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PROPAGATION OF WAVES AT THE LOOSELY BONDED INTERFACE OF TWO POROUS ELASTIC HALF-SPACES

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ABSTRACT

Employing Biot's theory for wave propagation in porous solids, the propagation of waves at the loosely bonded interface between two poroelastic half-spaces is examined theoretically. The analogous study of Stoneley waves for smooth interface and bonded interface form a limiting case. The results due to classical theory are shown as a special case.

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1 Introduction

Wave propagation in elastic media is of great interest in seismology and geophysics, and a substantial literature on this subject is available in Ewing *et al.* [1]. Several detailed accounts of wave propagation is given by Kolsky [2] and Miklowitz [3].

Surface waves will appear on the plane where two half–spaces of different elastic properties are joined, and such waves were usually referred to as Stoneley waves. It is usually assumed that the two half–spaces in contact with each other along a plane interface is one of bonded contact. This assumption implies that the displacements and stresses are continuous across the interface. The other case is of two half–spaces in smooth contact implies that the shearing stress vanishes at the interface while the normal displacement and normal stress are continuous. The continuity of displacement parallel to the interface cannot be imposed in view of the fact that a smooth interface will allow a finite amount of slip for the material motion. Stoneley waves were possible only if the material properties of two solids satisfy certain restrictive conditions [4]. The acoustic behaviour of loosely bonded interface of two elastic half–spaces is proposed by Murty [5] under an additional assumption that the slip at the interface is proportional to the local shearing stress.

In the present analysis, the study of propagation of waves at the loosely bonded interface of two poroelastic half–spaces is examined analytically following Murty [5]. The investigation is useful in non–destructive testing and in the study of dislocations in solids. This model is formulated using the basic formulation of stress waves in a liquid–filled porous media due to Biot [6]. Biot's model consists of an elastic matrix permeated by a net work of interconnected spaces saturated with liquid. An account of further researches based on this theory is given by Paria [7]. A historical review of the formulation of porous media theories is given by Boer *et al.* [8]. Using this theory, Tajuddin [9–13] examined some aspects of surface waves in porous solids, for example, Rayleigh waves, Stoneley waves and the dynamic interaction of a layer and a half–space.

In the considered problem, it is assumed that both the half–spaces are homogeneous, isotropic and linearly elastic. The frequency equation is derived and discussed in the limiting case. The analogous study of Stoneley waves for bonded and smooth interface form a limiting case of the present study, each for pervious and impervious surface [12]. Throughout our analysis, we show how the results of some earlier works follow as particular cases of the more general results presented here.

2 Governing Equations

The field equations of a poroelastic solid in the presence of dissipation b are [6]

$$\begin{aligned} N\nabla^2\vec{u} + (A+N)\nabla e + Q\nabla \in &= \frac{\partial^2}{\partial t^2}\,(\rho_{11}\,\vec{u} + \rho_{12}\,\vec{U}) + b\,\frac{\partial}{\partial t}\,(\vec{u}-\vec{U}), \\ Q\nabla e + R\nabla \in &= \frac{\partial^2}{\partial t^2}\,(\rho_{12}\,\vec{u} + \rho_{22}\,\vec{U}) - b\,\frac{\partial}{\partial t}\,(\vec{u}-\vec{U}). \end{aligned} \qquad (1)$$

In (1), ∇^2 is the Laplacian operator, $\vec{u}(u,w), \vec{U}(U,W)$ are solid and liquid displacements; $e, \in$ are the dilatations of solid and liquid; A, N, Q, R are poroelastic constants; and ρ_{ij} are mass coefficients following Biot [6]. The relevant solid stresses σ_{ij} and liquid pressure

s are

$$\begin{aligned} \sigma_{ij} &= 2Ne_{ij} + (Ae + Q \in)\, \delta_{ij}, \quad (i,j = 1,2,3) \\ s &= Qe + R \in, \end{aligned} \tag{2}$$

where δ_{ij} is the Kronecker delta function.

