IC/94/122

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

FINE STRUCTURE IN THE CLUSTER DECAYS OF THE TRANSLEAD NUCLEI

Ovidiu Dumitrescu

and

Constantin Cioacă

INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

MIRAMARE-TRIESTE

IC/94/122

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

FINE STRUCTURE IN THE CLUSTER DECAYS OF THE TRANSLEAD NUCLEI

Ovidiu Dumitrescu [1]
International Centre for Theoretical Physics, Trieste, Italy

and

Constantin Cioacă
Department of Physics, Chemistry Faculty, Bucharest University,
Boul. Carol nr. 13, Bucharest, Romania.

ABSTRACT

Within the one level R-matrix approach several hindrance factors for the radioactive decays in which are emitted α and other nuclei (such as ^{14}C and ^{20}O) are calculated. The interior wave functions are supposed to be given by the shell model with effective residual interactions. The exterior wave functions are calculated from a cluster - nucleus double - folding model potential obtained with the M3Y interaction. As examples of the cluster decay fine structure we analyzed the particular cases of α - decay of ^{255}Fm, ^{14}C - decay of ^{223}Ra and ^{20}O - decay of ^{229}Th and ^{255}Fm. Good agreement with the experimental data is obtained.

MIRAMARE – TRIESTE

June 1994

[1]Permanent address: Department of Theoretical Physics, Institute of Physics and Nuclear Engineering, Institute of Atomic Physics, P.O. Box MG-6, Măgurele, Bucharest, R-76900, Romania.

1 Introduction

Recently Hourani and his co - workers [1] experimentally discovered the fine structure in the ^{14}C radioactivity [2]. The theoretical studies of alpha [6], [7] (see also the review papers [3] [4], [5] and the references therein) and heavy cluster (e.g. ^{14}C) decay [7] (see also the recent review paper [8] and the references therein) have very much in common.

We view the decay process as composed of two main steps: First the mother nucleus makes a kind of phase transition from the initial state, which could be of any structure (Fermi liquid, superfluid, spherical or deformed, one or many alpha cluster state, one or many combined heavy cluster state etc), to the final state composed of at least one cluster, which is going to be emitted, and the residual nucleus, which may have also any structure as above. One mechanism of such a transition could be the cluster condensation (e.g. alpha condensation [9]), or what usually is assumed a formation of the cluster (e.g. the alpha cluster) from the already formed condensates of smaller clusters (e.g. Cooper pairs, IBM - bosons, etc.) [3 - 7] , [10 - 12]. Another (less studied) mechanism could be the slow shape deformation [13] from any initial shape configuration of the studied many particle system through shapes that are energetically very unfavored (a large amplitude collective motion) to a shape corresponding to the two daughter nuclei in contact.

Secondly, the two daughter nuclei tunnel through the potential barrier in their relative motion, without further change in shape.

Most of the theoretical models of heavy cluster decay [8] are based, essentially, on Gamov's theory [14] which was the first success of quantum mechanics when applied to the α - decay phenomenon, i.e. a detailed description of the second step - the tunneling through the potential barrier. The differences in approaches are related to the way of calculating the potential barrier defined by the (nuclear plus Coulomb) interaction potential acting between the emitted cluster and the residual nucleus. All these theoretical treatments fit to a law for favored cluster transitions, analogous to the Geiger - Nuttal [15] law for favored α - decay, which emerges directly from the simplest JWKB expression of the penetrability determined by the square well plus Coulomb interaction potential.

The unfavored transitions do not follow the Geiger - Nuttal law, because of the large variations of the reduced widths [3] [4], [5], [6], [7], which have a key role in the understanding of the decay process and require a precise knowledge of the structures of the initial and final quantum states. From such transitions we can learn much about the structure of atomic nuclei and the mechanism of the decay phenomenon. For instance, when treating the favored cluster decays, one assumes that the nucleons used to build the cluster are more or less strongly correlated in the initial state. This fact leads to the small hindrance factors [3], [4], [7]. On the contrary, the unfavored transitions (with large hindrance factors) are characterized by the fact that the nucleons used to build the cluster are collected from different strongly correlated groups of nucleons entering the structure of the initial state. In this last case it is necessary first, to breakup the correlated groups of nucleons and then to build the cluster, which is going to be emitted.

In the previous paper [7] the formal expressions for the theoretical hindrance factors are derived and calculations for few cases are performed.

In the present paper we continue this work and calculate several hindrance factors for the ^{4}He -, ^{14}C -, and ^{20}O - radioactivity. The calculations will be performed by using the approach given in Ref. [7].

2 Enlarged Superfluid Model

The enlarged superfluid model (ESM) Hamiltonian for nonrotational states of deformed nuclei includes an average field of neutron and proton systems in the form of the axial - symmetric Saxon - Woods (or Hartree - Fock), monopole pairing, isoscalar and isovector particle - hole and particle - particle multipole and spin - multipole interactions between quasiparticles as well as the so - called α - like four nucleon interaction [10]. For the particle - hole and particle - particle multipole and spin - multipole interaction part we use a separable interaction [10] of the rank N > 1 :

$$H = H_0 + H' \tag{1}$$

where

$$H_0 = \sum_\tau (H^{av}_{s.p.}(\tau) - G_\tau P^\dagger_\tau P_\tau) + H_4 \tag{2}$$

in which

$$H^{av}_{s.p.}(\tau) = \sum_{s\sigma} E_s a^\dagger_{s\sigma} a_{s\sigma} \tag{3}$$

$$P_\tau = \sum_s a_{s-} a_{s+} \tag{4}$$

$$H_4 = -G_4 P^\dagger_p P^\dagger_n P_n P_p \tag{5}$$

and

$$\begin{aligned} H' = & \\ & \sum_\tau [-\frac{1}{2} \sum_{\lambda\mu\sigma} \sum_{n=1}^{N} [\sum_{\eta=\pm 1} (\kappa^{\lambda\mu}_{0\tau} + \eta\kappa^{\lambda\mu}_{1\tau}) * \\ & Q^\dagger_{n\lambda\mu\sigma}(\tau) Q_{n\lambda\mu\sigma}(\eta\tau) + G^{\lambda\mu}_\tau P^\dagger_{n\lambda\mu\sigma}(\tau) P_{n\lambda\mu\sigma}(\tau)] - \\ & \frac{1}{2} \sum_{L\lambda\mu\sigma} \sum_{n=1}^{N} [\sum_{\eta=\pm 1} (\kappa^{L\lambda\mu}_{0\tau} + \eta\kappa^{L\lambda\mu}_{1\tau}) * T^\dagger_{nL\lambda\mu\sigma}(\tau) T_{nL\lambda\mu\sigma}(\eta\tau) + \\ & G^{L\lambda\mu}_\tau P^\dagger_{nL\lambda\mu\sigma}(\tau) P_{nL\lambda\mu\sigma}(\tau)]] \end{aligned} \tag{6}$$

