Optimization of Accelerator Control

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I. INTRODUCTION

Expensive exploitation of charged particle accelerators is inevitably concerned with requirements of effectively obtaining of the best characteristics of accelerated beams for physical experiments.

One of these characteristics is intensity. Increase of intensity is hindered by a number of effects, concerned with the influence of the volume charge field on a particle motion dynamics in accelerator's chamber. However, ultimate intensity, determined by a volume charge, is almost not achieved for the most of the operating accelerators. This fact is caused by losses of particles during injection, at the initial stage of acceleration and during extraction. These losses are caused by deviations the optimal from real characteristics of the accelerating and magnetic system. This is due to a number of circumstances, including technological tolerances on structural elements of systems, influence of measuring and auxilliary equipment and beam consumers' installations, placed in the closed proximity to magnets, and instability in operation of technological systems of accelerator.

Control task consists in compensation of deviations of characteristics of magnetic and electric fields by optimal selection of control actions. As for technical means, automatization of modern accelerators allows to solve optimal control problems in real time. Therefore, the report is devoted to optimal control methods and experimental results.

II. METHODS AND PRINCIPLES OF CONTROL ORGANIZATION

Tasks of the accelerating complex systems control are stated as tasks of extremal control. The following stages may be determined in solving of these tasks:

- study of accelerator as an object of automatic optimization;
- selection of methods of optimization and tracking of extremum;
- comparative study of methods, using models, which have the main peculiarities of the control object;
- synthesis of extremal control algorithm and procedure of estimations of automatic adjustment efficiency at operating accelerator.

Solution of a task can be shown as an example of extremal control of accelerated beam intensity for a proton synchrotron at the Institute of Theoretical and Experimental Physics (Moscow).

Intensity of a beam, injected into the ring, is a function of 11 independent variables, normalized with respect to injector current:

- electrostatic injector voltage;
- injection field intensity;
- radio frequency adjustment in the form of delay of the master clock start;
- correction currents of beam orbit.

Process of intensity change is characterized by spontaneous drift (10 – 12% shift), which can be compensated by varying of the above mentioned variables. Dispersion of an interference is selected in accordance with a noise level, reduced by averaging of beam intensity measurements at the accelerator to 3%.

Criteria of preliminary selection of optimization methods were algorithm discreteness, caused by cyclic processes in the accelerator, as well as convergence in conditions of substantial noises, high speed, minimality of spread in magnitudes of an output value during tuning, compactness of control program.

An important peculiarity, determining selection of a method, is a problem of creation of adequate mathematical description, that forces us to consider an object as a "black box". In this case it is necessary to use search step methods.

It should be noted, that for the use of these methods a necessary condition of object parametrization is satisfied. The condition consists in definiteness of controlled variables, whose varying enables reaching of extremum.

As competitive methods have been selected method of sequential simplex planning, including automatic selection of a step, and methods of random search in modifications:

- with estimation of gradient
- with self-learning
- with punishment of randomness

Values of methods parameters, ensuring a stable convergence and the highest speed in conditions of interferences at models (1) have been determined at the first stage of the studies. In this case a higher speed of the method of sequential simplex planning and higher reliability of extremum search may be noted. One should consider a higher sensitivity in estimation of direction near the extremal zone and complete set of an operating program as the advantages. It is obvious, that it is extremely important to know efficiency characteristics of a priority selected optimization methods, obtained in conditions, near to existing at the object. Comparative studies with the use of models were carried out in the following conditions:

Task of maximization of a single-extremum scalar function

$$I(X) = E_{\xi}\{Q(X,\xi)\}$$

in situation of noise is considered. Here, $X = (x_1, \cdots x_n)$ is a vector of controlled variables, which are subject to determination. Functional $Q(X,\xi) = I(X) + \xi$ is considered to be measured during optimization. Here ξ is a random value, distributed normally with the expectation, equal to zero, and dispersion σ^2.

Extremum of the function I(X) is determined in the specified region

$$X_i^{min} < X_i < X_i^{max} \quad , \qquad i = 1, 2 \cdots n$$

Assume, that measurement of a quality function and measurement of vector X are produced only in discrete moments of time

$$t^j = j\Delta t \qquad (j = 0, 1 \cdots n)$$

magnitudes of vector X of controlled variables on each j-th step are designated as X^j; $I^j = I(X^j)$, $Q^j = I(X^j) + \xi^j$.

