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QUENCHING OF THE GAMOW-TELLER MATRIX ELEMENT IN CLOSED LS-SHELL-PLUS-ONE NUCLEI

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ABSTRACT

It is evident that nuclear Gamow-Teller matrix elements determined from β-decay and charge-exchange reactions are significantly quenched compared to simple shell-model estimates based on one-body operators and free-nucleon coupling constants. Here we discuss the theoretical origins of this quenching giving examples from light nuclei near LS-closed shells, such as ^{16}O and ^{40}Ca.

Gamow-Teller matrix elements in mirror transitions in closed-LS-shell-plus (or minus)-one nuclei, ^{15}O, ^{17}F, ^{39}Ca and ^{41}Sc, have been experimentally determined[1]) from β-decay data. Compared to single-particle estimates using the free-nucleon axial-vector coupling constant, $g_A \simeq 1.26$, these matrix elements are quenched by roughly 13% at the A=16 closed shell and 30% at the A=40 closed shell. Furthermore (p,n) reactions[2–4]) have extended the number of data by measuring the spin-flip transitions, $j=\ell+\frac{1}{2} \rightarrow j=\ell-\frac{1}{2}$, which again, on comparison with single-particle estimates, indicate large quenching of order 40% in the matrix element.

Theoretically these reductions must stem either from the inadequacy of the single-particle description of the nuclear state or from the inadequacy of the impulse-approximation one-body GT operator. Calculations of the first effect, frequently called core polarization, estimate through perturbation theory the admixtures of $2p$-$1h$ and $3p$-$2h$ configurations in the basically single-particle wave function and evaluate the impact the admixtures have on the calculated matrix element. At LS closed shells there is no contribution to the core polarization in first order. The reason is that with no spatial dependence in the one-body GT

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operator it cannot excite a $1p$-$1h$ state from a closed LS shell. Thus calculations must be taken to second order in perturbation theory. Computationally this leads to a time-consuming calculation, as there is no selection rule to limit the intermediate-state summation and the convergence is slow. This is particularly true with tensor forces in the residual interaction as first stressed by Shimizu, Ichimura and Arima.[5]) This propensity of the tensor force to couple strongly to high-lying excited states has been called 'tensor correlations' and the phenomenon leads to a reduction in the GT matrix element.

The other inadequacy concerns the use of one-body operators. Corrections to the impulse approximation arise because nucleons in nuclei interact through the exchange of mesons, and this exchange can be perturbed by the action of the weak axial current. Since this perturbation requires at least two nucleons to be involved, corrections from this origin lead to two-body GT operators. The mathematical technique used to determine these operators follows from considering all Feynman graphs in which a meson is exchanged between two nucleons and linking the weak axial current in all possible places in the diagram. Consider first graphs involving pion exchange. These are the most important graphs because the pion, being the lightest mass meson, generates the longest-range exchange current operators. Heavy-meson exchange leads to short-range operators, which in nuclear physics are less important since the nuclear wave function goes rapidly to zero at short distances. The possible pion-exchange graphs can be classified into two types: Born graphs, involving only nucleons and pions, and non-Born graphs in which the axial current either excites a nucleon to an excited state, such as the isobar Δ, or converts the pion into a heavy meson, such as the ρ. The important point is that for axial currents the pion Born graphs are identically zero.[6]) This is in sharp distinction to electromagnetic currents where pion Born graphs give an important contribution to such properties as magnetic moments. Thus the intimate connection between the GT operator and the spin part of the isovector M1 operator, evident in the impulse approximation, is broken when meson-exchange current (MEC) corrections are considered. For the non-Born graphs, we will distinguish between diagrams involving nucleon excitations – referring to these as isobar graphs – and those in which a pion is converted to a ρ meson – referring to these as $\rho\pi$ or meson-exchange-current (MEC) graphs. In general, the latter MEC graph gives a small contribution and is not an important ingredient in the quenching of the GT matrix element.

There are two further points to consider: The core polarization calculation corrects the matrix element of a one-body operator evaluated in the closed-shell-plus-one configuration for the presence of $2p$-$1h$ and $3p$-$2h$ admixtures in the single-particle wave function. The perturbation calculation is carried out to second order in the residual interaction (or to the fourth power in the meson-nucleon coupling constants). It is logical, therefore, that the matrix elements of the two-body operators should likewise be corrected for $2p$-$1h$ and $3p$-$2h$ admixtures. Since the two-body operator itself involves the meson-nucleon coupling constants to the second power, it is sufficient to estimate this correction to first order in the residual interaction. These terms have been called 'crossing terms' in the work of the Tokyo group.[7,8])

The second point concerns the one-body GT operator, which is obtained as a leading term in the nonrelativistic reduction of a relativistic axial-vector current. There is a few per cent correction coming from the next-order terms in the nonrelativistic reduction. The GT operation, $1/2 g_A \underset{\sim}{\sigma} \tau_\pm$, is then modified to

$$g_A \left\{ \underset{\sim}{\sigma} - \frac{1}{2} p^2/M^2 (\underset{\sim}{\sigma} - (\underset{\sim}{\sigma} \cdot \underset{\sim}{\hat{p}}) \underset{\sim}{\hat{p}}) \right\} \frac{1}{2} \tau_\pm \ , \tag{1}$$

where $\underset{\sim}{p}$ is a nucleon momentum and M its mass. The correction depends on estimating $\langle p^2/M^2 \rangle$ for a nucleon in a nucleus.