Here τ = - $\frac{1}{2}$ stands for the proton system and τ = + $\frac{1}{2}$ stands for the neutron system, $a^\dagger_{s_\tau\sigma_\tau}(a_{s_\tau\sigma_\tau})$ are the fermion operators which create (destroy) a nucleon in (from) the single particle state $| s_\tau\sigma_\tau >$, where σ_τ is the sign of the projection of the angular momentum of the state onto the nuclear symmetry axis, s_τ being the rest $(N_\tau, n_{z_\tau}, \Omega_\tau, \pi_\tau, ...)$ of the quantum numbers that label the single particle energy levels. The term H_4 from the eq.(5) is an effective, coherent two - pairs (four - nucleon) interaction term, which induces the dynamical alpha - like four nucleon correlations in the superfluid phases of atomic nuclei [11]. G_τ are the pairing coupling strenghts, $G_\tau^{\lambda\mu}$ and $G_\tau^{L\lambda\mu}$ are the coupling constants of the particle - particle interaction [10] $\kappa_{0\tau}^{\lambda\mu}$, $\kappa_{1\tau}^{\lambda\mu}$ and $\kappa_{0\tau}^{L\lambda\mu}$, $\kappa_{1\tau}^{L\lambda\mu}$ are the isoscalar and isovector coupling constants of the particle - hole multipole - multipole and spin - multipole interactions [12]. G_4 is the four - nucleon interaction constant and $\sigma = \pm 1$.

To find the superfluid solutions we first should deal with the mean field. As a trial wave function for the ground state of the atomic nucleus we use the BCS - type wave function and the mean field is described by the H_0 - part of the Hamiltonian (1).

$$| BCS >= \prod_{\tau s_\tau}(u_{s_\tau} + v_{s_\tau} a^\dagger_{s_\tau +} a^\dagger_{s_\tau -}) \mid 0 > \quad (7)$$

where $u_s^2 + v_s^2 = 1$ and $| 0 >$ denotes the absolute vacuum.

Thus the constrained energy functional is:

$$W =< BCS \mid H_0 - \sum_\tau \lambda_\tau \hat{N}_\tau \mid BCS >=$$
$$\sum_\tau (\sum_{s_\tau} 2(\tilde{E}_{s_\tau} - \lambda_\tau) v_{s_\tau}^2) - G_\tau \chi_\tau^2) - G_4 \chi_p^2 \chi_n^2 \quad (8)$$

Here λ_τ denotes the nucleon Fermi level, $\hat{N}_\tau$ is the nucleon number operator and

$$\chi_\tau =< BCS \mid \sum_{s_\tau} a^\dagger_{s_\tau +} a^\dagger_{s_\tau -} \mid BCS >= \sum_{s_\tau} u_{s_\tau} v_{s_\tau} \quad (9)$$

is the so - called pairing correlation function or order parameter and

$$\tilde{E}_{s_{p(n)}} = E_{s_{p(n)}} - \frac{1}{2}(G_{p(n)} + G_4 \chi_{n(p)}^2) v_{s_{p(n)}}^2 - \frac{1}{4} G_4 v_{s_{p(n)}}^2 \sum_{s_{n(p)}} v_{s_{n(p)}}^4 \quad (10)$$

are the modified (from the values $E_{s_{p(n)}}$) single - particle energies. Usually these self - consistent field corrections are omitted [12].

The minimization of the function W given by eq. (8) with respect to the variational parameters leads to the following gap and constraint eqs.:

$$\frac{1}{2}(G_{p(n)} + G_4 \chi_{n(p)}^2) \sum_{s_{p(n)}} \epsilon_{s_{p(n)}}^{-1} = 1$$
$$\sum_{s_\tau} (1 - (\tilde{E}_{s_\tau} - \lambda_\tau) \epsilon_{s_\tau}^{-1}) = N_\tau$$
$$\epsilon_{s_\tau} = \sqrt{(\tilde{E}_{s_\tau} - \lambda_\tau)^2 + \Delta_\tau^2} \quad (11)$$

for the doubly even mass deformed superfluid nuclei. For odd- and odd - odd - mass deformed superfluid nuclei the above equations are modified according to the blocking effect [12].

The Bogoliubov - Valatin u_s and v_s - parameters are parametrized according to the following formulae:

$$\begin{pmatrix} u_{s_\tau}^2 \\ v_{s_\tau}^2 \end{pmatrix} = \frac{1}{2}(1 \pm \frac{\tilde{E}_{s_\tau} - \lambda_\tau}{\epsilon_{s_\tau}}) \quad (12)$$

and the correlation function becomes

$$\chi_\tau = \frac{1}{2} \Delta_\tau \sum_{s_\tau} \epsilon_{s_\tau}^{-1} \quad (13)$$

A simple inspection of the gap equations shows that the proton and neutron equations are coupled, i.e. it is possible that the superfluidities of the proton and neutron systems may be generated by one another, even in the case when for one system, in the absence of four nucleon interactions, the Belyaev's condition [11] is not satisfied. The aditional term $G_4 \chi^2$ may increase the pairing strengths in order to fulfil the Beliaev's condition. This mechanism explained [10] in several cases the origin of the odd - even staggering of the charge radii of isotopes of one element. Moreover, the gap equations can have [10] for some nuclei more than one set of solutions, fact which open a new area of research - the superfluid isomers.

To find the excitation spectrum and the corresponding wave functions we add to the H_0 the H' part and use the recepe from the Refs. [10], [12], [19], [20].

