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NUMERICAL SIMULATION METHODS FOR WAVE PROPAGATION THROUGH OPTICAL WAVEGUIDES

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MIRAMARE-TRIESTE

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NUMERICAL SIMULATION METHODS FOR WAVE PROPAGATION THROUGH OPTICAL WAVEGUIDES [1]

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Preface

The ICTP-LAMP reports consist of manuscripts relevant to seminars and discussions held at ICTP in the field of Laser, Atomic and Molecular Physics (LAMP).

These reports aim at informing LAMP researchers on the activity carried out at ICTP in their field of interest, with the specific purpose of stimulating scientific contacts and collaboration of physicists from Third World Countries.

If you are interested in receiving additional information on the Laser and Optical Fibre activities at ICTP, kindly contact Professor Gallieno Denardo, ICTP.

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1 Introduction

Optical fibers and integrated optical waveguides are the basic building blocks of photonic devices and systems for optical communication and information processing. In both optical fibers and integrated optical devices, the basic phenomenon is that of waveguidance, and in order to effectively analyse and design these waveguides devices, it is neceassary to understand the phenemonon of guidance through them. In the most basic form, this requires the solutions of Maxwell's equations for the boundary conditions represented by the waveguiding structure. Fortunately, for optical waveguides, in most cases of practical importance, the conditions are such that the vector nature of optical waves can be ignored, at least to a very good approximation, and then, it suffices to solve the much simpler Helmholtz equation. This "simpler" Helmholtz equation, however, is still difficult to solve paticulaly if the waveguides are not uniform along the direction of propagation which is the case for most of the devices. One then has to use approximate and/or numerical techniques to obtain solutions of the Helmholtz equation. The present report is concerned with some of such techniques that we have developed recently.

In the most general form, a guided-wave optical device can be represented by an index distribution of the form $n^2(x,y;z)$, where z is the general direction of propagation of waves and (x,y) represent the transverse coordinates. The modeling and analysis techniques aim at obtaining the solution of the wave equation (the Helmholtz equation) for the amplitude of the electric field $\Psi(x,y,z)$ as it propagates through the device. The most commonly used technique for obtaining the propagating field distribution through a given waveguiding structure for a given input field has been the Beam Propagation Method (BPM). However, using such a method for a full three dimensional (3-D) analysis is mathematically involved and computationally time consuming. Therefore, often, a 2-D transverse refractive index distribution is sought to be replaced by an equivalent 1-D index profile; the equivalence being defined differently for different methods. In this way, one can construct a 2-D equivalent guiding structure for a given 3-D optical device and simulate its characteristics. This 2-D guiding structure is then analysed using the 2-D BPM which is much less involved and comparatively quicker than 3-D BPM. A good majority of guided wave devices can be modelled in this way with a reasonably good accuracy. However, this procedure requires a method to obtain 1-D equivalents for a given 2-D index profiles. This has been a more difficult problem than propagating fields. A number of methods have been proposed, though the most commonly used method is the effective-index method (EIM). In fact, the combination of EIM with 2-D PBM has the most widely used technique for simulation of integrated optical (IO) structures. Recently, through a European cooperative study under COST 216 program, it has been concluded that this technique does not give sufficient accuracy and one has to look for a better alternative. It has also been shown explicitly that EIM always over-estimates the propagation constant and the error could be quite substantial.

We have been working on both the aspects of the problem and have developed a better method for obtaining the 1-D equivalent waveguides and a method, better than the BPM, to simulate propagation through waveguides. The development of the method for 1-D equivalents is based on the understanding of wave propagation through waveguides with 2-D index profiles. This work has earlier been reported in a LAMP Series Report [1].

The simulation of the field propagation through waveguides require numerical solution of the Helmholtz equation. We have recently developed a method for this purpose based

on the principle of orthogonal collocation and during the past few years have made several modifications/advancements in this method. The method is also applicable to nonlinear pulse propagation through optical fibers which has lately assumed great importance due the possibility of repeaterless communication over several thousands of kilometer through solitons.

In the present report, we discuss some of the salient features of this method and its application to both linear and nonlinear wave propagation through optical waveguides.

2 Wave Propagation through Optical Waveguides

A waveguide structure is defined by its refractive index distribution $n^2(x,y,z)$ which contains all information regarding interfaces also. The electromagnetic fields that propagate through such a dielectric structure must satisfy the Maxwell equations. However, in a majority of practical waveguiding structures (we will confine our discussion to such cases), the relative variation of the refractive index is sufficiently small to allow the scalar wave approximation. It, then, suffices to consider instead a much simpler Helmholtz equation:

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}+k_0^2n^2(x,y,z)\psi(x,y,z)=0 \qquad (1)$$

where $\psi(x,y,z)$ represents one of the cartesian components of the electric field (generally referred to as the scalar field). The time dependence of the field has been assumed to be $\exp(i\omega t)$ and $k_0=\omega/c$.

The problem that is addressed in this report is then to obtain the solutions $\psi(x,y,z)$ of Eq.1 given the field $\psi(x,y,z_0)$ at a plane $z=z_0$. Thus, we are dealing with an initial value problem with respect to the variable z (which is generally taken as the overall direction of propagation). However, the presence of the partial derivatives with respect to the transverse coordinates (x and y) makes this problem much more complex and one has to devise special methods even to obtain numerical solutions. For simplicity we shall confine our discussions, initially to 2-dimensional structures and return to 3-D structures later in Sec. 5. Thus, we consider for the present a planar refractive index distribution, $n^2(x\ z)$ for which the Helmholtz equation takes the form

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial z^2}+k_0^2n^2(x,z)\psi(x,z)=0 \qquad (2)$$

and z is assumed to be the general direction of propagation.

If the waveguide is uniform in the direction of propagation, the problem of wave propagation can be solved in terms of the modes of the waveguide [2],[3]. These modes, which have well defined transverse field patterns and have specific phase constants, form an orthogonal set of basis functions. An incident field at z=0 can be expressed as a linear combination of these modes, which then propagate along the waveguide with their phases changing in a definite way but differently, in general, for different modes. Therefore, at any other value of z, these would combine to give a field which represents the incident field after propagation through the length z of the waveguide. Thus, once the modes are known for a given uniform waveguide, one can, in general, propagate a given field through any length of the waveguide. It must, however, be mentioned that these modes–the guided and the radiation– are infinite in number and, in fact, the radiation modes form a continuum

of modes and, hence, the summation/integration over these modes has to be truncated leading to an inaccuracy in any propagation. The approach of total field propagation, that we discuss next, can therefore also be useful even for uniform waveguides.

A straight and uniform waveguide is, however, an idealization and practical waveguides do have variations along their lengths, such as bends in optical fibers or refractive index and/or geometric variations in fibers/integrated optic (IO) waveguides. In addition and more importantly, there are devices which inherently have structures which are nonuniform. These include, e.g., tapers, y-junctions, couplers, etc. The transverse cross-sections can in general be either rectangular or multilayer as in IO devices or circular/elliptical as in fiber devices. Wave propagation through such guides-wave structures cannot, in general, be modelled in terms of the modes, although in some structures it can be treated via coupled mode theory (which is usually limited to very few modes)[3]. This has led to the development of the approach in which the total fields or beams are propagated by numerically solving the Helmholtz equation directly and lately, a number of methods have been developed for this purpose.

There are several advantage of treating total fields, rather than indivisual modes; for instance, the analysis is not restricted to uniform or near uniform waveguides and one can handle, in principle, arbitrary index and/or geometry variations along the direction of propagation. Further, the radiation modes are included in the total field propagating along the waveguiding structure and it is not necessary to obtain them explicitly which is often not practical. These methods can be used to obtain the evolution of fields in a structure that are not uniform along the direction of propagation; an example is in Fig.1 where the evolution of the field in a y-junction is depicted. The most commonly used of these methods is the Beam Propagation Method (BPM) which was first developed to propagate laser beams through atmosphere [4]. It was later used for beam propagation through optical fibers [5]–[9] and through rib waveguides [10]. In the BPM, using the Fresnel approximation [5], the total propagation is separated into two parts: propagation in a uniform medium and the focussing effect resulting from the transverse inhomogeneity in the refractive index. The computation of propagation through the uniform medium is done by decomposing the field into its plane waves. This requires the computation of two Fourier transforms at each propagation step. The effect of focussing is introduced simply in the form of a multiplication by a phase factor at each propagation step. The BPM usually requires considerable computational time and can be limited in accuracy because of its inherent approximations. An extensive discussion of this method and its limitations can be found in [11] and [12].

