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HAMILTONIAN FORMULATION FOR THE MARTIN-TAYLOR MODEL

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1. Introduction

The Ergodic Magnetic Limiter (EML) concept plays a central role in the present study of plasma confinement in Tokamaks. It is an apparatus that yields an additional control over the external layers of the confined plasma column, by means of the creation of a locally stochastic region that can be modified through the variation of external parameters like limiter current and number of coils.

The existence of such a stochastic layer does uniformize the particle flux exiting the plasma column, as well as heat flux uniformization, thus avoiding localized heating on the Tokamak inner wall [1]. Our work concentrates on the study of this stochastic layer and its optimization. In order to accomplish this task we employ a hamitonian formulation of magnetic field line flow with a subsequent application of Escande-Doveil renormalization method [2], which have been extensively used to obtain accurate estimates of stochasticity thresholds in systems exibiting hamiltonian chaos [3].

2. The Martin-Taylor Model

The EML model to be studied here [4] consists of a set of m pairs of wires assembled in the shape of a grid that encircles the Tokamak torus (with major radius R_0 and minor radius a) along the poloidal curvature. The current flowing through each wire must have opposite sense to the adjacent wire. As long as the limiter magnetic field decreases from the edge to the plasma column centre, we consider a region near the Tokamak edge, so as to neglect the poloidal and toroidal curvature effects.

Within this region we use a slab (rectangular) geometry, such that x corresponds to the poloidal arc length, z to the toroidal one and y stands for the radial distance measured from the edge to the centre. Application of Laplace equation to the internal region under the limiter action results in the following field components [4]

$$B_x = -\frac{m\mu_0 I}{\pi a}\cos\left(\frac{mx}{a}\right)\exp\left(-\frac{my}{a}\right), \tag{2.1.1}$$

$$B_y = \frac{m\mu_0 I}{\pi a}\sin\left(\frac{mx}{a}\right)\exp\left(-\frac{my}{a}\right), \tag{2.1.2}$$

$$B_z = B_0. \tag{2.1.3}$$

3. Hamiltonian Formulation and Application of Renormalization Method

We define the following set of canonically conjugated variables (x, y) ,representing the coordinate and momentum respectively, and the "time" variable z. In such a geometry the magnetic field line equation

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_z} \tag{3.1}$$

can be rewritten in the following set of Hamilton's canonical equations:

$$\frac{dx}{dz} = \frac{B_x}{B_0} = \frac{\partial H}{\partial y} \tag{3.2.1}$$

$$\frac{dy}{dz} = \frac{B_y}{B_0} = -\frac{\partial H}{\partial x}. \tag{3.2.2}$$

The description following these equations is not actually the temporal dynamics of the system, but its evolution parametrized by z-coordinate at a fixed physical time. The integration of this system for the Martin-Taylor model enables us to obtain the following field line hamiltonian

$$H(x, y, z) = H_0(y) + H_1(x, y, z), \tag{3.3}$$

where

$$H_0(y) = \frac{1}{2\pi R_0}\left(\alpha y + \frac{3y^2}{2}\right) \tag{3.4}$$

is the unperturbed part, and

$$H_1(x, y, z) = A(y_1)\cos\left(\frac{my}{a}\right) + A(y_2)\cos\left(\frac{mx}{a} - \frac{z}{R_0}\right) \tag{3.5}$$

represents the EML effect in the two-resonance approximation. Here $\alpha = \frac{2\pi a}{q(a)}$ is the field line disnlacement due to the rotational transform, and $\beta = \alpha\left(\frac{dq}{dr}\right)_a$ is the displacement due to magnetic shear, where $q(r)$ is the safety factor and y_1, y_2 are radial locations of the primary resonances.

The application of the Escande-Doveil renormalization method requires the tranformation of the hamiltonian (3.3) into the so called paradigm hamiltonian

$$H_P(p, q, t) = \frac{1}{2}p^2 - M\cos q - P\cos k(q - t), \tag{3.6}$$

through a series of canonical transformations. The global stochasticity threshold is estimated by means of the criterion

$$MP^{g-1} \approx 3.10^{-3}, \tag{3.7}$$

where $g = \frac{1+\sqrt{5}}{2}$ is the golden number. The criterion (3.7) can be applied provided the parameters M, P and k belong to the central case, which corresponds to the intervals

$$k = 1 \text{ and } \frac{1}{25} < \frac{M}{P} < 25 \tag{3.8.1}$$

$$\frac{M}{P} = 1 \text{ and } \frac{1}{4} \leq k \leq 4. \tag{3.8.2}$$

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