PARTICLE INTERACTIONS IN CONCENTRATED SUSPENSIONS

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ABSTRACT

An overview is presented of research that focuses on slow flows of suspensions in which colloidal and inertial effects are negligibly small. We describe nuclear magnetic resonance imaging experiments to quantitatively measure particle migration occurring in concentrated suspensions undergoing a flow with a nonuniform shear rate. These experiments address the issue of how the flow field affects the microstructure of suspensions. In order to understand the local viscosity in a suspension with such a flow-induced, spatially varying concentration, one must know how the viscosity of a *homogeneous* suspension depends on such variables as solids concentration and particle orientation. We suggest the technique of falling ball viscometry, using small balls, as a method to determine the effective viscosity of a suspension without affecting the original microstructure significantly. We also describe data from experiments in which the detailed fluctuations of a falling ball's velocity indicate the noncontinuum nature of the suspension and may lead to more insights into the effects of suspension microstructure on macroscopic properties. Finally, we briefly describe other experiments that can be performed in quiescent suspensions (in contrast to the use of conventional shear rotational viscometers) in order to learn more about boundary effects in concentrated suspensions.

INTRODUCTION

Many industrial processes include the transport of suspensions of solid particles in liquids, such as coal and other solid feedstock slurries. Oil, gas, and geothermal energy production rely on the transport of suspensions such as muds, cements, proppant, and gravel slurries in the drilling and completion of a well Suspensions are not only ubiquitous in energy production, but also in high-energy-consumption industrial processes such as found in pulp and paper manufacturing. The complex rheological response of suspensions often limit the efficiency of the design of such processes, causing loss of productivity, increased cost, and increased energy consumption. Because of the importance of particulate two-phase flows in the applications described above, the study of suspension rheology remains an important component of the technical research directed by a national energy policy.

This overview of our recent research supported by the Department of Energy, Office of Basic Energy Sciences, will focus on slow flows of suspensions of relatively large particles, in which colloidal and inertial effects are negligibly small. There is growing evidence that even in this restricted range of flows, the rheology of a suspension with a nondilute particle concentration cannot be characterized by a single viscosity. Instead, the microstructure of the suspension determines the overall macroscopic properties, and the flow history of the suspension determines aspects of the microstructure. (A good overview of flow-induced microstructural changes can be found in an article by Acrivos [1].) Hence, conventional viscometers, which impose macroscopic flow fields, may not measure the viscosity of the homogeneous suspension originally introduced into the viscometer, but rather may represent a property governed by the nonhomogeneous structure created by the flow. Such structures may be intrinsically different for various classes of flow fields associated with different viscometers.

Advances in the ability to predict the rheological response of concentrated suspensions depend on answering three broad questions: 1) How does the macroscopically imposed flow field affect the microstructure of a suspension? 2) How does the microstructure of a suspension affect the rheological properties? 3) How do boundaries, such as walls, affect the microstructure and properties? Aspects of these questions are being addressed in our work.

In the following section we will illustrate the existence of flow-induced microstructural changes with data on the time evolution of concentration and velocity profiles in suspensions undergoing flow between counter-rotating concentric cylinders (similar to the geometry found in "cup and bob" or "Couette" viscometers). We will show that the resultant profiles in these one-dimensional flows can be predicted well by the expressions describing "hydrodynamic diffusion" originally developed by Leighton and Acrivos [2, 3, 4]. However, additional phenomena arise in more complex flows, such as the two-dimensional migration of particles seen in the eccentric annular gap of a "journal bearing" flow. This illustrates that the complex interaction of particles cannot be adequately described by the one-dimensional theory originally proposed by Leighton and Acrivos, and it suggests that other avenues be taken to relate the macroscopic behavior to the evolution of microstructure. One such avenue recently suggested in the literature is to use a kinetic theory approach, which has been applied successfully in granular flows [5, 6]. In this theory the intensity of the velocity fluctuations, caused by particle interactions, is characterized by a "granular temperature" analogous to the temperature in classical kinetic theories and governed by a balance of fluctuation energy. Kinetic theory approaches emphasize the importance of obtaining experimental data not only on average behavior of suspensions but also on the fluctuations about those averages.