3 Solution of the Problem

Consider the waves on the plane where two homogeneous, isotropic poroelastic half–spaces are joined. Let the half–spaces be separated by a horizontal plane $z = 0$ of rectangular coordinate system with the origin at the interface, x–axis in the direction of propagation and z–axis into the interior of the lower half–space. The solid displacement functions $\vec{u}(u, w)$ which can be readily evaluated from (1) representing the plane harmonic waves travelling in the x–direction, are

$$\begin{aligned} u &= -ik\left[Be^{pz} + De^{-pz} + Ee^{qz} + Fe^{-qz} - i\delta(Le^{\gamma z} - Me^{-\gamma z})\right] e^{i(\omega t - kx)}, \\ w &= k\left[\alpha(Be^{pz} - De^{-pz}) + \beta(e^{qz} - Fe^{-qz}) - i(Le^{\gamma z} + Me^{-\gamma z})\right] e^{i(\omega t - kx)}, \end{aligned} \tag{3}$$

where ω is the frequency of wave, k is the wavenumber, B, D, E, F, L, M are all constants and α, β, δ are

$$\alpha = pk^{-1} = (1 - c^2 c_{fd}^{-2})^{1/2}, \quad \beta = qk^{-1} = (1 - c^2 c_{sd}^{-2})^{1/2}, \quad \delta = \gamma k^{-1} = (1 - c^2 c_s^{-2})^{1/2}, \tag{4}$$

where $c = \omega k^{-1}$ and c_{fd}, c_{sd}, c_s are dilatational wave velocities of first and second kind and shear wave velocity, respectively [6]. By substituting the displacement solutions u, w in (2), the relevant stresses are

$$\begin{aligned} \sigma_{zz} &= k^2[G(Be^{pz} + De^{-pz}) + S(Ee^{qz} + Fe^{-qz}) - 2Ni\delta(Le^{\gamma z} - Me^{-\gamma z})]e^{i(\omega t - kx)}, \\ \sigma_{zx} &= -Nik^2[2\alpha(Be^{pz} - De^{-pz}) + 2\beta(Ee^{qz} - Fe^{-qz}) - i(1 + \delta^2)(Le^{\gamma z} + Me^{-\gamma z})]e^{i(\omega t - kx)}, \\ s &= k^2[\phi(Be^{pz} + De^{-pz}) + \chi(Ee^{qz} + Fe^{-qz})]e^{i(\omega t - kx)}, \end{aligned} \tag{5}$$

where G, S, ϕ, χ, ξ^2 and η^2 are

$$\begin{aligned} G &= 2N\alpha^2 + (\alpha^2 - 1)(P - 2N + Q\xi^2), \quad S = 2N\beta^2 + (\beta^2 - 1)(P - 2N + Q\eta^2), \\ \phi &= (\alpha^2 - 1)(Q + R\xi^2), \quad \chi = (\beta^2 - 1)(Q + R\eta^2), \\ \xi^2 &= \frac{(PR - Q^2) - c_{fd}^2(RM_{11} - QM_{12})}{c_{fd}^2(RM_{12} - QM_{22})}, \quad \eta^2 = \frac{(PR - Q^2) - c_{sd}^2(RM_{11} - QM_{12})}{c_{sd}^2(RM_{12} - QM_{22})}. \end{aligned} \tag{6}$$

In the above, $P(= A + 2N)$ is a poroelastic constant and M_{11}, M_{12}, M_{22} are

$$M_{11} = \rho_{11} - ib\omega^{-1}, \quad M_{12} = \rho_{12} + ib\omega^{-1}, \quad M_{22} = \rho_{22} - ib\omega^{-1}. \tag{7}$$

4 Boundary Conditions – Frequency Equations

In (3)–(7), the subscripts 1 and 2 are used for dependent variables and material parameters in the upper and lower half–spaces, respectively. Further, the displacements and stresses must decrease with increasing distances from the plane $z = 0$ in the lower half–space. Thus the equations relating phase velocities and wavenumbers can be obtained using (3) and (5) with aforesaid conditions to write the equations of both half–spaces into the boundary conditions. For simplicity, a case of a non–dissipative medium ($b = 0$) is considered, then the phase velocity c will be ωk^{-1}.