In summary, then, corrections to lowest-order shell model estimates of the GT matrix element in the impulse approximation come from: core polarization, isobar currents, MEC currents, crossing terms and relativistic corrections. All these ingredients for closed-LS-shell-plus-one nuclei have been calculated by the Tokyo group[7,8]) and by Towner and Khanna.[9]) We quote in Table I some results from a recent review by Towner,[10]) where more details can be found. It is useful to characterize the results of the calculation in terms of an equivalent effective one-body operator that contains three independent rank-one tensors

$$(GT)_{\rm eff} = \left\{ g_{LA,{\rm eff}}\ \underset{\sim}{L} + g_{A,{\rm eff}}\ \underset{\sim}{\sigma} + g_{PA,{\rm eff}} [Y_2(\underset{\sim}{\hat{r}}) \times \underset{\sim}{\sigma}]^{(1)} \right\} \frac{1}{2} \tau_\pm \ , \tag{2}$$

where $g_{A,{\rm eff}} = g_A + \delta g_A$, etc., with g_A the bare impulse-approximation value and δg_A the correction to it. Note that in the bare operator $g_{LA} = g_{PA} = 0$. From Table I we see the results are not good. In general, theory is underpredicting the degree of quenching in the experimental matrix element. Note also that the term δg_{LA} is small, while the term δg_{PA} has a rather subtle role to play. For diagonal matrix elements the quenching is proportional to $\delta g_A + K \delta g_{PA}/\sqrt{(8\pi)}$ where

Table I. Summary of all corrections to the ground-state diagonal GT matrix element, $\delta\langle GT\rangle_d$, and the off-diagonal spin-flip matrix element, $\delta\langle GT\rangle_f$, expressed as a percentage of the single-particle value for closed-shell-plus (or minus)-one configurations.

		δg_{LA}	δg_A	δg_{PA}	$\frac{\delta\langle GT\rangle_d}{\langle GT\rangle_d}$ calc	$\frac{\delta\langle GT\rangle_d}{\langle GT\rangle_d}$ expt[a]	$\frac{\delta\langle GT\rangle_f}{\langle GT\rangle_f}$ calc	$\frac{\delta\langle GT\rangle_f}{\langle GT\rangle_f}$ expt
$A=16$	$0p_{1/2}^{-1}$	0.013	−0.185	0.234	−1.9	−13.1±0.5	−17.0	∼−38[b,c]
$A=16$	$0d_{5/2}$	0.013	−0.175	0.179	−10.2	−13.8±0.3	−15.8	∼−33[b]
$A=40$	$0d_{3/2}^{-1}$	0.009	−0.255	0.167	−17.1	−33.7±1.0	−21.9	∼−45[b,d]
$A=40$	$0f_{7/2}$	0.010	−0.221	0.105	−14.2	−26.2±0.4	−18.8	

[a]Deduced from experimental β-decay data recorded in Ref. [1]).
[b]From (p,n) measurements of Watson *et al.*,[2]) where we have renormalized the (p,n) cross section so that the deduced GT matrix element for the ground-state transition agrees with β-decay measurements. Watson *et al.*, however, prefer to normalize their results to distorted-wave impulse approximation calculations and find for mass $A=15,39$ notable discrepancies between their deduced values of the ground-state GT matrix element and those deduced from β-decay.
[c]From Goodman *et al.*[3])
[d]From Rapaport *et al.*[4])

$K = 2\ell/(2\ell+3)$ for orbits with $j = \ell+\frac{1}{2}$, and $K = (2\ell+2)/(2\ell-1)$ for $j = \ell-\frac{1}{2}$. For the $0p_{1/2}$ orbit in particular, there is a very strong cancellation between these two terms resulting in a small predicted value for $\delta\langle GT\rangle_d/\langle GT\rangle_d$. For the spin-flip transitions, on the other hand, the quenching is proportional to $\delta g_A - \frac{1}{2}\delta g_{PA}/\sqrt{(8\pi)}$ and numerically the two terms add. Thus larger quenching is predicted for spin-flip transitions than for diagonal transitions, as experimentally observed, although the magnitude of the effect is underpredicted. In Table II we give a breakdown of the contributions to the effective operator from various sources, where it is seen the principal contribution comes from core polarization. Isobar currents give an important contribution to δg_{PA} but in this calculation its cancellation against δg_A leads to an overall effect that is small for diagonal matrix elements. This is somewhat controversial. Alternative theories[11]) using enhanced effective interactions in the isobar-hole channels claim much larger isobar contributions. Rho,[11]) in particular, argues that the crossing terms should roughly cancel the tensor correlations in the core-polarization calculation. We also show the result of the Tokyo group[7,8]) in Table II whose overall result is rather similar to ours.