In the third section we will discuss the use of falling ball rheometry as a means to circumvent the problems encountered with using conventional rotational devices to measure suspension viscosity. If the size of the falling ball is of the order of the characteristic length of the suspended particles, the ball disturbs the original microstructure of the quiescent suspension only slightly as it falls. Hence, one can use this technique to determine the viscosity of a homogeneous suspension (or likewise one with any set microstructure). One can then incorporate this information into a constitutive relationship to determine the *local* viscosity in a flow field, given that the local concentration is known [4]. Furthermore, falling ball rheometry is not limited to the measurement of macroscopic average viscosities. The velocity fluctuations experienced by the falling ball can also be measured and can give insights into the importance of particle interactions.

We have also proposed use of quiescent suspensions in other apparatus to provide further insights into the rheological behavior of concentrated suspensions, especially the effects of boundaries. Rolling ball rheometry could be explored as a means to determine the effect on the local viscosity of the microstructure imposed by the wall. Measuring the torque on a ball spinning in an otherwise quiescent suspension has been proposed as a sensitive measure of slip at the wall. These ideas will be addressed in the fourth section of this article, and the results of preliminary measurements will be discussed. In the final section we will summarize our conclusions.

EFFECTS OF FLOW ON THE MICROSTRUCTURE OF SUSPENSIONS

Flow-induced migration and ordering of suspended particles have been hypothesized to create viscosity measurements that vary with the total strain to which a given suspension has been subjected [7, 3]. This migration is thought to occur whenever particle interactions are stronger or more frequent in one part of a flow field than in another, as could occur in the presence of spatially varying shear rate, concentration, or viscosity fields. A Newtonian fluid in the annular domain between rotating concentric cylinders (i.e. wide-gap Couette apparatus) possesses perhaps the simplest flow field of any realizable *nonhomogeneous* shear flow. As such, this is a useful device in which to study the effects of nonhomogeneous shear on the microstructure of a concentrated suspension.

The spatial distribution of suspended particles present in concentrated suspensions is difficult to measure because most suspensions are opaque even at relatively low particle concentrations. However, under the auspices of the Department of Energy, Office of Basic Energy Sciences, noninvasive techniques based on nuclear magnetic resonance (NMR) imaging have been developed by Fukushima and coworkers to study both concentration and velocity profiles in multiphase flows [8, 9]. We have employed these NMR imaging techniques to study the flow-induced migration of particles in a suspension undergoing flow in a wide-gap Couette apparatus. The details of the experiments can be found elsewhere [10, 11]. However, some results of these studies will be briefly discussed here in order to illustrate how dramatically a suspension's microstructure can be affected during flow.

The primary data obtained from these experiments are NMR images of the concentration and velocity fields. Representative examples of the concentration images are shown in Figure 1. As shown on the left, the initial image of a bimodal suspension (60 vol% neutrally buoyant spheres, of which 65% are 3.175 mm and 35% are 780 μm in diameter, in a viscous Newtonian liquid) is essentially uniform. Dark areas represent areas of high solid concentration. Individual large spheres can almost be distinguished, although the thickness of the imaged volume (2.4 cm) results in a blurring of the particles. After 3600 revolutions of the inner cylinder, the final image was taken, shown on the right. In this image the fluid fraction is significantly higher near the inner rod (the region of highest shear rate) and lower near the outer cylinder. It is apparent that the particles have migrated from the region of highest shear to the region of lowest. Furthermore, distinct shells of larger spheres, interspersed with fluid and smaller spheres, can be seen in this image. From visual observations of the particles near the outer wall of the apparatus, the larger spheres appear approximately hexagonal close packed within the outermost shell. That is, the arrangement of the large spheres is two-dimensional hexagonal close packed in concentric sheets. This structure begins to appear very quickly: significant migration can be detected after only 50 revolutions of the inner cylinder. It is important to note that this migration does not appear to be caused by inertial effects, which are negligible at the rates of rotation, the viscosity of the suspending liquid, and the particle sizes involved here. Subsequent experiments with suspensions of unimodal distributions of large spheres revealed that this shear-induced structure was not unique to bimodal suspensions.