A way of determination of a step value $\Delta X^j = X^j - X^{j-1}$ is specified by a search algorithm. Summarization of the methods being compared:

Random search, including punishment of randomness. A next step in the space of controlled variables is determined according to the rule:

$$\Delta X^{j+1} = \begin{cases} \Delta X^j, & \text{if } Q^j > Q^{j-1} \\ \alpha \Xi^{j+1}, & \text{if } Q^j \leq Q^{j-1} \end{cases}$$

where $\Xi = (\xi_1, \cdots \xi_n)$ is a random vector uniformly distributed on a sphere of unit radius; α is a step scale.

Scale of a step is adapted during search according to the dependence

$$Q = A_0 \exp\{L \times [N^2(1 + \Delta n^2/T^2) + (\sup N)^2(1 - \Delta n^2/T^2)]\}$$

where A_0 is initial scale of a step; N is number of unsuccessful steps, implemented from a last point; supN is maximum number of unsuccessful steps, implemented from any point during all the process of search; Δn is difference between a number of a current point of search and number of a point, where supN of unsuccessful steps has been performed; T is maximum permissible distance between a current point of search and a point, from which sup N of unsuccessful steps has been performed; L is parameter, determining dynamics of step adaptation.

Random search with gradient estimation. Search algorithm has the following view:

$$\Delta X^{j+1} = \begin{cases} \Delta X^j, & \text{if } Q^j > Q^{j-1} \\ \alpha^{j+1} S^j / |S^j|, & \text{else} \end{cases}$$

where α is scale of a operating step; S is stochastic estimate of a gradient, determined by the following algorithm:

$$S^j = 1/(2mg^j)\Sigma_{\psi=1} \{Q(X^j + g^j\Xi^\psi) - Q(X^j - g^j\Xi^\psi)\Xi^\psi$$

where $m \leq n$ is the number of pairs of trials for gradient estimation; g^j is a value of operating step; Ξ^j is vector, uniformly distributed on the sphere of unit radius.

Scales of the operating and trial steps are adapted during search in accordance with the following dependences:

$$a^j = \begin{cases} a^j, & \text{if } Q^j > Q^{j-1} \\ a^j/N+1, & \text{if } Q^j \leq Q^{j-1} \end{cases}$$

$$g^j = \begin{cases} g^j, & \text{if } Q^j > Q^{j-1} \\ g^j/\sqrt[4]{N+1}, & \text{if } Q^j \leq Q^{j-1} \end{cases}$$

N is the number of unsuccessful steps for all the previous process of search. Besides:

a) If the process of gradient estimation reveals positive augments of quality function, transition to a trial point, that is

$$X^j + g^j S^j/|S^j|, \text{ if } \Delta Q^{j+1} = X^{j+1} = X^j + \Delta X^j = \{= \max(\Delta Q^{j+1}, \Delta Q^{j,\Psi})\}$$

$$X^j + g^j\Xi^\Psi, \text{ else}$$

b) After a step along estimation of a gradient a concluding state is determined by maximum increment of quality function at trial and operating steps, that is

$$X^j = \begin{cases} X^j + g^j\Xi^\Psi, & \text{if } \Delta Q^{j,\Psi} > 0 \\ X^j, & \text{else} \end{cases}$$

Random search with self-learning. Rule of step calculation this algorithm is the following one:

$$\Delta X^{j+1} = (W^{j+1} + R\Xi^{j+1}/|W^{j+1} + R\Xi^{j+1}|$$

where R = const > 0 is radius of guiding sphere; Ξ is vector, uniformly distributed on spere of unit radius; $W^j = (W_1{}^j, \cdots W_n{}^j)$ is vector of memory, $|W^j| < C$, C = const > 0.

$$W^j = C \operatorname{grad} I(X_0);$$

$$W^{j+1} = lW^j - \delta\Delta Q^j \Delta X^j$$

where $0 < l < 1$ is parameter of forgetting; $\delta \geq 0$ is parameter of self-learning; $\Delta Q^j = Q^j - Q^{j-1}$ is augment of functional at j-th step.

Sequential simplex planning. Essence of optimization by means of this method consists in the following.

Regular simplex, centre of which is at the start point, is constructed in the space of controlled variables, and quality function is estimated at all its vertices:

$$Q(X^j), \; j = 0,1,2,\cdots n.$$

Then a trial step - mirror reflection of a worse vertex, where quality function is minimum, is performed through a centre of the opposite face

$$X^{rfl} = 2X^c - X^{wt},$$

where X^{rfl} is reflected vertix position vector; X^c is a vector of the face centre position; X^{wt} is position vector of the worse vertix.

