IC/93/311

INTERNATIONAL CENTRE FOR
THEORETICAL PHYSICS

UNIFICATION OF ELECTROMAGNETIC, STRONG AND WEAK INTERACTION

Duong Van Phi

and

Duong Anh Duc

INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

MIRAMARE-TRIESTE

IC/93/311

International Atomic Energy Agency
and
United Nations Educational, Scientific and Cultural Organization
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

UNIFICATION OF ELECTROMAGMETIC, STRONG AND WEAK INTERACTION

Duong Van Phi [1]
International Centre for Theoretical Physics, Trieste, Italy

and

Duong Anh Duc
Department of Computer Science, Hochiminh City University,
227 Nguyen van Cu, Distr. 5, Hochiminh City, Vietnam.

ABSTRACT

The Unification of Electromagnetic, Strong and Weak Interactions is realized in the framework of the Quantum Field Theory, established in an 8-dimensional Unified Space. Two fundamental, spinor and vector field equations are considered. The first is of the matter particles and the second is of the gauge particles. Interaction Lagrangians are formed from the external and internal currents and the external and internal vector field operators. Generators of the local gauge transformations are the combinations of the matrices of the first field equation.

MIRAMARE – TRIESTE
September 1993

[1]On leave of absence from: Hochiminh City University, P.O.Box 657, Central Post Office, Hochiminh City, Vietnam.

1 INTRODUCTION

The great achievement of the Unification of Electromagnetic and Weak Interactions [1], [2] opened a new era in Physics to study the problems of Quantum Field Theory and Elementary Particles. It suggests to unify into a general scheme also the other interactions, Strong and Gravitational. For the Grand Unification, unification of Electromagnetic, Weak and Strong Interactions, which is related to our present paper, most research works is performed on basis of Quark-Lepton Models [3], [4], [5]. In this paper we shall present some results of this unification in another way. It is realized in framework of the Quantum field Theory established in an 8-dimensional Unified Space [6]. We shall consider two fundamental field equations in this space. The first is the first rank Spinor field equation. The second is the vector field equation. The spinor field equation describes a set of the fermions (prefermions) including the proton, electron, neutrino and neutron. The vector field equation describes a set of the bosons (prebosons), including the photon and the pions. The notion of preparticles is issued in consideration of the field operators in Unified Space. They are represented in the form of Spectral Expansions over the masses . Preparticle is the one having the characteristic numbers identical to the characteristic numbers of corresponding particle, but its mass can take any evalue between zero and infinity [7]. The preparticle should become a particles when its mass coincides to the mass of corresponding particle. For the interaction Lagrangians, we shall consider the currents, obtained by the local gauge transformations [8], [9] and with the generators formed by combinations of the matrices Γ of above mentioned spinor field equation. Because these matrices respect the determined algebra , and so the generators generally do not commute with the matrices in the spinor equation, and therefore we will make some generation of definition of the gauge transformations:

We shall introduce the signs R and L for the generators Q and this means that the generator Q^R should be posed on the right-hand side of the matrices $\Gamma^0\Gamma^\mu$ of the spinor field equation and the generator Q^L should be posed on the left-hand side of the these matrices.

The interaction Lagrangians are obtained by combination of the external and internal currents and the external and internal vector field operators: Electromagnetic Interaction is obtained by the product of the Electromagnmetic Currents and the external vector field operators $A^k(x,X)$, k=1,2,3,4; Strong Interaction is obtained by the product of the internal currents and the internal vector field operators $\Pi^a(x,X)$ a=5,6,7,8; Weak Interaction is obtained with the Weak Charged and Weak Neutral currents and the field operators $W^+_K(x,X)$, $W^-_k(x,X)$ and $Z^0_k(x,X)$, of the bosons W^+, W^- and Z^0. These bosons are supposed to be the composites of the prephotons and the prepions: the W^+ is composed from a prephoton and a pre-Π^+; the W^- is composed from a prephoton and a pre-Π^- and the Z^0 is composed from a prephoton and a pre$-\Pi^0$ [10].

In Section II, we shall represent the fundmental field equations. In Section III we will consider the definition of the currents . The Lagrangians of Electromagnetic, Strong and Weak Interactions are considerd in Sections IV, V and VI. In Section VII the Lagrangian for Unification of these three interactions is written. Some discussions are made in the section VIII.