The concentration can be quantified in any region of the image by recognizing that the fluid in the imaged slice gives a full-intensity signal and the particles give no signal. The normalized value of the image intensity is proportional to the density of the liquid phase protons in a volume element. By using an imaging sequence with a slice selective refocusing pulse, as proposed by Cho and coworkers [12], the relative phase shift can be made proportional to the velocity. By using such a technique we can also find the velocity in each region of an image of a flowing suspension. Figure 2 shows velocity measurements for a suspension of 50 vol% of 675 μm spheres undergoing flow in a wide-gap Couette apparatus after a steady-state microstructure has formed. The velocity profile falls off much more rapidly than in a Newtonian fluid (shown for comparison by the solid curve). The particle concentration approaches maximum packing near the outer wall, and the velocity profiles reveal that the suspension is almost stagnant in this region.

In addition to expanding our general understanding of the micromechanics of shear-induced migration, the primary purpose of the NMR imaging studies was to determine the dependence of the particle migration on a number of experimental parameters. These parameters included strain, shear rate, and viscosity of the suspending fluid, as well as concentration and diameter of the suspended particles. The results of a constitutive model based on Leighton and Acrivos' scaling arguments compared very favorably to these experimental results [4].

This constitutive model consists of both a Newtonian constitutive equation, in which the viscosity depends on the local particle volume fraction, and a diffusive equation that accounts for shear-induced particle migration. Two adjustable parameters arise in the diffusive equation, which describe the relative strength of the mechanisms for particle migration. These two rate parameters were taken to be constants and were evaluated by comparison to the experimental measurements of velocity and concentration profiles in the wide-gap Couette apparatus for one suspension at one strain. With these parameters fixed, predictions for particle concentration profiles were then compared to the experimental results for suspensions with a wide

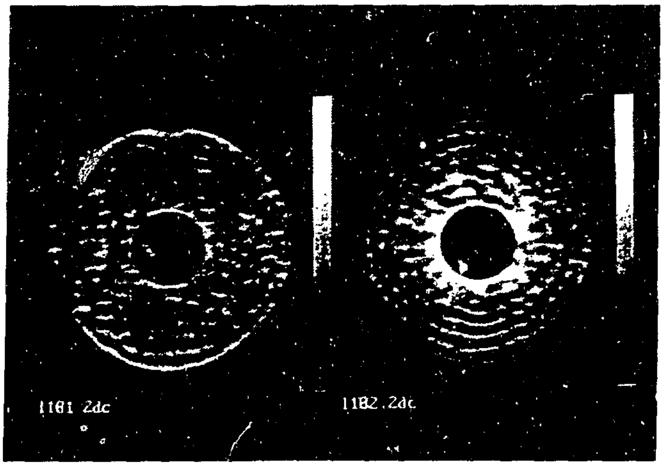

Figure 1: NMR images of a cross section of a suspension of 60 vol% bidisperse spheres between concentric cylinders. The image on the left represents the initially well dispersed state of the suspension. The image on the right was taken after rotating the inner cylinder until a steady state was achieved. The bright area near the inner cylinder represents a higher fluid fraction, indicating that the particles have migrated away from this area of higher shear rate.

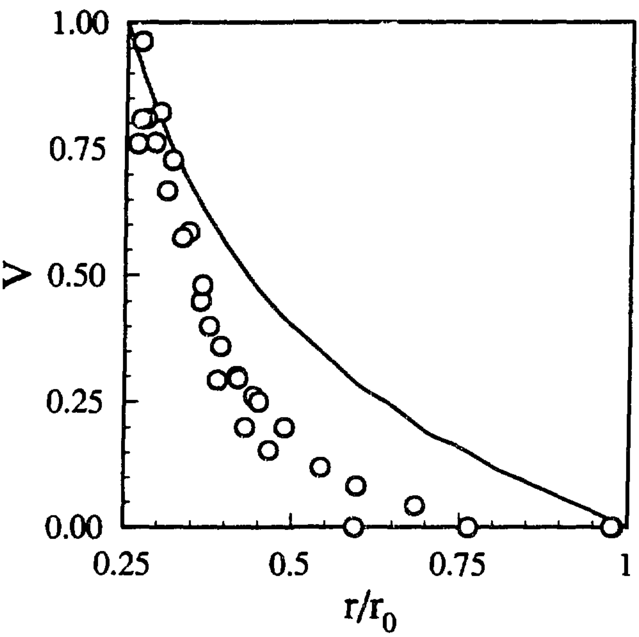

Figure 2: Steady-state velocity profile for a suspension of 50 vol % spheres with a mean diameter of 675 μm. The azimuthal velocity was measured along one diameter of the image. The solid curve is the velocity profile for a Newtonian fluid.

