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Meson Phase Space Density from Interferometry

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Abstract

The interferometric analysis of meson correlations provides a measure of the average phase space density of the mesons in the final state. This quantity is a useful indicator of the statistical properties of the system, and it can be extracted with a minimum of model assumptions. Values obtained from recent measurements are consistent with the thermal value, but do not rule out superradiance effects.

It would be interesting to know the average phase space density of the pions produced in ultrarelativistic heavy ion collisions. In the final state, the local phase space density is frozen (Liouville's theorem) and it gives a measure of the dynamics in the prior interacting region. If the system could be described by a local statistical equilibrium, the distribution function f would have the Bose-Einstein form, $f_T(p,r) = 1/(\exp(u(r) \cdot p/T) - 1)$. For massless particles the average of this quantity is a pure number,

$$\langle f \rangle_T = \frac{\int d^3p\, d^3r\, f_T^2}{\int d^3p\, d^3r\, f_T} = \frac{\zeta(2)}{\zeta(3)} - 1 \approx 0.37\,.$$

For massive bosons, the number is lower; for example, for pions at a chemical freezeout temperature of 200 MeV the average is

$$\langle f \rangle_{m,T} \approx 0.15\,. \tag{1}$$

If the experimental $\langle f \rangle$ were close to this, it would be welcome evidence for the existence of a local equilibrium. If $\langle f \rangle$ came out much larger, it would lend considerable support to the idea of superradiant pion states[1], which have been an object of renewed interest[2,3]. On the other hand, if $\langle f \rangle$ came out much smaller than thermal, it would point to entropy-generating processes such as the slow decay of heavy resonances or quark-gluon droplets.

In this letter I want to point out that the measured two-particle correlations yield direct information about the phase space density of the mesons, when interpreted according to Pratt's interferometric formula[4]. To see this, we begin from Pratt's formula written as

$$\frac{d^6 n^{(2)}}{d^3 p_1 d^3 p_2}\bigg|_{p+q,p-q} - \frac{d^3 n^{(1)}}{d^3 p}\bigg|_{p+q} \frac{d^3 n^{(1)}}{d^3 p}\bigg|_{p-q} = \int d^4 x_1 d^4 x_2 g(x_1, \vec{p}) g(x_2, \vec{p}) \cos q \cdot (x_1 - x_2) \tag{2}$$

where g is the source function for the mesons. We next convert the source function g to an equivalent source at a common time t_0 by the replacement

$$g(\vec{r}, t, \vec{p}) \rightarrow \delta(t - t_0) \int^{t_0} dt' g(\vec{r} - \vec{v}(t' - t_0), t', \vec{p}) \equiv \delta(t - t_0)(2\pi)^3 f(r, p)$$

where v is the velocity associated with the momentum vector $\vec{p}$. This replacement does not affect the correlation function if $\vec{v} \cdot (\vec{p_1} - \vec{p_2}) = (E_1 - E_2)$. The condition is satisfied for small momentum differences and for longitudinal motion of extreme relativistic particles, and seems rather safe for the present application. The coefficient of the δ-function is then the phase space density at t_0, extrapolating the positions of the final state mesons to that time. We next integrate over the momentum difference $d^3 q$, which produces a delta function $\delta^3(r_1 - r_2)$ to eliminate one of the spatial integrals. The result is

$$\int \frac{d^3 q}{(2\pi)^3} \int d^4 x_1 d^4 x_2 g(x_1, \vec{p}) g(x_2, \vec{p}) \cos q \cdot (x_1 - x_2) = (2\pi)^3 \int d^3 r f^2(\vec{r}, \vec{p}).$$

Finally we integrate over $d^3 p/(2\pi)^3$ and normalize to the number of particles to obtain the phase space average,

$$\langle f \rangle = \frac{1}{n} \int d^3 p \int d^3 q \left[\frac{d^6 n^{(2)}}{d^3 p_1 d^3 p_2}\bigg|_{p+q,p-q} - \frac{d^3 n^{(1)}}{dp^3}\bigg|_{p+q} \frac{d^3 n^{(1)}}{dp^3}\bigg|_{p-q} \right]. \tag{3}$$

