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A Coarse-Mesh Nodal Method–Diffusive-Mesh Finite Difference Method

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Introduction

Modern nodal methods have been successfully used for conventional light water reactor core analyses where the homogenized, node average cross sections (XSs) and the flux discontinuity factors (DFs) based on equivalence theory can reliably predict core behavior[1,2]. For other types of cores and other geometries characterized by tightly-coupled, heterogeneous core configurations, the intranodal flux shapes obtained from a homogenized nodal problem may not accurately portray steep flux gradients near fuel assembly interfaces or various reactivity control elements. This may require extreme values of DFs (either very large, very small, or even negative) to achieve a desired solution accuracy. Extreme values of DFs, however, can disrupt the convergence of the iterative methods used to solve for the node average fluxes[3], and can lead to a difficulty in interpolating adjacent DF values. Several attempts to remedy the problem have been made, but nothing has been satisfactory[3]. A new coarse-mesh nodal scheme called the Diffusive-Mesh Finite Difference (DMFD) technique, as contrasted with the coarse-mesh finite difference (CMFD) technique, has been developed to resolve this problem. This new technique and the development of a few-group, multidimensional kinetics computer program are described in this paper.

Description of the DMFD Method

In an attempt to accurately account for steep flux gradients, we chose a linear discontinuous finite difference diffusion formulation to provide a relationship that preserves the node average flux and its gradient (and consequently surface currents) at the node interface. A similar coarse-mesh finite difference formulation using a usual definition of DFs and bounded diffusion coefficients was introduced[4]. With extreme values of DFs encountered in the problems of steep flux gradients, we observed that this formulation caused a large spectral radius for the global flux iteration matrix and posed an extremely difficult convergence problem. To address this problem, we developed the DMFD technique that provides a means to confine or bound the magnitudes of DFs within a certain range through the use of a node average flux defined at diffusive-mesh points inside a node and coupled to neighboring nodes in a finite difference formulation of the spatial discretization of the leakage terms. The DMFD DFs are defined, for each energy group, by preserving the node average fluxes of two nodes (i,j,k) and $(i+1,j,k)$ and surface average net current $\bar{J}(x_i)$ at the nodal interface x_i as follows:

$$\begin{pmatrix} \dfrac{1}{f_i(x_i)} \\ \dfrac{1}{f_{i+1}(x_i)} \end{pmatrix} = \frac{1}{\bar{\phi}(x_i)} \begin{pmatrix} \dfrac{-d_i}{D_i} & \bar{\phi}(x_i - d_i) \\ \dfrac{d_{i+1}}{D_{i+1}} & \bar{\phi}(x_i + d_{i+1}) \end{pmatrix} \begin{pmatrix} J(x_i) \\ 1 \end{pmatrix} \quad (1)$$

where

d_i = a distance in the node (i,j,k) from the interface x_i defined as the "diffusive-mesh" distance,

$\bar{\phi}(x_i + d_{i+1})$ = the flux on the diffusive mesh point at $x_i + d_{i+1}$

= the node average flux of the node $(i+1,j,k)$,

$\bar{\phi}(x_i - d_i)$ = the flux on the diffusive mesh point at $x_i - d_i$

= the node average flux of the node (i,j,k),

$\bar{\phi}(x_i)$ = the surface flux at x_i based on the conventional mesh-centered CMFD formulation, and

D_i = the diffusion coefficient of the node (i,j,k).

This formula is distinct from other definitions of the DFs because of the ratio of the surface flux obtained from the CMFD formulation to each of the two DMFD surface fluxes which may be discontinuous at the interface to preserve the surface net current. The DMFD surface fluxes are determined by preserving the surface net currents and by using the two diffusive mesh distances as finite mesh intervals. The diffusive-mesh distance is determined by the magnitude of the group-wise diffusion length of each node, so that the inter-node leakage terms can be characterized by a finite difference formulation for mesh intervals on the order of the diffusion length. A choice of $d_i = 1.5D_i$ has produced no extreme values of DFs; consequently, it provides numerical stability in problems which have very steep flux gradients at node interfaces. The diffusive-mesh distances of a node may be direction-dependent. The radial DFs can be determined from the node average fluxes and surface currents obtainable from fine-mesh calculations for any fully heterogeneous two-dimensional problem that extends the geometry around the node of interest, such as a color-set, a partial-core, or even a full-core configuration.

