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LINEARIZING W-ALGEBRAS

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ABSTRACT

We show that the Zamolodchikov's and Polyakov-Bershadsky nonlinear algebras W_3 and $W_3^{(2)}$ can be embedded as subalgebras into some *linear* algebras with finite set of currents. Using these linear algebras we find new field realizations of $W_3^{(2)}$ and W_3 which could be a starting point for constructing new versions of W-string theories. We also reveal a number of hidden relationships between W_3 and $W_3^{(2)}$. We conjecture that similar linear algebras can exist for other W-algebras as well.

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1 Introduction

By now there exist many examples of extended conformal algebras, the overwhelming majority of which (the so called W-type algebras) is essentially nonlinear (see e.g. [1] and references therein). Because of the intrinsic nonlinearity of W-algebras, their study is a more difficult task compared to linear algebras. However, as was shown by Goddard and Schwimmer [2], there exists a deep relation between $SO(3)$ and $SO(4)$ extended nonlinear superconformal algebras of Knizhnik-Bershadsky [3] on the one hand and linear $N = 3$ and $N = 4$ superconformal algebras [4] on the other: the latter can be obtained from the nonlinear algebras by adding some spin 1/2 and spin 0 currents. So, there arises a natural question: whether in other cases there exist linear Lie algebras with finite number of currents, such that they contain some nonlinear W-algebras with an arbitrary central charge as subalgebras in a nonlinear basis. If existing, such linear algebras could be helpful at least for constructing new realizations of W-algebras. In the present letter we give the positive answer to the above question for the two simplest nonlinear algebras: Zamolodchikov W_3 algebra [5] and Polyakov-Bershadsky $W_3^{(2)}$ algebra [6]. We find that in both these cases the linear algebras indeed exist and contain the considered W-algebras as subalgebras. Using the linear algebras obtained we construct new field realizations of $W_3^{(2)}$ and W_3 algebras and find a number of intrinsic relationships between these algebras.

2 Linearizing W_3 algebra.

In this Section we will construct a linear algebra W_3^{lin} which contain W_3 algebra as a subalgebra in some basis for the defining currents. Hereafter, some nonlinear redefinition of the currents is called the change of the basis of the (non)linear algebra, if (i) it is invertible and (ii) both it and its inverse are polynomial in the currents and derivatives of the latter. A subset of the currents is meant to form a (non)linear subalgebra of given W-algebra if in some basis this subset is closed; all the algebras related by (nonlinear) transformations of the basis are treated as equivalent.

The W_3 algebra [5] contains the currents $\{T, W\}$ with spins $\{2, 3\}$ respectively, obeying the following operator product expansions (OPE): [3]

$$
\begin{aligned}
T(z_1)T(z_2) &= \frac{c}{2}\frac{1}{z_{12}^4} + \frac{2T}{z_{12}^2} + \frac{T'}{z_{12}} \quad , \quad T(z_1)W(z_2) = \frac{3W}{z_{12}^2} + \frac{W'}{z_{12}} , \\
W(z_1)W(z_2) &= \frac{c}{3}\frac{1}{z_{12}^6} + \frac{2T}{z_{12}^4} + \frac{T'}{z_{12}^3} + \left[2\kappa\Lambda + \frac{3}{10}T''\right]\frac{1}{z_{12}^2} + \left[\kappa\Lambda' + \frac{1}{15}T'''\right]\frac{1}{z_{12}} , \quad (2.1)
\end{aligned}
$$

[3]All the currents in the right-hand sides of the OPEs are evaluated at the point z_2, $z_{12} = z_1 - z_2$. Multiple composite operators (AB) are always ordered from the right to the left.

where

$$\Lambda \equiv (T\,T) - \frac{3}{10}T'' \quad , \quad \kappa \equiv \frac{16}{22+5c} \quad . \tag{2.2}$$

Our main idea is to extend a nonlinear algebra by some currents with *a priori* unknown properties to get a linear algebra after passing to another basis for the currents. The only assumption we will impose from the beginning is that the nonlinear algebra we started with still forms a subalgebra (in the above sense) in this extended algebra.