An operating step follows after a trial one to a point, which is determined according to the rule:

$$(1 + \gamma)X^{rfl} - \gamma X^c, \text{ if } Q(X^{rfl}) > Q(X^{wt})$$

is tension;

$$X^{end} = \{\beta X^{wt} + (1-\beta)X^{c}, \text{ if } Q(X^{rfl}) < Q(X^{wt})$$

is compression;

$$X^{rfl}, \text{ if } Q(X^{j}) < Q(X^{rfl}) < Q(X^{wt}); \; j \neq wt$$

(If $Q(X^{wt}) < Q(X^{rfl}) < Q(X^{j})$ $j \neq wt$ then X^{wt} is substituted for X^{rfl} and compression is produced). Here, the following designations are taken: X^{end} *is position vector of the* operating step end; X^{wt} is position vector of the best vertix; $\gamma > 0$ is coefficient of tension; $0 < \beta < 1$ *is coefficient of* compression.

Subsitution for a vertix, depending on an operating step result, is produced according to the rule;

$$X^{wt} = \begin{cases} X^{end}, \text{ if } Q(X^{end}) > Q(X^{rfl}) \\ X^{rfl}, \text{ if } Q(X^{rfl}) > Q(X^{end}) \quad \text{– in tension} \\ X^{end}, \text{ if } Q(X^{end}) > Q(X^{wt}) \quad \text{– in compression} \end{cases}$$

If after compression

$$Q(X^{end}) < Q(X^{wt}),$$

then initial simplex is drawn to the best vertix:

$$X^{j} = 0,5(X^{j} + X^{wt}), \quad j = 0,1, \cdots n$$

Then process is repeated since a moment of determination of a worse vertix.

Study of the methods was carried out with the use of a test nonlinear function in two modifications Q_1 – separable quadratic form, complicated by noise

$$Q_1(X,\xi) = X^{T}BX + B_0 + \xi,$$

where B is diagonal coefficient matrix, determined as approximation of results of statistical identification of an object. B_0 *is a free term,* ξ is noise addition; Q_2 – unseparable function, having a modular surface of "two-dimensional backbone" $Q_2(X,\xi) = \Sigma B_{ij} X_i^2 = 100(X_{11} - X_{10}^2) = (1 - X_{10}) + B_0 + \xi$

Correctness of statement of a problem on equality of conditions of methods comparison at a specified form of a model consists in identity of initial conditions for all selected methods and optimality of parameters of each method from a viewpoint of a specified characteristic speed of operation, number of quality function samples before reaching of a specified zone of extremum.

All selected methods have been preliminary optimized for parameters. Start points for all tests were a single value

$$X_0 = \{X_i\} \qquad i = 1,2, \cdots, 11$$

Studies were carried out in simulation of optimization process at computer. Results were averaged from ten "ascensions" for the determinate method (simplex) and from fifty "ascensions" for varieties of a random search.

All used criteria of methods comparison are divided into two classes: local and integral ones. The first class is concerned with a single elementary stage of search-operating step, the second one- with all the process of optimization since a start moment till operation of the rule of breakpoint. The following criteria of comparison were used:

- losses for search. An average local rate of optimization is determined and the following calculation is performed:

$$\Pi^{j} = E(K^{j})/E(\delta Q^{j})$$

where

K^{j} is the number of samples at j-th step;
$\delta Q^{j} = \Delta Q^{j}/Q^{j-1}$ is relative change of quality factor;

- error probability. It determines probability of an erroneous operating step $P = P\{Q(X^{j} + \Delta X^{j}) < Q(X^{j})\}$
- inaccuracy. Characteristic of method inaccuracy is integral one. It determines discrepancy ε of the obtained and unknown quatity $E(\varepsilon) = E(X^{*} - X^{ext})$, where X^{*} is solution, obtained as result of method operation. X^{ext} is solution, corresponding to extremum of quality function.
- number of samples of quality function for reaching of a specified level. This characteristic depends on initial conditions of X;
- reliability. Reliability $\rho(\varepsilon)$ of method is probability of reaching the specified χ-vicinity of extremum for a specified number of samples of the quality function.

Realiability is numerically estimated in the following way:

$$P(\varepsilon) = 1 - P_{\chi} = \int_0^{\chi} \rho(\varepsilon^{j})d\varepsilon^{j}$$

where $\varepsilon^{j} = |X^{j} - X^{ext}|$ is discrepancy at j-th step, distributed with density $\rho(\varepsilon)$, $P_{\chi} = \int_{\chi}\rho(\varepsilon^{j})d\varepsilon^{j}$ is probability of unachieved required accuracy of solution.