Following the analogy of Murty [5], the boundary conditions at the loosely bonded interface are as follows:

at $z = 0$,

$$\begin{aligned} (\sigma_{zz}+s)_1 =& \ (\sigma_{zz}+s)_2, (\sigma_{zx})_1 = (\sigma_{zx})_2, w_1 = w_2, \\ (\sigma_{xz})_2 =& \ ckN_2(u_2-u_1)\psi(1-\psi)^{-1}c_{s2}^{-1}, \quad (0 \le \psi \le 1) \end{aligned} \tag{8a}$$

$$(s)_1 = 0, (s)_1 = 0, \tag{8b}$$

$$\left(\frac{\partial s}{\partial z}\right)_1 = 0, \left(\frac{\partial s}{\partial z}\right)_2, \tag{8c}$$

where the subscripts 1 and 2 relate to the upper and lower half–spaces, respectively. Eqs.(8a) and (8b) are to be satisfied for a pervious surface, while Eqs.(8a) and (8c) are to be satisfied for an impervious surface. Employing (3) and (5) along with the prerequisite conditions to write the equations of both half–spaces into the boundary conditions relating to a pervious surface gives six homogeneous equations in six constants B, E, L, D, F, M. These have a nontrivial solution only when the matrix of the coefficients of these constants are singular. This determines the frequency equation to be

$$|A_{ij}| = 0, \qquad i,j = 1,2,\ldots,6 \tag{9}$$

where the elements of the determinant are

$$\begin{aligned} A_{11} =& \ G_2+\phi_2, A_{12} = S_2+\psi_2, A_{13} = 2N_2\delta_2, A_{14} = -(G_1+\phi_1), \\ & \ A_{15} = -(S_1+\psi_1), A_{16} = 2N_1\delta_1, \\ A_{21} =& \ 2N_2\alpha_2, A_{22} = 2N_2\beta_2, A_{23} = N_2(1+\delta_2^2), \\ & \ A_{24} = 2N_1\alpha_1, A_{25} = 2N_1\beta_1, A_{26} = -N_1(1+\delta_1^2), \\ A_{31} =& \ \alpha_2, A_{32} = \beta_2, A_{33} = A_{36} = -1, A_{34} = \alpha_1, A_{35} = \beta_1, \\ A_{41} =& \ (1-\psi)A_{21} - A_{44}, A_{42} = (1-\psi)A_{22} - A_{44}, A_{43} = (1-\psi)A_{23} - \delta_2 A_{44} \\ A_{44} =& \ A_{45} = -N_2\psi c c_{s2}^{-1}, A_{46} = -\delta_1 A_{44}, A_{51} = \phi_2, A_{52} = \chi_2, A_{64} = \phi_1, A_{65} = \chi_1, \\ A_{53} =& \ A_{54} = A_{55} = A_{56} = A_{61} = A_{62} = A_{63} = A_{66} = 0\ . \end{aligned} \tag{10}$$

In case of an impervious surface, we can write the frequency equation in the form

$$|B_{ij}| = 0, \qquad i,j = 1,2,\ldots,6 \tag{11}$$

where the elements of the determinant are

$$
\begin{aligned}
B_{11} = &\ A_{11}, B_{12} = A_{12}, B_{13} = A_{13}, B_{14} = A_{14}, B_{15} = A_{15}, B_{16} = A_{16}, \\
B_{21} = &\ A_{21}, B_{22} = A_{22}, B_{23} = A_{23}, B_{24} = A_{24}, B_{25} = A_{25}, B_{26} = A_{26}, \\
B_{31} = &\ A_{31}, B_{32} = A_{32}, B_{33} = B_{36} = -1, B_{34} = A_{34}, B_{35} = A_{35}, \\
B_{41} = &\ A_{41}, B_{42} = A_{42}, B_{43} = A_{43}, B_{44} = B_{45} = A_{44}, B_{46} = A_{46}, \\
B_{51} = &\ \alpha_2\phi_2, B_{52} = \beta_2\chi_2, B_{64} = \alpha_1\phi_1, B_{65} = \beta_1\chi_1, \\
B_{53} = &\ B_{54} = B_{55} = B_{56} = B_{61} = B_{62} = B_{63} = B_{66} = 0.
\end{aligned}
\tag{12}
$$

The frequency equations (9) and (11) are nondispersive.

5 Results and Discussion

The frequency equation for the case $\psi = 0$ correspond to smooth interface while $\psi = 1$ for bonded interface. In these limiting cases, the frequency equations (9) and (11) reduce to the analogous frequency equations, each for pervious and impervious surface studied by Tajuddin *et al.* [12]. Also these equations allow real values of phase velocity satisfying conditions for interfacial waves. Setting liquid effects to vanish in (9), the results of classical theory due to Murty [5] are obtained as a special case. When $0 < \psi < 1$, the frequency equations gives attenuated interfacial waves which will be studied separately.

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