AN ANALYTICAL THEORY OF TRANSMISSION LINE SHIELDING

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Abstract - The classical electrogeometric model of shielding failure flashovers on transmission lines is investigated by analytical methods. Most of the basic elements that has appeared in the literature on the subject have been incorporated and put into a comprehensive model. These elements are: tower top geometry, structure height above ground, line insulation, lateral slope of ground, probability distribution of lightning currents, ratio of striking distances to ground wire and earth relative to conductor, and probability distribution of lightning leader approach angle to ground. Departing from a basic idealistic case, the sensitivity of the model to variations in these parameters is studied. Numerical examples are given.

Key-words - Lightning, transmission line, shielding failures, electrogeometrics.

1. INTRODUCTION

The first firm evidence on the extent of lightning by-passing the shield provided by tower top ground wires and striking directly on the conductors, goes back to the time before World War II, [1]. It was found, by extensive application of magnetic links on the German HV-grid, that shielding angles about 45° gave inadequate protection. The definite recognition of shielding failures, however, did not come until the first 345 kV-lines were erected in US. The lightning trip-out rates of these lines, designed with shielding angles about 30°, were an order of magnitude higher than expected. Ensuing studies were mainly addressing the backflash failure mode, but [2], based on electrogeometrics, showed that shielding failures could as well be a substantial failure mode. Finally, failure mode indicating instruments were installed on some lines, [3] and [4], and mainly shielding failures were found to be at the root of the bad lightning performance. Shielding angles that were adequate for the earlier lower structures were found to be quite insufficient for the new double-circuit lines of heights about 50 m. Later on 15° was decided to be the criterion angle for such lines.

Though still much is unknown about this elusive phenomenon, current design praxis, based on empirical grounds, will handle the problem in normal cases. This does not, however, preclude further study. Anomalous lightning performance of some shielded lines around the world is still puzzling.

The aim of this paper is to review the basic elements of the electrogeometric model (EGM) as it has been applied to shielding failures. By the systematically analytical approach made here, it is intended that the subject will be better comprehended.

Through the years more knowledge has been gained on the probability distribution of lightning currents, which is an essential element of the problem. We now know that the distributions used in the earliest studies were compromised by upward flashes due to very high collecting locations, which acted exaggerating on the lower end tail of the distribution. Here the log-normal distribution will be used. Besides its being commonly recognized from improved measurements, this distribution lends itself superbly to analytical studies.

A glossary of symbols appears at the end of the paper.

2. CONCEPTS

2.1 Electrogeometrics

Classical EGM assumes the lightning leader tip to approach ground in a straight line manner until any grounded object comes within "striking distance". The first such object met will be the termination point of the flash. The striking distance is assumed to be related to the peak current of the ensuing return stroke through the charge in the lightning channel. This relation between striking distance and current is quantified in a functional form, and various expressions have been proposed through the years.

Some inherently statistical quantities have to be modelled. Firstly, the current, which from numerous measurements has been found to vary within a wide range. By its relation to current, the striking distance will also vary statistically. Thus two random variables, I for current and S for striking distance are introduced. Secondly, though there is actually not much known on its statistical distribution, the leader approach angle random variable Ψ also exerting an influence on shielding failures.

It is often assumed that a flash has different striking distances to conductor, ground wire and earth. The model considered here makes use of factors k_c, k_g and k_e so that

$$S_c=k_cS\ ,\ S_g=k_gS\ ,\ S_e=k_eS \qquad (2.1)$$

where S_c, S_g and S_e are the random striking distances to conductor, ground wire, and earth plane respectively. Without loss of generality we can set $k_c=1$ and postulate that $k_g\geq1$ and $k_e\leq1$.

2.2 Shielding failure flash-over calculations

Figure 1 illustrates the exposure of a line to lightning using EGM. Hits to the arc AB will hit the conductor. Hits to the arc to the left of A will hit the ground wire, and hits to the right of B will hit the earth plane. Leaders aiming into the ground zone, here called exposure zone, of width w indicated in the figure will hit the conductor. The quantity w is a function of $s=s_c$ and ψ.

For a fixed value on s, the mean value of w with respect to Ψ, $w(s,\cdot)$, is given by

$$w(s,\cdot)=\int_{-\pi/2}^{\delta}w(s,\psi)f_\Psi(\psi)d\psi \qquad (2.2)$$

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where δ, which is a function of s, is defined in the figure, and where $f_\Psi(\psi)$ is the probability density function of Ψ. For the case $k_g=1$, it is seen that $\delta=\theta$.

