EXPERIMENTS ON SCALAR MIXING AND TRANSPORT

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ABSTRACT

We provide an overview of our recent work on passive (temperature) scalar mixing in both homogeneous and inhomogeneous turbulent flows. We show that for homogeneous grid generated turbulence, in the presence of a linear temperature profile, the probability density function (pdf) of the temperature fluctuations has broad exponential tails, while the pdf of velocity is Gaussian. However in the absence of a scalar gradient the pdf of temperature is Gaussian. This new result sheds insight into the fundamentals of turbulent mixing as well as to the nature of the velocity field. We also show that the spectrum of the temperature fluctuations has a scaling region that is consistent with Kolmogorov scaling although a similar scaling region is absent for the velocity field in this low Reynolds number flow. Finally, we describe our results concerning the mixing and dispersion of scalars in a jet. We show that although initially the scalar mixing is strongly dependent on input conditions, the mixing is rapid and that the correlation coefficient asymptotes to unity by x/D ~ 20.

INTRODUCTION

The understanding of scalar mixing and transport in turbulent flows remains a vital issue because of its fundamental importance, both in its own right and also in the way it sheds light on the basic characteristics of the velocity field itself, and because of its obvious practical significance in combustors, chemical mixers and the environment. At the fundamental level, a full statistical description has not emerged, even for simple flows although very recently much progress has occurred in our understanding of the nature of the probability density function (pdf) of the scalar fluctuations. Our contribution to this will be described in part 1 below. There are, however, still complex problems concerning one of the oldest and most studied statistical descriptions; the one dimensional scalar spectrum. Both experiments (e.g. Warhaft and Lumley 1978) and computation, (Metais and Lesieur 1992) show anomalous scaling regions in the scalar spectrum in isotropic turbulence. They are anomalous since they occur in the absence of such regions in the velocity spectrum. Although the experimental observations are quite old there has been no systematic study of the scalar spectrum as a function of Reynolds number and thus it has not been possible to determine whether these scaling regions are artifacts of the initial conditions in these low Reynolds number flows or whether they are fundamental to a statistical description of the flow. Our recent experiments towards an understanding of this problem will be described in part 2. We will show there is indeed a scaling region of constant slope close to -5/3 and its width increases systematically with Reynolds number. We also continue to be interested in the effect of initial conditions on the rate of mixing of scalar fluctuations. We have recently been studying scalar dispersion from heated wires in a jet. We have examined both single and two scalar mixing. Here we will describe our results and relate them to mixing in grid turbulence.

Thus our paper is concerned with three distinct topics: the pdf of passive scalar fluctuations, the spectrum of scalar fluctuations, and mixing and transport in a jet. Necessarily, because of space, only a brief summary can be provided and the reader is referred to Jayesh and Warhaft (1992, 1993), Warhaft (1992) and Tong and Warhaft (1993) for details, including descriptions of experimental apparatus.

1. THE PROBABILITY DENSITY FUNCTION (PDF) OF A PASSIVE SCALAR IN GRID TURBULENCE

Until our recent study (Jayesh and Warhaft 1991, 1992) there appeared to be no published experimental data on the details of the passive scalar pdf in homogeneous isotropic turbulence, particularly its tails that describe the higher-order moments. Possibly, this is due to it having been assumed that the scalar fluctuations are purely Gaussian, reflecting the velocity field, which early on was shown to have a Gaussian pdf, at least up to the fourth moment. Perhaps more pertinently, there has been no theory (until recently, see below) that suggested universality in the tails of the scalar pdf and thus experimental motivation has been lacking.

