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ISOSPIN AND QUARKS IN NUCLEAR BETA-DECAY

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1. PURPOSE

The purpose of the present paper is to expose in some detail the technical problems relating to the extraction of the vector coupling constant from the beta decay of complex nuclei. The paper also considers the extraction of the axial coupling constant from the beta-decay of the neutron. The internal consistency of all data relating to beta-decay, including that of the muon, is also examined, within the standard model, with a view to the possible intervention of W_R.

2. INTRODUCTION

Ideally, allowed vector (Fermi) beta-decay just spirits away charge since the leptons are in a mutual $j = 0$ state and, furthermore, do not carry orbital angular momentum away from the nucleus; the nuclear wave function is unchanged; the nucleus is simply tilted in isospin space. Allowed axial (Gamow-Teller) beta-decay, on the other hand, flips nucleon spin with the leptons being in a mutual $j = 1$ state so that such decay may change the J-value of the nucleus, or may change the nuclear wave function, leaving J unchanged, or may leave the nuclear wave function unchanged in which case it simply tilts the nucleus in ordinary space as well as in isospin space. Evidently, if the transition is between states of $J = 0$ without change of parity only vector decay is possible and that only if the states are members of the same isomultiplet T. Furthermore, if we accept the hypothesis of the conserved vector current (CVC), at least in its weak form, we have:

$$ft = \frac{K}{G_V^2 \mid M \mid^2} \quad (1)$$

where

$$\begin{aligned} K &= 2\pi^3 \ ln2 \ \hbar^7/(m_e^5 c^4) \\ &= (8.120270 \pm 0.000012) \times 10^{-7} \mathrm{GeV}^{-4} \mathrm{sec} \end{aligned} \quad (2)$$

and where

$$| M |^2 = T(T+1) - T_Z(T_Z+1) \tag{3}$$

for the transition $T_Z \to T_Z + 1$.

3. THE REAL WORLD

Equation (1) and its concomitant G_V apply in an ideal, completely charge-independent, world, G_V there being the primitive constant of vector coupling for the nucleon. In practice, de facto charge dependences of several kinds subvert Eq. (1) at various levels. That such effects are of major importance is demonstrated in Fig. 1(a) where the experimental $(ft)_e$[1] of the eight accurately-measured $J^\pi = 0^+ \to 0^+$ transitions within $T = 1$ isomultiplets* (for which $| M |^2 = 2$ in Eq. (3)), as listed in Table 1, are displayed; these $(ft)_e$-values have been corrected for electron capture

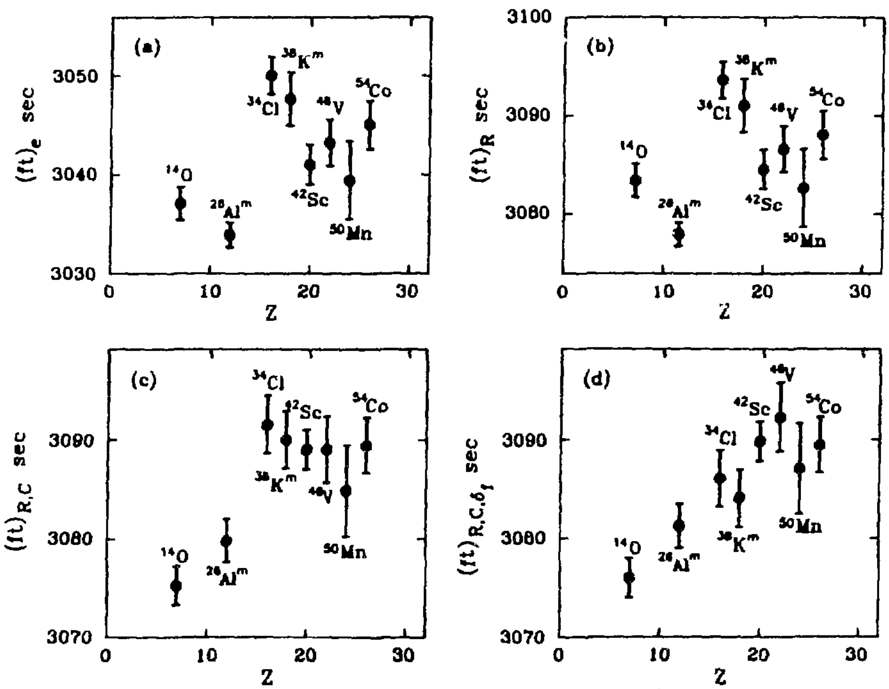

Fig. 1(a) Experimental $(ft)_e$ from the Chalk River compilation[1]; (b) $(ft)_e$ corrected for outer radiative corrections: $(ft)_R = (ft)_e(1+\delta_R)$; (c) As (b) with additional correction for the nuclear-structure-dependent part, C_{NS}, of the inner radiative correction: $(ft)_{R,C} = (ft)_e(1+\delta_R)(1+\frac{\alpha}{\pi}C_{NS})$; (d) As (c) with additional correction for the case-to-case fluctuations, δ_{cf}, of the nuclear mismatch: $(ft)_{R,C,\delta_f} = (ft)_e(1+\delta_R)(1+\frac{\alpha}{\pi}C_{NS})(1-\delta_{cf})$.

*It is amusing to note that the availability, for accurate study, of all eight members of this set is due to various "accidents": in the case of ^{14}O because the (Gamow-Teller) transition to the ground state of ^{14}N, which might have been expected to overwhelm that of the Fermi decay in question, is highly suppressed (cf. the analogue ^{14}C); in the cases of ^{26}Al^m and ^{38}K^m because, although the $J^\pi = 0^+, T = 1$ states of those bodies, from which the vector decay of Fig. 1(a) takes place, are excited states they are separated from the $T = 0$ ground states of high spin ($J = 5$ and 3 respectively) by such small energy differences (0.23 and 0.13 MeV respectively) that their gamma-decay is much slower than their beta-decay; in the remaining cases because although the nuclei are self-conjugate their ground states are not of $T = 0$ but of $T = 1, J^\pi = 0^+$.

Table 1. The accurately-measured vector transitions.

Body	$(ft)_e$	δ_f^{THH}	δ_f^{OB}	δ_1	δ_2	δ_3	C_{NS}	$(ft)^*$	N_q
^{14}O	3037.1 ± 1.6	-0.02	-0.04	1.29	0.22	0.01	-1.13 ± 0.17	3150.3 ± 2.0	3.1 ± 0.5
^{26}Al^m	3033.9 ± 1.2	-0.06	-0.03	1.11	0.32	0.02	0.26 ± 0.26	3155.1 ± 2.2	2.9 ± 0.5
^{34}Cl	3049.9 ± 1.9	0.19	0.18	1.01	0.39	0.03	-0.26 ± 0.26	3159.9 ± 2.8	3.2 ± 0.7
^{38}K^m	3047.7 ± 2.7	0.22	0.17	0.97	0.41	0.04	-0.14 ± 0.00	3157.8 ± 2.8	2.6 ± 0.5
^{42}Sc	3041.1 ± 2.0	-0.12	0.06	0.94	0.45	0.04	0.60 ± 0.10	3163.7 ± 2.2	3.4 ± 0.6
^{46}V	3043.1 ± 2.2	-0.08	-0.13	0.91	0.47	0.05	0.32 ± 0.32	3166.1 ± 3.3	3.9 ± 1.2
^{50}Mn	3039.4 ± 3.9	-0.05	-0.07	0.88	0.49	0.05	0.32 ± 0.32	3160.5 ± 4.6	2.5 ± 0.7
^{54}Co	3044.9 ± 2.3	0.01	-0.01	0.86	0.50	0.06	0.18 ± 0.18	3163.2 ± 2.8	2.9 ± 0.6

The ft columns are in s; the δ are in %.