3 Alpha Decay

By using the enlarged superfluid model (ESM) [10], we calculated the quasiparticle - phonon structure of the ground state of the ^{255}Fm nucleus, who emits alpha and ^{20}O clusters. We calculated also the ground and several excited states of the daughter nuclei: ^{251}Cf and ^{235}U , respectively. The results are reproduced in Table 1. The structures of these states are very close to the structures given in the Refs.[19] and [20]. Within the R-matrix approximation [7] we calculated the HF's for the favored and some unfavored α - decays of ^{255}Fm to ground and some excited states in ^{251}Cf nucleus. The expressions of the reduced widths within the superfluid model are given in Ref. [16]. The results have been compared with the calculations of Ref. [5] and the experimental data [24] (see Table 2). They are not far from our previous calculations [3]. A relatively good agreement with the experimental data is obtained. The data denoted by HF_{MPR} have been obtained by using the reduced widths from Ref. [5] and the penetrability ratio calculated with the M3Y double folding potential [7]. In calculating the ^{255}Fm - and ^{251}Cf - excited states structure the used ESM parameters are: $G_p = 0.14$ MeV, $G_n = 0.12$ MeV, $G_4 = 0.25$ keV. The parameters of the average field are taken from Ref. [19]. The used deformation parameters are: $\beta_{20} = 0.26$ and $\beta_{40} = 0.035$ (see Ref.[19]). The used particle - hole quadrupole and octupole parameters (see eq. 6) are: $\kappa_{n\tau}^{\lambda\mu} = \kappa_{0\tau}^{2\mu} = 0.664$ keV fm^{-4}; $\kappa_{n\tau}^{\lambda\mu} = \kappa_{1\tau}^{2\mu} = 62.4$ eV fm^{-4}; $\kappa_{n\tau}^{\lambda\mu} = \kappa_{0\tau}^{3\mu} = 8.6$ eV fm^{-6} $\kappa_{n\tau}^{\lambda\mu} = \kappa_{1\tau}^{3\mu} = 1.2$ eV fm^{-6}. The used particle - particle quadrupole parameter (see eq. 6) are: $G_{n\tau}^{L\lambda\mu} = G_{0\tau}^{L2\mu} = 12$ eV fm^{-4}. All the other coupling constants entering the eq. (6) and not mentioned here have been taken equal to zero.

In the calculated structures we included the quadrupole and octupole phonons with $\lambda\mu$ = 20,22,30,31,32 and i=1,2 (see Ref. [10]), following the results within the quasiparticle - phonon model developed in the Ref. [19].

From Tables 1 and 2 we conclude that the α - decay of ^{255}Fm ground state to ^{251}Cf $\frac{7}{2}^+$; $E_x = 106.33$ keV - state can be considered as favored α - transition. The explanation of small (close to unity) HF's in this case, is based on the picture according to, the cluster

(in this case an α - particle) is builded from the fermions just situated at the Fermi surface, where strong pairing correlations occur and, in addition, one may neglect the differences in structure of the parent and daughter states. On the contrary, for the other α - transitions (see Table 2), the hindrance factors are large, and this is explained by the fact that during the formation process of the α - cluster, at least one Cooper pair is destroyed and one nucleon from this Cooper pair is coupled with the uncoupled mother nucleus nucleon in order to participate in the formation of the α - cluster.

The channel radial regular and irregular wave functions have been calculated by using the Coulomb potential plus the realistic M3Y double folding potential [22], in which one uses an effective interaction derived from the G - matrix elements based on the Reid soft - core NN potential [29] in the form assuming only OPEP force between the states with odd relative angular momentum [30]. This potential is obtained numerically, and then is interpolated by cubic spline functions to improve the accuracy of the numerical integration. The radial scattering wave functions are calculated at the experimental resonance energies using the Numerov algorithm. At a distance of 15 fm the nuclear folding potential V_n has practically no contribution, and the regular solution is normalized to have the asymptotic behavior of the Coulomb functions [31]. The value of the irregular solution at this distance is obtained from the Wronskian relation and then the whole irregular solution is obtained integrating backwards to the origin. However at small distances the fragments interact strongly, and this asymptotic solution should be gradually replaced by the "internal" wave function supposed to describe the compound system before decay.

4 Cluster Decay

We also calculated the favored and weak unfavored ^{20}O - transitions from ^{255}Fm - nucleus to some excited states in ^{235}U nucleus by using the approximations suggested in Ref. [7]. In calculating the ^{235}U - excited states structure the used ESM parameters are: $G_p = 0.14$ MeV, $G_n = 0.10$ MeV, $G_4 = 0.26$ keV. The parameters of the average field are taken from

Ref. [20]. The used deformation parameters are: $\beta_{20} = 0.23$ and $\beta_{40} = 0.08$. The used particle - hole quadrupole and octupole parameters (see eq. 6) are: $\kappa^{\lambda\mu}_{n\tau} = \kappa^{2\mu}_{0\tau} = 0.667$ keV fm^{-4}; $\kappa^{\lambda\mu}_{n\tau} = \kappa^{2\mu}_{1\tau} = 0.062$ keV fm^{-4}; $\kappa^{\lambda\mu}_{n\tau} = \kappa^{3\mu}_{0\tau} = 0.011$ keV fm^{-6} $\kappa^{\lambda\mu}_{n\tau} = \kappa^{3\mu}_{1\tau} = 0.001$ keV fm^{-6}. The used particle - particle quadrupole parameter (see eq. 6) are: $G^{\lambda\mu}_{n\tau} = G^{2\mu}_{\tau} = 15$ eV fm^{-4}. All the other coupling constants entering the eq. (6) and not mentioned here have been taken equal to zero.

From the Table 1 we learn that within the ESM - model [10] the structure of the ^{255}Fm ground state contains contributions from two single quasiparticle states namely, 97.9 % - [613] $\frac{7}{2}^{+}$ and 2.1 % - [624] $\frac{7}{2}^{+}$ emerging from $1i_{\frac{11}{2}}$ and $2g_{\frac{7}{2}}$, respectively. These states occur also in the structure of ^{235}U excited states lying at 446 keV and 1236 keV excitation energy, respectively (see Table 1 and Ref. [20]). By using the ESM structure for the initial and final odd-mass nuclei, in the case of favored or weak unfavored radioactive decay with emission of a spherical double even mass cluster (e.g. ^{20}O) we may write for the hindrance factor the following expression:

$$HF\left[mother\ \ nucleus(I_i^{\pi_i}K_i) \rightarrow^{20} O + daughter\ \ nucleus(I_f^{\pi_f}K_f)\right] \approx \left\{\sum_l F_l \left|C^{I_i\ l\ I_f}_{K_i K K_f} C_{\rho_i} C_{\rho_f} (RSA)_l^{(i\rightarrow f)}\right|^2\right\}^{-1} \quad (14)$$

$C_{\rho_{i(f)}}$ are the weights of the single quasiparticle state in the structure of the $i(f)$ - state, which should be the dominant one, i.e. the other contributions (unfavored quasiparticles or quasiparticle - phonon contributions) should be negligible small. The quantities (RSA) replaces essentially the ratio of the favored intrinsic spectroscopic amplitudes [7] corresponding to the transitions between odd - mass and doubly even nuclei, respectively. The intrinsic spectroscopic amplitude (θ_{in}) is defined by the quantity:

$$\theta_{in} = \sum_{\nu_1..\nu_{14}} \sum_{\omega_3...\omega_{20}} A^{LM}(\nu_1..\nu_{20} \mid \omega_1..\omega_{20})\xi^{fav}(\nu_1..\nu_{14} \mid \omega_1..\omega_{20}) \quad (15)$$

analogous to the quasiparticle contribution in the matrix element from the eq. (11) of Ref.