Our method [13, 14] is based on the collocation principle and has some unique features in comparison to other methods. It can be used to obtain an arbitrary accuracy and, for a desired accuracy, it is faster than the BPM. In Sec.3, we discuss the basic collocation principle and its application to solve of Eq.2. Our method uses the orthogonal collocation method and hence, we refer to it as the OCM. In Sec. 4, an unconditionally stable form of the collocation method is developed using the split-step procedure and we refer to this method as the split-step collocation method (SSCM). Section 5 is devoted to the discussion on the application of the basic method, the OCM, to three-dimensional (3-D) propagation. Section 6 discusses applications of the collocation method to propagation through uniform waveguides. In this section, therefore, we discuss the use of our method for obtaining the propagation constant and the fields of the modes a given waveguide. On the other hand, Sec.6 gives some examples on the evolution of fields through waveguides

which vary along the direction of propagation. These include components like tapers and y-junctions.

Nonlinear pulse propagation through optical fibers has lately assumed great importance due to the possibility of repeaterless communication over several thousands of kilometer through solitons. These solitons are possible in fibers due to the balance of the dispersive nature of propagation by the nonlinearity in the fibers. However, there are several aspects of soliton propagation which have to be understood through numerical simulation of their propagation. In Sec.8, we discuss the application of the collocation method for nonlinear pulse propagation through optical fibers.

The work on the collocation method has been reported in [13]–[21].

3 Basic Collocation Method

Collocation methods have been used since the turn of this century for solving integral equations. These were first applied to the solution of differential equations by Frazer, Jones and Skan [22] in 1937 and independently by Lanczos [23],[24] in 1938. The collocation methods belong to the family of methods used for solving differential equations which can be grouped together under the common name – the method of weighted residuals [25],[26]. In the collocation method, the solution of a differential equation is sought in the form of a linear expansion as a polynomial or over a set of polynomials or functions. The coefficients of expansion are obtained by imposing the condition that the expansion satisfies the differential equation exactly at certain discrete points on the independent variable axis (or plane). These points are referred to as the collocation points. In earlier methods, these points were chosen to be equi-distant. However, Lanczos [24] showed that such a choice may lead to divergence in results (a phenomenon termed as 'Runge divergence') and suggested the use of orthogonal polynomials as the basis functions for the expansion. Later, Villadsen and Stewart [27] developed this concept further and called it "orthogonal collocation". Fletcher [28] has shown that the orthogonal collocation gives results with accuracies comparable to the Galerkin method and its implementation is much simpler. Further, the equidistant collocation yields poorer results in comparison to the othogonal collocation [28]. Orthogonal collocation has been applied using the Radau, Tchebycheff and Legendre polynomials to solve a variety of chemical engineering problems (see, e.g., [26]). In all these problems the range of the independent variables is finite. We have, for the first time, developed [13] the collocation method for the Helmholtz equation which is over an infinite range. We have used the Hermite-Gauss [13], the Laguerre-Gauss [14] and the sinusoidal functions [19] as the basis functions. We have used the orthogonal collocation method (OCM) which is outlined in Sec.3.1.

3.1 Orthogonal Collocation Method(OCM)

We begin with the Helmholtz equation for 2-D propagation, Eq.2, and seek its solution for $\psi(x,z)$ as a linear combination over a set of suitable orthogonal functions, $\phi_n(x)$:

$$\psi(x,z) = \sum_{n=1}^{N} c_n(z)\phi_n(x) \tag{3}$$

where $c_n(z)$ are the expansion coefficients. The choice of $\phi_n(x)$ depends on the boundary conditions and the symmetry of the guiding structure (see Sec.5.2). For a planar structure,

for example, the Hermite-Gauss functions are suitable while for a cylindrical structure the Laguerre-Gauss functions would be more appropriate. For the present, we have

$$\phi_n(x) = \mathcal{N}_{n-1} H_{n-1}(\alpha x) \exp(-\frac{1}{2}\alpha^2 x^2) \tag{4}$$

where $\mathcal{N}_{n-1}$ is the normalization constant and α is a parameter which can be chosen arbitrarily, but its choice can influence the accuracy for a given value of N [14]. Obviously, the accuracy of expansion in Eq.3 improves as N increases. Since Eq.2 cannot, in general, be solved exactly, an approximate solution is sought by imposing certain conditions which determine the coefficients, $c_n(z)$. In the collocation method, this is done by requiring that the differential equation, Eq.2, is satisfied *exactly* by the expansion in Eq.3 at N collocation points $x_j, j = 1, 2, \ldots, N$. This implies that we can uniquely determine only N coefficients in the expansion and thus the orthogonal functions used in the expansion are $\phi_1, \phi_2, \ldots, \phi_N$. In the orthogonal collocation method, the collocation points x_j are chosen such that these are the zeroes of ϕ_{N+1}. Thus,

$$H_N(\alpha x_j) = 0, \quad j = 1, 2, \ldots, N$$

The Hermite polynomials H_N has N distinct zeroes which are well documented in the tables for the Hermite-Gauss quadrature formulae [29],[30]. Writing the Helmholtz equation, Eq.2 at each of these collocation points, we obtain a set of N total differential equations

$$\left.\frac{\partial^2 \psi}{\partial x^2}\right|_{x=x_j} + \frac{d^2\psi_j}{dz^2} + k_0^2 n^2(x_j, z)\psi_j(z) = 0, \quad j = 1, 2, \ldots, N \tag{5}$$

where $\psi_j(z) = \psi(x = x_j, z)$. This set of equations can be written in the form of a matrix differential equation:

$$\frac{d^2\mathbf{\Psi}}{dz^2} + \mathbf{D} + \mathbf{R}\mathbf{\Psi} = 0 \tag{6}$$

where

$$\begin{aligned} \mathbf{\Psi}(z) &= \text{col.}[\psi(x_1, z)\ \ \psi(x_2, z)\ \ \ldots\ \ \psi(x_N, z)] \\ \mathbf{R}(z) &= k_0^2 \times \text{diag.}[n^2(x_1, z)\ \ n^2(x_2, z)\ \ \ldots\ \ n^2(x_N, z)] \\ \mathbf{D}(z) &= \text{col.}\left[\left.\frac{\partial^2\psi}{\partial x^2}\right|_{x=x_1} \left.\frac{\partial^2\psi}{\partial x^2}\right|_{x=x_2} \ \ldots\ \left.\frac{\partial^2\psi}{\partial x^2}\right|_{x=x_N}\right] \end{aligned} \tag{7}$$

Further, we can write the expansion in Eq.3 at the collocation points as

$$\psi(x_j, z) = \sum_{n=1}^{N} c_n(z)\phi_n(x_j), \quad j = 1, 2, \ldots, N$$

which can also be written in the matrix form as

$$\mathbf{\Psi}(z) = \mathbf{A}\mathbf{C}(z) \tag{8}$$

where

$$\mathbf{C}(z) = \begin{pmatrix} C_1(z) \\ C_2(z) \\ \vdots \\ C_N(z) \end{pmatrix}; \quad \mathbf{A} = \begin{pmatrix} \phi_1(x_1) & \phi_2(x_1) & \ldots & \phi_N(x_1) \\ \phi_1(x_2) & \phi_2(x_2) & \ldots & \phi_N(x_2) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_1(x_N) & \phi_2(x_N) & \ldots & \phi_N(x_N) \end{pmatrix} \tag{9}$$

Similarly, by differentiating Eq.3 twice with respect to x and writing the resulting equation at each of the collocation points, we obtain

$$\mathbf{D}(z) = \mathbf{B}\mathbf{C}(z) \tag{10}$$

where

$$\mathbf{B} = \begin{pmatrix} \left.\frac{\partial^2\phi_1}{\partial x^2}\right|_{x_1} & \left.\frac{\partial^2\phi_2}{\partial x^2}\right|_{x_1} & \ldots & \left.\frac{\partial^2\phi_N}{\partial x^2}\right|_{x_1} \\ \left.\frac{\partial^2\phi_1}{\partial x^2}\right|_{x_2} & \left.\frac{\partial^2\phi_2}{\partial x^2}\right|_{x_2} & \ldots & \left.\frac{\partial^2\phi_N}{\partial x^2}\right|_{x_2} \\ \vdots & \vdots & \ddots & \vdots \\ \left.\frac{\partial^2\phi_1}{\partial x^2}\right|_{x_N} & \left.\frac{\partial^2\phi_2}{\partial x^2}\right|_{x_N} & \ldots & \left.\frac{\partial^2\phi_N}{\partial x^2}\right|_{x_N} \end{pmatrix} \tag{11}$$

Substituting from Eq.8 $\mathbf{C} = \mathbf{A}^{-1}\mathbf{\Psi}$ into Eq.10, we get $\mathbf{D}(z) = \mathbf{B}\mathbf{A}^{-1}\mathbf{\Psi}(z)$ and Eq.6 then takes the form

$$\frac{d^2\mathbf{\Psi}}{dz^2} + \mathbf{S}\mathbf{\Psi}(z) = 0 \tag{12}$$

where

$$\mathbf{S} = \mathbf{B}\mathbf{A}^{-1} + \mathbf{R}(z) = \mathbf{S}_0 + \mathbf{R}(z) \tag{13}$$

We shall refer to Eq.12 as the collocation equation. In deriving this equation from the Helmholtz equation, Eq.2, no approximation has been made except that N is finite, and Eq.12 is exactly equivalent to Eq.2 as $N \to \infty$. Equation 12 is a matrix total differential equation and can be solved as an initial value problem using any standard method such as the Runge-Kutta method or the predictor-corrector method [29],[31].

