the Eckhaus instability. In this technique, small changes to the phase and amplitude of a slowly-varying sinusoidal signal can be computed. From this information, we can calculate the space-time evolution of the frequency, wave number and amplitude of an unstable state. By mapping the wave number of the pattern to a gray scale, the spatio-temporal dynamics can be easily visualized, as is shown for the data in Fig. 3.

In the state shown in Fig. 3(a) , which has an initial wave number $k_i > k_o$, a sinusoidal modulation in the wave number grows in from zero amplitude. The wavelength of this modulation is the longest wavelength which can fit into the annular cell. The evolution of this state, and all experimentally observed states with $k_i > k_o$ proceeds in this manner (i.e., that the instabilities grow from an initial modulation wave number $q \equiv 0$), which is identical to the Eckhaus behavior seen in pure fluid convection. However, in contrast to the behavior of the stationary wavelength modulation in the pure fluid system, the wavelength modulation in the binary fluid system propagates. The speed of the modulation is approximately twice the phase velocity of the underlying traveling waves and in the same direction as the phase velocity. The initially sinusoidal modulation grows in amplitude and narrows in spatial extent, until a roll pair is destroyed in the highly compressed region of the modulation, as shown in Fig. 3(b). Our experiments indicate that this is the generic mechanism by which wave number changes occur for cases where $k_i > k_o$.

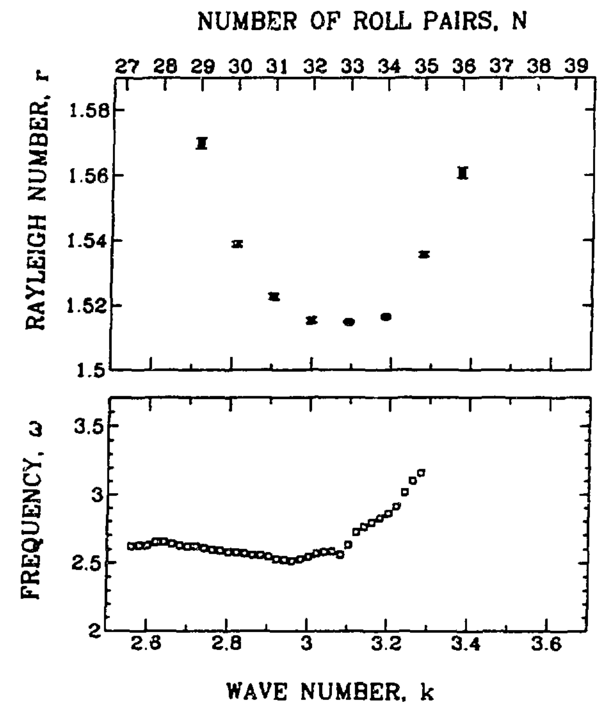
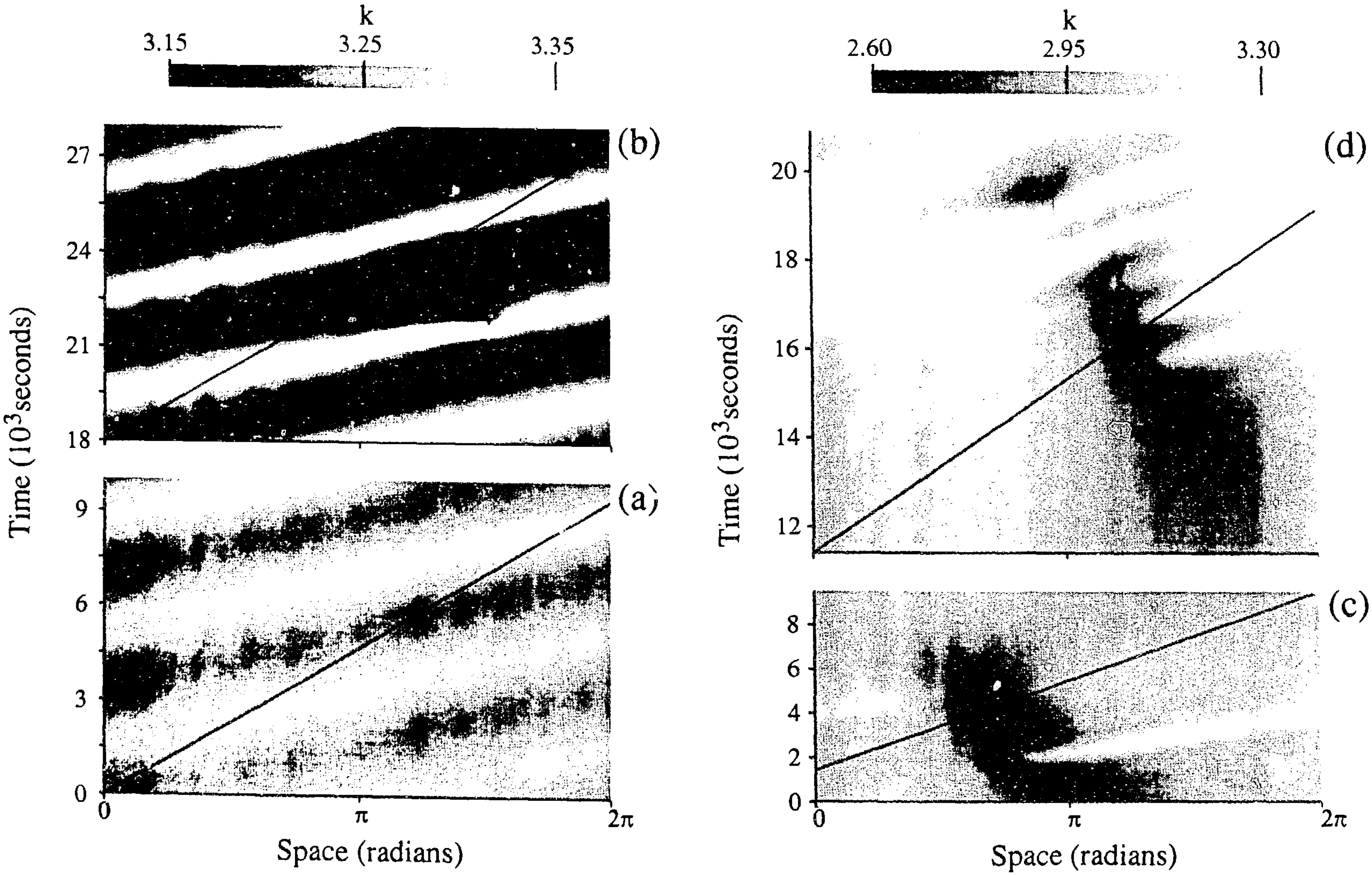