The spatial discretization of the flux along the axial direction in DMFD is a fine-mesh finite difference method, which has exhibited a few advantages over sophisticated coarse mesh alternatives. The advantages include accurate axial representations of neutronically sensitive regions such as zone interfaces of fuel or reactivity control elements, and absence of approximations for the transverse leakage shapes that are difficult to determine in tightly-coupled heterogeneous fuel assemblies. In addition to the fine-mesh capability, an axial fine-mesh collapsing scheme has been developed to employ large axial mesh intervals where solution

accuracy can be achieved by use of non-unity DFs in the axial direction. This capability, based on Joo's technique[5], is particularly useful in representing partially-rodded coarse nodes for rod motion transient problems.

Both the theta method[6] and a variant of the Stiffness Confinement Method (SCM)[7] have been implemented in the DMFD formulation to replace the time derivative term in the kinetic equations. The SCM has been generalized in an explicit few-group formulation where a prompt neutron dynamic frequency function becomes a space-, energy-, and time-dependent function. The frequency function is defined by the time derivative of the logarithm of the average flux of the node *(i,j,k)*, for group g, as

$$\omega_g^{i,j,k}(t) = \frac{d}{dt}\ln\left[\bar{\phi}_g^{i,j,k}(t)\right]. \qquad (2)$$

The precursor equations are solved analytically by direct integration over a time step assuming linear variations in time for the dynamic frequency function. The time-dependent DMFD equations are then transformed into equations formally identical to the static DMFD equations. The dynamic frequencies are calculated iteratively within each time step as part of the transient flux iteration process.

The DMFD algorithm has been incorporated into a nodal kinetics program where a successive block overrelaxation technique and a Gauss-Seidel iterative method were employed for transient analyses of cores of various geometries including hexagonal modules. The in-core thermal and xenon feedback models used in WIGL2[6] and TWIGL[8] have been implemented.

Results and Conclusion

A Diffusive-Mesh Finite Difference nodal method has been developed to analyze three-dimensional transient problems for a wide variety of core types. Its numerical stability, solution accuracy, and computational speed have been achieved. Compared to fine-mesh, fully heterogeneous, four energy-group, three-dimensional, diffusion theory PDQ08 solutions for several reactor core problems with module-size coarse nodes, the following accuracies have been achieved: Eigenvalues within 0.1%Δk/k, fuel module power fractions within 1%, and rod reactivity worths within 5% for limiting control-rod positions.

The accuracy and effectiveness of the DMFD technique when applied to several published benchmark problems have also been demonstrated. Among the test problems is the three-dimensional LMW transient problem with and without thermal feedback effects[9, 10]. In this problem, a transient is introduced by a withdrawal of one rod bank followed by an insertion of another bank. The XSs and radial DFs for twenty- four fuel modules were generated from fine-mesh, radial quarter-core PDQ08 solutions, and the radial water reflector was replaced by an albedo core boundary condition. Auxiliary axial fine-mesh DMFD calculations employed a 2.5 cm mesh to determine the XSs and axial DFs of a partially rodded coarse node whose axial dimension was 20cm. When the dynamic frequency method is employed, typical timestep sizes used over the 60-second transient are 0.8 s during the rod motion period and 4.2s thereafter. Results are shown in Figures 1 and 2. The total transient power levels are in good agreement with QUANDRY[10] and

SPNOVA-K[11]. It should be noted that the rod cusping problem exhibited in the QUANDRY result (Figure 2) has been eliminated in the DMFD solution. Each time step computation without thermal feedback takes approximately 0.5 s on a Cray-YMP/8 single processor when a 2.5 cm axial mesh is employed, and the execution becomes much faster when the fine mesh is collapsed into 20 cm coarse axial nodes and corresponding axial DFs are employed.