The field represented by $\mathbf{\Psi}(z)$ varies rapidly on account of its phase factor. In a homogeneous medium, this phase factor can be exactly taken out as $\mathbf{\Psi}(z) = \mathcal{X}_0 \exp(-ikz)$ where k is the wave number in that medium and $\mathcal{X}_0$ is a constant matrix. In a waveguiding structure, one can similarly write

$$\mathbf{\Psi}(z) = \mathcal{X}(z)e^{-ikz} \tag{14}$$

where $\mathcal{X}(z)$ is a slowly varying envelope of the wave and $k = k_0 n_{\text{ref}}$ with n_{ref} being the index of a reference medium (a convenient choice is the cladding or the substrate index). Substituting from Eq.14 into Eq.12, we obtain an equation satisfied by the envelope $\mathcal{X}(z)$:

$$\frac{d^2\mathcal{X}}{dz^2} - 2ik\frac{d\mathcal{X}}{dz} + (\mathbf{S} - k^2\mathbf{I})\mathcal{X}(z) = 0 \tag{15}$$

where $\mathbf{I}$ is a unit matrix. For a mode in a uniform waveguide, it is sufficient to consider the parabolic equation which is obtained by neglecting the second derivative term:

$$\frac{d\mathcal{X}}{dz} = (\mathbf{S} - k^2\mathbf{I})\mathcal{X}(z)/2ik, \tag{16}$$

since the propagation constants and the modal fields obtained from Eq.15 and Eq.16 are simply related [7]. Even for a non-uniform waveguide, Eq.16 is an excellant approximation which is equivalent to the Fresnel approximation [5]. This approximation has been extensively used in the study of waveguides and is in fact an essential approximation for the BPM. In our method, on the other hand, it is optional as one could solve either of Eqs.15

& 16. However, we have found that, in practical waveguiding problems, this approximation is extremely good except near the excitation planes where rapidly varying transient fields exist or in cases where reflections are important such as periodic structures. In our examples, we have solved Eq.16.

It may be pointed out that $\mathbf{S}(z)$ has the z-dependence only through the diagonal matrix $\mathbf{R}(z)$, and $\mathbf{BA}^{-1}$ is constant. Thus, $\mathbf{BA}^{-1}$ has to be computed just once to solve a waveguide design problem in which the refractive index distribution is changed to get a desired effect on the propagation. The matrix $\mathbf{A}$ always has an inverse as long as the collocation points are distinct and the basis functions are linearly independent. Further, in case of orthogonal collocation, $\mathbf{A}^{-1}$ can be obtained by simply multiplying its transpose $\mathbf{A}^T$ by a known diagonal matrix (see Appendix A).

3.2 Numerical Example

In this section, we consider an example to show the accuracy and efficiency of the collocation method. We also compare our results with those obtained using the BPM. In order to show the inherent accuracy of the method, we consider the propagation of the fundamental mode through a uniform waveguide. The modal field does not undergo any change except for a phase factor; hence, any change in the amplitude of the modal field would directly reflect the error in the method of propagation. We also choose an index profile for which the modal field and the propagation constant are analytically known. Thus, the numerical computations have been performed for a secant-hyperbolic profile [32]:

$$n^2(x) = n_2^2 + (n_1^2 - n_2^2)\ \mathrm{sech}^2(x/a) \tag{17}$$

with a=3μm, n_1=1.45 and n_2=1.4476. We assume the free space wavelength of the propagating wave to be λ_0=1.31μm so that the V-value is 1.2, where $V = k_0 a\sqrt{n_1^2 - n_2^2}$. We consider the incidence of the fundamental mode of the waveguide at z=0; thus [32],

$$\psi(x, z{=}\,0) = \cosh^{-W}(x/a) \tag{18}$$

where $W = a\sqrt{\beta^2 - k_0^2 n_2^2} = \frac{1}{2}\sqrt{1 + 4V^2}$. As a measure of accuracy we have computed the correlation factor (CF) of the propagating field at z=$z_f \equiv$100μm with the incident field:

$$CF = \frac{\int \psi^*(x, z=0)\ \psi(x, z=z_f)\ dz}{\sqrt{\{\int |\psi(x, z=0)|^2\, dz\}\{\int |\psi(x, z=z_f)|^2\, dz\}}} \tag{19}$$

The absolute value of the correlation factor should be unity, since only the phase changes as a mode propagates through the waveguide. Thus, any deviations in $|CF|$ from unity is a direct measure of the accuracy of the method used for computing the propagated field. This quantity is plotted in Fig.2. For comparison, we have also included the results obtained using the BPM with a grid of 128 points in the cross-section between -50μm to 50μm. The value of the extrapolation interval is 2.5μm for all calculations. Further, to show the computational efficiency of the method, we have also shown the relative computation time. The Runge-Kutta method has been used to solve Eq.16. The figure clearly shows that the collocation method is computationally more efficient than the BPM. For example, an accuracy of $\sim 10^{-9}$ is obtained using BPM with 128 points. The same accuracy is obtained using the collocation method with N=42 and the computation

time is reduced by about 40%. Further, for N-values between 42 and 55, the collocation method gives better accuracy with smaller computation time in comparison to the BPM with 128 points.

3.3 Salient Features of the Collocation Method

1. Both the BPM and the collocation method are based on the scalar wave approximation. However, in the formulation of the BPM it is necessary to make a further approximation, namely, the Fresnel approximation, which is justified in most cases of practical interest. However, there are cases where this approximation cannot be used (e.g. periodic waveguides). In the collocation method, the Fresnel approximation is not essential; but, its use, wherever applicable, reduces the computational effort. No physical or mathematical approximation is made in deriving Eq.12. The only approximation made is the finiteness of N. The accuracy improves indefinitely as N increases.

2. It is quite straightforward to obtain propagation constants and modal fields with the collocation method (see Sec.6). Finding eigenvalues and eigenvectors is much more tedious in the case of the BPM [8]. It is possible to develop a matrix eigenvalue equation using BPM for uniform waveguides using the split-step FFT method [33], but the matrix so obtained is complex with complex eigenvalues whereas in the collocation method, the matrix obtained is real and its eigenvalues are real (see Sec.6).

3. An important approximation in the BPM is the finite extent of the computational grid beyond which the FFT procedure assumes the field and the waveguide to be periodic. One has to assume the presence of strong absorbers at the edge of the grid. There is no need for such an assumption in the collocation method.

4. In the collocation method, it is possible to take computational advantage of the special symmetry of the waveguiding structure. For example, in a circularly symmetric fiber or fiber devices, the three dimensional problem can be treated as a two dimensional problem (see Sec.5.2).

5. The collocation equation, Eq.12, is equivalent to the Helmholtz equation except for the finiteness of N and this equivalence can be made as accurate as desired by making N large enough. However, computational effort increases at least as N^2 because of the matrices involved. Further, the approximations also come into play due to the numerical method adopted for the solution of the collocation equation. Thus, the results obtained in Fig.2 using the Runge-Kutta method do not show any saturation in accuracy as the error is sufficiently small for the Δz-values used due to the single step error in the Runge-Kutta method being of the order of $[(\Delta z)^5]$. However, other methods (see, e.g., Sec.4) of lower accuracy do bring in saturation in accuracy.

4 Unconditionally Stable Form of the Collocation Method

4.1 Problem of Stability

The matrix equation obtained from the Helmholtz equation in the Sec.3 can be solved using a variety of numerical methods such as the Runge-Kutta methods or the predictor-corrector methods. These methods are available in various orders, e.g., the most commonly used Runge-Kutta method is of the fourth order and the single step error term is proportional to $(\Delta z)^5$. This generally means that in comparison to a lower order method, this method gives comparable error with a larger step size, Δz. However, we are dealing with a matrix equation which is basically a set of simultaneous differential equations. In solving such a set of equations, one could encounter the problem of stiffness which means that a numerical solution of such a system of equations can become unstable unless the step size, Δz, is very small (see, e.g., [34]). For instance, in the numerical examples of Sec.3.2, we have used Δz=2.5μm. However, a value of $\Delta z > 5\mu$m results in blowing up of the solution after propagation through a few steps. Stiffness is a widely studied problem in applied mathematics (see, e.g., [34],[35] for a detailed discussion on this problem). A lower order method (e.g., second order Runge-Kutta method [29]) is more stable than a higher order method. Further, the stiffness gets evened out after a certain number of iterations and one could use a larger step size for further iterations. Although there are some special methods available for dealing with stiff systems [34]-[36], these are somewhat involved to use. To overcome the problem of stiffness, we have developed a different approach to solve the collocation differential equation. This approach is of the second order and uses the symmetrized splitting of an operator just as in the case of the BPM. It is also necessary, as in the BPM, to use the Fresnel approximation.