2 FUNDAMENTAL FIELD EQUATIONS

The field equations of this approach are established in an 8-dimensional pseudo-euclidean space [6], determined by the relation

$$d\zeta^2 = dx^2 - dX^2, = 0$$

where ζ is the 8-dimensional coordinate vector, $x = (x, t)$ is the usual coordinate-time 4-vector and X=(X,T) is the internal coordinate-time euclidean 4-vector. The space of the last relation is called the Unified Space

For nontrivial solutions of the field equations, established on symmetry of Unified Space, the momenta must satisfy the relation

$$p^2 - P^2 = 0,$$

where p is the usual 4-momentum and P is the internal Euclidean 4-momenta.

In the case of free motions of particles, the Unified Space is divided into two invariant subspaces, Minkowskian and Internal, and then the coordinate relation should be written as

$$dx^2 = dX^2 = ds^2,$$

and the momentum relation will be

$$p^2 = P^2 = m^2,$$

where m is the mass of the particle (preparticle)

In 8-dimensional Unified Space there are two fundamental field equations. The first belongs to the spinor group representation. It is the first rank spinor field equation. The second belongs to the tensor group representation and is the vector field equation [6].

2.1 Spinor Field Equation

The first equation takes the form

$$i\Gamma^{\mu}\partial_{\mu}\Psi(\zeta) = 0 \tag{2.1}$$

where Γ^{μ} are the $\{16 \times 16\}$-matrices, represented in the forms:

$$\Gamma^k = \begin{pmatrix} \gamma^k & 0 & 0 & 0 \\ 0 & \gamma^k & 0 & 0 \\ 0 & 0 & \gamma^k 0 & \\ 0 & 0 & 0 & \gamma^k \end{pmatrix} \qquad k = 1, 2, 3, 4$$

$$\Gamma^5 = -i\begin{pmatrix} 0 & 0 & 0 & \gamma^5 \\ 0 & 0 & \gamma^5 & 0 \\ 0 & \gamma^5 & 0 & 0 \\ \gamma^5 & 0 & 0 & 0 \end{pmatrix} \qquad \Gamma^6 = \begin{pmatrix} 0 & 0 & 0 & -\gamma^5 \\ 0 & 0 & \gamma^5 & 0 \\ 0 & -\gamma^5 & 0 & 0 \\ \gamma^5 & 0 & 0 & 0 \end{pmatrix} \tag{2.2}$$

$$\Gamma^7 = -i\begin{pmatrix} 0 & 0 & \gamma^5 & 0 \\ 0 & 0 & 0 & -\gamma^5 \\ \gamma^5 & 0 & 0 & 0 \\ 0 & -\gamma^5 & 0 & 0 \end{pmatrix} \qquad \Gamma^8 = \begin{pmatrix} 0 & 0 & -\gamma^5 & 0 \\ 0 & 0 & 0 & -\gamma^5 \\ \gamma^5 & 0 & 0 & 0 \\ 0 & \gamma^5 & 0 & 0 \end{pmatrix}$$

where $\gamma^k, \gamma^5, ((\gamma^5)^2 = 1)$ are the Dirac matrices; Γ^{μ} , $(\mu = 1, 2, ..., 7, 8)$ satisfying the relations

$$\Gamma^{\mu}\Gamma^{\nu} + \Gamma^{\nu}\Gamma\mu = 2g^{\mu\nu} \tag{2.3}$$

The $g^{\mu\nu}$ is equal to 0, if $\mu \neq \nu$, equals to 1 if $\mu = \nu = 4$ and equals to -1 if $\mu = \nu \neq 4$. As usual, we introduce the matrix

$$\Gamma^9 = \Gamma^1\Gamma^2\Gamma^3\Gamma^4\,\Gamma^5\Gamma^6\Gamma^7\Gamma^8$$

Then the relations (2.3) are valid also for $\mu = 9$.
We have $(\Gamma^{\mu})^{+} = -\Gamma^{\mu}$, $(\Gamma^{\mu})^2 = -I$. The Γ^9 is of form

$$\Gamma^9 = F^5\begin{pmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & -I & 0 \\ 0 & 0 & 0 & -I \end{pmatrix}, \quad F^5 = \begin{pmatrix} \gamma^5 & 0 & 0 & 0 \\ 0 & \gamma^5 & 0 & 0 \\ 0 & 0 & \gamma^5 & 0 \\ 0 & 0 & 0 & \gamma^5 \end{pmatrix} \tag{2.4}$$