$\rho(\varepsilon)$-noise immunity. One of the most important criteria of the search methods application for solution of the task of accelerator optimization is ability to orient oneself with respect to situation of noise.

Changing level of noise at the accelerator lays down a requirement of stable convergence of the search procedure in some range of noise. Therefore, it is important to study dependence of methods rate on a value of noise and to estimate an upper boundary of a noise level, at which correct orientation is still possible. For collection of noise immunity statistics a noise level is changed from 0 up to 25%, and for each concrete level an average number of steps was determined, necessary for coming to a zone, limited by the surface,

$$\{X: I(X)\} = C \quad C = 0,95 \, I_{max}$$

Main conclusions:

- at the initial stage of search a random search with self-learning is characterized by the smaller losses. At all the following stages the simplex method is characterized by minimum losses. A random search with gradient estimation is similar to it.

- simplex method has the smallest probablity of errors;
- simplex method has demonstrated a higher accuracy of search for extremum in comparison with methods of a random search, has turned out to be the best one in criteria of speed of responce, noise immunity and has demonstrated reliability criterium results, almost identical with the random search method.

III. STUDY OF EFFICIENCY AT AN OBJECT

The studies were carried out for criteria of speed of responce, search variance and reliability. Speed of responce was estimated according to the number of steps, which are to be performed since start till completion of search.

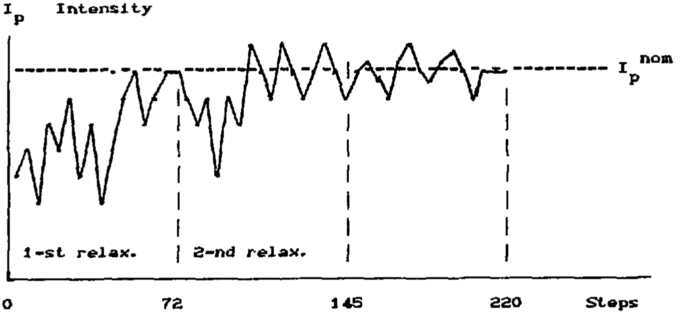

Fig. 1 The extremum search mode of the system

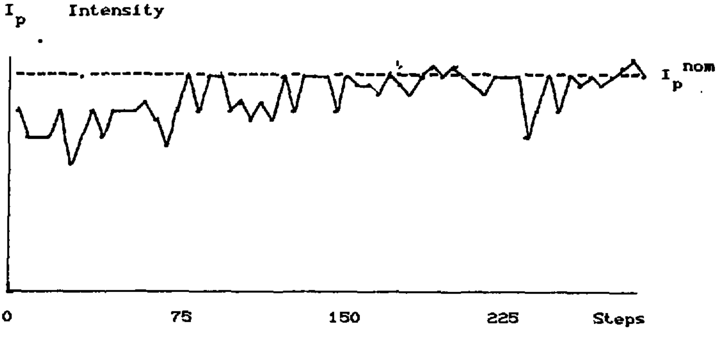

Fig. 2 The extremum follow mode of the system.

Search variance is meant as growth in the output parameter instability because of the search steps. Procedure of extremum search is considered to be reliable, if manual tuning after each series of ascensiens is not effective enough.

Estimation of methods efficiency was carried out in the following conditions:

- detuning in all controlled variables at initial level of intensity of 20 ..... 40% from maximum one;
- detuning of accelerator in two controls (two-dimensional backbone was reproduced artificially);
- nominal conditions of accelerator operation in the presence of the optimal state drift.

All methods turned out to be serviceable ones. However, efficiency of their use turned out to be not identical. In starts from peripheral points the best results were demonstrated by simplex method. An appreciable growth in intensity in the use of this method ends after 60 ..... 80 steps. However, search variance turned out to be high enough. The best average result among methods of random search is 100 ..... 150 steps to an object.

A main result of the carried out studies was proof of applicability of extremal control methods for control of accelerator, and efficiency of them has been demonstrated in practice. This became apparent, first of all, in decrease in time of reaching of the operating conditions of the accelerator (this time was 8 ..... 20 minutes) and in the improvement of the accelerator operation quality. The latter is characterized by process variance, reduced almost by a factor of two, in comparison with manual tuning, and by intensity level, increased by 5 ..... 10%. Fig. 1 and Fig. 2 demonstrate the search and follow modes of system.

IV. CONCLUSION

Though the obtained results are of particular character, they may be used for control of objects of the same class. As for study methodology, it may be assumed as a basis of the approach and organization of solution of the extremal control problems by other types of electrophysical installations.