[16] entering the α - decay rate of axially deformed odd - A nuclei. The only difference between the cases corresponding to the odd-mass and doubly - even nuclei, is that in the first case the sum in the above equation excludes the common quasiparticle state of both the mother and daughter nuclear states (e.g. $[613]\frac{7}{2}^{+}$ for the ^{255}Fm$\rightarrow^{20}$O+^{235}U). In the estimations we performed (see Table 3), the approximation $(RSA) \approx 0.4$ has been used, mainly determined by the overlap integral between the odd nucleon orbital wave functions in the mother and daughter nuclei. The suggestion given in Ref. [36] pages 406 - 407, according to the levels lying at 445.71 keV and 509.82 keV should belong to the rotational band built on the intrinsic state with the structure ground state $\otimes$ octupole phonon, does not fit to our results. The suggested structure contribute with the 9% only, the dominant contribution coming from the single quasiparticle state $[624]\frac{7}{2}^{+}$.

Unfortunately the above discussed ^{20}O radioactivity cases have the half - lives greater than the maximum half - life ($10^{25.75}$ sec.) among the experimentally measured [8] cluster decay half - lives and, hence, hard to be measured.

The ^{223}Ra nucleus belongs [28] to the well known region of soft nuclei with Z $\approx$ 88 and N $\approx$ 134, with strong octupole correlations in the ground and low lying excited states, where the $1j_{\frac{15}{2}}$ intruder orbital interacts strongly with the $2g_{\frac{9}{2}}$ natural parity orbital. The HF's for both the α - and ^{14}C - decays of the ground state of ^{223}Ra are very difficult to be calculated at the moment, due to the unknown accurate structure of the mother and daughter nuclei. Studying the experimental HF for α - decays to ^{219}Rn ground and low lying excited states [25] we learn that $\approx$ fifteen [25] transitions have small ($\leq$ 100) HF's and from these transitions five have HF's $\leq$ 10. The corresponding excited states have very different structure and this tells us that the structure of the ground state of ^{223}Ra is not as simple, as e.g. the ^{255}Fm case, and it may contain many more or less equal components of single quasi - particle or quasi - particle - phonon structure. Unfortunately, not all the spins and parities of the ^{219}Rn - excited states, populated by α - decay, are known. Thus, it is a real difficult problem to describe the quantum states involved in the α - and ^{14}C - decay of

^{223}Ra. In our opinion, it is not sufficient a description of these states within an independent particle model only [17], [18]. Residual interactions could play an important role [3]. The restrictions concerning the number of quasiparticles and phonons (as e.g. in the case of ^{255}Fm→α+^{251}Cf - decay, where only $\lambda\mu$ = 20,22,30,31,32 and i=1,2 - phonons have been used), lead to inaccurate structure of the ^{223}Ra - nucleus. First the valence single particle space should be extended and secondly, at the next step when incorporating the quasiparticle - phonon interaction, the number of quasiparticles and phonons should be increased. Such a task is as hard as to perform the calculations within the OXBASH shell model code with realistic residual interactions [7].

To understand this situation we construct a very simple model, which proves to deserve attention by itself and to suggest the highly nontrivial behavior of any realistic model.

Assume, for a moment, that the structure of the ground state of the ^{223}Ra - nucleus consist of spherical core described by an independent particle model. Above the core there exists a deformed single particle neutron orbital only. The wave function for this orbital can be expanded in terms of spherical orbitals. In this case the spectroscopic amplitude entering the expression of the HF can be factorized according to:

$$\theta_{Nl\,K_i-K_f}^{(K_i^{\pi_i}K_i\to K_f^{\pi_f}K_f)} = C_{\Omega_i}C_{\Omega_f}a_{N_il_ij_i}^{\Omega_i=K_i}a_{N_fl_fj_f}^{\Omega_f=K_f}\sqrt{2I_f+1}\begin{pmatrix} I_i & l & I_f \\ K_i & K & K_f \end{pmatrix}\theta_{core}^{(j_i\pi_i\to j_f\pi_f)} \qquad (16)$$

where $C_{\Omega_{i(f)}}$ are the weights of the single quasipalticle state in the structure of the $i(f)$ - state, a_{Nlj}^{Ω} are the Nilsson - like amplitudes, $\begin{pmatrix} I_i & l & I_f \\ K_i & K & K_f \end{pmatrix}$ stands for the 3-j symbol and $\theta_{core}^{(j_i\pi_i\to j_f\pi_f)}$ acts as a spectroscopic amplitude between many body core $\mid j_{i(f)}\pi_{i(f)} >$ states, including both the cluster overlaps [21], [7] and the intrinsic overlap integrals [7]. Now the ratio of the intrinsic spectroscopic amplitudes (RSA) from the eq. (1) is given by the ratio of $\theta_{core}^{(j_i\pi_i\to j_f\pi_f)}$ calculated for the ^{223}Ra →^{14}C+^{209}Pb transition and $\theta_{core}^{(j_i\pi_i\to j_f\pi_f)}$ calculated for the ^{222}Ra →^{14}C+^{208}Pb - transition. The expression of the hindrance factor becomes

$$HF\left[mother\ \ nucleus(I_i^{\pi_i}K_i) \to^{14} C + daughter\ \ nucleus(I_f^{\pi_f}K_f)\right] \approx \qquad (17)$$
$$\left\{\sum_l F_l\left|C_{K_iKK_f}^{I_i\ l\ I_f}C_{\rho_i}C_{\rho_f}a_{N_il_ij_i}^{\Omega_i=K_i}a_{N_fl_fj_f}^{\Omega_f=K_f}(RSA)_l^{(i\to f)}\right|^2\right\}^{-1}$$

Within such an approximation we calculated the hindrance factors for ^{223}Ra (g.s) → ^{14}C + ^{209}Pb - and ^{229}Th (g.s) → ^{20}O + ^{209}Pb - cluster transitions.