The principal impetus for our study came from the theory of Pumir, Shraiman and Siggia (1991). They argued, using a one-dimensional phenomenalogical model for a passive scalar advected by turbulence, that in the presence of a mean scalar gradient, the scalar pdf will have exponential tails but in the absence of the gradient (i.e. with uniform mean temperature) the scalar will have a Gaussian pdf. The two techniques we have developed over the years (the *mandoline*, Warhaft and Lumley (1978) and the *toaster* Sirivat and Warhaft 1983) were ideal to test their theory since the *mandoline* provides temperature fluctuations without a mean temperature gradient while the *toaster* can produce a linear temperature gradient. In our experiment (Jayesh and Warhaft 1991, 1992) we systematically varied the Reynolds number and other flow parameters.

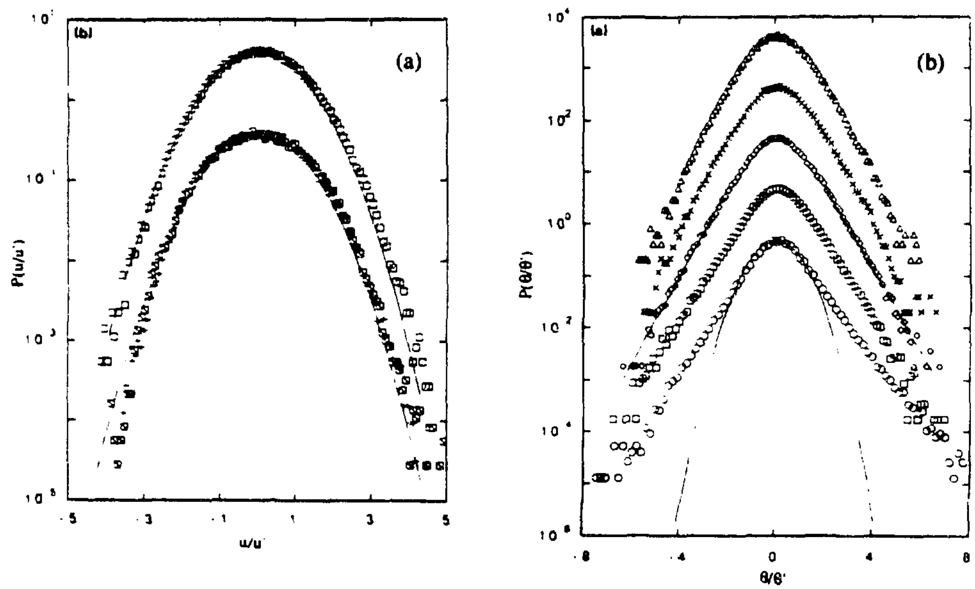

Figure 1 a) Pdf's of longitudinal velocity, u. Lower curves are at $x/M = 4$, upper curve is at $x/M = 62.4$. The solid line is Gaussian. The upper curve has been shifted one decade with respect to the lower one. b) Pdf of temperature at various x/M $U=8.9$ m/sec, $\beta=6.06$ K/m. $M=2.5$ cm. O:$x/M=36.4$;□ :$x/M=62.4$;◇ :$x/M=82.4$;✗ $x/M=102.4$;△ $x/M=132.4$. Each curve has been shifted one decade with respect to the lower one. A Gaussian curve is shown at $x/M=36.4$ and a straight line fit to the tails ($|\theta/\theta'|>2$) is shown at $x/M=82.4$. Here U is the mean speed, β is the temperature gradient and M is the mesh length. The pdf's have been nonnalized by the tempeature rms, θ'. Reproduced from Physics of Fluids A.

Figure 1 shows the velocity pdf and the temperature pdf in the presence of a mean scalar gradient. While the velocity pdf is Gaussian deep into its tails (note the logarithmic plot) the scalar pdf is distinctly non-Gaussian, showing exponential tails (linear on the log plot). Our study has shown that these tails weaken slightly with downstream distance (Figure 1b) but always remain broader than Gaussian. They were observed in the integral scale Reynolds number, Re_l, range $60 < Re_l < 1100$. On the other hand, in the absence of a mean gradient, the scalar pdf is close to Gaussian (Jayesh and Warhaft 1992). The qualitative difference between the gradient and no gradient case appears to provide confirmation of the Pumir, Shraiman, Siggia theory and, for the gradient case are consistent with recent work of Gollub et al. (1991) and Kerstein (1991).