(2.5)

The equation (2.1) has 16 independent solutions and the field operator $\Psi(\zeta)$ could be represented as [11]

$$\Psi(\zeta) = (\psi_1(\zeta) \;\; \psi_2(\zeta) \;\; \bar{\psi}_3(\zeta) \;\; \bar{\psi}_4(\zeta)) \tag{2.6}$$

Using the Nother theorem we can obtain the following internal conserved characteristic numbers for the particles belonging to the equation (2.1): third component of the isospin I and third component of the so called D-spin. We have for p , $I_3 = 1/2, D_3 = 1/2$, for e, $I_3 = -1/2,\ D_3 = -1/2$; for ν, $I_3 = 1/2, D_3 = -1/2$ and for n, $I_3 = -1/2, D_3 = 1/2$. And from these we established a classification of the fermions of equation (2.1)

$$\psi_1 = \psi_p,\ \psi_2 = \psi_e,\ \psi_3 = \psi_{\nu},\ \psi_4 = \psi_n \tag{2.7}$$

This is made using the generalized Gell-Mann-Nishijima Relation and accounting the internal characteristic numbers of the leptons. It is of the form [15]

$$Q = I_3 + D_3 = I_3 + (B - L_i + S_t)/2$$

The equation (2.1) can be transformed into the Dirac equation form using the following Unitary operator [12]

$$U = \frac{1}{\sqrt{2}}(I + \frac{\Gamma^a P^a}{\sqrt{P^2}}) \tag{2.8}$$

With this operator the equation (2.1) turns into

$$(i\Gamma^k\partial_k - \sqrt{-i\partial_a . - i\partial_a})\Psi(x, X) = 0,$$

where the radical operation is performed after the derivations. The operator $-i\partial_a.-i\partial_a$ is the one of the squared internal momentun $P^2 = P^a P^a$ which is equal to the squared mass m^2 of the preparticle. It takes the values between zero and infinity. The $\Psi(x, X)$ can be represented in Spectral Expansion form [7], [6]:

$$\Psi(x,X)=\sum_{J_1}\sum_{J_2}\int_0^\infty dm.h(J_1,J_2;m)\psi(x,J_1,m)\chi(X,J_2,m),$$

in which external spinor field function satisfies the equation

$$(i\Gamma^k-m)\psi(x,J_1,m)=0$$

and the internal spinor field function satisfies the equation

$$(iG_a\partial_a-m)\chi(X,J_2,m)=0$$

Here we should remark that the Quantization of the internal spinor field of the last equation must be made in accordance to the Bose-Einstein Statistics [13].

2.2 Vector field equation

The other fundamental field equation takes the form

$$\partial^\mu\partial_\mu V_\nu(\zeta)=0 \tag{2.9}$$

with the generalized Lorentz condition

$$\partial^\mu V_\mu(\zeta)=0 \tag{2.10}$$

where μ, $\nu=1,2,...,7,8$.

This equation is of the photons and the pions. In variable separation we have for the photons the equations

$$(\partial^k\partial_k+m^2)A_n(x,m)=0,\quad k,n=1,2,3,4 \tag{2.11}$$

$$(\partial^a\partial_a+m^2)\Phi(X,m)=0\quad a=5,6,7,8 \tag{2.12}$$

and the Lorentz condition

$$\partial^k A_k(x,m)=0 \tag{2.13}$$

In the last equations the masses are identical and they take the values between zero and infinity, $0\le m^2\le\infty$.

For the pions we have the equations

$$(\partial^a\partial_a+m^2)\Pi_b(X,m)=0,\quad a,b=5,6,7,8 \tag{2.14}$$

$$(\partial^k\partial_k+m^2)\phi(x,m)=0\quad k=1,2,3,4 \tag{2.15}$$

and the internal Lorentz condition

$$\partial^a\Pi_a(X,m)=0 \tag{2.16}$$

As in the last case, the masses in these equations are identical and $0\le m^2\le\infty$.

