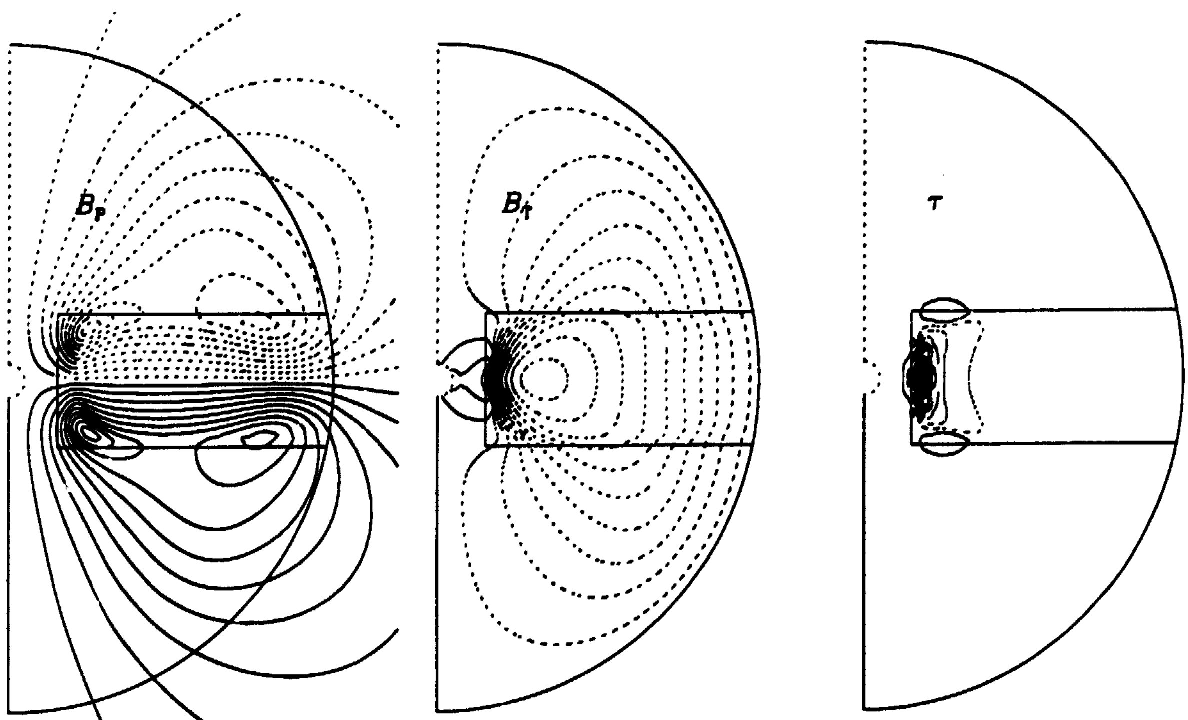
LUNFD6-(NFAS-1012)-1-23-1994

Magnetohydrodynamics of accretion disks

Ulf Torkelsson

Lund Observatory 1994



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Box 43, 221 00 Lund,
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Date of issue May 14, 1994

CODEN: LUNFD6/(NFAS-1012)/1-23/(1994)

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Sponsoring organization

Title and subtitle

Magnetohydrodynamics of accretion disks

Abstract

The thesis consists of an introduction and summary, and five research papers. The introduction and summary provides the background in accretion disk physics and magnetohydrodynamics. The research papers describe numerical studies of magnetohydrodynamical processes in accretion disks.

Paper I is a one-dimensional study of the effect of magnetic buoyancy on a flux tub in an accretion disk. The stabilising influence of an accretion disk corona on the fl tube is demonstrated.

Paper II-IV present numerical simulations of mean-field dynamos in accretion disks. Paper II verifies the correctness of the numerical code by comparing linear models to previous work by other groups. The results are also extended to somewhat modified dis models. A transition from an oscillatory mode of negative parity for thick disks to a steady mode of even parity for thin disks is found. Preliminary results for nonline dynamos at very high dynamo numbers are also presented.

Paper III describes the bifurcation behaviour of the nonlinear dynamos. For positi dynamo numbers it is found that the initial steady solution is replaced by an oscillat solution of odd parity. For negative dynamo numbers the solution becomes chaotic at sufficiently high dynamo numbers.

Paper IV continues the studies of nonlinear dynamos, and it is demonstrated that a chaotic solution appears even for positive dynamo numbers, but that it returns to a steady solution of mixed parity at very high dynamo numbers.