(of some 0.1%) and for branching (everywhere greater than 99% and known to ±0.01%); the phase space factors f incorporate the gross effect of the Coulomb field of the daughter nucleus Z on the departing positron through the Fermi function $F(Z,W)$ as precisely calculated - see section 3.1: the best-fit quadratic in Z has $\chi^2/\nu \simeq 11$.

We now discuss the ways in which Eq. (1) is affected by charge dependences bearing in mind that, as seen from Fig. 1(a) and Table 1, the experimental accuracy on the $(ft)_e$-values is a few times 0.01% which provides the measure for the confidence with which the various charge-dependent effects must be estimated.

3.1 The f-value

The phase space is profoundly affected by the Coulomb field of the daughter nucleus acting upon the departing positron; this introduces a factor of order 2 for the heaviest bodies of present concern in Table 1. At the level of precision here sought it is not adequate to use the usual analytical point-nucleus solution for the positron wave function: we must consider the effect of an extended charge distribution. This move from point to finite nucleus results in a further correction of up to 2% or so for our practical cases on top of that due to the use of the point-nucleus $F(Z,W)$ evaluated at $R = \sqrt{5/3}R_{rms}$ where R_{rms} refers to the extended charge distribution.[2] This finite-size correction is dominated by R_{rms} and is only weakly sensitive to the form of the charge distribution; however, differences of up to about 0.1% arise in the present cases as between a uniform spherical charge distribution and one of more realistic form of the same R_{rms} so this point must be taken.[3] An accurate approximation to this use of a more realistic form for the charge distribution in the present cases[4] is to multiply f by $1 + A - BW_o/2$ where $A = 1.8 \times 10^{-5} \times |Z|^{1.36}, B = 2.4 \times 10^{-6} \times |Z|$ and W_o is the total end-point energy in natural units. (The finite-size correction is not itself strongly sensitive to R_{rms} in our region of practical interest: knowledge of the nuclear size to about 2% is needed to determine the correction to 0.01%.[2]) Similarly, it is not fully adequate in effecting the lepton-nucleon convolution to use for the nucleons wave functions uniform through the nuclear volume: use of more realistic single-nucleon wave functions can entrain changes of order 0.1%.[5] (Note that we are not here concerned with charge-dependent differences between the initial proton wave function and that of the neutron into which the proton transforms itself, such as will be considered in section 3.3 in the context of the nuclear mismatch, the charge-dependent modification of $| M |^2$, but only with the role of the nucleonic wave function in weighting the convolution of the positron and neutrino wave functions.)

Two further corrections to the f-value relate not to the size of the nucleus but to its mass. The first recognizes that the nucleus recoils from the resultant of the lepton momenta; this converts the two body phase space for a nucleus of infinite mass into three body phase space and is worth up to 0.01% in the present context.[2] The second recognizes that the Coulomb field that affects the positron's wave function is not fixed in space but recoils with the nucleus; this correction is here less than 10^{-3}% and so may be ignored.[6]

Analytical expressions have been presented[2,7] that permit the evaluation, with an accuracy that significantly betters 0.01%, of the fundamental $F(Z, W)$ and its associated point-nucleus f-value together with the finite size and mass effects with the exception of those relating to the forms of the charge distribution and of the single-nucleonic wave functions; a numerical parameterization of similar content that betters 0.1% is also available[8] but without the desirable facility for change of R for a given Z that the analytical treatment has. However, even apart from effects associated with the forms of the charge distribution and of the single-nucleonic wave functions it must be recognized that tractable analytical expressions for the positron wave functions even of great elaboration and refinement,[9] are necessarily approximate in the degree to which they can accommodate higher terms in the radial behaviour and may not be wholly adequate at the level of precision to which we must currently aspire. Thus, two alternative analytical expressions that have been suggested[9,10] differ in the f-values that they generate by up to 0.26% in the present context[2]; this significantly exceeds the experimental error. It seems, therefore, that if we are to seek confidence in the f-value at the 0.01% level we must resort to direct numerical integration of the Dirac equation in the field of a charge distribution to represent the daughter nucleus that is synthesized from appropriate proton orbitals derived from some suitable Saxon-Woods or Hartree-Fock prescription that correctly reproduces the nuclear size and then to convolute the resultant positron wave functions with exact neutrino wave functions weighting that convolution by the appropriate single-nucleonic wave functions deriving from that same prescription. This is done in preparation of the Chalk River data base of Table 1.[1] It is then perfectly adequate to use the numerical parameterization[8] to adjust for possible subsequent changes to Q-values and the analytical expressions[2,7] for possible changes in nuclear dimensions. This procedure omits only the relativistic term that is written $\int \alpha.r$ in traditional notation; its evaluation is not unambiguous but its magnitude is very small; it occasions no significant anxiety.

In all computations of the f-value allowance must be made for the effect of the atomic electrons in partially screening the nuclear charge. For the decays in question this correction is only a little over 0.1% in all cases (see e.g. Ref.[11]) and may be made with great accuracy.

We may also note that the slowing-down of the beta-decay due to the rearrangement of the atomic electrons as between initial and final states is very slight: the correction is about $3.2 \times 10^{-4} \times | Z |^{-0.54}$, viz. 0.01% or less for the cases in question.[4]

Before leaving the f-value we should remark that we have so far considered only the electrostatic interaction of the positron with the charge of the daughter nucleus;

but there is also a magnetostatic interaction which is strong but of short range: it is incorporated into the term C of the inner radiative correction to be discussed in section 3.2.2. Overall, the f-value evaluation, for a given Q-value, is probably reliable to $\pm$ 0.01%.