4.2 Split-Step Collocation Method (SSCM)

We consider Eq.16 which can be expressed in the form

$$\frac{d\mathcal{X}}{dz} = (\mathbf{H}_1 - \mathbf{H}_2)\mathcal{X}(z)/2ip \tag{20}$$

where $\mathbf{H}_1 = \mathbf{R}(z) + \mathbf{D}_1 - p^2\mathbf{I}$ is a z-dependent diagonal matrix and $\mathbf{H}_2 = \mathbf{A}\mathbf{D}_2\mathbf{A}^{-1}$ is a constant square matrix. The matrices $\mathbf{D}_1$ and $\mathbf{D}_2$ are defined as (see Appendix A)

$$\mathbf{D}_1 = \alpha^4 \times \text{diag.}(x_1^2 \;\; x_2^2 \ldots x_N^2) \quad \text{and} \quad \mathbf{D}_2 = \alpha^2 \times \text{diag.}[1 \;\; 3 \;\; 5 \ldots (2N-1)] \tag{21}$$

A formal solution of Eq.20 can be written as

$$\mathcal{X}(z + \Delta z) = \exp[(\mathbf{H}_1(z) - \mathbf{H}_2)\Delta z/2ip]\mathcal{X}(z) \tag{22}$$

which, on using the symmetrized splitting, becomes

$$\mathcal{X}(z + \Delta z) = \mathbf{P}\mathbf{Q}(z)\mathbf{P}\mathcal{X}(z) + \bigcirc[(\Delta z)^3] \tag{23}$$

with

$$\mathbf{P} = \exp[-\mathbf{H}_2\Delta z/4ip] \quad \text{and} \quad \mathbf{Q}(z) = \exp[\mathbf{H}_1(z)\Delta z/2ip]$$

The matrix $\mathbf{P}$ is a constant matrix and is to be evaluated only once for a propagation problem. The evaluation of $\mathbf{P}$ can be directly done because of its special form

$$\mathbf{P} = \exp[-\mathbf{A}\mathbf{D}_3\mathbf{A}^{-1}]$$

where the matrix $\mathbf{D}_3 = \mathbf{D}_2\Delta z/4ip$ is a diagonal matrix. An expansion the exponential followed by some matrix manipulations, we obtain

$$\mathbf{P} = \mathbf{A}\exp(-\mathbf{D}_3)\mathbf{A}^{-1}$$

The $\exp(-\mathbf{D}_3)$ can be easily evaluated since $\mathbf{D}_2$ is a diagonal matrix and the exponential of a diagonal matrix is a diagonal matrix with the diagonal elements being simply the exponential of the corresponding diagonal elements of the argument matrix. Similarly, the matrix $\mathbf{Q}(z)$ can be easily evaluated since it is also the exponential of a diagonal matrix. Further, since the diagonal matrices commute with each other, we can separate the z-dependent part in $\mathbf{Q}(z)$ as

$$\mathbf{Q}(z) = \mathbf{Q}_1\,\mathbf{Q}_2(z)$$

with

$$\mathbf{Q}_1 = \exp[(\mathbf{D}_1 - p^2\mathbf{I})\Delta z/2ip] \quad \text{and} \quad \mathbf{Q}_2(z) = \exp[\mathbf{R}(z)\Delta z/2ip]$$

The evaluation of $\mathbf{Q}_1$ and $\mathbf{Q}_2(z)$ is simple since these are diagonal matrices.

Thus, a basic propagation step of the split-step collocation method (SSCM) is

$$\mathcal{X}(z + \Delta z) = \mathbf{P}\mathbf{Q}_1\mathbf{Q}_2(z)\mathbf{P}\mathcal{X}(z) + \bigcirc[(\Delta z)^3] \tag{24}$$

which requires two multiplications of a square matrix with a vector and one multiplication of a diagonal matrix with a vector, since $\mathbf{P}\mathbf{Q}_1$ can be evaluated once and treated as a single constant matrix for the propagation. Since all the matrices $\mathbf{P}, \mathbf{Q}_1$ and $\mathbf{Q}_2(z)$ are unitary for real indices, these do not blow up for any value of the arguments. Thus, this algorithm is unconditionally stable for any value of Δz.

However, for obtaining desirable accuracy, the value of Δz is typically only few microns whereas the propagations lengths could be several thousands of microns, indeed even millimeters. Thus, normally propagation over several thousands of steps is to be carried out. In such cases, one does not require to monitor the actual field profile at each step of propagation and in reality several hundred steps are evaluated without visualizing the field. In such cases, the computational effort could be further reduced by making the following transformation

$$\mathcal{Y}(z) = \mathbf{P}\mathcal{X}(z) \quad \text{and} \quad \mathbf{\Gamma} = \mathbf{P}^2\mathbf{Q}_1$$

wwere $\mathbf{\Gamma}$ is a constant matrix. Equation 24 then takes the form

$$\mathcal{Y}(z + \Delta z) = \mathbf{\Gamma}\mathbf{Q}_2(z)\mathcal{Y}(z)$$

Now, each propagtion step requires the multiplication of a diagonal matrix with a column vector and the multiplication of the resulting vector with a square matrix. Conversion from $\mathcal{X}$ to $\mathcal{Y}$ and from $\mathcal{Y}$ to $\mathcal{X}$ is required only at the initial point and the final point, respectively.

4.3 Example and Discussion

We again consider the example discussed in Sec.3.2 and results are given in Fig.3 where we have plotted the error in the correlation factor as a function of N. The results of Fig.2 are also included for comparison. The figure shows that the SSCM gives the same accuracy as that obtained by solving the collocation equation in the OCM using the Runge-Kutta method for smaller Δz-values, but the computation time is reduced by a factor of about two. The reduction in the computation time arises because, in the SSCM, the number of multiplications required to propagate the field over one step reduces to $2N(N+1)$ from $4N^2$ in the the Runge-Kutta method. For relatively larger N and Δz, the accuracy curve saturates. This is due to the $(\Delta z)^3$ error in the splitting of the operators (Eq.23). In the Runge-Kutta method, this error is of the order of $(\Delta z)^5$ [31] and hence, saturation in the correlation error does not appear unless very large N is used; however, one cannot use larger values of Δz, since the Runge-Kutta solution becomes unstable. The SSCM, however, involves only unitary operators (for real refractive indices)and is hence stable for arbitrary values of Δz. This fact is explicitly shown in Fig.4 where the error in the correlation factor is plotted as a function of z for different values of Δz. The method remains stable even for $\Delta z = 50\mu m$ though the error increases. On the other hand, the error in the OCM with the Runge-Kutta solution blows up for $\Delta z > 5\mu m$ after propagation through few steps only.

5 Application to 3-Dimensional Propagation

In the sections above, we have confined our discussion to two-dimensional guiding structures. In this section, we discuss the extension of the collocation method for 3-D guiding structures. A direct 3-D analysis is much more involved and time consuming and, therefore, in a number of cases modelling is done by reducing the 3-D structure to an equivalent 2-D structure using methods like the effective index method [39], [40] or the variational method [1],[41], [42] and studying the propagation through the 2-D structure [43]. In such cases, the methods discussed above are useful. However, a 2-D model may at times give erroneous results (See [44]). Thus it becomes necessary to have a general method to study three dimensional structures.

The collocation method can be easily extended to three dimensions, but, as in the case of the BPM, there is a substantial increase in the computational effort, which may be greater by one or two orders of magnitude in comparison to that in 2-D cases. In general, one has to use Cartesian coordinates for 3-D problems, except when the device is made from optical fibers and retains its circular symmetry (as, for example, in fiber tapers). In the latter case, one could use the circular cylindrical coordinates with substantial computational advantage (see Sec.5.2).