Paper V describes a first attempt at simulating the small-scale turbulence of an accretion disk in three dimensions. There is only find cases of decaying turbulence, but this is rather due to limitations of the simulations than that turbulence is reall absent in accretion disks

Key words accretion:accretion disks - magnetohydrodynamics - turbulence
chaotic phenomena - novae, cataclysmic variables - galaxies: active

Classification system and/or index terms (if any)

Supplementary bibliographical information

Language English

ISSN and key title

ISBN

Recipient's notes

Number of pages 94

Price

Security classification

Distribution by (name and address)

Ulf Torkelsson, Lund Observatory, Box 43, S-221 00 Lund, Sweden

Signature Ulf Torkelsson

Date April 18, 1994

Magnetohydrodynamics of accretion disks

av

Ulf Torkelsson

Akademisk avhandling, som för avläggande av filosofie doktorsexamen vid matematisk naturvetenskapliga fakulteten vid universitetet i Lund, kommer att offentligen försvaras å Palaestra hörsal, Universitetsplatsen, lördagen den 14 maj, 1994, kl 10.00

Institutionen för astronomi, Lunds universitet

Lund 1994

Magnetohydrodynamics of accretion disks

Ulf Torkelsson

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This thesis includes an introduction aiming at presenting the basics of both accretion disk physics. magnetohydrodynamics with special emphasis on dynamo theory. and the application of magnetohydrodynamics to problems regarding accretion disk. and the following papers:

Torkelsson. U.. 1993. *Magnetic buoyancy in accretion disks*, A&A, 274, 675

Torkelsson. U.. Brandenburg. A.. 1994a. *Turbulent accretion disk dynamos*, A&A, 283, 677

Torkelsson. U.. Brandenburg. A.. 1994b. *Chaos in accretion disk dynamos ?*, Chaos, Solitons & Fractals. in press

Torkelsson. U.. Brandenburg. A., 1994c, *The many incarnations of accretion disk dynamos: mixed parities and chaos for large dynamo numbers*, A&A. submitted, (revised)

Brandenburg. A.. Nordlund, Å., Stein, R. F., Torkelsson, U., 1994, *Simulating 3-D MHD Keplerian shear flows*, ApJ. to be submitted

Front cover

The magnetic field generated by a nonlinear dynamo in an accretion disk. To the left is shown the field lines of the poloidal magnetic field, in the middle a contour map of the toroidal field and to the right a contour map of the torque, that is exerted by the magnetic field on the disk. Negative values are denoted by broken lines. The inner and outer semicircles denote the inner and outer boundaries of the computational grid, respectively. The extent of the disk is indicated by solid lines. The large amount of flux outside the disk is due to the magnetic buoyancy, which leads to a vertical transport of magnetic flux, and thus limits the magnetic field strength.

1 Introduction

When I began my graduate studies in astronomy, I was uncertain concerning what I wanted to work on, but I definitely knew that I wanted to specialise in a theoretical topic, and as I had always been fascinated by extreme conditions and exotic places, compact objects seemed to be a good choice. Obviously it is difficult to make a theoretical thesis project at an institute specialising in observational astronomy, so you have better work in a field where you can either get away with publishing almost anything or the theory has not yet reached a mature state. Accretion disk theory in general still fulfills the second alternative to some extent, and the magnetohydrodynamic theory of accretion disks definitely does so (even though there has been one conference specifically devoted to the topic of magnetic fields in accretion disks (Belvedere 1989)). Concerning the first alternative, I will leave it to the reader to decide whether this thesis proves its validity.

So what is an accretion disk? It appears that most people outside astronomy, and even a fair fraction of the undergraduate students, have never heard of such beasts. On the other hand practically everyone knows what a star is, and probably most people also know that a star mainly gets it energy from fusing hydrogen to helium. From this fact two corollaries can be stated:

- It seems to be a common belief within the public that hydrogen fusion is the most efficient source of energy, which is the motivation for the huge financial investments in fusion research.
- Most people, if they think about it, will come to the conclusion that all the light that we see from the universe in some way originates from thermonuclear fusion.

Neither of these corollaries are true, and that is the story of accretion

Obviously we are also able to get energy from matter falling through a gravitational field, but we know that potentially we are able to get more energy from a certain amount of water by running it through a fusion reactor than through a hydroelectric powerstation. However the energy output from the hydroelectric powerstation can theoretically be increased by letting the water fall down 500 metres instead of 50 metres. One can then ask how far the water has to fall, in order that the kinetic energy exceeds what can be derived from nuclear fusion. It turns out that this is not possible for a power station located on the Earth or the Sun, and not even if it is relocated to a white dwarf, but it is actually possible on a neutron star or a black hole to derive more energy from infalling, accreting, matter, than from nuclear fusion (cf. Frank et al. 1992, Ch. 1.1).