3.2 The Radiative Corrections

The virtual photon exchanges that build up the effective electrostatic positron-nucleus potential, within which the Dirac equation is solved in deriving the f-value as above, represent the greater part of the electromagnetic intercourse involved in nuclear beta-decay. There are, however, other terms, both nucleon-dependent and nucleus-dependent that involve additional photons, both real (inner bremsstrahlung) and virtual and that also involve the mechanism of the weak interaction and that therefore entrain the W and Z bosons that are the vehicles of that interaction. These additional photonic and specifically electro-weak effects are the radiative corrections; they separate rather accurately, although only to an approximation to be discussed below, into two parts: inner and outer. Within this initial approximation the outer correction is decay-energy-dependent and Z-dependent and, slightly, nuclear-size-dependent but does not depend upon nucleon structure nor upon the anatomy of the weak interaction process; the inner correction is concerned with nucleon structure and the weak mechanism but not with the energy release nor, in lowest order, with the nuclear context within which the decaying nucleon is immersed (although, as we discuss in section 3.2.2, it does contain a very important small term of that nature) and so, to a first approximation, may be treated as a renormalization of the vector coupling constant converting the primitive G_V into the operational G_V^*.

3.2.1 The outer corrections. The outer radiative corrections are of various orders $Z^n\alpha^m$; they must be carefully defined so as to avoid double counting with $F(Z,W)$. Naive vertex counting involving n virtual photons would suggest terms of order $(Z^2\alpha)^n$ which would be disastrous but such terms, in fact, vanish: the lowest that survive are of the form $Z^n\alpha^m$ where $m \geq n$; furthermore, those remaining of order $(Z\alpha)^n$ are negligible following the construction of $F(Z,W)$ which is itself a function of $Z\alpha$.[12]

The correction of order α, relating to a single nucleon, is known exactly; call it $\delta_1 = \frac{\alpha}{2\pi}\overline{g(W,W_o)}$ where the bar indicates integration of the explicit function $g(W,W_o)$[13] over the positron spectrum to its end point W_o. The corrections of order $Z\alpha^2(\delta_2)$[14,15] and $Z^2\alpha^3(\delta_3)$[14] have recently been stabilized and may be written down explicitly in various orders of approximation; δ_2 depends significantly upon the nuclear size as well as upon Z; δ_3 is available only in lowest approximation, also involving the nuclear size. δ_1, δ_2 and δ_3 are listed in Table 1; all speed the decay and, being designed to be combined in this way (and with $F(Z,W)$), total $\delta_R = \delta_1 + \delta_2 + \delta_3$. δ_R, about 1.4–1.5%, is huge in relation to the experimental errors; however, δ_1 is "exact", δ_2 is probably secure to $\pm$0.02% or better and δ_3 to $\pm$0.01% or so. Higher terms are probably negligible and we conclude that the outer radiative corrections are under good control in relation to the experimtental errors. δ_R is traditionally applied to the experimental lifetime leading to the outer-radiative-corrected:

$$(ft)_R = (ft)_e(1 + \delta_R). \qquad (4)$$

Figure 1(b) displays the $(ft)_R$ which has $\chi^2/\nu \simeq 13$ for the best quadratic fit in Z viz. slightly poorer than for the raw $(ft)_e$ of Fig. 1(a).

3.2.2 The inner correction. For a point 4-fermion interaction the inner radiative correction is $\frac{\alpha}{2\pi}3ln\frac{\Lambda}{m_p}$ where Λ is a cut-off that must be imposed to eliminate the ultra-violet divergence; with the introduction of just the W-boson to mediate the interaction Λ is replaced by m_W but divergences remain elsewhere in the calculation; with the full electro-weak unification the residual divergences disappear[16] and we may define an inner radiative correction Δ^* for the nucleon (subsuming that for the muon):

$$G_V^{*2} = G_V^2(1 + \Delta^*) \tag{5}$$

$$\Delta^* = \frac{\alpha}{2\pi}\{3ln\frac{m_Z}{m_p} + 6\overline{Q}ln\frac{m_Z}{m_A} + 2C + \mathcal{A}\}. \tag{6}$$

In Eq. (6) the first, vector, term is exact by CVC; the second term is axial and is brought in by photon exchange, the infra-red divergence of the γW box diagram being cut off at some "axial mass" m_A which is usually taken to be that of the a_1-meson† viz. (1.262 ± 0.023) GeV which is close to the empirical mass, (1.032 ± 0.036) GeV, implied by the dipole parameterization of the axial form factor in νN interactions at low energy.[17] $\overline{Q}$ is the mean charge of the fundamental constituents of the nucleon i.e. $\overline{Q} = \frac{1}{2}$ for fundamental neutrons and protons and $\overline{Q} = \frac{1}{2N_q}$ for N_q quarks per nucleon. C is an asymptotic long-distance axial correction (which reaches out of the nucleon and which therefore entrains consideration of nuclear structure[18] and means that the inner radiative correction in the nuclear context cannot be thought of purely as a renormalization of the coupling constant according to Eq. (5)). $\mathcal{A}$ is a small QCD correction of magnitude -0.34.[19]

The total radiative correction is now $(1 + \delta_R)(1 + \Delta^*)$ which amounts to some 3.7%.

The importance of the nuclear-structure-dependent part of the asymptotic term C may be seen by writing $C = C_{BORN} + C_{NS}$ where

$$C_{BORN} = 0.788\lambda(\mu_p + \mu_n) = 0.885 \tag{7}$$

where λ is as defined in Eq. (30) and where the nuclear-structure-dependent C_{NS} is as listed in Table 1‡ and then removing C_{NS} from the inner radiative correction Δ^* and applying it as a correction to the individual transitions, defining:

$$(ft)_{R,C} = (ft)_e(1 + \delta_R)(1 + \frac{\alpha}{\pi}C_{NS}). \tag{8}$$

Figure 1(c) shows the substantial effect this correction has in bringing the data into smoother concordance: for the best quadratic fit in Z we now have $\chi^2/\nu \simeq 2.4$.

Previous analyses have not had regard for C_{NS} and it has been customary to use for C its nuclear-structure-independent part C_{BORN} which yields $\Delta^* = 0.02258$; this preserves the neat separation of inner and outer radiative corrections and one writes:

$$\mathcal{F}t = (ft)_R(1 - \delta_c) \tag{9}$$

†Some uncertainty should properly attach to this assumption but it is difficult to say what it might be.

‡The errors on the C_{NS}-values as listed in Table 1 reflect varying degrees of confidence in the wave functions used in making the estimates[18]; the use of better wave functions, as are now available, will result in smaller uncertainties.

where δ_c is the purely nuclear mismatch between initial and final nuclear wave functions induced by the various charge dependences that we shall consider in section 3.3. $\mathcal{F}t$, which should be the same for all eight transitions of Table 1, if weak CVC holds, now takes the place of ft in Eq. (1), G_V being there similarly replaced by G_V^*. However, this treatment relies on absolute theoretical estimates of the mismatch δ_c which, as will be discussed at length in section 3.3, we must regard with some reserve and is also no longer adequate in respect of the inner radiative correction which must be revised to incorporate: (i) higher power of α; (ii) the running QED coupling constant appropriate to the masses of all the real and virtual particles involved; (iii) the nuclear-structure-dependent part, $C_{\rm NS}$, of C the importance of which we have seen in Fig. 1(c).