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Figure 1
DMFD Result for the LMW Rod Motion Transient
(without thermal feedback effect)

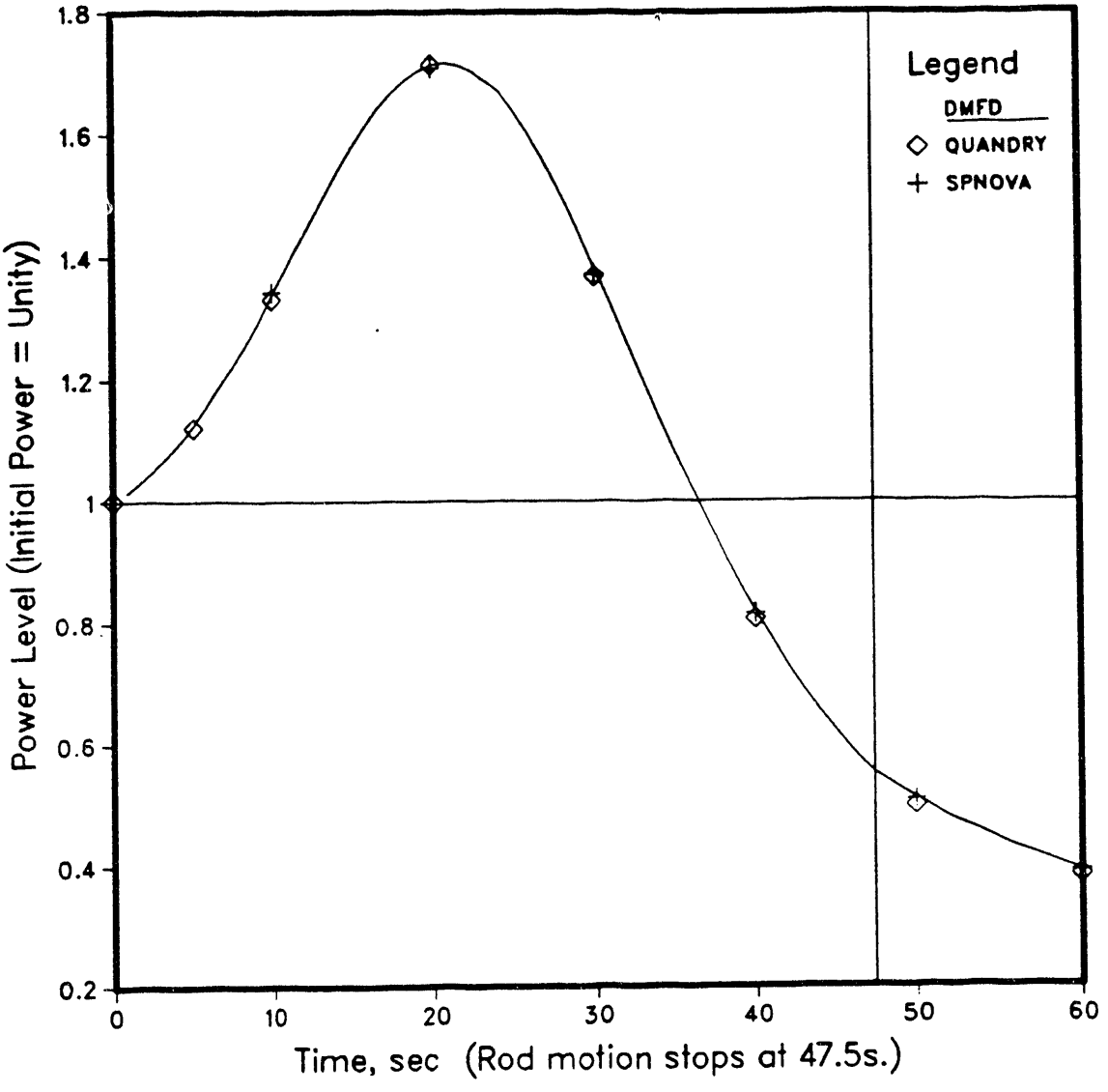