In the Table 5 and 6 the intrinsic spectroscopic amplitude ratios ((RSA)) has been estimated to be ≈ 0.52 and ≈ 0.42, respectively. In calculating the ^{223}Ra - and ^{229}Th - ground states structure (see Table 4) the used ESM parameters are: G_p = 0.14 MeV, G_n = 0.10 MeV, G_4 = 0.26 keV. The parameters of the average field (see Ref. [20]) are: V_{0p} = 55.53698 MeV, $r_{0,p}$ = 1.30975 fm, a_p = 0.70071, $\kappa_{s-o,p}$= 5.56479 MeV, V_{0n} = 37.78683 MeV, $r_{0,n}$ = 1.39628 fm, a_n = 0.70071, $\kappa_{s-o,n}$= 7.31907 MeV. The used deformation parameters are: β_{20} = 0.15, β_{40} = 0.10. The used particle - hole quadrupole and octupole parameters (see eq. 6) are: $\kappa_{n\tau}^{\lambda\mu} = \kappa_{0\tau}^{2\mu}$ = 0.67 keV fm^{-4}; $\kappa_{n\tau}^{\lambda\mu} = \kappa_{1\tau}^{2\mu}$ = 0.06 keV fm^{-4}; $\kappa_{n\tau}^{\lambda\mu} = \kappa_{0\tau}^{3\mu}$ = 0.01 keV fm^{-6} $\kappa_{n\tau}^{\lambda\mu} = \kappa_{1\tau}^{3\mu}$ = 1. eV fm^{-6}. The used particle - particle quadrupole parameter (see eq. 6) are: $G_{n\tau}^{\lambda\mu} = G_{\tau}^{2\mu}$ = 15 eV fm^{-4}. All the other coupling constants entering the eq. (6) and not mentioned here have been taken equal to zero.

A few more comments may be in order here. First of all our hybrid model with a spherical core and only one deformed orbital, when calculating the spectroscopic amplitudes is not to be taken too seriously for very complex structures, which should be the case of ^{223}Ra. This should be not true even for structures close to single quasiparticle states, because the assumption of the axial deformed core is not realistic [27]. On the other hand, when having realistic structures for both the initial and final states, calculations within shell model codes like OXBASH or ESM are practically impossible for nowadays computers. Therefore simple schematic models like above presented would be useful. In the presented calculations we estimated the core spectroscopic factor as in the case of the favored cluster decays, i.e. the magnitude of the core spectroscopic factor has been mainly evaluated by the overlap integral between the spherical wave functions describing the valence odd neutron in the mother and daughter nuclei, which does not participate in the cluster decay. This overlap integral is less than unity due to the fact that the two above orbitals are oscillator orbitals with different frequencies [7].

Within ESM the calculation of the hindrance factor HF=3, experimentally observed [1] in the case of the transition from the ground state of ^{223}Ra ($\frac{3}{2}^+$) to the $\frac{15}{2}^-$, 1423 keV excited state in ^{209}Pb can be performed by using the parity admixture only [33], [32]. There is an excited level $\frac{3}{2}^-$ in ^{223}Ra, lying at 50 keV excitation energy, which can be admixed [32] in the ground state. We roughly calculated (i.e. assuming a parity mixed doublet [32]) the admixture coefficient of this first excited state into the ground state of ^{223}Ra by using the technique developed in the Ref. [33] and the parity nonconserving potential used in Ref. [34]. This coefficient is of the order of 10^{-5}, but higher lying states could change this value. With this value, the hindrance factor for the mentioned transition is of the order of 10^6, far away from the experimental value. This simple estimation, however, should be changed by increasing the number of phonons and single quasiparticle states, used to describe the structure of above nuclear states.

5 Conclusion

In this work we reported some calculations performed, within the enlarged superfluid model [10], for some selected (favored and weak hindered) α transitions in the - ^{255}Fm (g.s) $\rightarrow \alpha$ + ^{251}Cf - process. A schematic model has been applied for ^{223}Ra (g.s) $\rightarrow$ ^{14}C + ^{209}Pb; ^{255}Fm (g.s) $\rightarrow$ ^{20}O + ^{235}U and ^{229}Th (g.s) $\rightarrow$ ^{20}O + ^{209}Pb processes. In these cases difficulties arise due to unknown structure of ^{223}Ra and ^{229}Th ground states and due to impossibility to calculate truly microscopically the spectroscopic amplitude. Nevertheless simple schematic models could help us in understanding the heavy cluster decay.

Assuming for the structure of the ground state of the ^{223}Ra - nucleus a hybrid model, with a core and above the core only one deformed single particle orbital, we could factorize the spectroscopic amplitude for the ^{14}C - decay into three factors; first one is the single

quasiparticle weight into the structure of the ground state of the ^{223}Ra - nucleus, the second one is the Nilsson - like amplitude of a spherical orbital into the deformed Nilsson - like orbital and the last one is the spectroscopic amplitude of the ^{14}C - decay from a *spherical configuration*. This last factor can be calculated by using an analogous recipe as given in Ref. [6], [7] for the case of α - decay. It may have large variations due to selection rules and internal structure of the core, when calculating its cluster overlap factor.

Our estimations of the HF's differ in magnitude from previous estimations [17], [18], [7].

Within ESM we overestimate the experimental HF corresponding to the ground state of ^{209}Pb and to the first excited state of ^{209}Pb. We cannot explain the experimental HF corresponding to the $\frac{15}{2}^-$ (1423 keV) state, but our approach does not use a very large basis of states. The HF corresponding to the $\frac{15}{2}^-$, 1423 keV state in ^{209}Pb, within ESM, has been calculated by using the parity admixture of $\frac{3}{2}^-$ first excited state in the ground state of ^{223}Ra.

Predictions have been done for the hindrance factors corresponding to the following cluster transitions (see Tables 3 and 6): ^{255}Fm (ground state) $\rightarrow$ ^{20}O + ^{235}U (445.716 keV, 1236 keV and their rotational bands) and ^{229}Th (ground state) $\rightarrow$ ^{20}O + ^{209}Pb (ground state, $\frac{11}{2}^+$ and 779 keV and $\frac{15}{2}^-$ excited state).