Figure 2. In panel (a), the experimentally determined Eckhaus stability boundary is shown. The wave number is measured in units of d, the cell height. In (b), the dispersion relation is shown. Units of ω are 10^{-2} rad/s.

When the initial wave number $k_i < k_o$, the evolution of the system is qualitatively different, as can be seen in Fig. 3(c). In this case, the modulation does not grow from long wavelength, but instead a region of localized dilation develops. This region becomes increasingly dilated and, in contrast to the case where $k_i > k_o$, propagates in a direction opposite the phase velocity with a very slow speed (approximately 1/10th the phase speed). When the amplitude of the wave number modulation grows sufficiently, a new roll pair is created. This newly created roll pair has a local wave number which is greater than k_o, and the resulting wave number modulation propagates away from the point where the new roll pair was created.

Caption for Figure 3 (figure appears on next page)
Figure 3. (a), (b) Evolution of the local wave number in space and time for $k_i > k_o$ at r=1.543, for two time intervals in the same data set. The local wave number is mapped to a gray scale, and the solid lines indicate the motion of a roll boundary, (i.e., the phase velocity of the rolls). The annihilation of one roll pair occurs at t=19.6 x 10^3 s. (c), (d) Evolution of the local wave number for values of initial wave numbers $k_i < k_o$ at r = 1.521, for two time intervals in the same data set. Roll pairs are created at several times, and a roll pair is lost at t = 19.6 x 10^3 s.


k
3.15
3.25
3.35
(b)
(a)
Time (10^3seconds)
27
24
21
18
9
6
3
0
0
π
2π
Space (radians)
k
2.60
2.95
3.30
(d)
(c)
Time (10^3seconds)
20
18
16
14
12
8
6
4
2
0
0
π
2π
Space (radians)

