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ABSTRACT

We search for the new kinds of classical potentials in two-dimensional induced gravity, which provide the triviality of the one-loop quantum corrections. First of all the gauge dependence of the effective potential is studied. The unique effective potential, introduced by Vilkovisky in 1984 is found to manifest the gauge dependence due to some unusual properties of the theory under consideration. Then we take the gauge of harmonical type, which provides the one-loop finiteness off shell, and then the solution for the required classical potential is found.

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1 Introduction

It is well-known that the choice of the action for two-dimensional gravity is concerned with some nontrivial problem. The action of Einstein type appears to be the integral of the topological invariant and hence does not conduct to the meaningful quantum theory. During the last decade there was considerable interest to induced 2D gravity, which is closely related to the theory of strings. The action of induced gravity was firstly proposed in the nonlocal form $\int R\Delta^{-1}R$ [1] and then it was studied from different points of view (see, for example, [2–16] and references therein).

This paper is devoted to the perturbative features of the local version of 2D induced quantum gravity. Following our previous works [10–12,16,18] we are dealing with the more general theory with the action including the arbitrary potential $V(\Phi)$.

$$S = \int d^2x\sqrt{g}\{\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + C_1 R\Phi + V(\Phi)\} \tag{1}$$

The theory (1), as well as the corresponding nonlocal version, possess a set of remarkable features. First of all the theories with $V(\Phi) = 0$ [2-5], and $V(\Phi) = \Omega\Phi$ [6-7] are exactly soluble. The last means that the effective action of the theory may be derived exactly and nonperturbatively and moreover is found to differ from the classical ones only by some finite reparametrization [17]. It looks very interesting to understand this results in the framework of usual perturbation theory [9-19]. The additional motivation for this task is related to the attempts to consider (1) as a toy model for some four-dimensional gravity theory [10,33–34].

In Refs. [10–13, 15–16, 18] the one-loop effective action was considered with the use of the harmonic gauges (of general type [18]). The main result of Refs. [18,16] is that the divergencies of the one-loop effective actions vanish when the special values of gauge parameters are chosen [18] and moreover the quantum corrections to the potential $V(\Phi)$ are equal to zero for the exactly soluble case $V(\Phi) = \Omega\Phi$ [16]. It is possible to use the expression for the effective potential for the search of any new form of $V(\Phi)$ which leads to the trivial quantum corrections. Here we shall try to solve this problem which is closely related to the gauge dependence. Note that the effective potential in the theory (1) exhibits the essential gauge dependence. In particular, the expression for effective potential of Ref. [16] contains only V and V' (here the dash denotes the differentiation on Φ), but the effective potential which was derived in conformal gauge depends also on V'' [14]. So it is not clear at once which gauge is more correct for our purposes.

The paper is organized as follows. The detailed analysis of the gauge dependence of the effective potential is given in Section 2. In Section 3 we discuss the gauge and parametrization independent effective action, proposed by Vilkovisky [19] (see also [20–33, 15]). As it was pointed out earlier [26, 28, 35], the gauge and parametrization independent effective action includes the dependence of the choice of the configuration space metric. It should be noted that the divergencies of the effective action in the theory (1) have been found to depend on the choice of configuration space metric in [15]. In the original paper of Vilkovisky there was some additional condition which allows to fix the mentioned dependence (within the Einstein $D = 4$ gravity) to obtain the unique result [19]. The unusual property of (1) is that here the configuration space metric may depend on the gauge parameters. Moreover, this dependence appears even in a "natural" metric and hence it is insensitive to the additional conditions like the one of Ref. [19]. Thus the effective potential contains the inherent gauge dependence and it is not clear how to choose the correct gauge. The only proper way is to take the gauge which provides the finiteness of the kinetic part of the effective action. In Section 4 we use such a gauge and find a new kind of $V(\Phi)$ with trivial quantum corrections. Section 5 is the conclusion, where the possible connection of our result with the exact solution of the theory is discussed.