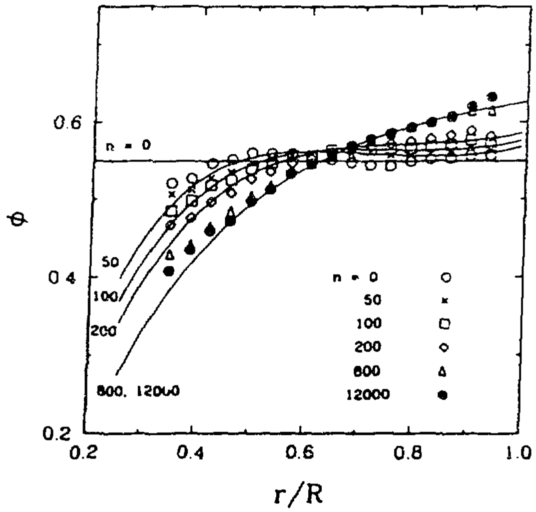

Figure 3: Transient profiles of the particle concentration for a suspension of 55 vol% particles with a mean diameter of 675 μm compared to predictions. Results are shown for the initial profile (o) and after the *number of revolutions of the inner* cylinder (n) equals 50 (×), 100 (□), 200 (◇), 800 (△), and 12000 (•). Model parameters used are those that best fit the data at n=200. Both model and data show that steady state is reached by the time the inner cylinder has rotated about 800 times.

range of particle sizes and concentrations.

Figure 3 shows the remarkably good comparison between the predictions of this model and the transient concentration profiles obtained for a suspension of 675 μm spheres at a volume fraction of 0.55. Figure 4 shows the steady-state concentration profiles for suspensions of either 100 or 675 μm particles compared with the predictions. The agreement between model and experiment is again excellent, with the theory fitting the experimental data for the suspension of 100 μm particles nearly as well as it did for the suspension of much larger particles used to calculate the two rate parameters.

Excitement generated by these results must be tempered by the results of ongoing research in more complicated two-dimensional flows. NMR imaging has also been used to study the flow of concentrated suspensions in the gap between a rotating inner cylinder placed eccentrically within an outer fixed cylinder (a journal bearing). With a Newtonian fluid, the majority of the flow will be in a cell concentric with the inner cylinder; however, with certain placements of the inner cylinder, a second cell, which rotates in the opposite direction, forms near the region of the outer wall furthest from the inner cylinder [13]. We have used NMR imaging to confirm that similar behavior occurs in concentrated suspensions. Here, particle migration creates a region of maximum solids concentration in the low-shear-rate region of the second cell (away from the wall).

The constitutive expression previously described was subsequently expanded to two-dimensional flows by describing the flow in terms of the strain rate tensor **D** and the migration in terms of gradients in the generalized shear rate $\dot{\gamma}=\sqrt{2\mathrm{tr}\mathbf{D}^2}$. The equation set was then again solved numerically and the predictions compared to the NMR imaging data. Unfortunately, this model failed to predict that the steady-state maximum concentration is not always at the outer wall, but in many cases is at a location within the gap.

The failure of the simple expression for one-dimensional hydrodynamic diffusion to capture the qualitative nature of this two-dimensional flow suggests that it has not been appropriately generalized to multiple dimensions and that other avenues should be explored in attempting to relate the macroscopic behavior to the evolution of microstructure. One such avenue recently suggested is to use a kinetic theory approach, which has been applied successfully in granular flows [5, 6]. In this theory the intensity of the velocity fluctuations, caused by particle interactions, is characterized by a "granular temperature" analogous to the