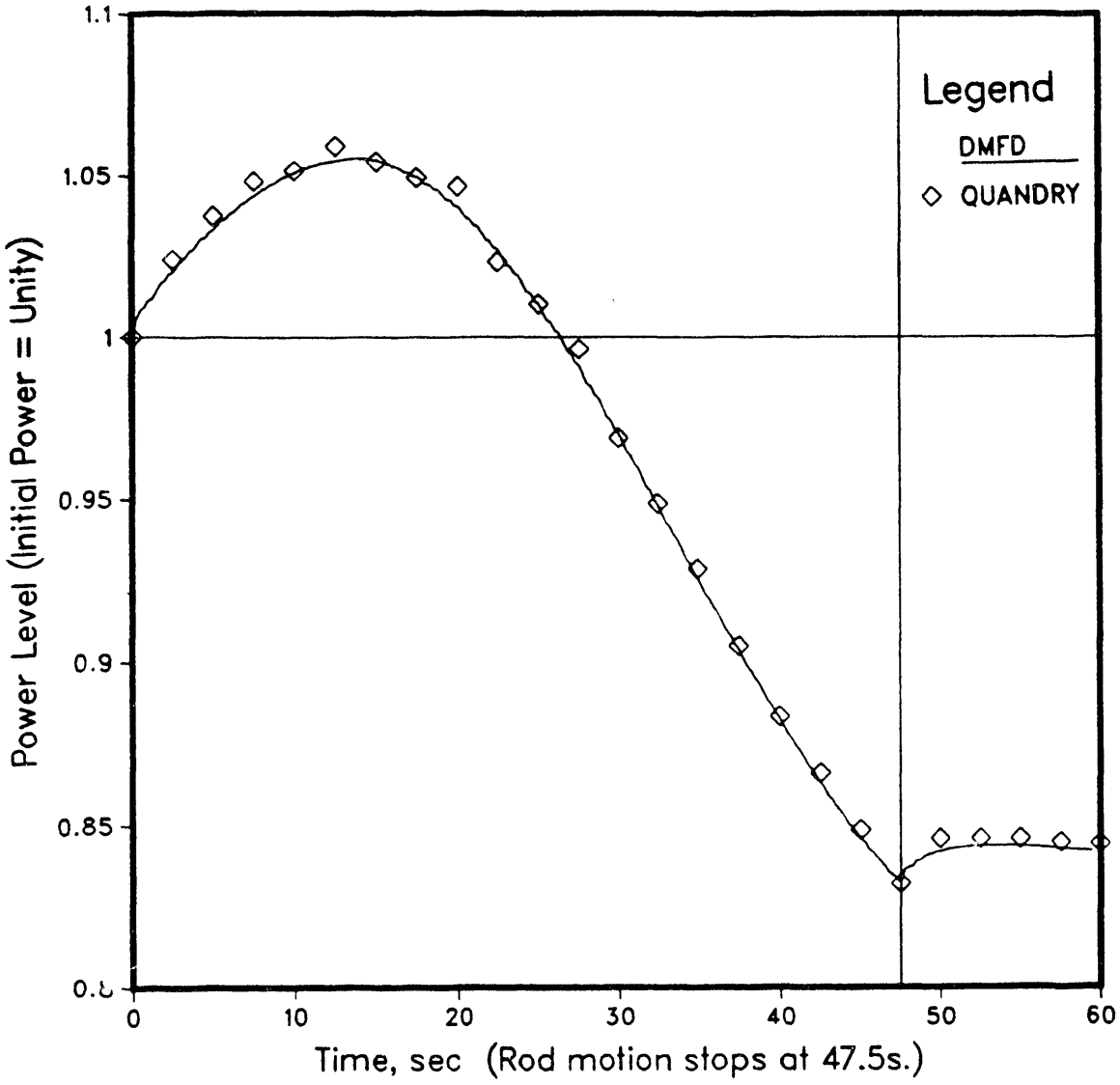


Figure 2
DMFD Result for the LMW Rod Motion Transient
(with thermal feedback effect)



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