2 Effective potential in a general harmonic gauge

In this section we shall obtain the general expression for the effective potential and the explicit renormalized result in some particular case. We use the background field method based on the expansion of the fields into background $g_{\mu\nu}, \Phi$ and quantum $h_{\mu\nu}, \phi$ ones.

$$g_{\mu\nu} \rightarrow g'_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu},$$
$$h_{\mu\nu} = \bar{h}_{\mu\nu} + \frac{1}{2} h g_{\mu\nu}, \bar{h}_{\mu\nu} g^{\mu\nu} = 0,$$
$$\Phi \rightarrow \Phi' = \Phi + \phi, \qquad (2)$$

Let us introduce the background gauge fixing action of the form

$$S_{gf} = -\frac{C_1}{2} \int d^2x \sqrt{g} \chi_\mu [\frac{\Phi}{\alpha}] \chi_\nu$$
$$\chi_\mu = \nabla_\nu \bar{h}^\nu_\mu - \frac{1}{2}(\beta + 1) \nabla_\mu h - \frac{1}{2} \gamma \nabla_\mu \phi \qquad (3)$$

where α, β, γ are gauge parameters which may depend on Φ. We did not include the terms with the derivatives of the background field Φ because these terms do not contribute to the effective action. The account of symmetries and the power counting show that the divergencies of the effective action have the following structure [18]

$$\Gamma_{div} = \int d^2x\sqrt{g}\{\frac{1}{2}A_1(\Phi)g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + C_1 R A_2(\Phi) + A_3(\Phi)\} \tag{4}$$

One–loop calculation of Ref. [18] gives the following values of $A_{1,2,3}$:

$$A_1 = -\frac{\alpha}{\varepsilon\Phi^2}, A_2 = 0, A_3 = \frac{1}{\varepsilon}[\frac{\alpha V}{C_1\Phi} + V'\frac{1}{C_1}]$$
$$(\gamma = \alpha, \beta = 0) \tag{5}$$

where $e = 2\pi(d-2)$ is the parameter of dimensional regularization. We define the effective potential $V(\Phi)$ as the part of effective action which survives on the constant background $\Phi = Const, R = 0$. So $A3$ in (2), (3) is the divergent part of $V(\Phi)$. One–loop effective action is given by the expression

$$\Gamma = -\frac{1}{2}\, Trln\hat{H}\Big|_{div} + Trln\hat{H}_{gh}\Big|_{div} \tag{6}$$

where $\hat{H}$ is the bilinear (with respect to quantum fields) form of the action $S + S_{gf}$, and $\hat{H}_{gh}$ is the ghost action operator. Further we shall consider only the effective potential terms and therefore we are interested only in the corresponding part of $\hat{H}$ and $\hat{H}_{gh}$. The expression for the effective potential may be presented in the form

$$V_{eff}^{(1-loop)} = V - \Delta V - \frac{1}{2}Tr\sum_{k=1}^{4} ln\lambda_k + Tr\sum_{l=1}^{2} ln\lambda'_l \tag{7}$$

where λ_k and λ'_l are the eigenvalues of the operators $\hat{H}$ and $\hat{H}_{gh}$ respectively. Tr include the integration over the momentums in the framework of some regularization scheme. As it was already pointed out in [16] all the second order operators in the traceless sector in $\hat{H}$ have minimal structure. The last means that

$$P_{\mu\nu,\rho\sigma}[k^\rho k^\tau \delta^{\sigma\chi} + k^\rho k^\chi \delta^{\sigma\tau} + k^\sigma k^\tau \delta^{\rho\chi} + k^\sigma k^\chi \delta^{\rho\tau}]P_{\sigma\chi,\alpha\beta} = 2P_{\mu\nu,\alpha\beta}k^2$$