Fortunately, for the case of W_3 algebra it proves enough to add only one extra spin 1 current J with the following OPE's:

$$\begin{aligned}
T(z_1)J(z_2) &= \frac{1+3c_1}{z_{12}^3} + \frac{J}{z_{12}^2} + \frac{J'}{z_{12}} \quad , \quad J(z_1)J(z_2) = \frac{c_1+1}{z_{12}^2} \, , \\
W(z_1)J(z_2) &= \sqrt{\frac{2}{3-22c_1-45c_1^2}}\Bigg\{ -\frac{(1+3c_1)^2}{2}\frac{1}{z_{12}^4} - \frac{(1+3c_1)J}{z_{12}^3} + \Big(2(1+c_1)T - 2(JJ) + \\
& \frac{1+3c_1}{2}J'\Big)\frac{1}{z_{12}^2} - \Bigg(\sqrt{\frac{3-22c_1-45c_1^2}{2}}W - 2(TJ) - \frac{3+c_1}{2}T' + \\
& \frac{4}{3(1+c_1)}(JJJ) - \frac{2(c_1-1)}{1+c_1}(J'J) - \frac{3c_1-1}{3(1+c_1)}J''\Bigg)\frac{1}{z_{12}}\Bigg\} \quad ,
\end{aligned} \tag{2.3}$$

where the central charges c and c_1 are connected as

$$c = 2 - \frac{4(3c_1+1)^2}{c_1+1} \quad . \tag{2.4}$$

Let us stress that the Jacobi identities completely fix all coefficients in the OPE's (2.3) as well as the relation between central charges (2.4). Thus, there is a unique possibility to extend the W_3 algebra by the spin 1 current under the natural assumption of preserving the structure of W_3.

Now it is a matter of straightforward calculation to show that this new algebra (2.1),(2.3) which we denote as W_3^{lin} is actually a linear algebra. To this end, we do the following *invertible* nonlinear transformation to the new basis $\{J, T_0, G^+\}$:[4]

$$\begin{aligned}
T_0 &= T + \frac{3c_1+1}{2(c_1+1)}J' \, , \\
G^+ &= W - \frac{2\sqrt{2}}{(8(c_1+1)-5(3c_1+1)^2)^{1/2}}\Bigg\{(JT_0) - \frac{2}{3(c_1+1)}(JJJ) + \frac{3c_1+1}{2(c_1+1)}(JJ') - \\
& \frac{3c_1+1}{4}\left(T_0 + \frac{3c_1+1}{6(c_1+1)}J'\right)'\Bigg\} .
\end{aligned} \tag{2.5}$$

In this new basis OPE's (2.1),(2.3) become linear

$$\begin{aligned}
T_0(z_1)T_0(z_2) &= \frac{1-4c_1-9c_1^2}{2(c_1+1)}\frac{1}{z_{12}^4} + \frac{2T_0}{z_{12}^2} + \frac{T_0'}{z_{12}} \quad , \quad T_0(z_1)G^+(z_2) = \left[\frac{3}{2} + \frac{1}{c_1+1}\right]\frac{G^+}{z_{12}^2} + \frac{G^{+\prime}}{z_{12}} \, , \\
T_0(z_1)J(z_2) &= \frac{J}{z_{12}^2} + \frac{J'}{z_{12}} \quad , \quad J(z_1)J(z_2) = \frac{c_1+1}{z_{12}^2} \quad , \quad J(z_1)G^+(z_2) = \frac{G^+}{z_{12}} .
\end{aligned} \tag{2.6}$$

[4]Let us note that transformation from T to T_0 is linear, but current J becomes primary with respect to T_0. This is the reason why we prefer to deal with T_0 rather than with T.

So we have explicitly shown that W_3^{lin} algebra (2.1)-(2.4) is linear and contains the W_3 algebra as a subalgebra in the nonlinear basis.

We close this Section with a few comments.