We have also studied other statistics such as the conditional scalar dissipation rate, the pdf of the temperature derivative and the effect of filtering on the scalar pdf. These are described in Jayesh and Warhaft (1992).

Our findings should have particular significance in the fields of turbulent mixing and combustion since they show, for the linear profile case, that enhanced thermal dissipation occurs in the presence of the large, rare temperature fluctuations that are responsible for the extended tails of the pdf. Thus more rapid smearing (mixing at the molecular level) will occur, enhancing reaction and combustion rates.

2. TEMPERATURE SPECTRA IN GRID TURBULENCE

The Corrsin-Obukhov (Corrsin 1957, Obukhov 1949) extension of the Kolmogorov (1941) similarity theory shows that for high Reynolds numbers the spectrum of a passive scalar in the inertial subrange has the form

$$E_\theta(k)=\beta\varepsilon^{-1/3}\varepsilon_\theta k^{-5/3} \quad (1)$$

Here $E_\theta(k)$ is the one dimensional spectrum defined by $\overline{\theta^2} = \int E_\theta(k)dk$ where $\overline{\theta^2}$ is the scalar variance and k is the wave number in the x direction; β is a universal constant and ε and ε_θ are the average dissipation rate of energy and average rate at which $\overline{\theta^2}$ is smeared at the molecular diffusive scale respectively. Measurements indeed show scaling regions but their slope is dependent on the type of flow and the Reynolds number. Sreenivasan (1991) has compiled various data from shear flow experiments (wakes and jets and boundary layers, both in the laboratory and in the field) and shows that the scalar scaling exponent increases from about 1.3 for a micro-scale (Taylor) Reynolds number, Re_λ, of about 200 to about 1.63 at $Re_\lambda = 2000$. The data seem to suggest an asymptotic limit of -5/3 although the Reynolds numbers have not been high enough to properly confirm this. It appears that for these strongly anisotropic shear flows R_λ must be significantly greater than 2000 before a locally isotropic region is sufficiently well established to fulfill the similarity requirements of the Kolmogorov-Corrsin-Obukhov theory.

Recently we (Jayesh and Warhaft 1991, 1992) have employed both the *toaster* and the *mandoline* to study passive scalar fluctuations (principally the scalar probability density function (pdf) and related statistics) in grid turbulence. In that work, for the mean gradient experiment, we varied the mean speed, U, and the mesh length M thereby varying the integral scale Reynolds number, Re_l from 60 to 1,100. Here $Re_l \equiv ul/\nu$ where u is the rms longitudinal velocity, l is the turbulence integral scale (close in value to the mesh length, M) and ν is the kinematic viscosity. This corresponds to a significant (but modest) variation of the micro-scale Reynolds number, R_λ, ($\equiv u\lambda/\nu$, where λ is the Taylor micro-scale) from approximately 30 to 130. These experiments have provided a broad data set from which passive temperature spectra, as a function of Reynolds number, can be studied.

Figure 2(a) shows four temperature spectra, for different Reynolds numbers, using the *toaster* to generate the passive thermal fluctuations (i.e., in the presence of a mean temperature gradient). The Reynolds numbers are given in the figure caption. The spectra show that there is a scaling region (a region of constant slope on a log-log plot) and that it increases with width as the Reynolds number increases.