From this follows that the "natural" configuration space metric in the theory (1) depends on the gauge parameters. After a little algebra, taking into account the results of [18] we find

$$det(\hat{H}) = a_0(z - z_4)(z^3 - S_1 z^2 + S_2 z - S_3) \tag{9}$$

where

$$z = k^2, z_4 = -\frac{\alpha V}{C_1\Phi},$$

$$a_0 = \beta^2\{\frac{C_1^2\Phi}{4\alpha^2}[(\frac{1}{2} - \frac{C_1\gamma^2}{2\alpha\Phi}) - (1 - \frac{C_1\gamma^2}{\alpha\Phi}) + \frac{C_1}{\Phi}(\frac{\gamma}{\alpha} - 1)^2]\}-$$

$$-\beta\{\frac{C_1^2\Phi}{4\alpha})(C_1 + 1)[\frac{\gamma}{\alpha} - 1][1 + \frac{\gamma\beta}{\alpha}] + \frac{C_1^3\Phi}{8\alpha}[1 + \frac{\gamma\beta}{\alpha}]^2$$

$$S_1 = -\frac{1}{a_0}\{\beta^2[\frac{C_1\Phi}{4\alpha}(\frac{1}{2} - \frac{C_1\gamma^2}{2\alpha\Phi})V' - \frac{C_1^2\Phi^2}{4\alpha^2}V'']-$$

$$-\beta[\frac{\gamma}{\alpha} - 1]\frac{C_1^2\Phi}{4\alpha}V' + \frac{C_1^2\Phi}{2\alpha}(\frac{1}{2}V' - V\frac{C_1\gamma^2}{2\alpha\Phi})\}$$

$$S_2 = \frac{1}{a_0}\{\frac{C_1\Phi}{2\alpha}(V')^2 + +VV'(\frac{1}{2} + \frac{\gamma\beta}{2\alpha}) + \eta^2\frac{C_1\Phi}{4\alpha}VV''\}$$

$$S_3 = -\frac{V(V')^2}{8a_0} \tag{10}$$

The expression (10) may be rewritten in the form

$$det(\hat{H}) = a_0\prod_{i=1}^{4}(z - z_i) \tag{11}$$

where the values of the roots z_i are defined by the well-known Cardano formula

$$z_1 = \frac{S_1}{3} + A_1 + A_2,$$

$$z_{2,3} = \frac{S_1}{3} - \frac{A_1 + A_2}{2} \pm \sqrt{3}(\frac{A_1 + A_2}{2}),$$

$$A_{1,2} = (-\frac{q}{2} \pm \sqrt{u})^{\frac{1}{3}},$$

$$q = -\frac{2}{9}(S_1)^3 + \frac{1}{3}S_1S_2 - S_3,$$

$$u = \frac{1}{27}p^3 + \frac{1}{4}q^2, p = -\frac{1}{3}(S_1)^2 + S_2.$$

After the integration we get

$$V_{eff}^{(1-loop)} = V - \Delta V +$$

$$+\frac{1}{4\pi}\{\Lambda^2 ln(a_0) + \sum_{i=1}^{4}[z_i ln(\frac{\Lambda^2}{z_i}) + z_i]\} \tag{12}$$

where Λ is cut-off regularization parameter. From (12) follows that the logarithmical divergencies of the effective potential depends on V'' as well as the finite part. Hence there is no crucial difference between conformal and harmonical gauges. If $\beta = 0$, (12) coincides with the result of [18, 16] which was obtained within the dimensional and cut-off regularizations. Taking the counter term in form

$$\Delta V = +\frac{1}{4\pi}\mu^2 A(\Phi) ln(\frac{\Lambda^2}{\mu_2}) + \frac{1}{4\pi}\Lambda^2 B(\Phi), \tag{13}$$

where μ^2 is the dimensional parameter of renormalization, and A, B are some unknown functions, we require $V_{eff}^{(1-loop)}$ to be finite and after determination of A and B we finally get

$$V_{eff}^{(1-loop)} = V - \frac{1}{4\pi}\{\frac{V'}{C_1}[1 - ln(\frac{V'}{C_1\mu^2})] + \frac{\alpha V}{C_1\Phi}[1 - ln(\frac{\alpha V}{C_1\Phi\mu^2})]\} \quad (14)$$

The expression (14) has to be supplemented by the normalization conditions. Let us, for example, consider the exactly soluble case $V(\Phi) = \Omega\Phi$. Then the quantum correction in (14) is just the constant which does not depend on Φ. Introducing the condition

$$V_{eff}^{(1-loop)}(\Phi = 0) = 0 \quad (15)$$

we find $V_{eff}^{(1-loop)} = V$ that is the absence of all the quantum corrections. For the other interesting case $V_{eff}^{(1-loop)} = \Omega exp(\lambda\Phi)$ we take the normalization condition of the form

$$V_{eff}^{(1-loop)}(\Phi = 0) = \Omega \quad (16)$$

and find that quantum corrections do not vanish even for the value $\alpha = 0$, and do not repeat the structure of V.