First of all, let us note that the linear W_3^{lin} algebra (2.6) is homogeneous in the current G^+. So G^+ is the typical null field and we can consider the limit $G^+ = 0$. In this limit the expressions (2.5) give the well known realization of W_3 algebra in terms of the spin 1 current J and an arbitrary stress-tensor T_0 [7].

Secondly, we would like to stress that, strictly speaking, it make sense to fix the conformal spins of the currents only in the basis where all of them are at least quasi-primary. So, while we consider the W_3 algebra (2.1), the current W has spin 3, but in the case of extended algebra (2.1),(2.3), where the current J is not quasi-primary, we must fix all spins with respect to the new stress-tensor T_0 (2.5). In this basis the conformal spin of current W (as well as of the current G^+ related to W by the invertible transformation (2.5)) is just[5]

$$\frac{3}{2} + \frac{1}{c_1+1} .$$

We have explicitly checked that there exist no other non-trivial extensions of W_3 algebra by a primary spin 1 current, such that the structure of W_3 itself remains intact.

Finally, we can reverse our arguments. We may start from the linear algebra (2.6) and show that the currents T and W defined through the transformations (2.5) form the W_3 algebra (2.1). Thus, all the remarkable nonlinear features of W_3 algebra can be traced to the choice of a nonlinear basis in the linear algebra (2.6).

In the next Section we will use this approach to construct the linear algebra $W_3^{(2)lin}$ which contains quantum $W_3^{(2)}$ algebra as a subalgebra.

3 Linearizing $W_3^{(2)}$ algebra.

In this Section we explicitly demonstrate that the quantum $W_3^{(2)}$ algebra is a subalgebra of very simple *linear* algebra.

The algebra $W_3^{(2)}$ contains the bosonic currents $\{J_w, G^+, G^-, T_w\}$ with the spins $\{1, 3/2, 3/2, 2\}$, respectively [6], and supplies the simplest nontrivial example of algebras with fractional bosonic spins. The quantum OPE's for its currents are as follows

$$\begin{aligned}
T_w(z_1)T_w(z_2) &= \left[\frac{(7-9c_1)c_1}{2(c_1+1)}\right]\frac{1}{z_{12}^4} + \frac{2T_w}{z_{12}^2} + \frac{T_w'}{z_{12}} \quad , \quad T_w(z_1)G^\pm(z_2) = \frac{3}{2}\frac{G^\pm}{z_{12}^2} + \frac{G^{\pm\prime}}{z_{12}} \\
T_w(z_1)J_w(z_2) &= \frac{J_w}{z_{12}^2} + \frac{J_w'}{z_{12}} \, , \; J_w(z_1)J_w(z_2) = \frac{c_1}{z_{12}^2} \, , \; J_w(z_1)G^\pm(z_2) = \pm\frac{G^\pm}{z_{12}} \qquad (3.1) \\
G^+(z_1)G^-(z_2) &= \left[\frac{(3c_1-1)c_1}{c_1+1}\right]\frac{1}{z_{12}^3} + \left[\frac{3c_1-1}{c_1+1}\right]\frac{J_w}{z_{12}^2} -
\end{aligned}$$

[5]It becomes 3/2 in the classical limit $c_1 \to \infty$.

$$\left[T_w - \frac{2}{c_1+1}(J_wJ_w) - \frac{3c_1-1}{2(c_1+1)}J'_w\right]\frac{1}{z_{12}}. \tag{3.2}$$

Without going into details, let us show that the linear algebra, which we will denote as $W_3^{(2)lin}$ and which is defined by the following set of OPE's