The above revisions lead to the replacement of $(1+\delta_R)(1+\Delta^*)$ of the traditional approach by $(1+\Delta+\delta_2+\delta_3)$ where[19]:

$$1+\Delta = \{1+\frac{\alpha}{2\pi}\left[ln\frac{m_p}{m_A}+2C\right]+\frac{\alpha(m_p)}{2\pi}(\overline{g(W,W_o)}+\mathcal{A})\}S(m_p,m_Z). \tag{10}$$

$S(m_p,m_Z)$ is derived by renormalization group methods which sum up all leading terms of the form $(\alpha\ \ ln m_Z)^n$ and so constitutes an approximation to the incorporation of all powers of α. Explicitly[19,20]:

For $m_t < m_W$:

$$S(m_p,m_Z)=\left[\frac{\alpha(m_c)}{\alpha(m_p)}\right]^{\frac{3}{4}}\left[\frac{\alpha(m_\tau)}{\alpha(m_c)}\right]^{\frac{9}{16}}\left[\frac{\alpha(m_b)}{\alpha(m_\tau)}\right]^{\frac{9}{19}}\left[\frac{\alpha(m_t)}{\alpha(m_b)}\right]^{\frac{9}{20}}\left[\frac{\alpha(m_W)}{\alpha(m_t)}\right]^{\frac{3}{8}}\left[\frac{\alpha(m_Z)}{\alpha(m_W)}\right]^{\frac{12}{11}} \tag{11a}$$

For $m_W < m_t < m_Z$:

$$S(m_p,m_Z)=\left[\frac{\alpha(m_c)}{\alpha(m_p)}\right]^{\frac{3}{4}}\left[\frac{\alpha(m_\tau)}{\alpha(m_c)}\right]^{\frac{9}{16}}\left[\frac{\alpha(m_b)}{\alpha(m_\tau)}\right]^{\frac{9}{19}}\left[\frac{\alpha(m_W)}{\alpha(m_b)}\right]^{\frac{9}{20}}\left[\frac{\alpha(m_t)}{\alpha(m_W)}\right]^{\frac{36}{17}}\left[\frac{\alpha(m_Z)}{\alpha(m_t)}\right]^{\frac{12}{11}} \tag{11b}$$

For $m_Z < m_t$:

$$S(m_p,m_Z)=\left[\frac{\alpha(m_c)}{\alpha(m_p)}\right]^{\frac{3}{4}}\left[\frac{\alpha(m_\tau)}{\alpha(m_c)}\right]^{\frac{9}{16}}\left[\frac{\alpha(m_b)}{\alpha(m_\tau)}\right]^{\frac{9}{19}}\left[\frac{\alpha(m_W)}{\alpha(m_b)}\right]^{\frac{9}{20}}\left[\frac{\alpha(m_Z)}{\alpha(m_W)}\right]^{\frac{36}{17}} \tag{11c}$$

The QED running coupling constants $\alpha(\mu)$ are defined by modified minimal subtraction:

$$\alpha^{-1}(\mu)=\alpha^{-1}(0)-\frac{2}{3\pi}\sum_f Q_f^2 ln\frac{\mu}{m_f}+\frac{7}{2\pi}ln\frac{\mu}{m_W} \tag{11d}$$

$$\alpha^{-1}(0)=\alpha^{-1}+\frac{1}{6\pi}=137.089 \tag{11e}$$

In Eq. (11d) f refers to all elementary fermions, the second term on the RHS being evaluated for $m_f < \mu$ and the third for $\mu > m_W$.

$S(m_p,m_Z)$, evaluated using $m_Z = 91.16$ GeV and $m_W = 80.6$ GeV[21] (and $m_{u,d,s} = 0.07$ GeV[19]; $m_c = 1.35$ GeV; $m_b = 5$ GeV[21]) has the value 1.0225 almost independently of m_t in the experimentally-permitted range $m_t > 80$ GeV.[22]

The full radiative corrections are now expressed through the factors:

$$\text{Rad}_{\text{OLD}} = (1+\delta_R)(1+\Delta^*)$$
$$\text{Rad}_{\text{NEW}} = 1+\Delta+\delta_2+\delta_3$$

Rad_{NEW} - Rad_{OLD} ranges from 0.00152 for ^{14}O to 0.00134 for ^{54}Co and so it is important to use the full treatment represented by Rad_{NEW} since the change that it entrains is significant in relation to experimental accuracy.

3.3 The Nuclear Mismatch

There remains to be discussed δ_c, the nuclear mismatch due to charge-dependences, which causes the square of the matrix element in the denominator of Eq. (1) to fall below its value given in Eq. (3) by the factor $(1 - \delta_c)$. Since, as will be seen, δ_c is of the order of ten times the experimental error on individual decays its adequate treatment is a matter of high concern. Unfortunately there is no unambiguous direct procedure through which δ_c may be confidently calculated. One reason for this is that the overlap that defines δ_c involves all the nucleons, neutrons and protons, of the entire nucleus and not just the nucleon nominally responsible for making the beta-transition so that, for example, a misattribution of only 0.001% to the contribution of individual nucleons to δ_c would, in the heavier bodies of our concern, result in an error in the overall δ_c equal to the experimental error in the data to be analyzed. However, as will be seen, uncertainties in the estimation of δ_c go beyond that just indicated and make the matter additionally unsure in respect of dead reckoning.

The nuclear mismatch δ_c can be estimated via global considerations relating to the general behaviour of nucleons in charge-dependent potentials[23] or via detailed microscopic shell model wave functions case by case. It is clear from Fig. 1(c) that sufficient case-to-case fluctuations remain in $(ft)_{R,C}$ to demand consideration by the latter method.

It is usual, in shell model calculations of δ_c, to divide the effect into two parts: $\delta_c = \delta_{c1} + \delta_{c2}$. δ_{c1} is due to the T_Z-dependence of the configurational mix out of which the detailed wave functions are constructed (which contains a component that can be described as isospin mixing although it is not very useful to use that term in this context). δ_{c2} is due to the fact that the change of T_Z between initial and final states gives rise to different binding energies for the "decaying" proton and the "resultant" neutron; this, together with the Coulomb effect on the proton, results in different wave functions ψ_p and ψ_n for the proton and neutron states respectively so that their radial overlap:

$$\Omega_\pi = \int_o^\infty \psi_{p\pi}\psi_{n\pi} dr \tag{12}$$

is less than unity. Here the subscripts π remind us that when we deal with many-particle shell model wave functions the decaying proton does not observe the parent nucleus of the $A-1$ system solely, if at all, in its ground state, to which the nominal binding energy is referred, but rather in an extensive spectrum of parent states π in relative abundances described by the respective spectroscopic factors S_π and with isospins $\pi_< = T - \frac{1}{2}$ and $\pi_> = T + \frac{1}{2}$; and similarly for the resultant neutron. At this point one introduces the conventional and dubious fiction that the effective binding energies, to be used in generating the $\psi_{p\pi}$, $\psi_{n\pi}$ via some fancied overall nuclear potential, are the nominal binding energies, as referred to the ground states of the respective $A-1$ nuclei, plus the excitation energies of the parent states π in the respective $A-1$ nuclei; these excitation energies are taken from experiment if available, and if sufficiently confident association can be made between the experimental states and those of the model, or from the theoretical shell model wave functions if not. The associated spectroscopic factors S_π are similarly derived from theory or experiment following which we have[24]:

$$(1-\delta_{c2})^{\frac{1}{2}} = \frac{1}{2}\left[\frac{1}{T}\sum_{\pi_<} S_\pi \Omega_\pi - \frac{1}{T+1}\sum_{\pi_>} S_\pi \Omega_\pi\right]. \tag{13}$$