Additional experimental work on the ^{14}C fine structure decay of ^{223}Ra with higher resolution would be very valuable. This might allow the resolution of: 1) the groups populating the $\frac{15}{2}^-$ and $\frac{5}{2}^+$ states in ^{209}Pb and 2) the groups leaving from ground $\frac{3}{2}^+$ and excited $\frac{3}{2}^-$ (50 KeV) states of ^{223}Ra - nucleus, in order to determine more conclusively the HF for populating the $\frac{15}{2}^-$ state in ^{209}Pb.

Acknowledgements

One of the authors (O.D.) would like to thank Professor Luciano Fonda for his permanent encouragement. He would also like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

References

[1] E. Hourani *et al* Phys. Rev., **C 44** (1991) 1424; L. Brillard *et al* C.R. Acad. Sci. Paris, **309** (1989) 1105

[2] H.J. Rose and G.A. Jones, Nature, **307** (1984) 245

[3] O. Dumitrescu, Fiz. Elem. Chastits At.Yadra **10** (1979) 377 (Sov.J.Part.Nucl., **10** (1979) 147) O. Dumitrescu and A. Sandulescu, Nucl. Phys., **A 100** (1967) 456; Phys. Lett., **B 19** (1965) 404; Phys. Lett., **B 24** (1967) 212; Annales Acad. Sci. Fennicae, Series A, VI.Physica **265** (1968); M.I. Cristu, O. Dumitrescu, N.Y. Pyatov and A. Sandulescu, Nucl. Phys., **A 130** (1969) 31; O. Dumitrescu and D.G. Popescu, Revue Roumaine de Physique **18** (1973) 1065 and **21** (1976) 323; O. Dumitrescu, M. Petrascu, D.G. Popescu and A. Sandulescu, Revue Roumaine de Physique **18** (1973) 1231;

[4] H.J. Mang, Ann. Rev. Nucl. Sci., **14** (1964) 1, Z. Phys., **148** (1957) 572, Phys.Rev., **119** (1960) 1069, Proc. Second International Conf. Clust. Phen. Nucl. College Park, Maryland (1975);

[5] H.J. Mang, J.K. Poggenburg and J.O. Rasmussen, Phys. Rev.,**181** (1969) 1697; J.K. Poggenburg UCRL - 16187 (1965)

[6] M. Grigorescu, B.A. Brown and O. Dumitrescu, Phys. Rev., **C 47** (1992) 2666

[7] O. Dumitrescu, Phys. Rev. C 49 (1994) 1466; Preprint ICTP Trieste IC/93/164 (1993); O. Dumitrescu, *"Fine Structure in Cluster Decays"* Proc. International NATO Advanced Study Institute, Predeal (Sept. 1993), Romania, *"Frontier Topics in Nuclear Physics"*, Eds. W. Scheid and A. Sandulescu, in press

[8] A. Sandulescu and W. Greiner, Rep. Progr. Phys. **55** (1992) 1423

[9] Y.K. Gambhir, P. Ring and P. Schuck, Phys. Rev. Lett. **51** (1983) 1235

[10] O. Dumitrescu, Fiz. Elem. Chastits At. Yadra 23 (1992) 430 (Sov.J. Part. Nucl., **23** (1992) 187); Nuovo Cimento **A 104** (1991) 1057 Preprint ICTP - Trieste **IC/91/72** (1991); O. Dumitrescu and M. Horoi, Nuovo Cimento, **A 103** (1990) 653

[11] M. Apostol, I. Bulboaca, F. Carstoiu, O. Dumitrescu and M. Horoi, Nucl. Phys., **A 470** (1987) 64; M. Apostol, I. Bulboaca, F. Carstoiu, O. Dumitrescu and M. Horoi, Europhys. Letters, **4** (1987) 197;

[12] V.G.Soloviev, *Theory of Complex Nuclei*, (Nauka,Moscow,1971; PergamonPress, New York,1976)

[13] F. Barranco, G.F. Bertsch, R.A. Broglia, E. Vigezzi, Nucl. Phys., **A 512**; F. Barranco, R.A. Broglia and G.F. Bertsch, Phys. Rev. Lett., **60** (1988) 507; G.F. Bertsch, F. Barranco and R.A. Broglia Windsurfing the Fermi Sea, Vol. 1, Eds. T.T.S. Kuo and J. Speth, Elesevier Science Publ. B. V., (1987); G.F. Bertsch, *"Collective Motion in Fermi Droplets"* in "Frontiers and Borderlines in Many Particle Physics", Eds. R.A. Broglia and J.R. Schrieffer, Amsterdam - North Holland (1989).

[14] G. Gamov, Zeit. Phys. **51** (1928) 24

[15] H. Geiger and J. M. Nuttal, Phylos. Mag. **22** (1911) 613; **23** (1911) 613;

[16] O. Dumitrescu and A. Sandulescu, Nucl. Phys., **A 100** (1967) 456;

[17] R.S. Sheline and I. Ragnarsson, Phys. Rev., **C 43** (1991) 1476

[18] M. Hussonnois *et al* Phys. Rev., **C 44** (1991) 2884; G. Ardisson *et al* Proc. Int. School Seminar Heavy Ion Physics, Dubna 1989, p.336 (1990); G. Ardisson and M. Hussonnois, C.R. Acad. Sci. Paris, **310** (1990) 367; M. Hussonnois *et al* J. Phys. G. **16** (1990) 177

[19] S.P. Ivanova, A.L. Komov, L.A. Malov and V.G. Soloviev, Izv. Akad. Nauk. SSSR (ser. Fiz.) **A 37** (1973) 911

[20] F.A. Gareev, S.P. Ivanova, L.A. Malov and V.G. Soloviev, Nucl. Phys., **A 171** (1971) 134: JINR - Dubna, **P4 - 5457** (1970)

[21] B.A. Brown, Nucl. Phys., **A 522** (1991) 221c; B.A. Brown and B. H. Wildenthal, Ann. Rev. Nucl. Part. Sci., **38** (1988) 29; B.A. Brown *et al* M S U - N S C L Report **524** (1985).