$$\begin{aligned}
T_w(z_1)T_w(z_2) &= \left[\frac{(7-9c_1)c_1}{2(c_1+1)}\right]\frac{1}{z_{12}^4} + \frac{2T_w}{z_{12}^2} + \frac{T'_w}{z_{12}} \quad , \quad T_w(z_1)J_w(z_2) = \frac{J_w}{z_{12}^2} + \frac{J'_w}{z_{12}} \, , \\
T_w(z_1)\tilde{G}^+(z_2) &= \frac{3}{2}\frac{\tilde{G}^+}{z_{12}^2} + \frac{\tilde{G}^{+\prime}}{z_{12}} \quad , \quad T_w(z_1)G^-(z_2) = \frac{3}{2}\frac{G^-}{z_{12}^2} + \frac{G^{-\prime}}{z_{12}} \, , \\
J_w(z_1)J_w(z_2) &= \frac{c_1}{z_{12}^2} \quad , \quad J_w(z_1)\tilde{G}^+(z_2) = \frac{\tilde{G}^+}{z_{12}} \quad , \quad J_w(z_1)G^-(z_2) = -\frac{G^-}{z_{12}} \, , \\
\tilde{G}^+(z_1)G^-(z_2) &= \text{regular} \, , \qquad (3.3) \\
T_w(z_1)\gamma(z_2) &= -\frac{1}{2}\frac{\gamma}{z_{12}^2} + \frac{\gamma'}{z_{12}} \quad , \quad J_w(z_1)\gamma(z_2) = \frac{\gamma}{z_{12}} \, , \qquad (3.4) \\
G^-(z_1)\gamma(z_2) &= \frac{1}{z_{12}} \qquad (3.5)
\end{aligned}$$

for the bosonic currents $\left\{\gamma, J_w, G^-, \tilde{G}^+, T_w\right\}$ with the spins $\{-1/2, 1, 3/2, 3/2, 2\}$, respectively, contains $W_3^{(2)}$ algebra (3.1),(3.2) as a subalgebra.

In order to prove this, we do the following *invertible* nonlinear transformation to the new basis $\{\gamma, J_w, G^-, G^+, T_w\}$, where

$$\begin{aligned}
G^+ &= \tilde{G}^+ + (T_w\gamma) - \frac{2}{c_1+1}(J_wJ_w\gamma) - \frac{3c_1+7}{2(1+c_1)}(J'_w\gamma) + \frac{2}{1+c_1}(J_wG^-\gamma\gamma) - \\
&\quad \frac{2}{3(c_1+1)}(G^-G^-\gamma\gamma\gamma) - 33(G^-\gamma'\gamma) + \frac{1-c_1}{1+c_1}(G^{-\prime}\gamma\gamma) + 3\left((J_w\gamma) - \frac{c_1+1}{2}\gamma'\right)' .
\end{aligned} \tag{3.6}$$

In this basis the complete set of OPE's of $W_3^{(2)lin}$ algebra is given by eqs. (3.1),(3.2),(3.4),(3.5) and by the following OPE

$$G^+(z_1)\gamma(z_2) = -\frac{(\gamma\gamma)}{z_{12}^2} + \left[\frac{2}{3(1+c_1)}(G^-\gamma\gamma\gamma) - \frac{2}{1+c_1}(J_w\gamma\gamma) + (\gamma'\gamma)\right]\frac{1}{z_{12}} . \tag{3.7}$$

So one observes that $W_3^{(2)lin}$ indeed contains $W_3^{(2)}$ as a subalgebra.

Let us remark that two currents G^- and γ looks like "ghost–anti- ghost" fields and so $W_3^{(2)lin}$ algebra can be simplified by means of the standard ghost decoupling transformations

$$\begin{aligned}
J &= J_w - (G^-\gamma) \quad , \\
T_0 &= T_w - \frac{3}{2}(G^-\gamma') - \frac{1}{2}(G^{-\prime}\gamma) - \frac{1}{c_1+1}\left(J_w - (G^-\gamma)\right)' .
\end{aligned} \tag{3.8}$$

After this $W_3^{(2)lin}$ algebra splits into the direct product of the bosonic ghost–anti-ghost $\{\gamma, G^-\}$ algebra (3.5) and the algebra of the currents $\{J, T_0, G^+\}$ which coincides with W_3^{lin} algebra (2.6). So

$$W_3^{(2)lin} = \Gamma \otimes W_3^{lin} \quad , \quad \Gamma = \left\{\gamma, G^-\right\} \, , \; W_3^{lin} = \left\{J, T_0, G^+\right\} \, . \tag{3.9}$$