Obviously, the more reliable the shell model wave functions, from the point of view of the parentage spectrum with its associated S_π and excitation energies, the more confidence we may repose in the δ_{c2} so computed. However, we still have to reckon with: (i) the uncertainties in the generation of the $\psi_{N\pi}$, which include the effects of the deformation, as well as the form, of the optical model potential; (ii) most particularly the uncertainty associated with the effective π-dependent binding energy fiction to which reference has been made; (iii) the fact that Eq. (13) concerns itself only with the mismatch between the initial proton and final neutron states of the transforming nucleon whereas there will also be mismatch as between the parent states themselves, represented by the remainder of the valence nucleons of the shell model wave functions, in the initial and final nuclei; (iv) the fact that the core nucleons, that are not involved in the generation of the shell model wave functions, being outside the basis of the shell model calculation, will also suffer mismatch as between initial and final nuclei. It is not possible, at this time, accurately to quantify these uncertainties and so direct computation of δ_{c2} must be viewed with appropriate reserve.

Some orientation into the dependence of the calculated δ_c on the details of its generation is given by comparing the two most recent computations due to Towner, Hardy and Harvey (THH)[1,25] and to Ormand and Brown (OB)[26]; the former use single-nucleonic ψ_N generated in a standard Saxon-Woods potential while the latter use a Hartree-Fock mean field approach. Table 2 lists the respective δ_{c1}, δ_{c2} and $\delta = \delta_{c1} + \delta_{c2}$. (The "errors" quoted for the δ_{c2} are somewhat impressionistic figures thought by the authors to be reasonable reflections of uncertainties in their respective procedures.) It is seen: (i) that the δ_{c1} are relatively small and although the differences are considerable as between THH and OB those differences are, for the most part, not too worrying in relation to the experimental uncertainties although in some cases comparable with them; (ii) that the δ_{c2} are considerably larger than the δ_{c1} and differ unacceptably as between THH and OB in relation to the experimental errors: the mean value of $\delta_c^{\rm THH} - \delta_c^{\rm OB}$ is 0.16% (as is that of the modulus of that quantity).

In regard to the mismatch of the core: (i) if an estimate of the core contribution to δ_{c2} is made in the literal Saxon-Woods spirit of THH a very substantial figure, of the percentage order, is obtained; (ii) the Hartree-Fock approach of OB suffers from the fact that such wave functions do not respect isospin even when the two-body force is an isospin scalar and so cannot be trusted for a direct evaluation of the core mismatch. OB overcame this difficulty by an ingenious if ad hoc stratagem following which the core mismatch was estimated to be negligible. However, the ad hoc nature of the remedy must leave some doubt as to the reliability of the conclusion.

The upshot of this discussion is that present direct computation of δ_c cannot be relied upon to better than the difference, 0.2% or so, between the $\delta_c^{\rm THH}$ and $\delta_c^{\rm OB}$ of Table 2 and that, lurking behind this figure, there are the additional uncertainties associated with the core nucleons and other considerations to which reference has been made.

However, there is an alternative approach to the assessment of δ_c[27] that seeks to by-pass much of the above uncertainty by looking to the experimental data themselves to help reveal the mismatch. This approach considers that δ_c will have two

Table 2. Calculated and inferred nuclear mismatch in %.

	THH			OB			
Body	δ_{c1}	δ_{c2}	δ_c	δ_{c1}	δ_{c2}	δ_c	δ_{cINF}
^{14}O	0.00	0.28 ± 0.03	0.28	0.01	0.18 ± 0.05	0.19	0.32
^{26}Al^m	0.06	0.27 ± 0.04	0.33	0.01	0.23 ± 0.06	0.24	0.48
^{34}Cl	0.02	0.62 ± 0.07	0.64	0.06	0.42 ± 0.07	0.48	0.84
^{38}K^m	0.16	0.54 ± 0.07	0.70	0.11	0.38 ± 0.12	0.49	0.90
^{42}Sc	0.04	0.35 ± 0.06	0.39	0.11	0.28 ± 0.05	0.39	0.71
^{46}V	0.09	0.36 ± 0.06	0.45	0.01	0.20 ± 0.06	0.21	0.65
^{50}Mn	0.10	0.40 ± 0.09	0.50	0.004	0.28 ± 0.06	0.28	0.72
^{54}Co	0.03	0.56 ± 0.06	0.59	0.005	0.34 ± 0.06	0.35	0.79

components: $\delta_c = \delta_{cu} + \delta_{cf}$; δ_{cu} is a smooth (monotonically increasing) function of Z, the underlying mismatch, whose form and magnitude are unknown; δ_{cf} are the case-to-case fluctuations about δ_{cu} associated with explicit shell model effects, case-to-case irregularities in binding energies and so on. All that we know, a priori, is that δ_{cu} and δ_{cf} must, presumably, separately go to zero as Z goes to zero. We might now hope that although different direct computations of δ_c may differ considerably in magnitude, and might be afflicted by the various uncertainties that we have considered, yet they might be more reliable in respect of, and agree better with each other in respect of, the case-to-case fluctuations δ_{cf} since these are more nearly due just to the valence nucleons of the shell model wave functions and to the de facto binding energy fluctuations that are taken into account in the computational procedures. Indeed[27] if we extract the δ_{cf} from the THH and OB calculations by separately best-fitting the arbitrary function AZ^B to each set of δ_c taken from Table 2 (the fitting form chosen matters little and the parameters have no significance) we find the δ_{cf}-values listed in Table 1 from which it is seen that the mean value of $\delta_{cf}^{\rm THH} - \delta_{cf}^{\rm OB}$ is -0.005% and that of $\mid \delta_{cf}^{\rm THH} - \delta_{cf}^{\rm OB} \mid$ is only 0.05%: $\delta_c^{\rm THH}$ and $\delta_c^{\rm OB}$ tend strongly to fluctuate up and down synchronously.

If we now apply this case-to-case correction δ_{cf} to the experimental data (using for δ_{cf} the mean of $\delta_{cf}^{\rm THH}$ and $\delta_{cf}^{\rm OB}$), defining:

$$(ft)_{R,C,\delta_f} = (ft)_e(1+\delta_R)(1+\frac{\alpha}{\pi}C_{\rm NS})(1-\delta_{cf}) \qquad (14)$$

we should be left with data requiring further correction only for the smooth underlying δ_{cu}, which the data themselves should then reveal, such that the appropriate extrapoation of $(ft)_{R,C,\delta_f}$ to $Z=0$ should define $(ft)_o$ which would then yield:

$$G_V^{*2} = K/2(ft)_o \qquad (15)$$

Figure 1(d) shows the $(ft)_{R,C,\delta_f}$, the best-fitting of which to a quadratic in Z has $\chi^2/\nu \simeq 0.7$ which is wholly satisfactory and justifies our procedure.