The dynamics of wave number changes can be quite complicated, as can be seen in Fig. 3(d). In this case, the creation event produces a localized region of increased wave number which propagates out of the dilated region. This pulse does not reach the dilated region again until it propagates around the entire length of the cell. For this reason, the dilation continues to be a source for the creation of new roll pairs until they propagate back to the dilation. Thus the dynamics of this wave number-unstable state is dependent on the aspect ratio of the system and the growth rate of the instability. The fact that the wave number compression pulse is advected out of the dilated region gives rise to complicated dynamics. This is evident in the state shown in Fig. 3(d), in which the system over corrects the wave number when the Eckhaus instability is encountered. Instead of increasing the overall wave number by adding one roll pair and bringing the state back into the Eckhaus stable regime, the system over corrects by adding three roll pairs, as the compression pulse propagates out of the dilated region. In an infinite system, we conclude that it is likely that this mechanism will not successfully readjust the wave number, and thus, in such a system, the Eckhaus unstable states with $k_i < k_o$ could not be stabilized by this mechanism. In the infinite case, the dilated region is likely to continue to create a series of wave number compression pulses.

The dispersion relation which characterizes the system can be determined experimentally by computing the wave number and frequency at each space-time point and calculating $\omega(k)$ by averaging the value of ω within small bins in k [5]. A dispersion relation calculated in this manner for the data shown in Fig. 3(c) is shown in Fig. 2(b). The group velocities predicted by the dispersion relation are consistent with the observed modulation group velocities. The qualitatively different behavior for $k > k_o$ and $k < k_o$ can be attributed to the nature of the dispersion relation, which has different characteristic slopes for $k > k_o$ and $k < k_o$.

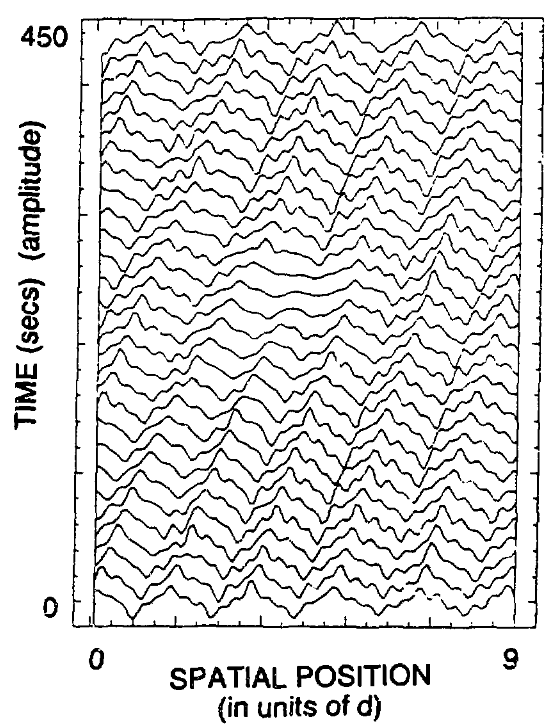


Figure 4. The shadowgraph signal (which is related to the convective amplitude) is shown during an annihilation event. This figure illustrates the rapid time scales on which the annihilation takes place.

It is also interesting to note that the time scale on which creation and annihilation events occur is quite short, as can be seen in Fig. 4, which shows the shadowgraph amplitude when a roll pair is annihilated. For the data shown, this is of the order of one half of the period of the traveling waves. This indicates that diffusion of concentration plays at most a minor role in the annihilation of a roll pair. The creation of a roll pair (e.g., Fig. 3(c) near time 2,000 sec) occurs on a similar time scale. We note in passing that there is information about the concentration field [14] in the shadowgraph signals (e.g., Fig. 4) regarding the concentration field. We have not yet studied this in detail. In contrast to the experimental results of Janiaud et al, we do not see a standing wave near an annihilation event.

SUMMARY AND CONCLUDING REMARKS

In this paper we have presented an overview of our experimental results on wavelength selection and wavelength changing events in traveling-wave convection in a binary fluid mixture. We have found that the wave number changes via propagating wave number modulations. The behavior of wave number selection is qualitatively different depending on the relationship of the initial wave number relative to the most stable wave number, k_o. For initial wave numbers $k_i > k_o$, the wave number modulation grows from zero amplitude at the largest wavelength that can fit into the system. This modulation is propagates at a group velocity that is consistent with the measured dispersion relation. The wave number modulation increases in amplitude and narrows in spatial extent and eventually triggers the annihilation of a roll pair. For initial wave numbers, $k_i < k_o$, the evolution is markedly different. A localized dilated region grows in amplitude and propagates slowly at a speed consistent with the measured dispersion relation in a direction opposite the phase velocity. This region eventually becomes so dilated that a new roll pair is created.