5.1 3-D Structures in Cartesian Coordinates

We now consider the 3-D Helmholtz equation, Eq.1, and use a double linear expansion of the field in terms of two sets of the Hermite-Gauss functions:

$$\psi(x,y,z) = \sum_{n=1}^{N}\sum_{m=1}^{M} K_{nm}\phi_n(x)\eta_m(y) \tag{25}$$

Table-I: Number of Multiplications in One Propagation Step*

	The BPM	The Collocation Method
$2-D$	$m(4\log_2 m+1)$	$4N^2$
$3-D$	$m^2(8\log_2 m+1)$	$4N^2(2N+1)$
Increment Factor	$2m$	$2N$
$(m, N \gg 1)$		

* m is the number of sample points in the BPM and N is the number of collocation points.

where $\phi_n(x)$ are defined in Eq. 4 and $\eta_m(y)$ are defined in an identical way with y and κ instead of x and α. Following the procedure described in Sec.3.1, one can define the collocation points on x and y axes and convert Eq.1 into the following matrix differential equation (see [14] details)

$$\frac{d^2\mathbf{\Psi}}{dz^2} + \mathbf{S}_0\mathbf{\Psi} + \mathbf{\Psi}\mathbf{T}^T + \mathbf{R}(\mathbf{\Psi}) = 0 \tag{26}$$

where $\mathbf{\Psi}$ is now a $N \times M$ matrix defining the field $\psi(x,y)$ at a matrix of collocation points, $\mathbf{S}_0$ is defined in Eq.13 and $\mathbf{T}$ is defined in an identical way with $\eta_m(y)$ replacing $\phi_n(x)$. $\mathbf{T}^T$ represents the transpose of $\mathbf{T}$. Further, $\mathbf{R}(\mathbf{\Psi})$ is a $N \times M$ matrix with its elements defined as

$$[\mathbf{R}(\mathbf{\Psi})]_{nm} = k_0^2 n^2(x_n, y_m, z)\psi(x_n, y_m, z)$$

Under the Fresnel approximation, we can reduce Eq.26 to a first order differential equation

$$\frac{d\mathcal{X}}{dz} = [\mathbf{S}\mathcal{X} + \mathcal{X}\mathbf{T}^T + \mathbf{R}(\mathcal{X}) - k^2\mathcal{X}]/2ik \tag{27}$$

where $\mathcal{X}$ and k are defined in Eq.14. To give an estimate of the increase in computational effort while going over from 2 to 3 dimensions, we have shown, in Table-I, the number of multiplications to be performed in each propagation step for both the BPM and the collocation method(OCM). As can be seen from Table-I, the increase in the number of multiplications while going over from 2 to 3 dimensions is two times the number of sample points in the transverse cross-section for both the cases. Thus, the increase in the computational effort can be expected to be less in the collocation method, since, as we have seen in Sec.3.2, the number of collocation points needed to achieve a certain degree of accuracy is substantially less than the number of points needed in the BPM. A numerical example is included in Sec.5.3.

5.2 3-D Structures in Circular-Cylindrical Coordinates

Optical fibers and fiber based devices fall in this category of guiding structures. If the device is made in such a way that the structure retains its azimuthal symmetry, as in the case of fiber tapers and expanders, then it is more advantageous to work in the circular-cylindrical co-ordinate system. Further, if one considers field patterns of one kind of azimuthal symmetry at a time, it is possible to write the scalar Helmholtz equation as:

$$\frac{\partial^2\psi}{\partial z^2} + \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} - \frac{l^2}{r^2}\psi + k_0^2 n^2(r,z)\psi(r,z) = 0 \tag{28}$$

where l is the azimuthal symmetry parameter of the field. The above equation is a partial differential equation and using the collocation method it can be converted into a matrix differential equation. As an illustration, we consider the case of circularly symmetric modes, l=0, and we can expand ψ in terms of the Laguerre-Gauss functions [29] which form a complete set of circularly symmetric functions in the domain $0 \leq r \leq \infty$, and go to zero as $r \to \infty$. Thus, we write

$$\psi(r,z) = \sum_{n=1}^{N} K_n \Theta_n(r)$$

where

$$\Theta_n(r) = L_{n-1}(b^2r^2)\exp(-\frac{1}{2}b^2r^2) \tag{29}$$

The corresponding matrix differential equation is

$$\frac{d^2\mathbf{\Psi}}{dz^2} + \bar{\mathbf{B}}\bar{\mathbf{A}}^{-1}\mathbf{\Psi} + \mathbf{R}(z)\mathbf{\Psi}(z) = 0 \tag{30}$$

where the elements of $\bar{\mathbf{A}}, \bar{\mathbf{B}}$ and $\mathbf{R}$ are defined as

$$\begin{aligned} \bar{A}_{jn} &= \Theta_n(r_j), \\ \bar{B}_{jn} &= \frac{d^2\Theta_n}{dr^2} + \frac{1}{r}\frac{d\Theta_n}{dr}\bigg|_{r_j}, \\ R_{jn} &= k_0^2 n^2(r_j,z)\delta_{jn} \end{aligned} \tag{31}$$

r_j's denote the collocation points defined as the zeroes of $\Theta_{N+1}(r)$. Under the Fresnel approximation, we then have (*cf.* Eq.16)

$$\frac{d\mathcal{X}}{dz} = [\bar{\mathbf{B}}\bar{\mathbf{A}}^{-1} + \mathbf{R}(z) - k^2\mathbf{I}]\mathcal{X}(z)/2ik \tag{32}$$

Thus, the computational effort involved in solving Eq.32 would be of the same order as that in case of planar waveguides.

5.3 Numerical Example

We now consider an example to show the effectiveness of the collocation method for 3-D propagation. We have performed calculations for a circularly symmetric waveguide with a Gaussian profile. We have chosen a circularly symmetric refractive index profile so that we can compare the results obtained using the methods developed in Sec.5.1 and Sec.5.2. The refractive index profile is given by

$$\begin{aligned} n^2(x,y,z) &= n_2^2 + (n_1^2 - n_2^2)\exp[-(x^2+y^2)/a^2] \\ \text{or,} \quad n^2(r,z) &= n_2^2 + (n_1^2 - n_2^2)\exp[-r^2/a^2] \end{aligned} \tag{33}$$

with n_1=1.45, n_2=1.435, λ=1.3μm and a=2.5μm. We again consider the propagation of the fundamental mode and after a propagation through a distance of z=10μm, we obtain the error in the correlation factor (defined in an analogous way as in Eq.19) between the field at z=0 and the field at z=10μm. This error is plotted, in Fig.5, as a function of

number of N, (we have taken M=N). The continuous curve is obtained using the method of Sec.5.1, while the dash-dot curve is obtained using the method of Sec.5.2. The order of accuracy obtained is nearly the same for a given N in the two cases; however, in the case of circularly symmetric basis functions, the computational effort is much less (about 1/40 for N=30). The figure also shows that the accuracy obtained using the 3D-BPM with 60 points can be achieved by using 30 collocation points (the method of Sec.5.1) and the time taken is about one third (the dashed curve shows relative computation time for method of Sec.5.1). Thus, we see that even in the case of three dimensional waveguides our method works better than the BPM, as can be expected from Table-I.

6 Analysis of Uniform Waveguides

Although the main application of the collocation method is in the study of structures nonuniform in the direction of propagation, it can also be used simply and effectively for obtaining the modes of a waveguide. In case of uniform waveguides, the collocation method results in a matrix eigenvalue equation, the solution of which yields the propagation constants and modal field distributions. We have solved this eigenvalue problem for different planar waveguides and have shown that the accuracies obtained using the collocation method are comparable to those obtained using the Galerkin method for the same number of basis functions and the computational effort taken is much less [14]. However, in case of refractive index profiles with a discontinuity, the collocation method yields poorer results for the propagation constant which can be improved by a simple perturbative correction [14].

We again consider the collocation equation, Eq.12. For a mode in a uniform waveguide, $\mathbf{\Psi}(z)$ can be written as $\mathbf{\Psi}_0\exp(-i\beta z)$, where $\mathbf{\Psi}_0$ represents the modal field pattern at the collocation points and remains unchanged along the length of the waveguide, and β is the propagation constant. Substituting this form for $\mathbf{\Psi(z)}$ in the matrix differential equation, Eq.12, we obtain

$$\mathbf{S}\mathbf{\Psi}_0 = \beta^2\mathbf{\Psi}_0 \tag{34}$$

which is a standard matrix eigenvalue equation. $\mathbf{S}$ is a real but not a symmetric matrix. However, by a similarity transformation, which leaves the eigenvalues of $\mathbf{S}$ unchanged, it can be transformed into a real symmetric matrix (See Appendix A for details). Thus, the eigenvalues and eigenvectors are all real and since the evaluation of eigenvalues for a symmetric matrix is much simpler, the computational effort required is also considerably reduced.