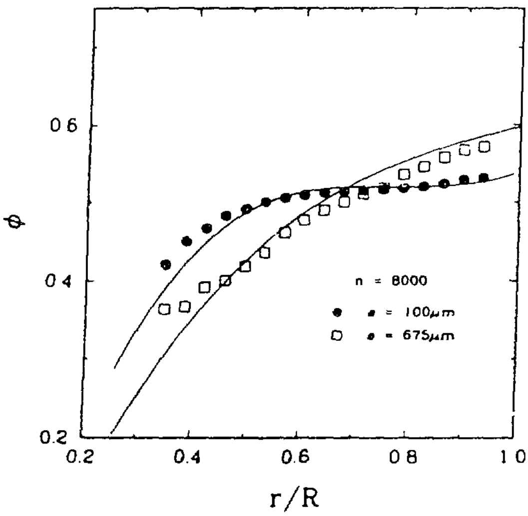

Figure 4: Measurements and predictions of the concentration profiles for suspensions of 50 vol% of either 100 μm ($\bullet$) or 675 μm ($\square$) particles. Both results are shown for 8000 rotations of the inner cylinder. Model parameters are unchanged from those used in Figure 3.

temperature in classical kinetic theories and governed by a balance of fluctuation energy. This approach emphasizes the importance of measuring not only average behavior of suspensions but the details of the fluctuations about those averages.

FALLING-BALL RHEOMETRY IN SUSPENSIONS

In previous work, we have shown that falling ball rheometry is an excellent tool to probe the rheological properties of a suspension without changing the properties through the very act of measuring them. Unlike conventional viscometers, which employ flow fields that tend to influence the microstructure of the suspension, falling ball rheometry can be used to determine the macroscopic viscosity of a suspension with little effect on the microstructure [14].

The discrete nature of the suspension is readily apparent in falling ball experiments. One expects a very large ball to fall smoothly through a suspension of tiny particles and its velocity to appear fairly constant. However, when we observe the passage of a ball of the same diameter as large suspended particles, we see that actually the velocity is not constant. Periods of almost no motion, as the falling ball approaches and "rolls off" suspended particles, alternate with periods of almost free fall in the interstices between suspended spheres. However, a statistical analysis reveals that the *average* terminal velocity of the ball, measured over a distance usually between 100 and 1000 suspended particle diameters, is reproducible. Furthermore, if this average terminal velocity, corrected for Newtonian wall effects, is translated into a viscosity, this viscosity is independent of the diameter of the falling ball relative to the diameter of the suspended particles over a wide range of falling-ball sizes.

We are also exploring the possibility of using the fluctuations in the terminal velocity, as the ball interacts with individual suspended particles or clusters of suspended particles, to give information about the suspension microstructure. Whereas the mean settling velocity predicts the continuum behavior of the suspension, the dispersivity around the mean velocity allows insight into the non-continuum behavior of the suspension caused by the presence of the macroscopic suspended spheres.

We performed experiments which focused on the three-dimensional dispersion of a single ball settling through a suspension of neutrally buoyant particles. The detailed paths of falling balls were recorded from direct observations in transparent suspensions (in which the refractive index of the suspending fluid was matched to that of the suspended particles) and using real-time radiography in opaque suspensions. The primary experimental parameters were the relative size of the settling ball and suspended particles, and the concentration and geometry of the suspended particles.

A principal objective of these experiments was to test whether the observed variations in the ball's settling velocity were the result of a Fickian (random) process. For sufficiently long times, the variances grew linearly with time, as predicted for a Fickian process. Because the falling ball in these experiments was similar in size to the suspended particles, it was possible to see the transition between the deterministic effects of a sphere settling past a particular arrangement of particles and the random process associated with a sphere settling past many such arrangements. The deterministic effects resulted in a quadratic growth in the variances for short times. The short-time nonlinear variances of Brownian tracer particles can be described in terms of particle inertia [15]. However, the short-time behavior of a ball falling in a suspension was not caused by the inertia of the falling ball, but rather by the time needed for the ball to change its local environment. This conclusion was supported experimentally by the observed insensitivity of the dispersive behavior to the falling ball's density. To determine the dispersivity when the variances were deterministic, we estimated the time scale on which it takes the settling ball to change its local neighborhood. (Because the settling ball tends to drag the suspension along with it, this time scale is greater than the time to travel just a few ball diameters.) This time scale was then used in a model, similar to that for Brownian tracers, relating the measured short-time variances to the dispersivity.