In the spectral expansion form, the field functions of (2.11) and (2.12) and those of (2.14) and (2.15) are written as

$$A_k(x,X)=\sum_{J_1}\sum_{J_2}\int dm^2 f(J_1,J_2);m^2)A_k(x,J_1,m^2)\Phi(X,J_2m^2) \tag{2.17}$$

for the case of $k=1,2,3,4$, and

$$\Pi_a(x,X)=\sum_{J_1}\sum_{J_2}\int dm^2 g(J_1,J_2;m^2)\Pi_a(x,J_2,m^2)\phi(X,J_1,m^2) \tag{2.18}$$

for the case of $k=5,6,7,8$ where the f and g are the are the expansion coefficients. They are proportional to the $<J_1,J_2|m^2>$.

3 LOCAL GAUGE TRANSFORMATIONS

As is well known, to obtain the currents corresponding to a Lagrangian $\mathcal{L}$ of the free particles, one can consider the local gauge transformations of the type [8]:

$$\Psi(x)\to\Psi'(x)=\exp\{i\Lambda(x)Q\}\Psi$$

where $\Lambda(x)$ is an arbitrary infinitesimal function, Q is a some matrix. Then the currents are determined by

$$J^\mu=\lim_{\Lambda\to 0}\frac{\delta\mathcal{L}'}{\delta\Lambda_{,\mu}(x)}$$

where $\mathcal{L}'$ is the Lagrangian with the field operator Ψ'.

Related to the form of Q the current could be either conserved or unconserved. Because the $\Lambda(x)$ is an infinitesimal function so

$$\partial_\mu J^\mu(x)=\lim_{\Lambda\to 0}\frac{\delta\mathcal{L}'}{\delta\Lambda}$$

and this means that the current will be conserved if , in the case of $\Lambda(x)$ becomes constant, the Lagrangian remains invariant. In the following consideration, related to the spinor field equation, we must consider some matrices Q_i and then the determination of the orders of these is necessary to made. For some illustration of the problem let us consider now the case of the Lagrangian of the Dirac equation

$$\mathcal{L}=-\frac{i}{2}\bar\Psi(x)\gamma^\mu\partial_\mu\Psi(x)+\frac{i}{2}\partial_{mu}\bar\Psi(x)\gamma^\mu\Psi(x)+m\bar\Psi(x)\Psi(x)$$

and a gauge transformation

$$\Psi(x)\to\Psi'(x)=e^{i\Lambda(x)Q}\Psi(x)$$

It is clear that in the Dirac equation we can write $\gamma^0\gamma^\mu\partial_\mu$ as well as $\partial_\mu\gamma^0\gamma^\mu$, so the factor $\Lambda(x)Q$ after derivation could be posed on the right-hand side and as well as on the left-hand side of and of course the results for the currents generally should be different. Therefore we would like to make some generalization for the definition of gauge transformation operation:

$$\Psi(x)\to\Psi'(x)=e^{i\Lambda(x)(Q_1^L,Q_2^R)}\Psi(x) \tag{3.19}$$

where the sign R means that the matrix Q_2^R should be on the right-hand side of $\gamma^0\gamma^\mu$ and the sign L is that the matrix Q_2^L should be on the left-hand side of $\gamma^0\gamma^\mu$. Then for the Hermitian conjugate we have:

$$\Psi^+(x) \to \Psi'^+(x) = \Psi^+(x)e^{-i\Lambda(x)(Q_1^{+R}, Q_2^{+L})}$$

It is easy to see that if the matrix Q_1 is Hermitian and commutes with the matrix $\gamma^0\gamma^\mu$, the new definition of the currents return to the usually used one.

It can be seen also that the determination of the conservation of the currents, which was mentioned above, remains valid.

4 ELECTROMAGNETIC INTERACTION

We shall use the Lagrangian of the spinor field (2.1) and the vector field equation (2.10) for the next consideration.

The Lagrangian of the equation (2.1) is

$$\mathcal{L} = -\frac{i}{2}\bar{\Psi}(x,X)\Gamma^\mu\partial_\mu\Psi(x,X) + \frac{i}{2}\partial_\mu\bar{\Psi}(x,X)\Gamma^\mu\Psi(x,X) \tag{4.20}$$

Now we introduce a gauge transformation

$$\Psi(x,X) \to \Psi'(x,X) = e^{ie\Lambda_e(x)Q_e^R}\Psi(x,X) \tag{4.21}$$

with

$$Q_e = \frac{i}{2}(\Gamma^5\Gamma^6 + \Gamma^7\Gamma^8 + \Gamma^5\Gamma^8 + \Gamma^6\Gamma^7) \tag{4.22}$$

where I is the 16×16 unit matrix.