Two specific s-values are now introduced. Firstly, there is a specific lightning current i_1 which, if injected into the conductor, is required for flash-over to occur. Through the relationship between current and striking distance, this insulation coordination current, or critical current, will correspond to a certain striking distance s_1. Secondly, there will be a certain s-value, s_2, for which points A and B meet. Clearly w is zero for s larger than the limit s_2. The current i_2 corresponding to s_2 is the upper limit for the currents that can possibly be injected into the conductor at a shielding failure occasion. If $i_1 \geq i_2$, then the line will be perfectly shielded.

The shielding failure outage rate N_{sf} per 100 km of line and year for a ground flash density of N_g strikes per km^2 and year is given by

$$N_{sf}=0.2N_g w(\cdot,\cdot) \quad (2.3)$$

where the mean width $w(\cdot,\cdot)$ is given by

$$w(\cdot,\cdot)=\int_{s_1}^{s_2} w(s,\cdot)f_S(s)ds \quad (2.4)$$

where f_S is the probability density function of S. It has been assumed that the tower is symmetric. The general case can be handled in an obvious way.

3. GEOMETRY

3.1 Exposure zone width

The auxiliary quantities u_e and u_g are introduced in Figure 2. Here u_e is associated with the shielding effect of the earth plane and u_g with that of the ground wire. It is seen that

$$w(s,\psi)=[c\sin(\theta-\psi)+u_e(s,\psi)-u_g(s,\psi)]/\cos\psi \quad (3.1)$$

It thus remains to establish u_e and u_g.

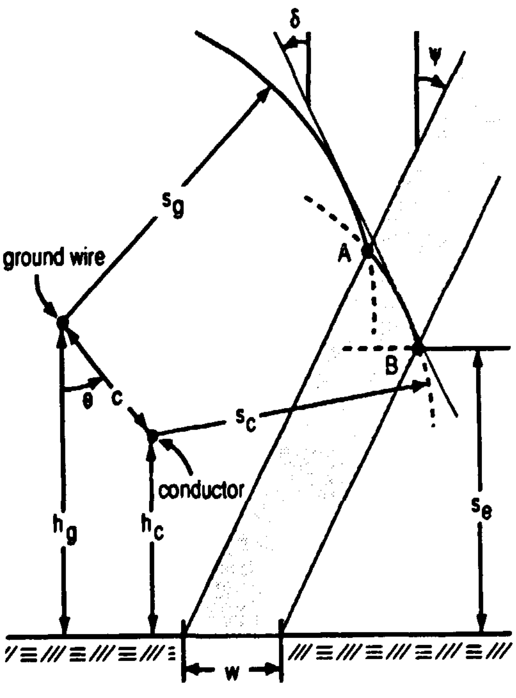

Figure 1 Definition of exposure width

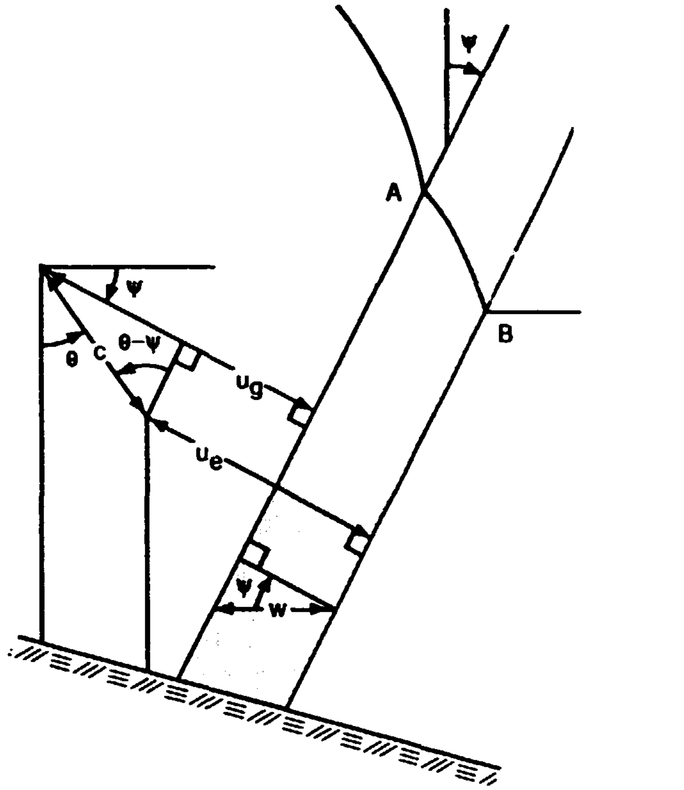

Figure 2 Decomposition of exposure zone width

3.2 Ground wire shielding effect

Figure 3 illustrates the situation. The location of end point A of the exposure arc is, for fixed values on s_g, s_c and ψ, given by the system of equations