The resulting dimensionless vertical dispersivities are shown in Figure 5 as functions of falling ball size and volume concentration of suspended spheres. For moderately concentrated suspensions, the vertical dispersivity decreased with increasing ball size, but always less rapidly than predicted by Davis and Hill for dilute suspensions [16]. This effect decreased with concentration, until, for a suspension with a solids volume concentration of 50%, the dispersivity was independent of ball size. At a constant size of falling ball relative to the suspended spheres, the vertical dispersivity increased approximately linearly with concentration. For example, for falling balls the same size as the suspended spheres, the dimensionless vertical dispersivity D^* was observed to depend on the volume fraction of solids, ϕ, as $D^* = 0.60\phi^{1.08}$. The measured horizontal dispersivity was at least 25 times smaller than its vertical counterpart (and below the experimental resolution for suspensions with only 15 vol% solids).

Vertical dispersivity was also measured in suspensions of randomly oriented rods. Here, the dispersivity was always virtually independent of ball size (however, one should note that the balls tested were always significantly larger than the rod diameter). The vertical dispersivity in these suspensions increased linearly with the specific viscosity. Because the viscosity was approximately linear with volume fraction for the suspensions tested [17], this parallels the theoretical behavior in dilute suspensions of spheres [16].

We have recently begun to observe not only the velocity of, but also the pressure drop across a ball falling in a quiescent suspension. For a ball falling in a single-phase Newtonian liquid, this pressure is constant, independent of viscosity (at low Reynolds numbers), and can be described analytically [18]. Although the pressure drop is independent of viscosity in a Newtonian liquid, it is reasonable to speculate that in a suspension the pressure drop may be dependent on the microstructure. Therefore, like the local viscosity, it may vary due to the discrete nature of the material.

We first tested this speculation by modeling a falling ball rheometer numerically with the boundary element method [19]. In this technique, the boundary integral equations for Stokes flow, coupled with the equilibrium equations for the solid particles, are discretized and solved numerically. Fully three-dimensional simulations of suspensions of spheres in a Newtonian liquid bounded by cylindrical walls were performed. The number and size of individual suspended spheres were varied to give volume concentrations ranging from zero to 5%. In these simulations, the pressure drop was influenced only weakly, if at all, by the introduction of neutrally buoyant particles. Furthermore, the arrangement of particles affected the pressure drop negligibly. In contrast, the relative viscosity in these simulations varied by over 10%. These results were consistent with the hypothesis that, despite the noncontinuum nature of these suspensions, each suspension could be treated as a Newtonian liquid with an effective relative viscosity. It also implied that the fluctuations in the velocity of the falling ball would be far more indicative of microstructural variations than would the corresponding fluctuations in pressure drop. However, the suspensions studied were all relatively dilute, and

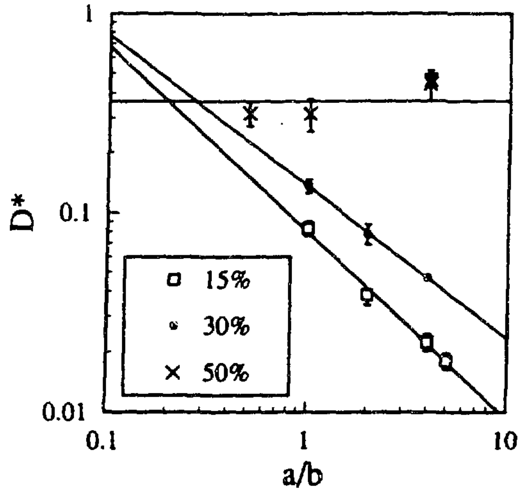

Figure 5: The dimensionless dispersivity as a function of the ratio of the radius of the settling ball (a) to the radius of the suspended spheres (b) for various volume concentrations of solids: 15 vol% (□), 30 vol% (•), or 50 vol% (×). The error limits shown are based on the nonlinear analysis which assumes that the time scale is known exactly, so they represent a minimum for the actual 95% confidence limits.

this behavior may not be present at high concentrations.