In order to demonstrate the accuracy and the computational efficiency of the collocation method in evaluating the modes of a waveguide, we include here an example [14] on a cladded prabolic profile:

$$\begin{aligned} n_2(x) &= n_1^2 - (n_1^2 - n_2^2)\,x^2/a^2, \quad |x| < a_0 \\ &= n_2^2, \qquad\qquad\qquad\quad |x| > a_0 \end{aligned}$$

where n_1=1.45, n_2=1.447636, a_0=3μm and the wavelength is varied to obtain different values of the V parameter. The results for the first two modes for different V-values are given in Table-II. The results obtained using the Ritz-Galerkin method are also included. The table also shows a relative computation time to obtain these results. It can be seen that for such profiles, which are continuous, it is greatly advantageous to use the

Table-II: Convergence of Numerical Results for β/k_0 for a Cladded Parabolic Profile Waveguide

			Collocation Method		Ritz-Galerkin Method	
V	m	Exact	N=20	N=30	N=20	N=30
1.2	0	1.44843	1.44844	1.44843	1.44843	1.44843
	1	1.44742	1.44720	1.44734	1.44721	1.44735
2.8	0	1.44918	1.44918	1.44918	1.44918	1.44918
	1	1.44778	1.44777	1.44778	1.44778	1.44778
3.2	1	1.44795	1.44794	1.44795	1.44795	1.44795
Computation time (arbitrary units)			5836	11435	18354	36978

collocation method, since the degree of convergence is the same for both the methods for a given N, whereas the computation time taken by our method is considerable less. The main reason for this difference is that there is no integration involved in our method, while in the Ritz-Galerkin method one must evaluate $N \times N$ integrals numerically.

A number of additional numerical examples are given in Ref.[14].

7 Evolution of Fields through Nonuniform Waveguiding Structures

In this section, we include few examples to show the applicability of the collocation method to the waveguiding structures that vary along the direction of propagation. In such cases, as mentioned earlier, the modal picture is no longer applicable, and the methods like the collocation method and the BPM offer the only possible solution to such propagation problems.

In the first example, we consider the propagation through a linear taper with index profile given by Eq.17 with the half-width a now becoming z-dependent as

$$a(z) = a_0 - \zeta z$$

with a_0=3μm and ζ=0.1. We have carried out calculations for a taper of length 10μm. Thus, the half width of the guide reduces linearly from 3μm to 2μm. The incident field is taken to be the normalized local fundamental mode at z=0. After propagation through the length of the taper, the power lost from the fundamental mode is calculated by taking the difference from unity of the square of the absolute value of the overlap integral between the output field at the end of the taper and the local normalized mode field of the waveguide with half-width a=a(10μm). The convergence of results is depicted in Fig.6 where we have plotted the fractional power lost from the fundamental mode as a function of computation time (normalized with respect to the time taken for the BPM with 128 points). The curves are obtained by varying the number of points in both the collocation method and the BPM. The figure shows that the time taken to reach the convergence is about 1/2 in the Runge-Kutta solution (OCM) and about 1/3 with the SSCM in comparison to the BPM. Similar results have been obtained for tapers with cladded parabolic index profile [14].

As the next example, we consider propagation through a Y-junction shown in the inset of Fig.1. Such a device serves the purpose of a beam divider in the integrated optical circuits. In our example, the index profile of each of the waveguide is given by $n(x) = 2.1512 + 0.0006\text{sech}^2(2x/3)$ with the separation of PORT 2 and PORT 3 being 10μm and the length of the coupling region is 1mm. The fundamental mode at λ=1.15μm is launched in PORT 1. Figure 1 shows evolution of the field as it propagates through the coupling region. The power coupled into the mode of PORT 2 is 49.55% of the incident power which means that there is a loss of 0.90% power. The convergence is obtained with 50 collocation points in these calculations.

As the final example, we consider propagation through a fiber taper. The profile of the fiber is given by Eq.33, but now the width parameter a varies linearly with z as $a(z) = 2.5 - 0.05z$ in μm. The loss from the fundamental mode of the fiber after a propagation through a length of 10μm is computed. The results are given in Fig.7 which shows that the convergence in the loss calculation is obtained with the collocation method in about half the computation time in comparison to that in the case of the BPM. For these calculation, we have used the Cartesian coordinates and the fourth-order Runge-Kutta method.

8 Application to Nonlinear Pulse Propagation through Optical Fibers

8.1 Generalized Nonlinear Schrödinger Equation

It is well known that the effect of pulse dispersion in optical fibers can be reduced by the use of the nonlinear Kerr effect as proposed by Hasegawa and Tappert [45] and experimentally verified by Mollenauer et al.[46]. A large number of interesting nonlinear phenomena occur in optical fibers due to the interplay of nonlinear and dispersive effects. Under some specific conditions solitons exist; solitons are pulses with specific shapes which travel undistorted for extremely large distances as a result of balance between nonlinear and dispersive effects in a fiber (see, e.g.,[47],[48]).

The nonlinear evolution of short pulses in an optical fiber is usually described by the nonlinear Schrödinger equation (NLS) which has been analytically solved [49]. The NLS equation holds good for propagation of pulses of picosecond duration through a lossless fiber. However, some assumptions implicit in the equation are no longer valid for pulses of smaller duration, particularly, in the femtosecond regime. Hence, the NLS equation has to be modified to include a host of other phenomena, such as higher order dispersion, higher order nonlinearities and self-steepening. Thus, in presence of Kerr-like nonlinearity, pulse propagation through a real (lossy) fiber can be described by the Generalised Nonlinear Schrödinger Equation (GNLSE) [47], [50]

$$\frac{\partial E}{\partial \xi} + \gamma E - i\frac{\partial^2 E}{\partial \tau^2} - \delta\frac{\partial^3 E}{\partial \tau^3} = i[|E|^2 E + is\frac{\partial}{\partial \tau}(|E|^2 E) - \tau_R E\frac{\partial}{\partial \tau}(|E|^2)] \qquad (35)$$

where it is assumed that the pulse is propapagting in the region of anomalous dispersion ($\beta_2 < 0$). The space and time variables are normalized such that

$$\xi = z\,|\beta_2|\,/T_0^2; \qquad \tau = \sqrt{2}\,T/T_0; \qquad T = t - \beta_1 z; \qquad \beta_n \equiv \left.\frac{d^n\beta}{d\omega^n}\right|_{\omega=\omega_0} \qquad (36)$$

where T represents the reduced time (measured from the pulse center), T_0 is the width of the pulse and β_1 is the inverse of the group velocity of the pulse. In Eq.35, $E(z,t)$ denotes the envelope of the electric field and is assumed to be a slowly varying function of z and t:

$$\psi(z,t) = E(z,t)e^{[i(\omega t - \beta z)]} \tag{37}$$

Further, in Eq.35, γ is the attenuation coefficient and the parameters δ, s and τ_R represent, respectively, the effect of higher order dispersion, self-steepening and the retarded nonlinear response. Explicitly, these can be written as

$$\delta = \sqrt{2}\beta_3/3\,|\beta_2|\,T_0, \quad s = 2\sqrt{2}/\omega_0 T_0 \quad \text{and} \quad \tau_R = \sqrt{2}T_R/T_0 \tag{38}$$

where β_3 is the third order dispersion parameter and T_R is related to the slope of the Raman gain in the fiber [47]. All these parameters δ, s and τ_R vary inversely with the pulse width and are negligible for pulses with T_0 >1ps, and become appreciable for pulses in the femtosecond regime.

Analytic solutions of Eq.35 are possible only for few specific cases and in most cases, the GNLSE has to be solved numerically. The numerical method commonly used to solve the GNLSE is the split-step Fourier method [47], [51], which is in fact similar to the BPM. This type of procedure has some inherent drawbacks. One has to use numerical differentiation in evaluating terms containing time derivatives of the pulse envelope. Further, the effects of nonlinearity and dispersion are assumed to be separated in space, whereas in reality both act simultaneously. The collocation method does not suffer from these drawbacks and the numerical examples show that the collocation method is considerably more efficient even for solving GNLSE.

8.2 Collocation Method for the GNLSE

We begin by expressing the pulse envelope $E(\xi,\tau)$ as a linear combination of the Hermite-Gauss functions, $\varphi_n(\tau) = H_{n-1}(\alpha\tau)\exp(-\alpha^2\tau^2/2)$:

$$E(\xi,\tau) = \sum_{n=1}^{N} c_n(\xi)\varphi_n(\tau) \tag{39}$$

α being the width parameter. The collocation points are now defined as the N zeroes of $H_N(\alpha\tau)$. Applying the collocation principle as described in Sec.3.1, Eq.35 can be converted into the following matrix differential equation:

$$\begin{aligned} \frac{d\mathbf{E}}{d\xi} + \gamma\mathbf{E} - i\hat{\mathbf{B}}\hat{\mathbf{A}}^{-1}\mathbf{E} - \delta\hat{\mathbf{D}}\hat{\mathbf{A}}^{-1}\mathbf{E} &= \\ &+ i\hat{\mathbf{P}}\mathbf{E} + s[\hat{\mathbf{Q}}\hat{\mathbf{F}}\hat{\mathbf{A}}^{-1}\mathbf{E}^* + 2\hat{\mathbf{P}}\hat{\mathbf{F}}\hat{\mathbf{A}}^{-1}\mathbf{E}] \\ &- i\tau_R[\hat{\mathbf{Q}}\hat{\mathbf{F}}\hat{\mathbf{A}}^{-1}\mathbf{E}^* + \hat{\mathbf{P}}\hat{\mathbf{F}}\hat{\mathbf{A}}^{-1}\mathbf{E}] \end{aligned} \tag{40}$$

where $\mathbf{E}(\xi)$ is a column vector of the values of $E(\xi,\tau_j)$ at consecutive collocation points τ_j and $\hat{\mathbf{P}}$ and $\hat{\mathbf{Q}}$ are diagonal matrices with the successive diagonal elements being $|E(\xi,\tau_j)|^2$ and $E^2(\xi,\tau_j)$, respectively. The elements of the $N\times N$ matrices $\hat{\mathbf{A}}, \hat{\mathbf{B}}, \hat{\mathbf{D}}$ and $\hat{\mathbf{F}}$ are defined as:

$$\hat{A}_{jn} = \varphi_n(\tau_j)$$

$$\begin{aligned} \hat{B}_{jn} &= \left.\frac{d^2\varphi_n}{d\tau^2}\right|_{\tau=\tau_j} \\ \hat{D}_{jn} &= \left.\frac{d^3\varphi_n}{d\tau^3}\right|_{\tau=\tau_j} \\ \hat{F}_{jn} &= \left.\frac{d\varphi_n}{d\tau}\right|_{\tau=\tau_j} \end{aligned} \tag{41}$$