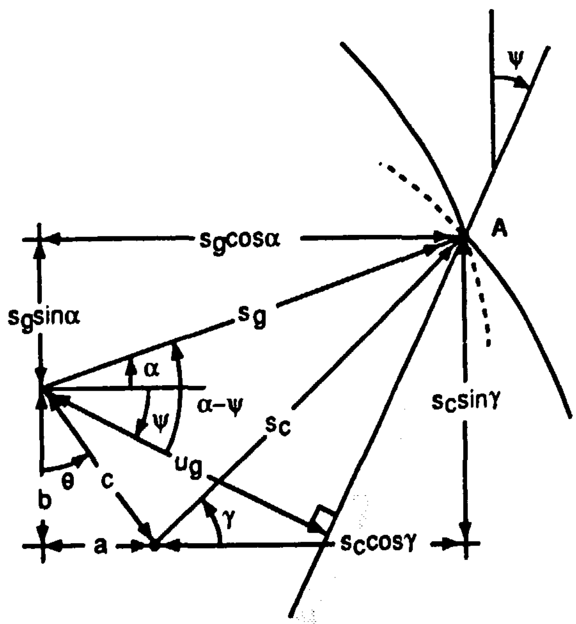

Figure 3 Ground wire shielding effect analysis

$$a+s_c\cos\gamma=s_g\cos\alpha \qquad (3.2)$$

$$b+s_g\sin\alpha=s_c\sin\gamma$$

with the solution

$$\sin(\theta-\alpha)=(c/2s_g)[1+(s_g/c)^2-(s_c/c)^2] \qquad (3.3)$$

$$\sin(\theta-\gamma)=-(c/2s_c)[1-(s_g/c)^2+(s_c/c)^2]$$

For $k_g=1$, which is assumed in the body of the analysis, the bracket factors are equal to unity. The simplified equations that result can in fact be deduced directly by observing that $\theta-\alpha$ and $\gamma-\theta$ must equal half the angle of the apex of the isosceles triangula formed by conductor, ground wire, and point A. With α so determined, it is seen that

$$u_g=s_g\cos(\alpha-\psi)\,,\ -\pi/2<\psi<\alpha \qquad (3.4)$$

$$u_g=s_g\ ,\ \alpha<\psi<\delta$$

3.3 Earth surface shielding effect

For the sake of generality, the earth surface in the vicinity of the line is supposed to have a lateral slope. From Figure 4, the location of end point B of the exposure arc must satisfy the equation

$$h_c\cos\eta+s_c\sin(\beta-\eta)=s_e \qquad (3.5)$$

which can be written

$$\sin(\beta-\eta)=(s_e-h_c\cos\eta)/s_c \qquad (3.6)$$

from which β is determined. Further

$$u_e=s_c\cos(\beta-\psi)\,,\ -\pi/2<\psi<\beta \qquad (3.7)$$

$$u_c=s_c\ ,\ \beta<\psi<\delta$$

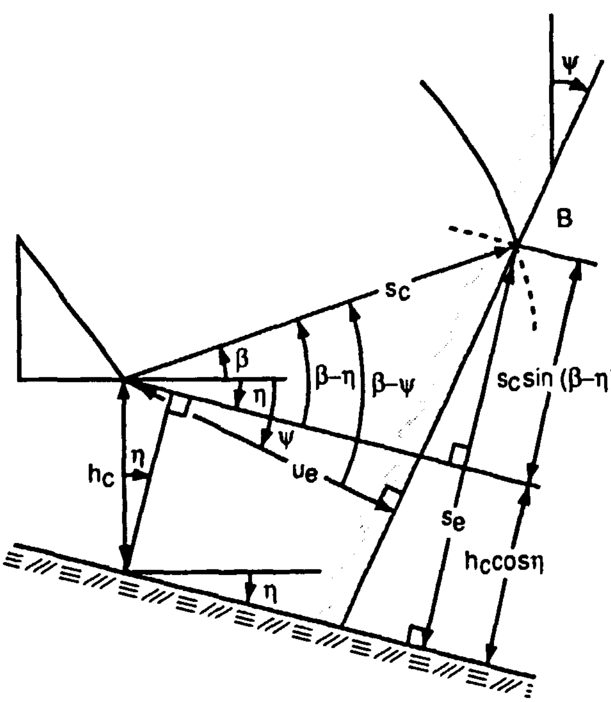

Figure 4 Earth surface shielding effect analysis

3.4 Condition for vanishing exposure arc

It is noted that β is increasing in s, tending to $\eta+\arcsin k_e$, while γ is decreasing with limit θ. Thus, assuming $\eta+\arcsin k_e>\theta$, an s-value s_2 is granted such that $\gamma(s_2)=\beta(s_2)$. This means that points A and B coincide whereby the exposure arc vanishes implying that $w(s_2,\psi)=0$ for all ψ.