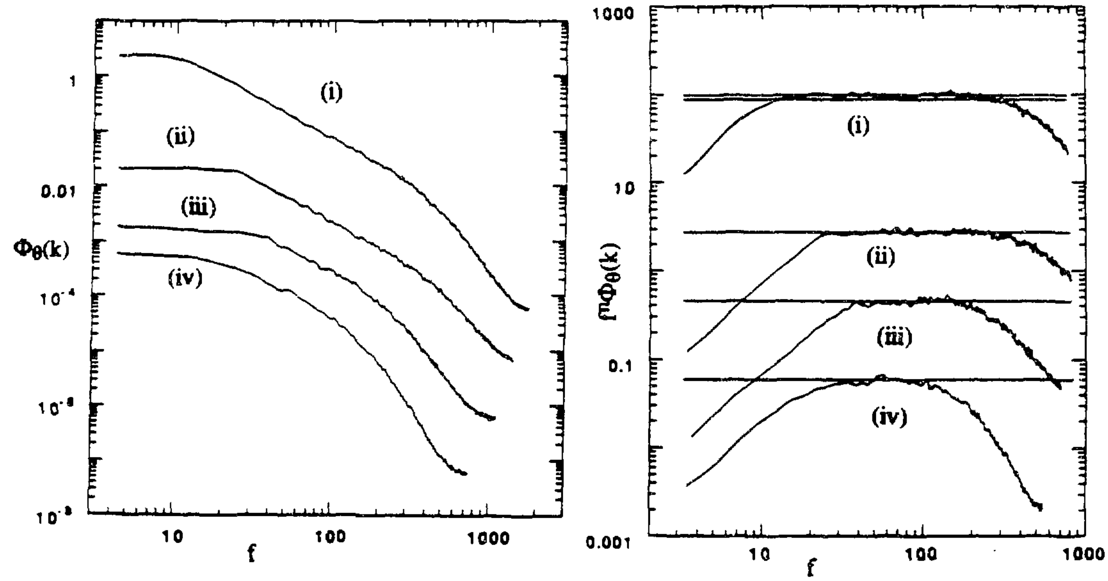

Figure 2 Temperature spectra in grid turbulence with a linear (passive) tempeature gradient. a) "Raw" spectra, b) spectra of a) multiplied by f^n where n is the slope of the scaling region in a). The Reynolds number for the four spectra are (i) $Re_l = 856$, (ii) $Re_l = 282$, (iii) $Re_l = 67$, (iv) $Re_l = 59$.

In order to more clearly display the scaling region we multiplied the ordinate of the raw spectra of Figure 1(a) by f^n (where f is the frequency and n is the scaling exponent). These spectra are shown in Figure 2(b). We determined n by fitting a least square best fit straight line to the scaling region of the raw spectra of Figure 2(a). The scaling region must be horizontal in the plot $f^n\phi_\theta(f)$ if the choice of n is correct. Figure 2(b) shows a clear scaling region of more than a decade for the high Re_l cases and no scaling region for the lowest Re_l case.

Figure 3 shows n as a function of Reynolds number for all of our experiments. Although there is quite a bit of scatter within each experiment (for a fixed Re_M), it is quite apparent that n does not have an overall variation with Reynolds number: its value for all the data was found to be 1.58 with a standard deviation of 0.07. Given the scatter the result is not inconsistent with the Kolmogorov scaling value of 1.67.

The data set of Figure 3 is mainly from the *toaster* experiments, for which there is a linear temperature profile. However we also obtained one data set for the *mandoline* and this gives the same scaling exponent as the toaster data (Figure 3). Note that although the velocity field is isotropic in both cases, the thermal field is not; for the mean gradient there is a heat flux and thus large scale anisotropy in the thermal field (Sirivat and Warhaft, 1983) while for the *mandoline* there is no heat flux (no mean gradient), suggesting approximate isotropy for the large scale thermal field (Warhaft and Lumley 1978). The same value of n obtained from these two different ways of creating the thermal field suggest it is the large scale structure of the velocity field (rather than the thermal field) that is relevant in determining the slope in the scaling region.