Various dervatives of $\varphi_n(\tau)$ can be analytically evaluated using the recurrence formulae for $H_n(\tau)$ [29]. The matrix equation, Eq.40, is solved using the Runge-Kutta method starting from the given pulse shape $E(\xi=0,\tau)$ at $\xi=0$.

8.3 Numerical Example

To establish the accuracies obtainable using the collocation method, we consider an example for which an analytical solution is known. Neglecting the effects of loss, third order dispersion, self steepening and the retarded nonlinear response, Eq.35 reduces to the nonlinear Schrödinger (NLS) equation:

$$i\frac{\partial E}{\partial \xi} + \frac{\partial^2 E}{\partial \tau^2} + |E|^2 E = 0 \tag{42}$$

which gives solitons as its solutions. The fundamental soliton has an initial pulse shape defined by (see, e.g., [47])

$$E(\xi=0,\tau) = \mathrm{sech}(\tau/\sqrt{2}) \tag{43}$$

In our example, we have propagated this pulse using the collocation method upto $\xi=2$ and have calculated the correlation factor (CF) of the output pulse amplitude with the input pulse amplitude:

$$CF = \frac{\int E^*(\xi=2,\tau)\;E(\xi=0,\tau)\;d\tau}{\sqrt{\int |E(\xi=2,\tau)|^2\,d\tau . \int |E(\xi=0,\tau)|^2\,d\tau}} \tag{44}$$

Ideally the absolute value of the correlation factor should be equal to unity since only phase changes with ξ. The deviation of the absolute value of the correlation factor from unity gives a measure of the error in the method of propagation. We have plotted this error as a function of the number of collocation points in Fig.8. We have also performed the same computations using the split-step Fourier method (BPM). We see that the accuracy of little over 10^{-9}, obtained by using 64 BPM points, can be obtained by using 35 collocation points and the computation time required is about half. On the other hand, in about the same amount of computation time (i.e., with $N=50$, since, in the collocation method the computation time inceases as N^2), an improvement in accuracy by about two orders of magnitude can be obtained.

More examples on pulse propagation including the effects of higher order dispersion, self-steepening and retarded response are included in Refs. [20], [21].

9 Summary

In conclusion, we have described the collocation method for propagation of optical fields through waveguiding structures and have presented some numerical examples for demonstrating the validity of the methods presented and for comparing wherever possible with the commonly used Beam Propagation Method (BPM).

We have shown through examples that the collocation method has a substantial computational advantage over the BPM. The collocation method converts the Helmholtz method into a matrix differential equation and the accuracy of this matrix representation can, in principle, be improved in an unlimited fashion as the size of the matrices involved increases. This, however, increases the computational effort involved and one generally has to make a compromise between the accuracy obtained and the computional effort put in. For a desired accuracy, the computational effort in the collocation method is substantially smaller in comparison to the BPM. In addition, there are several advantageous features in the collocation method that cannot be implemented in the methods like the BPM. For example, the collocation method retains the second order differentials in the Helmholtz equation. This allows the use of the collocation equation for waveguiding problems in which reflections are involved.

As mentioned above, the collocation method results in a matrix differential equation (which has been termed as the collocation equation); this is a great advantage, since this equation can be solved in a variety of ways depending on the problem at hand. One could solve it using a direct method like the Runge-Kutta method or the predictor-corrector method [29], [31], or, by using matrix operator algebra, or, by using a perturbative approach wherever applicable. For homogeneous media, even analytical solutions are possible.

The collocation method is general enough to be applicable to other propagation problem not based on the Helmholtz equation. We have used this method to solve the generalized nonlinear Schrödinger equation to model nonlinear pulse propagation through optical fibers. It can also be similarly applied to nonlinear propagation problems in planar waveguides.

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Appendix–A: Properties of Matrices A and S

We derive, in this Appendix, some of the proprties of matrices **A** and **S**. The matrix **A** is defined in Eq.9. The Hermite-Gauss functions, $\phi_n(x)$ are the functions used in the Gaussian quadrature procedure over the interval ∞ to $-\infty$[29],[30]. In this procedure, an integral over an infinite domain can be *exactly* converted into a finite summation if the integrand satisfies certain conditions. Using this property, we can write

$$\int_{-\infty}^{\infty} \phi_n(u)\phi_m(u)du = \sum_{j=1}^{N} W_j^2 \phi_n(u_j)\phi_m(u_j)du = \delta_{ij}$$

where $u = \alpha x$ and W_j^2 are the weight functions tabulated in the Gaussian quadrature tables [29], [30]. The integral on the left side is equal to the Kronecker delta, δ_{ij} because the functions $\phi_n(u)$ are orthonormal to each other. The summation in the above equation can be written in terms of matrices as

$$(\mathbf{WA}^T)(\mathbf{WA}) = \mathbf{I}$$

where **W** is a diagonal matrix with $W_1, W_2, \ldots, W_N$ as its diagonal elements. With simple manipulations, this equation can be written as

$$\mathbf{AA}^T = (\mathbf{WW}^T)^{-1}$$

which gives an analytical expression for the inverse of **A**

$$\mathbf{A}^{-1} = \mathbf{A}^T(\mathbf{WW}^T) = \mathbf{A}^T\mathbf{W}^2$$

Thus, the inverse of **A** can be found by simply multiplying the transpose of **A** by a known diagonal matrix $\mathbf{W}^2$.

The functions $\phi_n(u)$ satisfy the following differential equation [29]

$$\frac{d^2\phi_n}{du^2} = [u^2 - (2n-1)]\phi_n(u)$$

and, hence, the elements of matrix **B**, defined in Eq.11, can be written as

$$B_{jn} = \alpha^2[u_j^2 - (2n-1)]A_{jn}$$

which, in the matrix form, can be written as

$$\mathbf{B} = \mathbf{D}_1\mathbf{A} - \mathbf{AD}_2$$

where $\mathbf{D}_1$ and $\mathbf{D}_2$ are defined in Eq.21. The matrix **S** can now be written as

$$\mathbf{S} = \mathbf{D}_1 + \mathbf{R} - \mathbf{AD}_2\mathbf{A}^{-1} = \mathbf{D}_4 - \mathbf{AD}_2\mathbf{A}^{-1}$$

where $\mathbf{D}_4$ is a diagonal matrix. Now, we make a similarity transformation of **S** by **A** to obtain

$$\mathbf{S}' \equiv \mathbf{A}^{-1}\mathbf{SA} = \mathbf{A}^{-1}\mathbf{D}_4\mathbf{A} - \mathbf{D}_2$$

We have seen above that $\mathbf{AA}^T$ is a diagonal matrix and since all diagonal matrices commute with each other, we can write

$$\begin{aligned} \mathbf{D}_4 &= (\mathbf{AA}^T)\mathbf{D}_4(\mathbf{AA}^T)^{-1} \\ \mathbf{A}^{-1}\mathbf{D}_4\mathbf{A} &= \mathbf{A}^T\mathbf{D}_4(\mathbf{A}^T)^{-1} \\ &= (\mathbf{A}^{-1}\mathbf{D}_4\mathbf{A})^T \end{aligned}$$

which shows that $\mathbf{A}^{-1}\mathbf{D}_4\mathbf{A}$ is a symmetric matrix and hence, $\mathbf{S}'$ is also a real symmtric matrix. Thus, the eigenvalues and eigenvectors of $\mathbf{S}'$ and **S** are all real.

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Figure Captions

Fig.1 Evolution of the fundamental mode incident at a Y-junction shown in the inset. (see Sec.6 for other details)

Fig.2 The error in the correlation factor, 1-$|CF|$, as a function of the number of collocation points, N, using the OCM for a uniform waveguide with refractive index distribution defined in Eq.17. The dashed curve shows the relative computation time. Also indicated are the corresponding quantities for the BPM with 128 sample points.