Thus, W_3 is also a subalgebra of $W_3^{(2)lin}$ since $W_3^{lin} \subset W_3^{(2)lin}$. In this sense $W_3^{(2)lin}$ is analogous to the affine $sl(3)$ algebra, which can be reduced via the DS hamiltonian reduction procedure to either W_3 or $W_3^{(2)}$. However, unlike it, $W_3^{(2)lin}$ contains W_3 and $W_3^{(2)}$ as subalgebras. This means that every realization of $W_3^{(2)lin}$ is a realization of the W_3 and $W_3^{(2)}$ algebras simultaneously [6]. So the problem of the construction of realizations of these subalgebras is reduced to the problem of constructing realizations of $W_3^{(2)lin}$. Owing to the very simple structure of $W_3^{(2)lin}$ (3.10), this task actually amounts to the construction of realizations of W_3^{lin}. In the rest of this Section we will present an example of such a realization.

From the simple structure of the W_3^{lin} algebra OPEs (2.6) it is evident that its most general realization includes at least two free bosonic scalar fields ϕ_i ($i = 1, 2$) with OPEs

$$\phi_i(z_1)\phi_j(z_2) = -\delta_{ij} ln(z) \, , \tag{3.10}$$

as well as a commuting with them Virasoro stress tensor $\tilde{T}$ having a nonzero central charge which we denote c_T. Representing the bosonic primary field G^+ in the standard way by an exponential of ϕ_i, J by the derivatives of ϕ_i and T_0 by the sum of $\tilde{T}$ and the standard stress-tensors of ϕ_i with background charges, and requiring them to satisfy the OPEs (2.6), we find the following expressions

$$\begin{aligned}
G^+ &= s \cdot \exp\left(i\sqrt{N - \frac{1}{c_1+1}}\phi_2 + \frac{i}{\sqrt{c_1+1}}\phi_1 \right) , N \in Z \quad , \\
J &= i\sqrt{c_1+1}\phi_1' \quad , \\
T_0 &= \tilde{T} - \frac{1}{2}(\phi_1')^2 - -\frac{1}{2}(\phi_2')^2 - \frac{i\left(3 - N + \frac{2}{c_1+1}\right)}{2\sqrt{N - \frac{1}{c_1+1}}}\phi_2'' \quad , \\
c_T &= 3\frac{\left(3 - N + \frac{2}{c_1+1}\right)^2}{N - \frac{1}{c_1+1}} - \frac{(3c_1+1)^2}{c_1+1} \quad ,
\end{aligned} \tag{3.11}$$

where s is an arbitrary parameter. Its arbitrariness reflects the invariance of the OPEs (2.6) with respect to rescaling of the null field G^+. If $s \neq 0$, it can always be chosen, e. g., equal to unity by a constant shift of the field ϕ_1.

[6]Of course, the inverse statement is not correct in general.

In the case of $s=0$ the obtained realization can be simplified by introducing a new Virasoro stress-tensor T_n which absorbs the field ϕ_2

$$\begin{aligned} T_n &= \tilde{T} - \frac{1}{2}(\phi_2')^2 - \frac{i\left(3 - N + \frac{2}{c_1+1}\right)}{2\sqrt{N - \frac{1}{c_1+1}}}\phi_2'' \quad , & (3.12) \\ c_{T_n} &= 1 - \frac{(3c_1+1)^2}{c_1+1} & (3.13) \end{aligned}$$

and ϕ_2-dependence disappears altogether. In this notation the expressions (3.12) are given by

$$\begin{aligned} G^+ &= 0 \quad , \\ J &= i\sqrt{c_1+1}\phi_1' \quad , \\ T_0 &= T_n - \frac{1}{2}(\phi_1')^2 \quad . \qquad (3.14) \end{aligned}$$

After substituting eqs. (3.12) into (2.5), we get a realization of the W_3 algebra which generalizes the realization obtained in [7] and is reduced to it at $s=0$. After substituting (3.12) into (3.6),(3.8) we get a new realization of the $W_3^{(2)}$ algebra. The realizations constructed here may be important in the W_3 ($W_3^{(2)}$)-string theories, but detailed consideration of them is beyond the scope of the present letter.