For heavy ion collisions it is more useful to make the average over a fixed rapidity interval, because the system evolves to produce a spatial separation between particles of different rapidities. The different rapidity groups equilibrate independently. The formula for a small rapidity interval reads

$$\langle f \rangle_{dy} = \frac{1}{dn/dy} \int \frac{d^2 p_t}{p_0} \int d^3 q \left[\frac{d^6 n^{(2)}}{d^2 p_{t1} dy_1 d^2 p_{t2} dy_2}\bigg|_{p+q,p-q} - \frac{d^3 n^{(1)}}{d^2 p_t dy}\bigg|_{p+q} \frac{d^3 n^{(1)}}{d^2 p_t dy}\bigg|_{p-q} \right]. \tag{4}$$

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Eq. (3) and (4) just require integral properties of the correlation function, so they should be less dependent on the accuracy of the momentum measurements than other observables.

The integral should undoubtedly be evaluated directly from the experimental data, but for an orientation I shall try to evaluate it from published Na44 parameterized distributions[5]. The Na35 experiment[6] obtained similar information, but did not quote the entire parameterization. One of the common parameterizations for these correlations is Gaussian with parameters λ and source sizes R_L, R_s and R_o,

$$\frac{d^6 n^{(2)}}{d^2 p_{t1} dy_1 d^2 p_{t2} dy_2} = \left(1 + \lambda \exp(-\frac{1}{2}(q_L^2 R_L^2 + q_s^2 R_s^2 + q_o^2 R_o^2))\right) \frac{d^3 n^{(1)}}{d^2 p_t dy}\bigg|_{p_1} \frac{d^3 n^{(1)}}{d^2 p_t dy}\bigg|_{p_2}.$$

The single-particle transverse momentum spectra can be parameterized by an exponential function,

$$\frac{d^3 n^{(1)}}{d^2 p_t dy} = \frac{dn}{dy} \frac{exp(-p_t/T_t)}{2\pi T_t^2}.$$

With this parameterization, the average phase space density is given by

$$\langle f \rangle_{dy} = \sqrt{\frac{\pi}{2}} \frac{1}{R_L R_s R_o T_t^3}.$$

Ref. [5] quotes the following numbers for S + Pb$\to$ π^++X at midrapidity: $\lambda \approx 0.4$, $R_t \approx 6.0$ fm, $R_L \approx 6.0$ fm and $dn/dy \approx 40$. From their Fig. 7 can be deduced $T_t \approx 187$ MeV/c. Taking $R_s = R_o = R_t$ and inserting these numbers in eq. (3), I obtain

$$\langle f \rangle_{dy} \approx 0.07 - 0.16.$$

The range is obtained from the quoted experimental errors combined quadratically. I note that the NA35 experiment found a somewhat smaller source size; the lack of agreement between experiments is a caution not to draw strong conclusions from the present data.

One should also be reminded that the assumptions going into the interferometric formula, eq. (2), may not be well satisfied. The most critical assumption is that the one-particle distribution function is not affected by the Bose symmetrization, which is only satisfied for low phase space densities.

Given these caveats, what does one conclude? The extracted phase space density is consistent with local statistical equilibrium for a chemical freezeout temperature in the

100-200 MeV range. The analysis includes pions from long-lived resonance decays such as $\omega \to 3\pi$ and $\eta \to 3\pi$. If these could be subtracted out, the phase space density would be somewhat higher. Thus there may be an excess of pions produced in the source, and the possibility of coherent pion effects should not be ruled out.

The extracted phase space density appears high enough to make unlikely that long-lived intermediates such as droplets of quark-gluon plasma are produced abundantly. This is consistent with the present theoretical expectation of no strong first-order phase transition in the quark-gluon plasma[7].

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