In laboratory experiments we have begun to measure the pressure drop across balls of various sizes falling in more concentrated suspensions. Preliminary data indicate that the pressure drop occurring in a suspension made with 30 vol% uniformly sized spheres is identical to that predicted to occur in a single-phase Newtonian liquid. Like the viscosity measurements in moderately concentrated suspensions, the pressure drop behavior is independent of the relative sizes of the falling ball and suspended spheres. (However, the absolute measure of pressure drop in both single- and two-phase fluids is directly proportional to the weight of the falling ball.) Experiments are planned to look at suspensions of higher concentrations, as well as at falling balls much smaller than the suspended spheres.

OTHER EXPERIMENTS IN QUIESCENT SUSPENSIONS

In the falling ball experiments described in the section above, the drag on the ball appeared to be that found in a Newtonian liquid with *no slip* at the boundaries. Instead of measuring the mean velocity of a falling ball, we could instead measure the mean torque on a spinning ball. This geometry is more sensitive to slip at the ball boundary. Whereas the force F on a ball moving slowly through an unbounded Newtonian liquid without slip can be described as $F = 6\pi\mu av$ (where μ is the viscosity of the liquid and a and v are the radius and velocity of the ball, respectively), the force with perfect slip is $4\pi\mu av$. In contrast, the torque T on a ball spinning slowly in a Newtonian liquid is $8\pi\mu a^3\Omega$ (where Ω is the angular velocity of the ball): however, the torque on a ball with perfect slip at the boundaries is zero [20].

Kunesh and coworkers studied the torque on balls spinning in single-phase Newtonian liquids, verified the formula above, and quantified the effects of the free surface [21]. We propose similar experiments to measure the torque on balls spinning in otherwise quiescent suspensions. We will analyze the data for any apparent slip at the balls' boundaries.

Figure 6 is an illustration of the apparatus we have built with this goal in mind. A calibrated torque wire holds a ball on a rod while the suspension is rotated on a motorized platform. The number of rotations

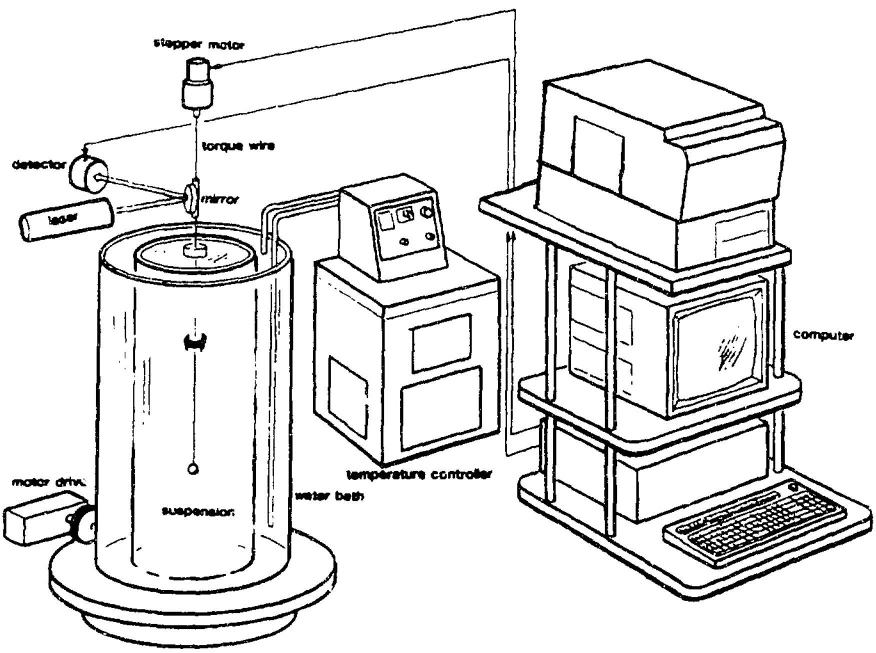

Figure 6: Apparatus to measure the torque on a ball spinning in a suspension.

of the platform is recorded automatically. A mirror at the bottom of the torque wire reflects a laser beam to a detector. The data from the detector is fed to a stepper motor holding the torque wire from above. The controller on the stepper motor uses this information to correct the position of the stepper motor and torque wire so that the reflected laser beam is maintained at the center of the detector. The corrections are recorded so that one knows how much the torque wire has twisted as a function of time. The twist can be directly related to the torque experienced by the ball and rod. We plan to measure this torque as a function of Ω on balls of various sizes relative to the suspended particles and in suspensions of various concentrations.