Table II. Contributions to the effective one-body GT operator from various sources for a 0d configuration at $A=16$, and a comparison with the 1983 Towner-Khanna calculation (TK), the Arima *et al.* calculation (ASBH) and the empirical values deduced by Brown and Wildenthal (BW).

	δg_{LA}	δg_A	δg_{PA}	$\frac{\delta \langle GT \rangle_d}{\langle GT \rangle_d}$ % calc
Core polarization	0.011	−0.136	0.005	−9.0
Isobar currents	0.002	−0.046	0.264	−0.9
MEC, $\rho\pi$	−0.002	−0.004	−0.065	−1.2
Crossing term	0.001	0.032	0.026	3.0
Relativistic	0.000	−0.021	−0.052	−2.1
Sum	0.013	−0.175	0.179	−10.2
TK (Ref. [9])	0.012	−0.191	0.103	−12.2
ASBH (Ref. [8])	0.013	−0.180	0.224	−10.3
BW (Ref. [12])	0.01(1)	−0.26(1)	0.09(4)	−18.7(16)

A different approach to the determination of the effective one-body operator is that of Brown and Wildenthal.[12]) The effective coupling constants are determined in a fit to a large number of data in the sd-shell using shell-model wave functions calculated without truncation in the complete sd-shell model space. The assumption is that the effective one-body operator is only a weakly varying function of nuclear mass; a proportionality of $A^{0.35}$ is assumed. From Table II we see that our calculated value of $\delta g_A \simeq -0.18$ is only two-thirds the empirically deduced value. If we assume that the core-polarization calculation is about right (because it is the principal correction for isoscalar magnetic moments which are well described in similar calculations[7−10]), then we can use the empirically determined value to solve for the isobar correction. The MEC, crossing term and relativistic corrections are small for GT transitions. Thus we write, in an obvious notation:

$$\begin{aligned} \delta g_A(\Delta) &= \delta g_A(\mathrm{BW}) - \delta g_A(\mathrm{CP}) - \delta g_A(\mathrm{MEC}) - \delta g_A(\mathrm{CT}) - \delta g_A(\mathrm{Rel}) \\ &= -0.13 \pm 0.01 \ . \end{aligned} \tag{3}$$

Our calculated value from Table II is $\delta g_A(\Delta) = -0.046$, nearly three times smaller.

This strongly suggests an enhanced effective interaction is required in the isobar-hole channels compared to the one-boson potential used by us.

REFERENCES

1. Raman, S., Houser, C.A., Walkiewicz, T.A. and Towner, I.S., Atomic Data and Nuclear Data Tables **21**, 567 (1978).
2. Watson, J.W. *et al.*, Phys. Rev. Lett. **55**, 1369 (1985).
3. Goodman, C.D. *et al.*, Phys. Rev. Lett. **54**, 877 (1985).
4. Rapaport, J. *et al.*, Nucl. Phys. **A431**, 301 (1984).
5. Shimizu, K., Ichimura, M. and Arima, A., Nucl. Phys. **A226**, 282 (1974).
6. Chemtob, M. and Rho, M., Nucl. Phys. **A163**, 1 (1971).
7. Hyuga, H., Arima, A. and Shimizu, K., Nucl. Phys. **A336**, 363 (1980); Arima, A. and Hyuga, H., in Mesons in Nuclei, ed. Wilkinson, D.H. and Rho, M. (North-Holland, Amsterdam, 1979), p. 685.
8. Arima, A., Shimizu, K., Bentz, W. and Hyuga, H., Adv. Nucl. Phys. **18**, 1 (1987).
9. Towner, I.S. and Khanna, F.C., Nucl. Phys. **A399**, 334 (1983); Towner, I.S. and Khanna, F.C., Phys. Rev. Lett. **42**, 51 (1979).
10. Towner, I.S., Phys. Reports **155**, 264 (1987).
11. Oset, E. and Rho, M., Phys. Rev. Lett. **42**, 47 (1979); Rho, M., Annu. Rev. Nucl. Part. Sci. **34**, 531 (1984).
12. Brown, B.A. and Wildenthal, B.H., Phys. Rev. C **28**, 2397 (1983); Brown, B.A. and Wildenthal, B.H., Atomic Data Nucl. Data Tables **33**, 347 (1985); Brown, B.A. and Wildenthal, B.H., Annu. Rev. Nucl. Part. Sci. **38**, 29 (1988).