Assuming that $k_g=1$ and $\eta=0$, combining (3.3) with (3.6) gives, after some elementary calculations, a quadratic equation in s with the solution

$$s_2=[k_e h_{am}+h_{gm}\sin\theta(1+r)^{1/2}]/(k_e^2-\sin^2\theta) \qquad (3.8)$$

$$h_{am}=(h_c+h_g)/2\ ,\ h_{gm}=(h_c h_g)^{1/2}$$

$$r=(1-k_e^2)(c/2h_{gm})^2$$

Similarly for the case with $k_g=1$, $c=0$ and $\eta\neq 0$ we have the solution

$$s_2=h\cos\eta/[k_e-\sin(\theta-\eta)] \qquad (3.9)$$

where $h=h_c=h_g$. The cases with $k_g>1$ and/or $c\neq 0$ and $\eta\neq 0$ simultaneously are more difficult to treat since fourth degree equations in s are involved. Though such equations are possible to solve analytically, the expressions for the roots of them will be quite voluminous, and further analysis will not be pursued here.

It should be pointed out that the condition $c=0$ is practically impossible, of course, but only serves to simplify the analysis by eliminating a second order factor, as will be shown later.

4. PROBABILITY

4.1 Leader approach angle

Here only the case $k_g=1$ will be studied. Combining (2.2) and (3.1), using (3.4) and (3.7), $w(s,\cdot)$ can be expressed as

$$w(s,\cdot)=cd+s[g(\alpha)-g(\beta)] \qquad (4.1)$$

$$d=\int_{-\pi/2}^{\theta}[\sin(\theta-\psi)/\cos\psi]f_\Psi(\psi)d\psi$$

$$g(\varepsilon)=\int_{-\pi/2}^{\varepsilon}\{[1-\cos(\varepsilon-\psi)]/\cos\psi\}f_\Psi(\psi)d\psi$$

For a Dirac delta-function type $f_\Psi(\psi)$ centered at any fixed point ψ, this is seen by inspection of (3.4) and (3.7). Then (4.1) will hold for any $f_\Psi(\psi)$, because any function can be approximated by a linear combinations of delta-functions.

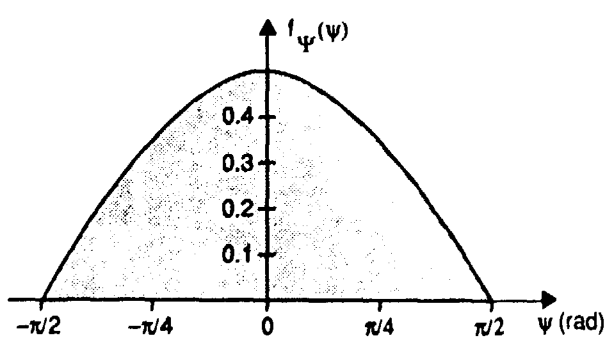

Figure 5 Probability density function of leader ground approach angle

Now, d and $g(\varepsilon)$ can be expressed analytically for any probability density function of the type

$$f_\psi(\psi)=k_m\cos^m\psi\ ,\ -\pi/2<\psi<\pi/2 \tag{4.2}$$

where m is a positive real number and k_m a normalization constant making the total probability equal to unity. In fact, this is the standard function used in this context, though its excellent analytical properties do not seem to have been utilized hitherto. The case $m=\infty$ corresponds to identically vertical approach angles. Beside this case, only the case m=1 with $k_1=1/2$ will be regarded here. These two cases are commonly considered to be the extreme cases of the actually not very well known angle distribution. Figure 5 shows $f_\psi(\psi)$ for m=1.

It is found that for $m=\infty$

$$d=\sin\theta \tag{4.3}$$

$$g(\varepsilon)=1-\cos\theta\ ,\ -\pi/2<\varepsilon<0$$

$$g(\varepsilon)=1-\cos\varepsilon\ ,\ 0<\varepsilon<\theta$$

and for m=1

$$d=(1+\sin\theta)/2 \tag{4.4}$$

$$g(\varepsilon)=(\varepsilon-\cos\varepsilon)/2\ ,\ -\pi/2<\varepsilon<\theta$$

with ε given in radians.