It can be seen that the Q_e is a hermitian matrix and commutes with the matrices Γ^k, $k = 1, 2, 3, 4$.

The currents then are of the form:

$$J_e^k = e\Psi(x,X)\Gamma^k Q_e\Psi(x,X) \tag{4.23}$$

The matrix Q_e in (4.21) with the above mentioned matrices Γ^μ is

$$Q_e = \begin{pmatrix} I & I & 0 & 0 \\ I & -I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

where I is 4×4 unit matrix.

And the electromagnetic interaction Lagrangian should be:

$$\mathcal{L}_I^e = J^\mu(x,X)A_\mu(x,X) = e\bar{\Psi}(x,X)\Gamma^k Q_e\Psi(x,X)A_\mu(x,X) \tag{4.24}$$

Introducing into (4.24) the matrix Γ^μ in (2.6) and the concrete form of Q_e we shall have the result:

$$\begin{aligned} \mathcal{L}_I^e &= e[\bar{\Psi}_p(x,X)\gamma^\mu\Psi_p(x,X)A_\mu(x,X) + \bar{\Psi}_p(x,X)\gamma^\mu\Psi_e(x,X)A_\mu(x,X) \\ &+\bar{\Psi}_e(x,X)\gamma^\mu\Psi_p(x,X)A_\mu(x,X) + \bar{\Psi}_e(x,X)\gamma^\mu\Psi_e(x,X)]A_\mu(x,X)] \end{aligned} \tag{4.25}$$

which is the expected Lagrangian for the electromagnetic interaction.

5 STRONG INTERACTION

We consider now the transformation:

$$\Psi(x,X) \to \Psi'(x,X) = e^{-ig\Lambda_s(X)(Q_1^L, Q_2^R)} \tag{5.26}$$

with the operators:

$$Q_1 = \frac{i}{2}(I + i\Gamma^7\Gamma^8) \tag{5.27}$$

and

$$Q_2 = \frac{i}{2}(I + i\Gamma^7\Gamma^8)(I + i\Gamma^9\Gamma^8) \tag{5.28}$$

Then we have for the internal currents:

$$J_s^\alpha = g\bar{\Psi}(x,X)[Q_1\Gamma^\alpha Q_2 + Q_2^+\Gamma^\alpha Q_1^+]\Psi(x,X) \tag{5.29}$$

The interaction Lagrangian should be of the form:

$$\mathcal{L}_I^s = iJ_s^\alpha\Pi^\alpha(x,X) = g\bar{\Psi}(x,X)[Q_1\Gamma^\alpha Q_2 + Q_2^+\Gamma^\alpha Q_1^+]\Psi(x,X)\Pi^\alpha(x,X) \tag{5.30}$$

And because

$$\Gamma^\alpha\Pi^\alpha = -iF^5 \begin{pmatrix} 0 & 0 & (\Pi^7 - i\Pi^8) & (\Pi^5 - i\Pi^6) \\ 0 & 0 & (\Pi^5 + i\Pi^6) & -(\Pi^7 + i\Pi^8) \\ (\Pi^7 + i\Pi^8) & (\Pi^5 - i\Pi^6) & 0 & 0 \\ (\Pi^5 + i\Pi^6) & -(\Pi^7 - i\Pi^8) & 0 & 0 \end{pmatrix},$$

$$Q_1 = \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I \end{pmatrix}$$

$$Q_2 = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix},$$

so the strong interaction Lagrangian should be

$$\begin{aligned}\mathcal{L}_I^s = & e[\bar{\Psi}_p(x,X)\gamma^5\Psi_p(x,X)\Pi^0(x,X)+\sqrt{2}\bar{\Psi}_p(x,X)\gamma^5\Psi_n(x,X)\Pi^-(x,X) \\ & +\sqrt{2}\bar{\Psi}_n(x,X)\gamma^5\Psi_p(x,X)\Pi^+(x,X)-\bar{\Psi}_n(x,X)\gamma^5\Psi_n(x,X)\Pi^0(x,X)]\end{aligned} \quad (5.31)$$

where $\Pi^{\pm}=\frac{1}{\sqrt{2}}(\Pi^5 \pm i\Pi^6)$ and $\Pi^0=\Pi^7$. The components $\Pi^{8(x,X)}$ are not figured in the last formula and this is in accordance with the internal Lorent condition, presented in the section II.