4. ANALYSIS OF THE FERMI TRANSITIONS

The preceding discussion has led to the point at which, from Fig. 1(d), we could extract $(ft)_o$, hence derive G_V^* by Eq. (15), hence G_V by Eq. (5) using Δ^* from Eq. (6) (setting $C = C_{\rm BOHN}$). However, as has been emphasized, such a procedure is no longer adequate in view of the development of the superior Δ of Eq. (10). The better plan is to construct from the experimental data:

$$(ft)^* = (ft)_e(1 - \delta_{cf})(1 + \Delta + \delta_2 + \delta_3) \tag{16}$$

and extrapolate $(ft)^*$ to $(ft)_o^*$ at $Z = 0$ then deriving:

$$G_V^2 = K/2(ft)_o^*. \tag{17}$$

This construction of $(ft)^*$ from Eq. (16) is given in Fig. 2 following the listing in Table 1 (where the increase in the errors of the $(ft)^*$ above those of the $(ft)_e$ is due to the theoretical uncertainties listed for the $C_{\rm NS}$). We find by extrapolation quadratic in Z[§]:

$$(ft)_o^* = (3139.1 \pm 2.6)\text{sec} \tag{18}$$

hence:

$$G_V^2 = (1.2934 \pm 0.0011) \times 10^{-10}\text{GeV}^{-4}. \tag{19}$$

Before proceding to further discussion note that the best way to extract the operational G_V^* is to start from the G_V of Eq. (19), to note the equivalence:

$$(1 + \Delta^*)(1 + \delta_1) \simeq 1 + \Delta \tag{20}$$

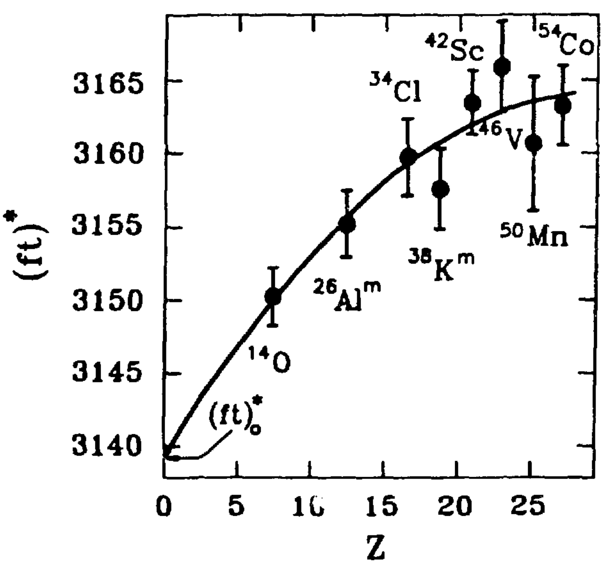

Fig. 2. $(ft)^* = (ft)_e(1 - \delta_{cf})(1 + \Delta + \delta_2 + \delta_3)$ as a function of Z extrapolated quadratically to $(ft)_o^*$ at $Z = 0$.

§No theoretical justification attaches to this fitting; it is simply the lowest-order adequate polynomial in Z.

from which derive Δ^* on a case-to-case basis and average them to gain the best effective Δ^* (it is 0.02397) and then use Eq. (5) to find:

$$G_V^* = (1.1508 \pm 0.0005) \times 10^{-5}\text{GeV}^{-2}. \tag{21}$$

It is also of interest to use the $(ft)^*$-fit of Fig. 2 to derive the δ_{cuINF} inferred from our present procedure, then to combine them with the δ_{cf}-values that we have used in obtaining the $(ft)^*$ thereby gaining the overall inferred $\delta_{cINF} = \delta_{cuINF} + \delta_{cf}$. Figure 2 lists these δ_{cINF}; it is seen that they are broadly of the same order as the δ_c of the direct computations but are, as we anticipated, somewhat larger: by a factor, on average of 1.4 than δ_c^{THH} and of 2.1 than δ_c^{OB}.

5. UNITARITY OF THE CABIBBO-KOBAYASHI-MASKAWA MATRIX

An immediate use for the G_V of Eq. (19) is in a test for the unitarity condition relating to the first row of the Cabibbo-Kobayashi-Maskawa (CKM) matrix:

$$| V_{ud} |^2 + | V_{us} |^2 + | V_{ub} |^2 = 1. \tag{22}$$

From muon decay we have:

$$G_\mu^2 = (1.36046 \pm 0.00005) \times 10^{-10}\text{GeV}^{-4} \tag{23}$$

(where the outer radiative correction, here also extended to higher orders in α by renormalization group methods,[28] has been applied to the experimental data[21] and where we remember that the muon's inner radiative correction has been subsumed into Δ).

With:

$$| V_{ud} |^2 = (G_V/G_\mu)^2 \tag{24}$$

we have:

$$| V_{ud} |^2 = 0.9507 \pm 0.0008. \tag{25}$$

This we may now combine with[29]:

$$| V_{us} |^2 = 0.0481 \pm 0.0008 \tag{26}$$

and[30]:

$$| V_{ub} |^2 \simeq 3 \times 10^{-5} \tag{27}$$

to find:

$$| V_{ud} |^2 + | V_{us} |^2 + | V_{ub} |^2 = 0.9989 \pm 0.0012 \tag{28}$$

which constitutes a satisfactory test.

6. THE NUMBER OF QUARKS PER NUCLEON

For an amusement in passing we can:

(i) Assume CKM unitarity;

(ii) Use $|V_{us}|$ (Eq. (26)) and $|V_{ub}|$ (Eq. (27)) to infer $|V_{ud}|$;

(iii) Use G_μ (Eq. (23)) and $|V_{ud}|$ to find G_V by Eq. (24);

(iv) Use G_V from (iii) to derive Δ^- (namely Δ from which C_{NS} has been extracted) on a case-by-case basis from:

$$K/2G_V^2 = (ft)_c(1-\delta_{cINF})(1+\frac{\alpha}{\pi}C_{NS})(1+\Delta^- + \delta_2 + \delta_3)$$

(v) Use the equivalence of Eq. (20) to infer Δ^{*-} on a case-by-case basis from the Δ^- from (iv);

(vi) Infer $\overline{Q}$ (viz. N_q) on a case-by-case basis from the Δ^{*-} from (v) using $C = C_{\mathrm{BORN}}$ in Eq. (6).

When this is done[31] we find the N_q-values listed in Table 1 which average $N_q = 3.02 \pm 0.20$ without regard for the error in $|V_{us}|$ (Eq. (26)); when that is included we find $N_q = 3.0 \pm 0.6$.