There are numerous open questions raised by our work. Powell and Bernoff have investigated the properties of traveling waves near a saddle-node bifurcation [15]. In an extension of this work, M.C. Cross has developed an amplitude equation analysis based on an expansion about saddle-node solutions [16]. This analysis makes several predictions which can be tested experimentally such as predictions for the growth rate of the instability and the group velocity. The non-linear evolution of the wave number modulation deserves further study and there are speculations that it may exhibit the amplitude-width scaling of a Korteweg - de Vries soliton [4]. The fact that dilations do not grow from $q = 0$ when $k_i < k_o$ but are spatially localized is presently unexplained theoretically and deserves further study. Finally, as we have noted above, another area yet to be explored is the relationship of the wave number modulations to modulations of the underlying concentration field [14].

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REFERENCES

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1. M. LOWE and J. GOLLUB, "Pattern Selection Nea. the Onset of Convection: The Eckhaus Instability," Phys. Rev. Lett. 55, 2575 (1985).

2. M. DOMINGUEZ-LERMA, D. CANNELL, and G. AHLERS, "Eckhaus Boundary and Wave Number Selection in Rotating Taylor-Couette Flow," Phys. Rev A 34, 4956 (1986).

3. L. KRAMER and W. ZIMMERMANN, "On the Eckhaus Instability for Spatially Periodic Patterns," Physica D 16, 221 (1985).

4. B. JANIAUD, A. PUMIR, D. BENSIMON, V. CROQUETTE, H. RICHTER, and L. KRAMER, "The Eckhaus Instability for Traveling Waves," *Physica D* 55, 269 (1992).

5. G.W. BAXTER, K.D. EATON, and C.M. SURKO, "Eckhaus instability for traveling waves," *Phys. Rev. A* 46, 1735 (1992).

6. P. KOLODNER, "Extended States of Nonlinear Traveling-Wave Convection: I. The Eckhaus Instability," *Phys. Rev. A* 46, 6431 (1992) and "Observations of the Eckhaus Instability in One-Dimensional Traveling-Wave Convection," Phys. Rev. A 46, 1739 (1992).

7. D. HURLE and E. JAKEMAN, " Soret-Driven Thermosolutal Convection," J. Fluid Mech. 47, 667 (1971).

8. W. BARTEN, M. LÜCKE, and M. KAMPS, "Structure and Dynamics of Nonlinear Convective States in Binary Fluid Mixtures," in *Nonlinear Evolution of Spatiotemporal Structures in Dissipative Continuous Systems* , Plenum Press, New York (1990).

9. W. BARTEN, M. LÜCKE, W. HORT, and M. KAMPS, "Fully Developed Traveling-Wave Convection in Binary Fluid Mixtures," *Phys. Rev. Lett.* 63, 376 (1989).

10. D. BENSIMON, A. PUMIR, and B.I. SHRAIMAN, "Nonlinear Theory of Traveling Wave Convection in Binary Mixtures," *J. Phys. (Paris)* 50, 3089 (1989).

11. D.R. OHLSEN, S.Y. YAMAMOTO, C.M. SURKO, and P. KOLODNER, "Transition from Traveling-Wave to Stationary Convection in Fluid Mixtures," *Phys. Rev. Lett.* 65, 1431 (1990).

12. P. KOLODNER, H. WILLIAMS, and C. MOE, "Optical Measurement of the Soret Coefficient of Ethanol/Water Solutions," *J. Chem. Phys.* 88, 6512 (1988).

13. P. BLOOMFIELD, "Fourier Analysis of Time Series: An Introduction," (Wiley, New York, 1976), pp. 118-150.

14. K.D. EATON, D.R. OHLSEN, S.Y. YAMAMOTO, C.M. SURKO, W. BARTEN, M. LÜCKE, M. KAMPS, and P. KOLODNER, "Concentration Field in Traveling-Wave and Stationary Convection in Fluid Mixtures," *Phys. Rev. A* 43, 7105 (1991).

15. J. POWELL and A. BERNOFF, "Saddle-Node Bifurcation of Slowly-Varying Nonlinear Traveling Waves," unpublished.

16. M. CROSS, unpublished.