6 WEAK INTERACTION

For the electromagnetic and strong interactions, which were considered above, the exchange factors for interactions are the photon, belonging to the isosinglet and the pions, belonging to the isotriplet. However, for weak interaction, as is proved theoretically in [1], [2] and experimentally in [14], the exchange factors are the bosons $W^{\pm}$ ans Z having the spin $J=1$ but for their isospins I, up to now, the confirmation is still unclosed. From experimental data [14] we can see a difficulty to classify these boson into any isomultiplet. Really, the difference between the masses of neutral boson and charged boson is about 10GeV, which is very large for a group of the particles belonging to an isotopic multiplet. In a recent paper [10] we have supposed that the intermediate vector bosons are the resonance composites of the photons and pions: the W^+ is a composite of the γ and the Π^+, the W^- is a composite of the γ and the Π^- and the Z^0 is a composite of γ and Π^0 . Below, we shall use this hypothesis for establishing the gauge transformation. Accounting for the absence of the internal invariance we shall represent the currents in two parts, one for the charged currents and another for the neutral currents.

Let us now consider the charged currents. Because they are complex, we shall consider then separately: the vector currents and axial currents.

For the vector currents we introduce the gauge transformation

$$\Psi(x,X)\to\Psi'(x,X)=\exp\{iG\Lambda_w^V(x)i\Gamma^a\}\Psi(x,X), \quad a=5,6 \quad (6.32)$$

And from here we have the vector charged currents

$$J_{cV}^{ka}(x,X)=iG\bar{\Psi}(x,X)\Gamma^k\Gamma^a\Psi(x,X) \quad (6.33)$$

For the second currents we consider the gauge transformation

$$\Psi(x,X)\to\Psi'(x,X)=\exp\{iG\Lambda_w^A(x)F^5\Gamma^a\}\Psi(x,X), \quad a=5,6 \quad (6.34)$$

and then we have the charged axial currents

$$J_A^{ka}(x,X)=G\bar{\Psi}(x,X)\Gamma^kF^5\Gamma^a\Psi(x,X) \quad (6.35)$$

It is easy to see that the charged (V-A)-Currents must be of form

$$J_{cw}^{ka}(x,X)=J_{cA}^{ka}(x,X)-iJ_{cV}^{ka}(x,X)=G\bar{\Psi}(x,X)\Gamma^k(I+F^5)\Gamma^a\Pi^a\Psi(x,X) \quad (6.36)$$

Now let us consider the neutral currents. For these we can introduce the gauge transformation

$$\Psi(x,X)\to\Psi'(x,X)=\exp\{iG\Lambda_w^n(x)q(I+F^5)i\Gamma^9\Gamma^8\Gamma^b, b=7,8 \quad (6.37)$$

where $b=7,8$ and q is a number. Then we have the neutral currents

$$J_{nw}^{kb}(x,X)=-\frac{i}{2}\bar{\Psi}(x,X)\Gamma^k(I+F^5)[\Gamma^9\Gamma^8,\Gamma^b]_+\Psi(x,X) \quad (6.38)$$

The weak interaction Lagrangian should then be written as

$$\mathcal{L}_I^w(x,X)=J_{cw}^{ka}(x,X)A_k(x,X)\Pi^a(x,X)+J_{nw}^{kb}(x,X)A_k(x,X)\Pi^b(x,X) \quad (6.39)$$