7. NEUTRON DECAY

From the G_V^* of Eq. (21) and the best "post 1986" (directly measured) neutron lifetime of[32,33]:

$$\tau_m = (888.8 \pm 2.4)\mathrm{sec} \tag{29}$$

we derive[6]:

$$G_V^{*2} + 3G_A^{*2} = (7.687 \pm 0.021) \times 10^{-10}\mathrm{GeV}^{-4} \tag{30}$$

(G_A^* here being the operational axial coupling constant analogous to the vector G_V^*).

Another important quantity is A_o, the asymmetry parameter for the beta-decay of polarized neutrons which is, in lowest order, given by:

$$A_o = -2\frac{\lambda(\lambda-1)}{1+3\lambda^2} \tag{31}$$

where:

$$\lambda = | G_A^*/G_V^* | \, . \tag{32}$$

Two recent accurate measurements of A_o exist yielding, after appropriate correction for weak magnetism (strong CVC), recoil and Coulomb effects[6] and after assurance that radiative corrections are negligible in their effect upon λ[34]:

$$G_A^*/G_V^* = -1.262 \pm 0.004 \quad \text{(Ref. 32,35)} \tag{33}$$

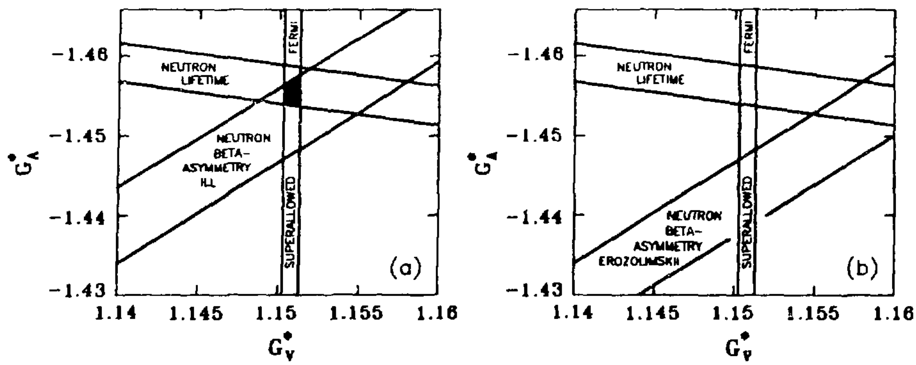

Fig. 3(a) G_V^* derived from vector (superallowed Fermi) decay (Eq. (21)); $G_V^{*2} + 3G_A^{*2}$ derived from the neutron lifetime (Eq. (30)) and G_A^*/G_V^* derived from the ILL measurement of neutron decay asymmetry (Eq. (33)); (b) As (a) but with the G_A^*/G_V^* measurement of Erozolimskii *et al.*, (Eq. (34)) replacing the ILL measurement.

$$G_A^*/G_V^* = -1.254 \pm 0.004 \quad \text{(Ref. 36)}. \tag{34}$$

We may now confront our values for $G_V^*, G_V^{*2} + 3G_A^{*2}$ and G_A^*/G_V^* which we do in Fig. 3(a) using Eqs. (21), (30) and (33) for these three quantities respectively and in Fig. 3(b) where we replace Eq. (33) by Eq. (34). Figure 3(a) shows good mutual accord and permits us to derive the overall:

$$G_A^* = -(1.4554 \pm 0.0016) \times 10^{-5}\text{GeV}^{-2} \tag{35}$$

or:

$$G_A^*/G_V^* = -1.2647 \pm 0.0014. \tag{36}$$

If we choose to derive G_V via unitarity of the CKM matrix as indicated in section 5 followed by allowance for inner radiative corrections via Δ and Δ^* as indicated in section 3 to gain G_V^* we should find:

$$G_A^* = -(1.4557 \pm 0.0020) \times 10^{-5}\text{GeV}^{-2} \tag{37}$$

$$G_A^*/G_V^* = -1.2649 \pm 0.0017. \tag{38}$$

Figure 3(b) shows no area of mutual consistency so does not provide a fit within the standard model.

8. LEFT-RIGHT SYMMETRIC MODELS

The lack of internal consistency of Fig. 3(b) suggests that we should explore the consequences of the intervention of right hand currents as a natural extension of the standard model[37] in the form of the introduction of a right handed partner, W_R, to the W_L of the standard model with the mass parameter $\delta = (m_L/m_R)^2$ and with the mixing angle ζ between the L and R mass eigenstates.

Such $L - R$ symmetry has a number of consequences for beta-decay that we now examine:

(i) The beta-decay asymmetry of polarized neutrons becomes[38]:

$$A_o = -2\frac{\lambda_p(\lambda_p - 1) - \lambda_p y(\lambda_p y - x)}{1 + 3\lambda_p^2 + x^2 + 3\lambda_p^2 y^2} \tag{39}$$

where, very nearly, $x = \delta - \zeta$ and $y = \delta + \zeta$ and where λ_p must be determined from parity-conserving variables only, viz. G_V^* and τ_m which, via Eqs. (21) and (29), yield:

$$\lambda_p = 1.2655 \pm 0.0022. \tag{40}$$

This analysis, using the A_o-value that lies behind Eq. (33) namely

$$A_o = -0.1146 \pm 0.0015 \tag{41}$$

gives only a useful constraint in the δ, ζ-plane such that the permitted area lies, at 90% CL, to the left of the line in Fig. 4(a) labelled "neutron/Fermi". Using the A_o-value that lies behind Eq. (34), namely

$$A_o = -0.1114 \pm 0.0015 \tag{42}$$

we find, at 90% CL, the allowed band of Fig. 4(b).

(ii) The longitudinal polarization of beta-particles will manifestly be affected by the presence of right hand currents: (a) absolutely, as has been sought in the case of Gamow-Teller transitions,[39] which yield the 90% CL upper constraint labelled "Gamow-Teller" in Fig. 4(a); (b) in the relative polarization in Fermi and Gamow-Teller transitions[40] which prescribe the 90% CL region between the curves of Fig. 4(a) labelled "F/GT".

(iii) Powerful constraints derive from muon decay: (a) via the Michel parameter[21] which, at 90% CL, limits the area available to that between the lines of Fig. 4(a) labelled "Michel"; (b) via the combinations of decay parameters $P_\mu\xi\delta/\rho$[41] and $P_\mu\xi$[42] which, at 90% CL, limit the allowed regions to those below their respectively-labelled curves in Fig. 4(a).

(iv) If we assume CKM unitarity the existence of W_R induces a shortfall of 2ζ in the test applied to the first row of the matrix[43] as quoted in Eq. (22) and reported in Eq.