The width of the scaling region for all of our data as a function of Reynolds number is shown in Figure 4. The monotonic increase of the width with Reynolds number is consistent with fundamental notions of scaling (e.g. Tennekes and Lumley, Chapter 8) and implies that our data are not "anomalous" as was earlier thought from measurements done at a single Reynolds number. Note that here too, the *mandoline* data are consistent with the *toaster* data. Kolmogorov scaling shows that the width of the scaling region should increase as $Re_l^{3/4}$ since $l/\eta \sim Re_l^{3/4}$ where l is the integral scale and η is the dissipation scale (note for our experiment of Prandtl number 0.7, $\eta_\theta \sim \eta$ where is the thermal smearing scale). We have plotted a line of slope 0.75 on Figure 4 and it reasonably represents the trend in the data, given its scatter.

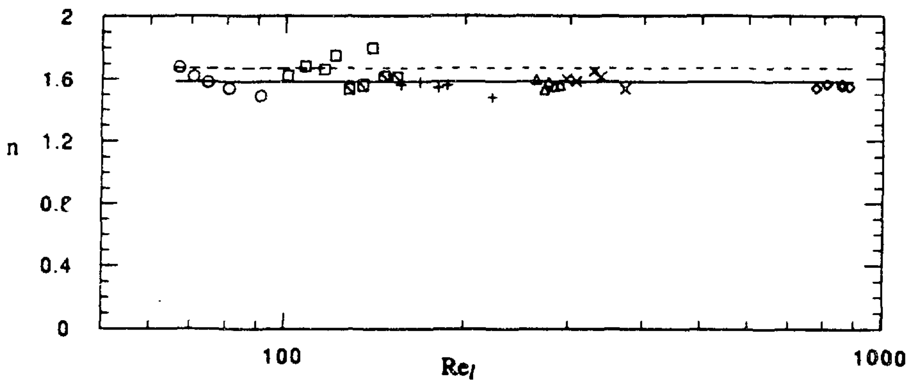

Figure 3. The slope of the scaling region for all experiments as a function of Re_l. The mean value is 1.58 (solid line). It is within one standard deviation of the Kolmogorov value of 5/3. The ◘ symbols are for no tempeature gradient (mandoline), all other symbols are for the toaster.

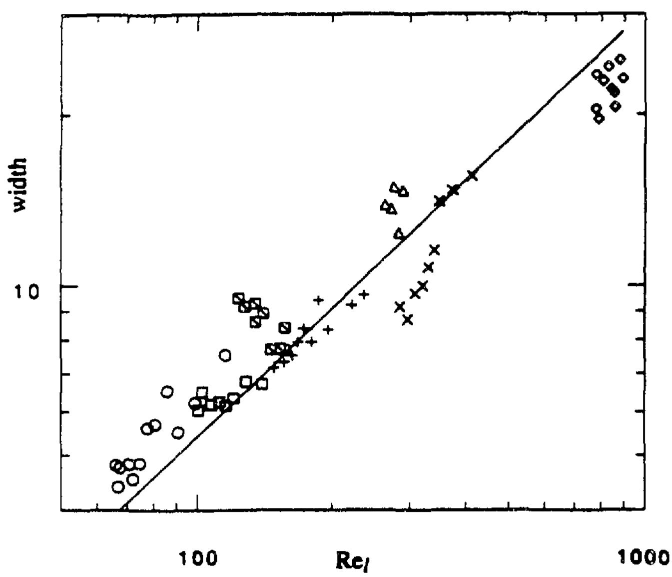

Figure 4. The width of the scaling region as a function of Re_l. The solid line has a slope of 0.75, the value predicted using Kolmogorov scaling: $l/\eta \sim Re_l^{0.75}$.