[22] G. Bertsch, J. Borisowicz, H. McManus and W.G. Love, Nucl. Phys. **A 284** (1977) 399

[23] J.O. Rasmussen, Phys.Rev.**113**(1959)1593; I. Perlman, J.O. Rasmussen, In *"Handbuch der Physik"* **42**(1957)109

[24] Y. A. Ellis and M. R. Schmorak, Nuclear Data Sheets, **B 8** (1972) 345

[25] C. Maples, Nuclear Data Sheets, **22** (1977) 223

[26] H.J. Daley and F. Iachello, Phys. Lett., **B 131** (1983) 281;

[27] G.A. Leander *et al* Nucl. Phys. **A 388** (1982) 452; Nucl. Phys. **A 413** (1984) 375;

[28] S. Aberg, H. Flocard and W. Nazarewicz, Ann. Rev. Nucl. Part. Sci., **40** (1990) 439,

[29] J.P. Elliot, A.D. Jackson, H. A. Mavromatis, E.A. Sanderson and B. Singh Nucl. Phys. **A 121** (1968) 241

[30] G.R. Satchler and W.G. Love, Phys. Reports **55** (1979) 123

[31] M. Abramowitz, I. A. Stegun, Handbook of Mathematical Functions, National Bureau of Standards, Washington D.C., 1964

[32] O. Dumitrescu and G. Clausnitzer Nucl. Phys. **A 552** (1992) 306.

[33] O. Dumitrescu, M. Gari, H. Kuemmel and J. G. Zabolitzky Zeit. Naturforschung **27 A** (1972) 733; Phys. Lett. **35 B** (1971) 19

[34] B. Desplanques and O. Dumitrescu, Nucl. Phys., **A 552** (1993) 306

[35] M.R. Schmorak, Nuclear Data Sheets **59** (1990) 507

[36] M.R. Schmorak, Nuclear Data Sheets **69** (1993) 375

[37] M.J. Martin, Nuclear Data Sheets **63** (1991) 723

[38] I. Ahmad, F. T. Porter, M.S. Freedman, R.F. Barnes, R.K. Sjoblon, F. Wagner Jr, J. Miested and P.R. Fields, Phys. Rev., C 3 (1971) 390; I. Ahmad and J. Milsed, Nucl. Phys. **A 239** (1975) 1

Table Captions

Table 1 : *The calculated, within ESM [10], structure of some ground and excited states entering the α transitions: ^{255}Fm (g.s) → α + ^{251}Cf and the ^{20}O transitions: ^{255}Fm (g.s) → ^{20}O + ^{235}U. These results are compared with the experimental data [35], [36]*

Table 2 : *The calculated, within ESM [10], hindrance factors for favored, weak unfavored and unfavored α - transitions from ^{255}Fm (g.s.) to the members of the rotational bands of several intrinsic states of ^{251}Cf. These results are compared with the calculated HF's by Mang, Poggenburg and Rasmussen [5] and experimental data [23], [24], [35] [38].*

Table 3 : *The calculated, within ESM [10], hindrance factors for the ^{20}O transition: ^{255}Fm (g.s) → ^{20}O + ^{235}U. The experimental energies are taken from Ref. [36]*

Table 4 : *The calculated, within ESM [10], structure of the ground and some excited states entering the cluster transitions: ^{229}Fm (g.s) → ^{20}O + ^{209}Pb and ^{223}Ra (g.s) → ^{14}C + ^{209}Pb. The experimental energies are taken from Ref. [37].*

Table 5 : *The calculated, within ESM [10], hindrance factors for the ^{14}C transition: ^{223}Ra (g.s) → ^{14}C + ^{209}Pb. The abbreviation DSPC means the dominant single particle configuration*

Table 6 : *The calculated, within ESM [10], hindrance factors for the ^{20}O transition: ^{229}Th (g.s) → ^{20}O + ^{209}Pb. The abbreviation DSPC means the dominant single particle configuration*

Table 1 : *The calculated, within ESM [10], structure of some ground and excited states entering the α transitions: ^{255}Fm (g.s) → α + ^{251}Cf and the ^{20}O transitions: ^{255}Fm (g.s) → ^{20}O + ^{235}U*

Nucleus	I^π K	E_{exp} (MeV)	E_{theo} (MeV)	Structure
^{255}Fm	$\frac{7}{2}^+$ $\frac{7}{2}$	0.	0.	**97.91** % [613] $\frac{7}{2}^+$ + 2.1 % [624] $\frac{7}{2}^+$ + 2.1 % [613] $\frac{7}{2}^+$ Q_{20} + 2.5 % [611] $\frac{3}{2}^+$ Q_{22}
^{251}Cf	$\frac{1}{2}^+$ $\frac{1}{2}$	0.	0.	**84.21** % [620] $\frac{1}{2}^+$ + 0.04 % [631] $\frac{1}{2}^+$ + 4.1 % [622] $\frac{3}{2}^+$ Q_{22} + 2.5 % [620] $\frac{1}{2}^+$ Q_{20}
^{251}Cf	$\frac{7}{2}^+$ $\frac{7}{2}$	0.10633	0.116	**87.23** % [613] $\frac{7}{2}^+$ + 6.04 % [624] $\frac{7}{2}^+$ + 2.1 % [611] $\frac{3}{2}^+$ Q_{22} + 2.5 % [725] $\frac{11}{2}^-$ Q_{32}
^{251}Cf	$\frac{3}{2}^+$ $\frac{3}{2}$	0.1777	0.176	**82.92** % [622] $\frac{3}{2}^+$ + 1.09 % [611] $\frac{3}{2}^+$ + 8.04 % [620] $\frac{1}{2}^+$ Q_{22} + 2.5 % [752] $\frac{3}{2}^-$ Q_{30}
^{251}Cf	$\frac{11}{2}^-$ $\frac{11}{2}$	0.3704	0.380	**87.88** % [725] $\frac{11}{2}^-$ + 6 % [613] $\frac{7}{2}^+$ Q_{32} + 4.04 % [615] $\frac{9}{2}^+$ Q_{31}
^{235}U	$\frac{7}{2}^+$ $\frac{7}{2}$	0.445716	0.458	**71.03** % [624] $\frac{7}{2}^+$ + 7.09 % [613] $\frac{7}{2}^+$ + 9.04 % [743] $\frac{7}{2}^-$ Q_{30} + 1.05 % [725] $\frac{11}{2}^-$ Q_{32}
^{235}U	$\frac{7}{2}^+$ $\frac{7}{2}$	1.236	1.458	**61.02** % [613] $\frac{7}{2}^+$ + 9.09 % [624] $\frac{7}{2}^+$ + 19.04 % [624] $\frac{7}{2}^+$ Q_{20} + 1.05 % [725] $\frac{11}{2}^-$ Q_{32}