3 Vilkovisky corrections to V

Our main purpose is to find the general form of the theory (1) which provides the triviality of the one-loop corrections. Since V_{eff} is gauge dependent it is not clear what means the correct choice of gauge. Thus it is natural to try to use the unique effective action of Vilkovisky where one can hope to get the gauge-independent result. At the same time there is the well-known problem in unique effective action which contains the dependence on the metric in the space of fields (configuration space) [23, 28, 31]. The additional condition of Vilkovisky [19] fix this metric to be natural (that is the bilinear form of the action in a minimal gauge). This condition implies the essential use of the classical action of the theory. However in the theory (1) even the natural configuration space metric depends on the gauge parameter and therefore such dependence may arise in the effective potential.

Our calculations have shown, that the gauge dependence of the Vilkovisky effective potential really takes place. Note that it also follows from the calculations of Kantowski and Marzban [15] with the natural metric of the form:

$$G_{ij} = \begin{pmatrix} \frac{C_1\Phi}{2\alpha} & 0 & 0 \\ 0 & 0 & -\frac{C_1}{2} \\ 0 & -\frac{C_1}{2} & (1 - \frac{C_1\alpha}{\Phi}) \end{pmatrix} \quad (17)$$

4 The solution of triviality equation

Let us now turn to the search of the theories with the trivial quantum corrections to $V(\Phi)$. We shall take the condition of triviality in the form:

$$V_{eff}^{(1-loop)} = (1+\tau)V + \eta \tag{18}$$

where τ, η are some constants. Our general purpose is to construct the recurrent procedure which may help to prove the triviality in higher loops. So we need the vanishing kinetic (at least) divergent contributions to the effective action to do this.

Thus there are only three appropriate gauges: i) light–cone, ii) conformal and iii) harmonical (3) with $\alpha = 0$ (see (5)). Since in the last case the loop corrections to the effective potential looks more simple let us consider (3). Then (16) is rewritten in the form

$$\tau V + \eta = \frac{V'}{4\pi C_1} ln(\frac{V'}{eC_1\mu^2}) \tag{19}$$

(19) is the ordinary differential equation which can easily be solved. In the case of $\tau = 1$ there is only one (well-known) solution $V(\Phi) = \Omega\Phi + \Omega_1$, where Ω, Ω_1 are some constants. However if τ is not equal to 1 there are two additional solutions of the form:

$$V = -\frac{\eta}{\tau} +$$

$$+\frac{\mu^2}{4\pi\tau}[-1 \pm \sqrt{8\pi C_1 \tau(\Phi - \Phi^*)}]e^{\pm\sqrt{8\pi C_1 \tau(\Phi-\Phi^*)}} \tag{20}$$

Here Φ^* is the constant, which is just nonessential because of invariance of the action (1) under the shifts of the field Φ.

The expression (20) contains the arbitrary parameters η, τ and moreover the extra dependence on the renormalization parameter μ. The last dependence has to be removed by the introduction of some normalization conditions. If one uses the condition (16) then the nontrivial part of the renormalized effective potential appears to be zero. On the other hand it is much more natural to take the more generally normalization condition of the form

$$V_{eff}^{(1-loop)}(0) = \tau^* V(0) + \eta^* \tag{21}$$

where τ^*, η^* are some constants (which may coincide with τ, η). Then it is possible to solve (21) and to express μ^2 (20) in terms of $\tau^*, \eta^*, \tau, \eta$.

5 Conclusion

We have considered some problems concerning the quantum $d = 2$ induced gravity theory (1). The theory with the special form of classical potential (20) allows only trivial one–loop corrections. Of course it is not for sure that the theory (1), (20) is exactly soluble. At the same time it seems promising to examine such theory with the help of nonperturbative methods.

Note that the perturbative exact solution of the theory requires the proof of triviality of the one–loop contributions to the kinetic part of the effective action.

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