3. THERMAL DISPERSION AND MIXING IN A JET

Despite the importance of turbulent mixing in a jet, there appears to be no previous work on diffusion and dispersion from point or line sources; all previous experiments have introduced the scalar evenly throughout the flow (e.g. heating the jet (Corrsin and Uberoi 1980) or having a jet of pure species A mixing with the surroundings of pure species B (Dowling and Dimotakis 1990)). Thus there has been no information on how quickly one or two species mix in such a flow, an issue of fundamental importance. Here we extend the inference method of Warhaft 1984, which consisted of placing two line sources in grid turbulence, to placing two circular thin heated wire rings in a jet (Figure 5a). A single wire ring is analagous to a single line source; from it we can determine how long the fluctuations take to smear and fill the whole jet. Two line sources (or rings) enable us to determine, by inference, the cross correlation between the fluctuations, thereby providing information on the mixing rate of two independent species (Fig. 5b).

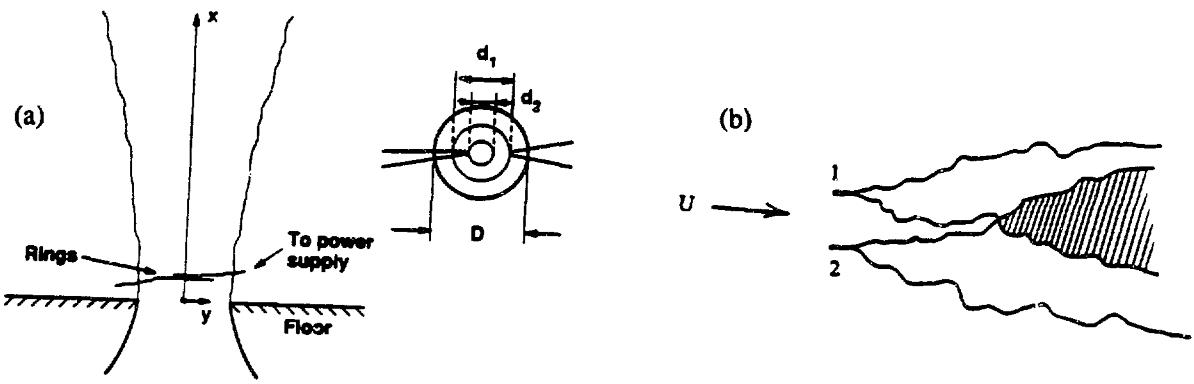

Figure 5a) Side and plan view of the jet of diameter D showing the fine wire rings (of diameters d_1 and d_2). The rings were placed concentrically above the jet, both in the same plane. They were suspended in the flow by means of their leads which were held by a clamp, outside the jet. The wires for the rings were 0.127 mm nichrome. b) Instantaneous thermal fields for two thermal line sources in a turbulent flow showing the region of overlap or interference (hatched region). This situation was studied by Warhaft 1984. Here we present the jet analogue using two fine wire rings.

As in our previous studies (Warhaft 1981, 1984) the cross correlation, ρ, between the thermal fields coming from each ring is determined by operating ring 1 and ring 2 separately and then operating them together. Under the assumptions of statistical stationarity, and that the scalar fields are passive, we determine the cross correlation from the relation

$$\rho = (\overline{\theta_b^2} - \overline{\theta_1^2} - \overline{\theta_2^2}) \Big/ 2\,(\overline{\theta_1^2})^{1/2}\,(\overline{\theta_2^2})^{1/2} \qquad (2)$$

where $\overline{\theta_b^2} = \overline{(\theta_1 + \theta_2)^2}$ is the variance with both rings operating and $\overline{\theta_1^2}$ and $\overline{\theta_2^2}$ are the variances of each ring operating separately. Figure 5b shows a sketch of a region of overlap (mixing of 2 scalars) for 2 line sources. Our interest is in determining ρ for this region, when two rings are used to generate the thermal field (Figure 5a). A practical realization of our experiment would be two species in concentric pipes, mixing in a jet formation (Kerstein 1990).