4.2 Striking distance

The lightning peak current is assumed to be log-normally distributed, i.e.

$$F_I(i)=\Phi[\sigma_I^{-1}\ln(i/\bar{I})] \tag{4.5}$$

where $F_I(i)$ is the probability distribution function of I, $\bar{I}$ the median value of I, and σ_I the standard deviation of lnI.

Further the striking distance is assumed to be related to the current through a power function relationship i.e.

$$S=vI^n \tag{4.6}$$

where v and n are constants. Then also S will be a log-normally distributed random variable with median value $\bar{S}$ and standard deviation of lnS, σ_S, given by

$$\bar{S}=v\bar{I}^n\ ;\ \sigma_S=n\sigma_I \tag{4.7}$$

In the numerical examples to come, for I given in kA and S in m, we use v=8 and n=0.65, [5], together with $\bar{I}$=26kA and σ_I=0.6, [6]. This gives $\bar{S}$=67 m and σ_S=0.39. The functions $F_S(s)$ and $f_S(s)$ are shown in Figure 6.

To calculate $w(\cdot,\cdot)$, integrals of the following type have to be evaluated

$$\int_{s_1}^{s_2} s\,g[\varepsilon(s)]f_S(s)ds \tag{4.8}$$

with $\varepsilon=\alpha$ or $\varepsilon=\beta$ and g as of above. As (4.8) cannot be integrated analytically for all the actual functions, resort to approximations has to be made. Actually only power functions can be handled, and for these the following relation applies, [6]

$$\int_{s_1}^{s_2} s^q f_S(s)ds=\bar{S}^q\exp[(q\sigma_S)^2/2][\Phi(z_2)-\Phi(z_1)] \tag{4.9}$$

$$z_j=\sigma_S^{-1}\ln(s_j/\bar{S})-q\sigma_S\ ,\ j=1,2$$

where q is any real number.

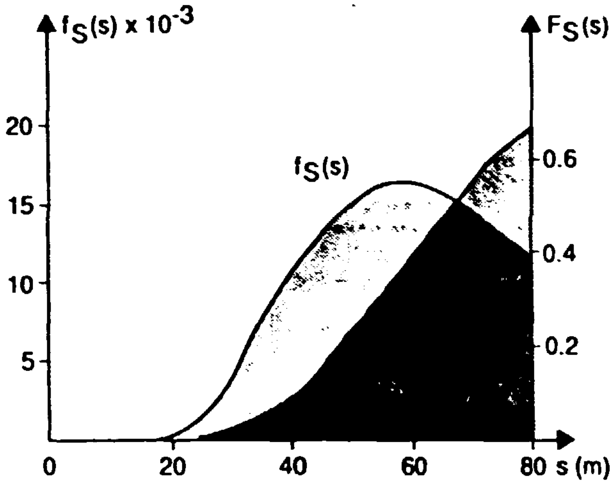

Figure 6 Probability density and distribution functions of striking distance

Then, once $g[\varepsilon(s)]$ has been approximated by a sum of power functions in s, there will be a corresponding approximation of the initial integral. The approach made here is to design an approximation that is asymptotically correct, in some sense, for s-values near s_2, the point at which w(s) vanishes. Here w(s) is an abbreviated notation for $w(s,\cdot)$. The simplest approximation satisfying these requirements is

$$w(s)\approx w'(s_2)(s-s_2) \tag{4.10}$$

which gives the correct derivative at s_2. This is the first non-vanishing term of the Taylor expansion, and more terms could be added if a higher accuracy is required. In the applications to follow, it will be shown that this will not normally be necessary.

In the sequel $w'(s_2)$ will be calculated for various cases. It will be simplifying to note that

$$w'(s_2)=-cd/s_2+s_2\{\alpha'(s_2)g'[\alpha(s_2)]-\beta'(s_2)g'[\beta(s_2)]\} \tag{4.11}$$

which follows from (4.1) by differentiation and insertion of s_2, noting the condition $w(s_2)=0$. The derivatives of α and β are determined by derivation with respect to s of their constitutive relations in Section 3.

5. BASIC CASE

By the terminology introduced above, the case with $m=\infty$, $\eta=0$, $k_g=k_e=1$, c=0 will be referred to as the basic case. This means vertical leader approach, level ground, equal striking distances, and that the separation between conductor and ground wire is neglected. The study will be confined to the case where the coordination striking distance $s_1>h$, $h=h_c=h_g$. For a coordination current of i_1=8kA, (4.7) gives s_1=31m, which is a typical tower height for single-circuit EHV lines.