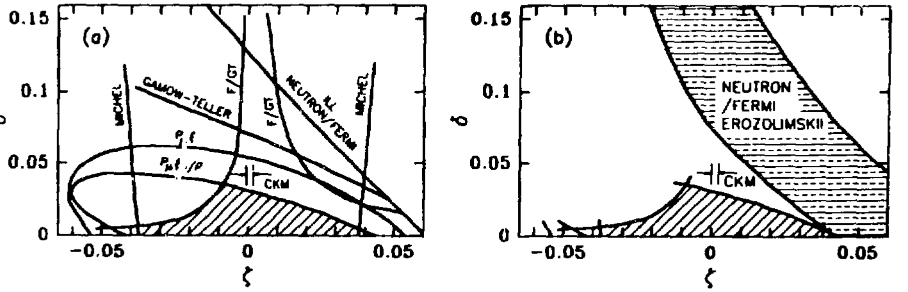

Fig. 4(a) constraints at 90% CL on the δ, ζ parameters of the left-right symmetric model derived as described in the text; the neutron/Fermi constraint is that deriving from the ILL measurement of A_o (Eq. (41)); (b) The shaded area is transferred from (a); the shaded band derives from the A_o-measurement of Erozolimskii *et al.*, (Eq. (42)).

(28) so that we should, from the latter, infer, at 90% CL, $\zeta = 0.0006 \pm 0.0010$; these limits are shown in Fig. 4(a).

The ensemble of these constraints gathered in Fig. 4(a) limit the allowed region of δ, ζ-space to the shaded area at 90% CL without reference to the sharp constraint in ζ afforded bythe CKM test of (iv) above. We derive $m_R/m_L > 5.5$ at 90% CL from the shaded area, i.e. a W_R mass of greater than 440 GeV, or $m_R/m_L > 5.8$ at 90% CL if the CKM constraint is accepted, i.e. a W_R mass of greater than 470 GeV.

We now transfer the shaded allowed area of Fig. 4(a) to Fig. 4(b) there to confront the shaded band that represents the δ, ζ-region permitted at 90% CL by the A_o-value of Eq. (42). We see that there is only marginal compatibility between the area and the band without admitting the CKM constraint, which is also shown in Fig. 4(b), and none at all if that constraint is accepted.

9. OTHER TESTS

We note that the method of analysis of vector beta-decay presented in sections 2 and 3 is implicitly based not only upon the assumption of weak CVC, which therefore cannot simultaneously be tested, but also upon the assumption of the absence of scalar and induced-scalar couplings for which the data could be analyzed were we to adopt directly-computed δ_c and to construct the $\mathcal{F}t$ of Eq. (8)[1,44]; a similar remark applies to testing for the possible intervention of massive neutrinos.[1,45] In the latter context, however, we may note a curious feature of Fig. 2 namely that the inferred underlying δ_{cu} of the nuclear mismatch increases approximately linearly with Z, indeed the best quadratic fit in Z, as seen, increases more slowly than linearly, whereas, offhand, we might have expected, following the Behrends-Sirlin-Ademollo-Gatto theorem,[46,47] a mismatch going approximately as Z^2 as has also been remarked elsewhere[48]; the sense of the difference is as would be expected from the intervention of massive neutrinos.[49]

10. G_V^*: FUTURE DEVELOPMENTS

10.1 ^{10}C Decay

The analysis presented here depends critically upon extrapolation of the experimental data, after allowance for radiative corrections (including C_{NS}) and for fluctuations in the nuclear mismatch, to $Z = 0$. Obviously any further $(ft)_e$ at lower Z than those presently available, and of comparable accuracy to the present set, would be most precious. The only such body is ^{10}C, at $Z = 5$, which has a sufficiently-accurately determined lifetime and Q-value and decays by a pure vector transition, of the type considered here, to the second excited state of ^{10}B. Unfortunately this desired transition shows only a 1.5% branch against the dominant (Gamow-Teller) decay to the first excited state of ^{10}B. The present[50] determination of the Fermi branch with an accuracy of ±0.4% of its own value, which yields an $(ft)_e$-value in accord with those of Fig. 1(a) but with an error enveloping the entirety of that figure, is already a considerable experimental tour de force; it seems unreasonable to look for the further improvement by an order of magnitude such as would be necessary to constitute a significant contribution to the sharpening of our analysis.

10.2 Neutron Decay

As we have seen, the problems of confidently analyzing the vector decays of complex nuclei are formidable despite the high precision of the experimental data. If the neutron lifetime could be measured with adequate precision and also, by A_o-measurement, the G_A^*/G_V^* ratio λ (both quantities now being known, as reported above, to about $\pm 0.3\%$) then we could access G_V^* independently of all the uncertainties entrained by the use of complex nuclei; this would be highly desirable. (It would also, of course, depend on the assumption of the standard, W_L-only, model.) The precision to which we should aspire in the extraction of G_V^* must match that ($\pm 0.05\%$) reported above from complex nuclei. This demands measurements of neutron lifetime and of λ whose mutual accuracy lies within an ellipse with semi-axes of $\pm 0.06\%$ on the λ-axis and $\pm 0.1\%$ on the τ_m-axis; this is very hard.

10.3 Pion Beta-Decay

Pion beta-decay, $\pi^{\pm} \rightarrow \pi^o + e^{\pm} + \overset{(-)}{\nu}_e$, is the archetypal test of weak CVC, directly yielding G_V (after application of the appropriate inner and outer radiative corrections both of which are, in lowest order, the same as for nucleon-decay[16]) without primary concern for structural effects (but see section 11). The present lifetime measurement for this branch of 10^{-8} against $\pi \rightarrow \mu + \nu_\mu$ has an accuracy of $\pm 3\%$[21] so that an improvement of more than an order of magnitude is needed. (Present uncertainty in the $\pi^{\pm} - \pi^o$ mass difference[21] corresponds to a $\pm 0.03\%$ uncertainty in the extracted G_V.)

11. PARTICLE STRUCTURE

We have tacitly assumed that the G_V of the nucleon or of the pion, arrived at after confident removal of the inner radiative corrections (which are themselves, as we have seen, particle-structure-sensitive), relate directly to the V_{ud} of the CKM matrix by Eq. (24) viz. reflect directly the fundamental weak quark-quark couplings. However, this is not the case: just as the vector decay of a complex nucleus is affected by charge-dependent effects within its structure so that of a "fundamental" particle such as a pion or a nucleon is affected by the fact that it is not a truly fundamental particle, such as quarks and leptons putatively are, but is itself structured and that that structure entrains the charge dependences, such as the mass differences, inherent to the truly fundamental elements of that structure, namely the quarks, so that, for example, hadrons are not pure in isospin, although that is only part of the problem. The magnitude of this effect, perhaps to be ascribed specifically to $\rho - \omega$ mixing, is a matter of contention it being argued: (i) that it could have a reflection on $|V_{ud}|$ as large as 0.2%[51]; (ii) that, as a consequence of the Behrends-Sirlin-Ademollo-Gatto theorem,[47] namely that renormalization of vector coupling constants goes only as the square of the mass splittings between the de facto (complex) particles involved so that there is here, for example, no first-order term in $m_u - m_d$, the effect is less than 0.001%.[52] But it is clear that, at some level, we shall have to wrestle with particle structure just as we are now wrestling with nuclear structure.

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