One must note that this flow geometry has the potential to induce structure over time. This will result in a decrease of the measured torque over time. However, the apparatus will allow us to record any time history associated with the torque, and the short-time behavior should still be an indication of any apparent slip at the ball's surface. The time history obtained will also allow us to study the fluctuations of the torque about the mean, which again may be indicative of the suspension microstructure.

The presence of the walls of a container can also induce structure. Another proposed study is of the behavior of a ball rolling down the wall of a inclined container holding a concentrated suspension. In rolling ball viscometry, a dense ball is allowed to roll/slide down an inclined surface, and its rate of travel is compared to that in a fluid of known viscosity. In a suspension, we can ratio the time it takes for the ball to travel a known distance in the suspension to that in the suspending liquid alone and, from this ratio, estimate the apparent relative viscosity. This procedure is similar to estimating the apparent viscosity with falling ball viscometry; however, the immediate region of the suspension seen by the moving ball is not uniform but has structure determined by the proximity of the bounding wall.

Preliminary studies have been performed with a suspension of 30 vol% 0.318-cm-diameter spheres neutrally buoyant in a viscous Newtonian liquid. The mean velocity of balls of three sizes ranging from 0.238 to 1.905 cm were first measured as they rolled down an 11° incline in the suspending liquid alone. Then the suspended particles were added, the suspension mixed well, and the measurements repeated. Again, as in the falling ball study, the moderately concentrated suspension behaved as a single-phase Newtonian liquid with an effective viscosity. The viscosity implied by the mean velocities of the rolling balls was independent of the ball size and was statistically indistinguishable from that measured in the falling ball experiments.

If one assumes that the suspension microstructure closest to the ball influences most the ball's velocity, then these results imply that in a moderately concentrated suspension, the microstructure near the bounding walls is similar to that in the bulk suspension. (The assumption of nearest-neighbor domination has been shown to be a good one in boundary element method calculations of the effect of neutrally buoyant particles various distances from a falling ball. Here, particles beyond about 5 ball diameters away exerted negligible influence on the ball's velocity. In other words, the ball fell at the same velocity whether or not the far-field particles were present [19].) Further experiments in very concentrated suspensions are planned. Here, the walls are more likely to induce structure (as seen with NMR imaging near the outer walls of the wide-gap Couette apparatus after the particles have migrated and concentrated to near maximum packing).

CONCLUSIONS

We have performed a variety of experimental and numerical studies to elucidate the linkage between the microstructure and the macroscopically observed responses of suspensions of particles in liquids. NMR imaging studies and visual observations have confirmed that a suspension's microstructure can change dramatically during flow: large concentration gradients can be formed from regions of low shear rate to regions of high shear rate, ordered structure can form at the walls in regions of high concentration, particles of aspect ratio greater than 1.0 can align, etc. Conventional rotational viscometers may induce such changes in the microstructure over time, and, therefore, the data from them may not be accurate measurements of the viscosity of the suspension originally introduced into the viscometer. In fact, a suspension cannot be simply described by a single effective viscosity.

Falling ball viscometers, on the other hand, can be used (with small falling balls) to determine an apparent viscosity of a *homogeneous* suspension, without significantly affecting the microstructure during the measurement. Such a measurement can then be combined with information about the evolving microstructure in a flow to predict the spatial variations in viscosity and the global behavior. However, further studies of the details of particle interactions are needed before definitive predictive capabilities can be developed. Measurement of the detailed fluctuations of the velocity of a ball falling through a suspension is an example of one such study.

Quiescent suspensions can also be used to examine effects of boundaries. We propose to complete two such studies: measurements of the torque on a spinning ball and the drag on a ball rolling down a wall. The former should be a more sensitive measure of apparent slip at the ball boundaries. The latter may elucidate the effect of structure induced by the proximity of walls.

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