From above we have $\alpha(s)=\theta$ identically and that

$$s_2=h/(1-\sin\theta) \tag{5.1}$$

$$w(s)=s(\cos\beta-\cos\theta)\ ,\ \sin\beta=1-h/s$$

$$w'(s_2)=-h\tan\theta/s_2$$

Figure 7 demonstrates the validity of the approximation of w(s) introduced in the preceding section for the case $\theta=30^\circ$. The linear approximation is obviously very good for s>h, which is the interesting range.

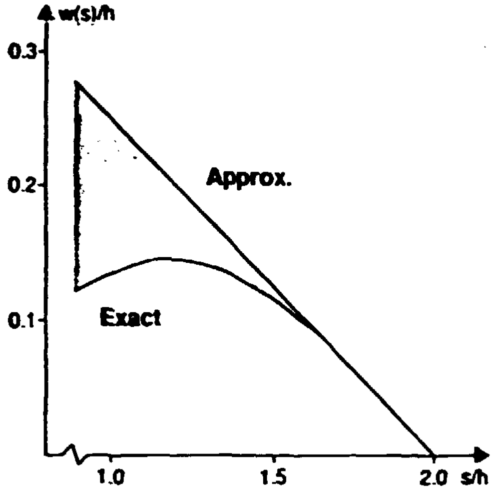

Figure 7 Validity of the linear approximation of the exposure width. $\theta=30°$ and $c=0$

The resulting approximation for w, using (4.9), is

$$w \approx h\tan\theta\{[\Phi(z_2)-\Phi(z_1)]-(\bar{S}/s_2)\exp(\sigma_S^2/2)[\Phi(z_2-\sigma_S)-\Phi(z_1-\sigma_S)]\} \quad (5.2)$$

$$z_j=\sigma_S^{-1}\ln(s_j/\bar{S})\ ,\ j=1;2$$

Now, we are in a position to calculate N_{sf} from (2.3). The result is shown in Table 1 for some combinations of θ, h and i_1. It is seen that the effect of variation of height is huge, actually more prominent than that of shielding angle. Insulation is influencing N_{sf} very little.

Table 1 Basic model estimates on shielding failure flashover rate per 100 km and year. Ground flash density: 1 per km^2 and year

Height (m)		25		37.5	
Critical current (kA)		6	8	6	8
Shielding angle	15°	0.02	0.01	0.21	0.19
	30°	0.11	0.09	0.58	0.51

It should be pointed out that only a simple hand-calculator and a Normal distribution table are needed for the calculations.

6. VARIATIONS OF THE BASIC CASE

6.1 General approach

The line followed here is initially to choose a combination of h and θ for the basic case so that its s_2 matches the one of the perturbed case. Using this combination for evaluating $w'(s_2)$ of the basic case will not then result in the correct value. This is remedied by application of a correction factor. Then, up to the validity of the Taylor approximation (4.10), N_{sf} of the two cases will agree, since the integrals in (2.4) will be identical.

6.2 Approach angle distribution

Since s_2 is independent of the angle distribution, the value for the basic case applies. Further, since the shielding provided by the earth plane is not affected by the distribution, the $\beta(s)$ of the basic case applies. We find

$$s_2=h/(1-\sin\theta) \quad (6.1)$$

$$w(s)=s[(\cos\beta-\cos\theta)+(\theta-\beta)]/2\ ,\ \sin\beta=1-h/s$$

$$w'(s_2)=-h\tan\theta[½(1+\sin\theta)/\sin\theta]/s_2$$

Thus, compared to the basic case, the shielding failure outage rate will be greater by a factor about $½(1+\sin\theta)/\sin\theta$. For $\theta=30°$, this factor is exactly 1.5, meaning 50% more failures. The ratio is progressively increasing for decreasing θ and vice versa.

6.3 Striking distance to earth plane

This affects the earth plane shielding and thereby $\beta(s)$. Here

$$s_2=h/(k_e-\sin\theta) \quad (6.2)$$

$$w(s)=s(\cos\beta-\cos\theta)\ ,\ \sin\beta=k_e-h/s$$

$$w'(s_2)=-h\tan\theta/s_2$$

To investigate the effect of the striking distance to earth plane, we are led to use the height $h(1-\sin\theta)/(k_e-\sin\theta)$ in the basic case. However, $w'(s_2)$ of the basic case will then be too large and the estimation would consequently have to be reduced by the given factor to match the correct value. Practically this reduction is a second order factor and the main impact of k_e comes from the effective height expansion. For example, if $k_e=0.9$ and $\theta=30°$, the effective height will be 25% larger than the physical one, which means much more to N_{sf} than just an increase by 25%.