Table 2 : *The calculated, within ESM [10], hindrance factors for favored, weak unfavored and unfavored α - transitions from ^{255}Fm (g.s.) to the members of the rotational bands of several intrinsic states of ^{251}Cf. These results are compared with the calculated HF's by Mang, Poggenburg and Rasmussen [5] and experimental data [23], [24].*

E_f (keV)	I_f^π	HF_{exp}	HF_{MPR}	HF_{ESM}	E_f (keV)	I_f^π	HF_{exp}	HF_{MPR}	HF_{ESM}
106.33	$\frac{7}{2}^+$	1.24	0.62	0.95	0.	$\frac{1}{2}^+$	4500	1621	2100
166.31	$\frac{9}{2}^+$	12.9	6.34	8.75	24.82	$\frac{3}{2}^+$	2800	1003	1300
239.33	$\frac{11}{2}^+$	52	21.07	28	47.83	$\frac{5}{2}^+$	500	246	265
325.3	$\frac{13}{2}^+$	125	178.3	210	105.7	$\frac{7}{2}^+$	120	416	475
424.1	$\frac{15}{2}^+$	390	544	570	146.5	$\frac{9}{2}^+$	610	399	455
-	$\frac{17}{2}^+$	-	-	-	237.7	$\frac{11}{2}^+$	3300	1416	1710
177.7	$\frac{3}{2}^+$	2700	15604	18150	370.4	$\frac{11}{2}^-$	540	3179	3355
211.6	$\frac{5}{2}^+$	2500	14719	17305	442.	$\frac{13}{2}^-$	840	3704	3950
258.4	$\frac{7}{2}^+$	3300	16084	18955	-	$\frac{15}{2}^-$	-	804687	847315
319.4	$\frac{9}{2}^+$	7300	24440	28540	-	$\frac{17}{2}^-$	-		

Table 3 : *The calculated, within ESM [10], hindrance factors for the ^{20}O transition: ^{255}Fm (g.s) → ^{20}O + ^{235}U*

E_f (keV)	$I_f^{\pi f}$	HF_{ESM}	E_f (keV)	$I_f^{\pi f}$	HF_{ESM}
445.716	$\frac{7}{2}^+$	≈ 185	1236.	$\frac{7}{2}^+$	≈ 5
509.92	$\frac{9}{2}^+$	≈ 428		$\frac{9}{2}^+$	≈ 11
587.82	$\frac{11}{2}^+$	≈ 729		$\frac{11}{2}^+$	≈ 18
682.57	$\frac{13}{2}^+$	≈ 1224		$\frac{13}{2}^+$	≈ 31

Table 4 : *The calculated, within ESM [10], structure of the ground and some excited states entering the cluster transitions: ^{229}Th (g.s) → ^{20}O + ^{209}Pb and ^{223}Ra (g.s) → ^{14}C + ^{209}Pb*

Nucleus	I^π K	E_{exp} (MeV)	E_{theo} (MeV)	Structure
^{229}Th	$\frac{5}{2}^+$ $\frac{5}{2}$	0.	0.	**87.91** % [633] $\frac{5}{2}^+$ + 1.1 % [622] $\frac{5}{2}^+$ + 2.1 % [743] $\frac{7}{2}^-$ Q_{31} + 2.5 % [631] $\frac{1}{2}^+$ Q_{22}
^{223}Ra	$\frac{3}{2}^+$ $\frac{3}{2}$	0.	0.	**78.21** % [631] $\frac{3}{2}^+$ + 2.04 % [642] $\frac{3}{2}^+$ + 13.1 % [752] $\frac{5}{2}^-$ Q_{31} + 2.5 % [761] $\frac{3}{2}^-$ Q_{30} 3.1 % [631] $\frac{1}{2}^+$ Q_{22} + 2.5 % [501] $\frac{1}{2}^-$ Q_{31}
^{209}Pb	$\frac{11}{2}^+$ $\frac{11}{2}$	0.7788	1.116	**97.23** % [606] $\frac{11}{2}^+$ + 1.04 % [615] $\frac{11}{2}^+$ + 2.1 % [743] $\frac{7}{2}^-$ Q_{32} + 2.5 % [725] $\frac{11}{2}^-$ Q_{30}
^{209}Pb	$\frac{9}{2}^+$ $\frac{9}{2}$	0.0	0.0	**92.92** % [615] $\frac{9}{2}^+$ + 1.09 % [624] $\frac{9}{2}^+$ + 1.04 % [615] $\frac{9}{2}^+$ Q_{20} + 2.5 % [624] $\frac{9}{2}^+$ Q_{20}

Table 5 : *The calculated, within ESM [10], hindrance factors for the ^{14}C transition: ^{223}Ra (g.s) → ^{14}C + ^{209}Pb. The abbreviation DSPC means the dominant single particle configuration*

E_f (keV)	$I_f^{\pi f}(DSPC)$	$[Nn_z\Lambda]_i$	$[Nn_z\Lambda]_f$	$a_{nlj}^{\Omega_i}$	$a_{nlj}^{\Omega_f}$	C_{Ω_i}	C_{Ω_f}	HF_{exp}	HF_{ESM}
0.	$\frac{9}{2}^+$ $(2g_{9/2})$	[642]	[615]	0.8	1.0	2 %	98 %	600.	≈ 668.
779.	$\frac{11}{2}^+$ $(1i_{11/2})$	[631]	[606]	0.8	1.0	78 %	97 %	3.	≈ 28.

Table 6 : *The calculated, within ESM [10], hindrance factors for the ^{20}O transition: ^{229}Th (g.s) → ^{20}O + ^{209}Pb. The abbreviation DSPC means the dominant single particle configuration*

E_f (keV)	$I_f^{\pi f}(DSPC)$	$[Nn_z\Lambda]_i$	$[Nn_z\Lambda]_f$	$a_{nlj}^{\Omega_i}$	$a_{nlj}^{\Omega_f}$	C_{Ω_i}	C_{Ω_f}	HF_{exp}	HF_{ESM}
0.	$\frac{9}{2}^+$ $(2g_{9/2})$	[633]	[615]	0.72	1.0	1 %	98 %	-	≈ 1070
779.	$\frac{11}{2}^+$ $(1i_{11/2})$	[622]	[606]	0.70	1.0	87 %	97 %		≈ 20