We close this Section with several comments.

First, in the case of $G^+=0$, the algebra W_3^{lin} (2.6) reduces to a direct product of the $u(1)$ algebra and the Virasoro one with the central charge (3.14). Then, the minimal Virasoro models [8] which correspond to the central charge

$$c_m = 1 - 6\frac{(p-q)^2}{pq} \Rightarrow c_1 = \frac{2}{3}\frac{q}{p} - 1 \qquad (3.15)$$

give rise to the following induced central charges

$$\begin{aligned} c_{W_3} &= 2\left(1 - 12\frac{(p-q)^2}{pq}\right) & (3.16) \\ c_{W_{3/2}} &= \left(1 - 6\frac{(q-2p)^2}{pq}\right) & (3.17) \end{aligned}$$

for the W_3 and $W_3^{(2)}$ subalgebras of $W_3^{(2)lin}$, respectively. The expression (3.17) exactly coincides with the central charges of W_3 minimal models [9], eq. (3.18) reproduces the minimal $W_3^{(2)}$ models [10] at $q=2\tilde{q}$.

Secondly, it is easy to obtain the classical analogues of all quantum expressions constructed here. We will not write all these relations except for the OPE's between the currents of classical version of the algebra W_3^{lin} (2.6) :

$$\begin{aligned} T_0(z_1)T_0(z_2) &= \frac{-\frac{9}{2}c_1}{z_{12}^4} + \frac{2T_0}{z_{12}^2} + \frac{T_0'}{z_{12}} \quad , \quad T_0(z_1)J(z_2) = \frac{J}{z_{12}^2} + \frac{J'}{z_{12}} \, , \\ T_0(z_1)G^+(z_2) &= \frac{3}{2}\frac{G^+}{z_{12}^2} + \frac{G^{+\prime}}{z_{12}}, \; J(z_1)J(z_2) = \frac{c_1}{z_{12}^2}, \; J(z_1)G^+(z_2) = \frac{G^+}{z_{12}}. \qquad (3.18) \end{aligned}$$

It is interesting to note that these OPE's form a subalgebra of the classical $W_3^{(2)}$ following from (3.1)-(3.2) in the limit $c_1 \to \infty$. So, in the classical case both W_3^{lin} and its nonlinear subalgebra W_3 are subalgebras of $W_3^{(2)}$. This statement fails to be true in the quantum case. Only a weaker property $W_3^{lin} \subset W_3^{(2)lin}$ is retained.

4 Conclusions and outlook.

In this paper we have constructed the linear conformal algebras with finite set of currents which contain the $W_3^{(2)}$ and W_3 algebras as subalgebras in some nonlinear basis. We have also found new realizations of $W_3^{(2)}$ and W_3 which can be a starting point for construction of new versions of W- string theories. The study of these linear algebras allowed us to reveal hidden intrinsic relations between them as well as between $W_3^{(2)}$ and W_3 algebras. The linear algebras constructed are similar in many aspects to the affine Kac-Moody algebra $sl(3)$ which is reducible both to the $W_3^{(2)}$ and W_3 algebras. However, the essential difference consists in that the linear algebras include $W_3^{(2)}$ and W_3 as subalgebras. We believe that the most of properties of nonlinear algebras and the theories constructed on their basis, could be understood in a simpler way by studying their linear counterparts.

We do not still know whether the existence of such linear algebras is a general property of the W-type algebras or an artifact of some exceptional cases like W_3 and $W_3^{(2)}$. If this property is general, there must exist an algorithmic procedure for constructing the linear algebras. In any case, it seems important to have more examples of linearization as well as real applications of these linear algebras. In the forthcoming paper we will consider more examples of nonlinear algebras admitting a linearization.

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