6.4 Slope of earth plane

Only the earth surface shielding is of concern. Here

$$s_2=h\cos\eta/[1-\sin(\theta-\eta)] \quad (6.3)$$

$$w(s)=s(\cos\beta-\cos\theta)\ ,\ \sin(\beta-\eta)=1-h\cos\eta/s$$

$$w'(s_2)=-h\cos\eta\tan(\theta-\eta)[\sin\theta/\sin(\theta-\eta)]/s_2$$

We are led to try the basic case with shielding angle $\theta-\eta$ and height $h\cos\eta$, since these values results in the right s_2. However, the resulting $w'(s_2)$ for the basic case will, for negative η, be too large by a factor of $\sin(\theta-\eta)/\sin\theta$. Thus for negative η, which is the most interesting case, the basic case will give an overestimation of the shielding failure outages, which however is practically negligible as the change in shielding angle means much more. In the same vein the actual height does not have to be adjusted since $\cos\eta$ is very close to unity for practical slopes. In summary, earth plane slope means a simple adjustment of the shielding angle by the slope angle.

6.5 Conductor to ground wire separation

Combining the results of Section 3 and 4 we have

$$s_2=\cos^{-2}\theta(h_{am}+h_{gm}\sin\theta)\approx h_{am}/(1-\sin\theta) \quad (6.4)$$

$$w(s)=a+s(\cos\beta-\cos\alpha)$$

$$\sin\beta=1-h_c/s\ ;\ \sin(\theta-\alpha)=c/2s$$

$$w'(s_2)=\{-a+½\sin\alpha(s_2)/\cos[\theta-\alpha(s_2)]-h_c\tan\beta(s_2)\}/s_2\approx$$

$$\approx-h_{am}\tan\theta\{1+½b/[h_{am}\sin\theta(1+\sin\theta)]\}/s_2$$

This suggests that the case be handled by the basic case using $h=h_{am}$ together with the actual θ. The correction of concern will be minor because $\frac{1}{2}b/[h_{am}\sin\theta(1+\sin\theta)]$ is much smaller than unity for practical lines. For $\theta=30°$, $b=7$ m, and $h_{am}=25$ m, the correction will be only 19%. The correction is progressively increasing for decreasing θ.

7. OTHER MODELS

In recent years efforts have been made to improve the classical EGM by giving a better representation of the electrical phenomena, especially by introducing an upward counter-leader starting from the grounded object. Some of these models have been formulated within the frame of EGM by letting the striking distance be dependent on height, and some of the models have been applied to shielding of overhead lines. We will look at two recent approaches.

In Eriksson [7] and Rizk [8], the current and height dependent striking distance has the form

$$s=\kappa i^n h^p \quad (7.1)$$

with κ, n, and p equal to 0.67, 0.74, and 0.6 and respectively 1.57, 0.69, and 0.45. The w-functions, however, are different. In [7] effectively

$$w(s)=a-s[(h_g/h_c)^p-1], \; \alpha<0 \quad (7.2)$$

$$w(s)=a-s[(h_g/h_c)^p\cos\alpha-1], \; \alpha>0$$

where α is given by (3.3). In [8]

$$w(s)=a-s[(h_g/h_c)^p-1] \quad (7.3)$$

independent of α.

Certainly also these models lend themselves to simplified analyses, basicly along the same lines as for the classical model. It is expecially interesting to note that if theoretically $c\to 0$ (corresponding to the basic case for the classical model) then the common expression

$$s_2 = h\tan\theta/p \quad (7.4)$$

will result. As this is much lower than $s_2 = h/(1-\sin\theta)$ for the classical model, a closer comparison of the three models feels warranted. In Figure 8 the actual w-functions are drawn alongside for the case $\theta=30°$, $c=8$ m and $h_{am}=37.5$ m.

The surprising conclusion drawn is that both of the newer models lead to much lower w-functions compared to that of the classical model. This would lead to much lower N_{sf} estimates. For coordination current $i_1=8$ kA, s_1 would be 26 m for Eriksson's model, 32 m for Rizk's model, and 31 m for the classical model, which does not mean a major difference. Comparing the f_S-functions that were actually used in the papers, no significant difference from the one used here is found. The newer models then ought to give much lower N_{sf} estimates than the classical model, which is known to be reasonably accurate. The relative difference would be an order of magnitude.