Substituting the matrices Γ in the operators Q, we shall have

$$\begin{aligned}\mathcal{L}_I^w = & \\ & G\{\sqrt{2}\bar{\Psi}_p(x,X)\gamma^k(I+\gamma^5)\Psi_n(x,X)W_k^-(x,X)+ \\ & \sqrt{2}\bar{\Psi}_n(x,X)\gamma^k(I+\gamma^5)\Psi_p(x,X)W_k^+(x,X)+ \\ & \sqrt{2}\bar{\Psi}_e(x,X)\gamma^k(I+\gamma^5)\Psi_\nu(x,X)W_k^+(x,X)+ \\ & \sqrt{2}\bar{\Psi}_\nu(x,X)\gamma^k(I+\gamma^5)\Psi_e(x,X)W_k^-(x,X)+ \\ & q\bar{\Psi}_p(x,X)\gamma^k(I+\gamma^5)\Psi_p(x,X)Z_k^0(x,X)- \\ & q\bar{\Psi}_n(x,X)\gamma^k(I+\gamma^5)\Psi_n(x,X)Z_k^0(x,X)+ \\ & q\bar{\Psi}_\nu(x,X)\gamma^k(I+\gamma^5)\Psi_\nu(x,X)Z_k^0(x,X)- \\ & q\bar{\Psi}_e(x,X)\gamma^k(I+\gamma^5)\Psi_e(x,X)Z_k^0(x,X)\}\end{aligned} \quad (6.40)$$

where $W_k^{\pm}(x,X)=A_k(x,X)\Pi^{\pm}(x,X)$ and $W^0(x,X)=A_k(x,X)\Pi^0(x,X)$.

7 UNIFICATION OF INTERACTIONS

We have the electromagnetic currents in (4.23), internal strong currents in (5.29) and the external-internal weak currents in (6.36) and (6.38). Now we can form from two last currents the external currents:

$$J_{cw}^k(x,X)=J_{cw}^{ka}(x,X)\Pi^a(x,X) \quad (7.41)$$

$$J_{nw}^K(x,X)=J_{nw}^{kb}(x,X)\Pi^b(x,X) \quad (7.42)$$

The electromagnetic as well as the currents in (7.41) and (7.42) transform as the vector functions in the usual Minkowskian Space. And, we can introduce the currents:

$$J_{ew}^k(x,X)=J_e^k(x,X)+J_{cw}^k(x,X)+J_{nw}^k(x,X) \quad (7.43)$$

The Lagrangian of Electromagnetic and Weak Interactions should take the form:

$$\mathcal{L}_I^{ew}=J_{ew}^k(x,X)A_k(x,X) \quad (7.44)$$

Now, formally, we can introduce the "8-dimensional vector currents":

$$J(x,X)=(J_{ew}^k(x,X),J^\nu(x,X)), \quad k=1,2,3,4; \quad \nu=5,6,7,8 \quad (7.45)$$

And the Lagrangian of Electromagnetic, Weak and Strong Interactions should be written as

$$\mathcal{L}_I^{ews}(x,X) = J^\mu(x,X)V_\mu(x,X), \mu = 1,2,3,4,5,6,8 \quad (7.46)$$

where $V(x,X)$ is the 8-dimensional vector field operator presented in section II.

8 DISCUSSIONS AND CONCLUSION

Thus, the Quantum Field Theory established in the Unified Space can give the unification of three interactions, electromagnetic, weak and strong. From the Lagrangian of electromagnetic interaction we can see that, the terms of charged particles, leptons as well as nucleons, are simultaneously figured. And, as it has been, for obtaining these we have introduced into the gauge transformation only one generator Q_e, formed from the matrices Γ of of the spinor field equation of these particles.

The Lagrangian of strong interaction is obtained by the same manner, but with the internal currents, deduced from a gauge transformation with the infinitesimal function of the internal coordinates. It is the product of these currents and the field operator $\Pi^a(x,X)$, a = 5,6,7,8. But, in the Lagrangian the field operator $\Pi^8(x,X)$ is self eliminated, and the interaction really is the one of nucleons and three pions, Π^+, Π^- and Π^0. This also is in accordance with the internal Lorent condition for the field function of the pions.

The Lagrangian of weak interaction is obtained with the "bicurrents", which have the external indices k = 1,2,3,4 and the internal indices α. For the charged currents $\alpha = 5,6$ and for the neutral currents $\alpha = 7,8$ The gauge particle field operators are the combinations of the photon and pion field operators. Therefore the gauge particles are the composites of two these particles: $W^\pm$ is the composite of γ and $\Pi^\pm$, Z^0 is the composite of γ and Π^0 [10]. The division of the term with W-bosons and the term with Z-boson shows that these intermediate bosons do not belong to an isotriplet.