We have carried out a systematic investigation of ring placement which was varied relative to the jet exit, and of Reynolds number. We have also studied the effect of the rings on the flow since they tend to slightly suppress the velocity fluctuations by inhibiting vortex paring. Our studies will be reported in Tong and Warhaft 1993. Here we show the effect of initial conditions on the evolution of ρ and compare it to grid turbulence.

Figure 6a shows ρ vs. x/D (where D is the jet diameter) for rings placed concentrically at x/D=9, i.e. at the beginning of the turbulent region. D for the jet was 30mm and the Reynolds number was 18,000. Notice that in spite of the large early differences in ρ, they all asymptote to unity very quickly, by about x/D = 20, which is equivalent to an eddy turn over time $S\left(\equiv \int_{x_0}^{x} \frac{dx/U}{l/u}\right)$ of about 2. (Here U and u are the mean and fluctuating velocity respectively l, is the integral scale and x_0 is the virtual jet origin). On the other hand, in grid generated turbulence, without mean shear, the evolution time for ρ is very long. Figure 6b compares the jet to the grid turbulence. Notice for comparable wire spacing complete mixing has not occurred by 4 eddy turn over times for the grid turbulence showing the important role of mean shear (and anisotropy) in the mixing process.

A systematic study of the evolution of ρ as well as temperature spectra and cospectra, pdf's and conditional dissipation is given in Tong and Warhaft 1993.

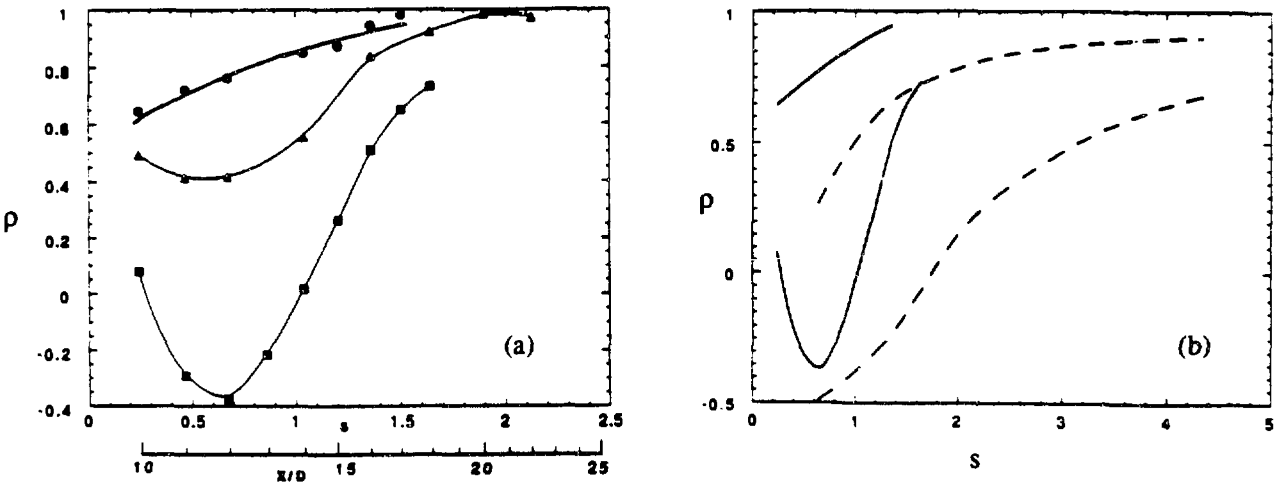

Figure 6a) ρ vs. x/D and S (the eddy turnover time) for the rings placed at x/D=9 for a 3 cm jet, Re_j= 1.8 x 10^4. The ring diameters for the circles are 35 and 40 mm, for the triangles they are 30 and 40 mm, and for the squares they are 20 and 40 mm. b) The data for the circles and squares of part a) compared with experiments carried out in grid turbulence for comparable wire spacing i.e. for comparable d/l where d is the distance between the wires and l is the integral scale of the turbulence (Warhaft 1984).

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