Checking the named papers, it appears that none of them actually did calculate N_{sf} using the methods advocated. In [7] it seems that only the condition for perfect shielding was calculated by the model, and that the N_{sf}-calculations were based on an extraneous formula, which effectively means that N_{sf} is set equal to the frequency of all hits to the line having current between i_1 and i_2. In [8] only the condition for perfect shielding was addressed.

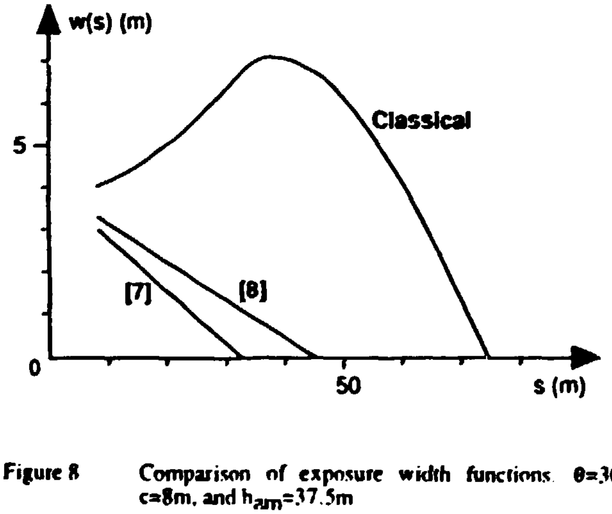

Figure 8 Comparison of exposure width functions. $\theta=30°$, $c=8$m, and $h_{am}=37.5$m

8. CONCLUSION

It is found that the study of shielding failure flash-overs using the classical electrogeometric model (EGM) can be conducted by entirely analytical methods. This makes possible a systematic and comprehensive investigation of the implications on the flash-over rate of the many parameters involved. These parameters are mainly: tower top geometry, structure height above ground, line insulation, lateral ground slope, probability distribution of lightning currents, ratio of striking distances to ground wire and earth relative to conductor, and probability distribution of lightning leader ground approach angle.

The various cases are studied departing from a basic case where the lightning leader approaches a level and flat ground, having equal striking distances to conductor, ground wire and earth, and where the separation of conductor and shield wire is neglected. All cases were found to be possible to be brought back to this situation. The study showed the effect on the flash-over rate from assumption on a rather adverse leader approach angle distribution to be quite moderate. Moreover, assumption on a lower striking distance to the earth surface was found to increase the effective height of the line, while a sloping surface was found to affect the effective shielding angle. For all practical cases, the separation of conductor and ground wire is a second order factor and can be neglected without a significant loss of accuracy. The effects of all these factors were possible to assess quantitatively.

Calculations for typical EHV-lines have been made on the basic case. It is found that height is a more dominant factor than shielding angle. The influence of insulation was found minor.

Two recently proposed variants of the classical EGM have been investigated. Both of them seemed to give much lower outage rate predictions compared to those of the classical model.

GLOSSARY OF SYMBOLS

a	Tower top distance defined in Figure 3
b	Do
c	Conductor to ground wire separation
$f_X(x)$	Probability density function of random variable X
$F_X(x)$	Probability distribution function of random variable X
h	Height above the earth
h_{am}	Arithmetic mean of heights
h_{gm}	Geometric mean of heights
i	Variable current value
i_1	Insulation coordination current
i_2	Current corresponding to s_2
I	Lightning current random variable
k	Striking distance coefficient
N_g	Ground flash density (per km^2 and year)
N_{sf}	Shielding failure flashover rate (per 100 km and year)
s	Variable striking distance value
s_1	Striking distance corresponding to current i_1
s_2	Maximum striking distance for shield penetration
S	Striking distance random variable
u	Auxiliary distance defined in Figure 2
w	Exposure zone width

α	Auxiliary angle defined in Figure 3
β	Auxiliary angle defined in Figure 4
γ	Auxiliary angle defined in Figure 3
δ	Auxiliary angle defined in Figure 1
Φ	Normal probability distribution function
η	Angle of lateral slope
θ	Shielding angle defined in Figure 1
ψ	Variable leader ground approach angle
Ψ	Leader ground approach angle random variable

For log-normally distributed variables X we denote

$\bar{X}$	Median value of X
σ_X	Standard deviation of ln X

Indices c, g and e denote conductor, ground wire and earth surface respectively.

In the figures, angles are positive if indicated anti-clockwise, otherwise negative.

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