Fig.3 The error in the correlation factor, 1-$|CF|$, and the relative computation time (for Δz=2.5μm) as a function of the number of collocation points, N, using the SSCM for a uniform waveguide with refractive index distribution defined in Eq.17. The dashed curves correspond to the OCM. Also indicated are the corresponding quantities for the BPM with 128 sample points.

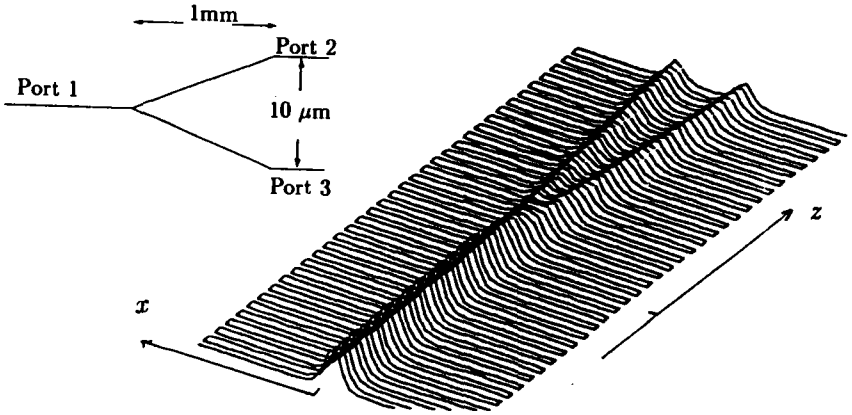
Fig.4 The error in the correlation factor, 1-$|CF|$, as a function of the propagation distance using the SSCM with N=50 for different values of Δz for a uniform waveguide with refractive index distribution defined in Eq.17. The dashed curves correspond to the OCM (using the Runge-Kutta method).

Fig.5 The error in the correlation factor and the relative computation time as a function of the number of collocation points after propagation by a distance of 10μm through a fiber with the refractive index profile given by Eq.33.

Fig.6 Power lost from the fundamental mode propagating in a linear taper as a function of relative computation time showing the convergence of various methods. (see Sec.6 for details).

Fig.7 Fractional power lost from a tapered fiber after propagation through a distance of 10μm as a function of computation time. The index distribution is defined in Sec.6.

Fig.8 The error in the correlation factor, 1-$|CF|$, as a function of the number of collocation points, N, after propagation of the fundamental soliton through a distance, ξ =2, using the collocation method (OCM). Also shown is the error in the BPM with 64 sample points.



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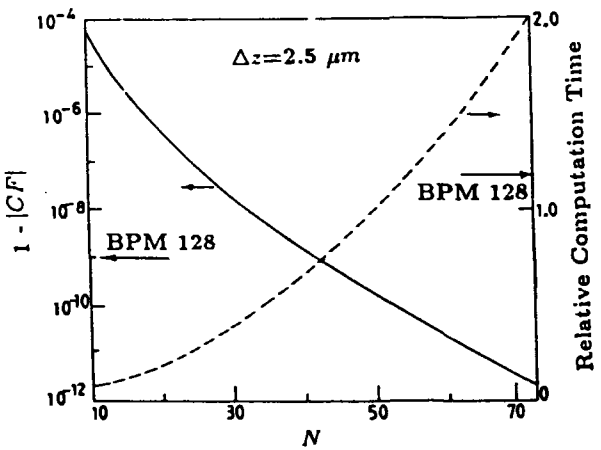
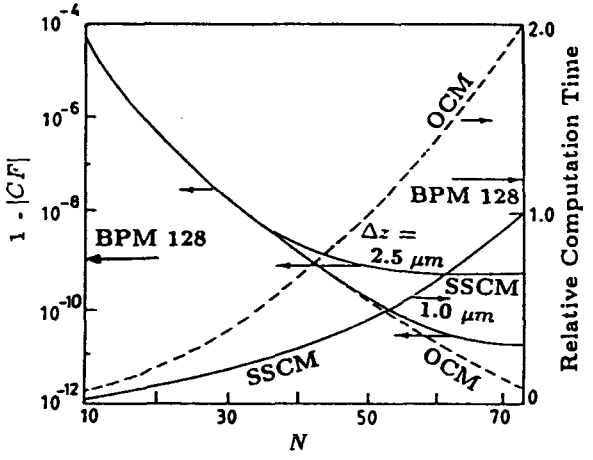


Fig.2 The error in the correlation factor, 1-$|CF|$, as a function of the number of collocation points, N, using the OCM for a uniform waveguide with refractive index distribution defined in Eq.17. The dashed curve shows the relative computation time. Also indicated are the corresponding quantities for the BPM with 128 sample points.



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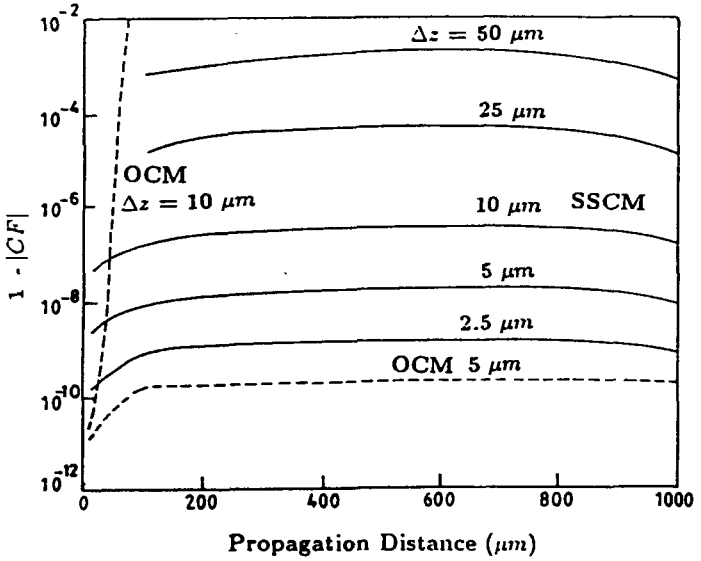

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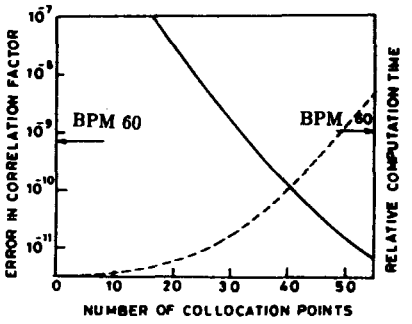

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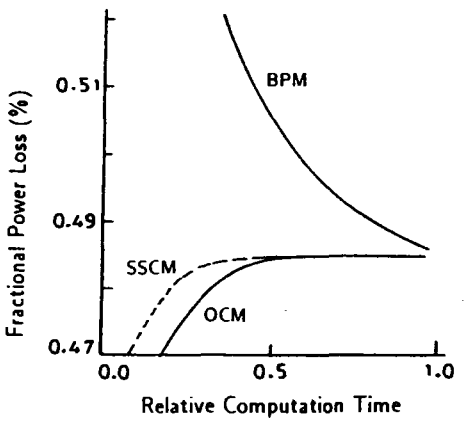

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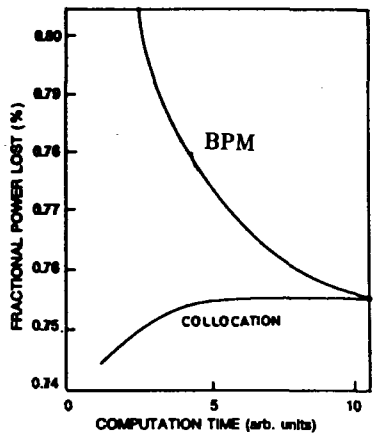
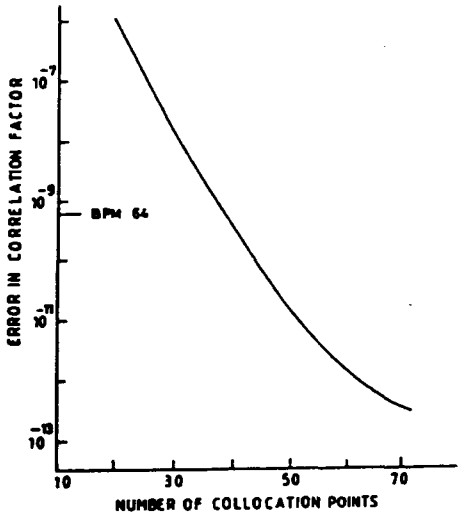


Fig.7 Fractional power lost from a tapered fiber after propagation through a distance of 10μm as a function of computation time. The index distribution is defined in Sec.6.



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