This could explain the experimental facts [14] that the difference between the mass of the Z and mass of the $W^\pm$ is very large, about 10Gev . Furthermore, because their full widths are also very large (about 10 GeV) we suppose that the observed intermediate bosons are the resonance composites of the prephotons and prepions, which have the great masses [10].

In section II we have presented the fiel operatots in Spectral Expansion form. This form is available to operate in the interaction theory in Unified Space. The S-matrix formulated in this space takes the form:

$$S = T\exp\{i\int \mathcal{L}_I^s(x,X)dxdX\},$$

where the interaction Lagrangian is formed from the field operators written in Spectral Expansion forms.

The elements of the S-matrix then contains the superpositions of the states of the preparticles. And for a given process, one can extract from these superpositions the states of observed particles, in initial and final states. This could be performed using the mass orthogonality relations : $< m^2|m'^2 >= \delta(m^2 - m'^2)$, for the boson field operators and $< m|m' >= \delta(m - m')$, for the fermion field operators [6]. For the vector field operators the extraction could be made directly from their spectral expansion expressions, however, for the spinor field operators this should be made after the reciprocal transformation U^{-1}, presented in section II.

In the probability expressions, together with the usual momemta p_k, will be also the internal momenta P_k of the observed particles k. Where the P_k^2 is equal to m_k^2, and m_k is the masses of these particles. And, for comparison with the experiments, one must take the averages of these expressions over the internal momenta on the mass surfaces [15].

ACKNOWLEDGMENTS

One of the authors, Duong Van Phi, expresses his deep gratitude to Prof. Adus Salam, the International Atomic Energy Agency, UNESCO and the International Centre for Theoretical Physics for hospitality at ICTP, Trieste, Italy. He also expresses deep thanks to Prof. J Strathdee for discussions on this paper.

REFERENCES

1. Abdus Salam, Gauge Unification of foundamental forces, Rev. Mod. Phys., V.52, p.525, 1980

2. S. Weinberg, Conceptual foundamentions of the unified theory of weak and electromagnetic interactions, Rev. Mod. Phys., V.5, p.515, 1980

3. Abdus Salam, Unification of the Fundamental Forces, Cambridge University Press, 1988.

4. R. Barloutaud, Nineth Workshop on Grand Unification, World Scientific, 1988

5. Graham G. Ross, Grand Unified Theories, The Benjamin Cummings Publishing Company, Inc. 1984.

6. Duong Van Phi and Duong Anh Duc, A formulation of the Quantum Field Theory in an 8-dimensional Unified Space, (will be published in IL Nuovo Cimento A)

7. Duong Van Phi, Field Theory in Unified Space and the Spectral Representations, Bul. of Hochiminh City University, 6, p.33, 1987

8. J. Bernstein, Elementary Particles and their Currents, W. H. Freeman and Company San Francisco and London, 1968

9. Duong Van Phi and Nguyen Mong Giao, Weak Currents in the Field Theory in Unified Space, JINR, Dubna, E2-81-519, 1981

10. Duong Van Phi and Duong Anh Duc, A Theory Approach for the Creation of the Matter of Universe, Preprint ICTP, Trieste, IC/93/284, 1993 (submitted to publication)

11. Duong Van Phi, Interaction des particules elementaires dans l'espace a huit dimensions, Cahier de Physique, Inst. Henrie Pointcare, T.21, p.101, 1967

12. Duong Van Phi, Interaction des particules elementaire dans l'espace a huit dimensions, C. R. Acad. , Paris, T. 266, p. 1545, 1968

13. Duong Van Phi and Chieu Lan, Quantization of the Fields in the internal 4-dimensional Euclidean Space, Bul. of Hochiminh City University,

14. M. Aguilar-Benitez et al., Review of Particle Properties, Phys. Rev., D45, Part 2, 1992

15. Duong Van Phi, Tran Tat Dat, et al. Decays of the Hyperons, Internal Report, ICTP, Trieste,IC/90/127, 1990 Duong Van Phi and Duong Anh Duc, Decay of J/Ψ, IL Nuovo Cimento A, V.105,1703,1992; Decay of the D-mesons (will be published in IL Nuovo Cimento A); Decay of the Bottom Mesons (will be published in IL Nuovo Cimento A)