Thus accretion is an attractive explanation for the source of energy in extremely energetic objects, like X-ray binaries and active galactic nuclei, but also for less energetic objects such as cataclysmic variables. The prototype object can be considered to be a dwarf nova, because they are the most well-observed and frequent ones. Dwarf novae are binaries consisting of a low-mass main-sequence star and a white dwarf. Because the main-sequence star is filling its Roche lobe, matter is flowing over into the Roche lobe of the white dwarf. However because of the orbital motion of the secondary, this material possesses a *too* high angular momentum to be able to fall directly onto the white dwarf, and instead it forms a disk, an accretion disk, around the white dwarf. The standard theory of accretion disks was developed in the early 70s, and reached a certain stage of maturity with the work of Shakura & Sunyaev (1973). In the standard model the disk is considered to be geometrically thin and optically thick. A detailed description of accretion disk theory will be given in Sect. 2.

Around the time I was born, Steenbeck, Krause, & Rädler (1966) laid the foundations for the theory of mean-field dynamos. From the beginning the aim was to explain the solar magnetic field and the sunspot cycle. The central idea is that one can split the velocity and magnetic field into two parts, a mean field and a fluctuating field. The production and destruction of the mean magnetic field is then described by the correlation between the fluctuating fields. Physically two different mechanisms participate in the production of the mean field. Differential rotation produces a *toroidal field* by winding up the field lines, and convective turbulence, through its helicity, creates both a toroidal and a poloidal field. Sect. 3 is dedicated to an overview of magnetohydrodynamics in general, and in particular dynamo theory.

These two threads are then brought together in Sect. 4, where I discuss dynamo action in accretion disks, and how the magnetic field affects the disk. In this context I will e[illegible]ally address how the magnetic field can make the disk turbulent, and be responsible for the transport of angular momentum. I will also spend some time pointing out which may be the most interesting paths to continue along in the future. Finally in Sect. 5 I will present the journal papers included in this thesis. The rest of this introduction will present some of the more typical classes of accreting objects, and also the conventions used in this thesis, and more generally in astronomy.

1.1 Cataclysmic variables

These are probably the most well-studied accretion disk-laboratories. One reason is that they are quite common. The typical cataclysmic variable consists of a late-type dwarf and a white dwarf. The late-type star fills its Roche lobe, and thus matter streams over towards the white dwarf. In general there will form accretion disks in these systems, but there is at least one subclass, where no accretion disks form. These are the so-called AM Hers, where the white dwarf is strongly magnetized. The magnetic field will then control the overflowing matter, which will be channelled into a narrow accretion column. The DQ Hers are commonly believed to be an intermediate case with weaker magnetic fields and accretion disks, but both properties have been questioned (see King 1986; Norton et al. 1992).

Also the non-magnetic cataclysmic variables can be divided into several subclasses, dwarf novae, novalike variables, recurrent novae, and novae. The most spectacular subclass is the novae, which have each undergone one outburst, during which the object was increasing in brightness with approximately 10 magnitudes (for the non-astronomers, for some time the star was 10 000 times brighter than it used to be). These outbursts are believed to be thermonuclear explosions in the matter that has gathered on the surface of the white dwarf as a result of the accretion. Because the nova explosion is only indirectly connected to the accretion process, I will not spend more words on it. The recurrent novae undergo weaker outbursts with an interval of some decades, and I will not discuss them either. The novalike variables look similar to novae, that are not in the outburst state. Finally the dwarf novae undergo considerably smaller outbursts, around 5 magnitudes, with intervals of weeks or a month. These outbursts are due to instabilities in the accretion disk, and they are our best source of information regarding the mechanism being responsible for the transport of angular momentum in the disk. They have been intensely studied both photometrically and spectroscopically, in some cases leading to the production of images of the accretion disk (Horne 1991). The main reason that dwarf novae and novae behave differently appears to be that they are accreting at different rates. A dwarf nova is accreting at a low rate, where the accretion disk is intrinsically unstable, whereas novae and novalike variables are accreting at a higher rate where the disks are stable.

1.2 X-ray binaries

In X-ray binaries the compact accreting object is a neutron star, and as a neutron star is somewhat more massive than a white dwarf, and much smaller (you might be able to fit in a neutron star between Malmö and Lund), much more energy is released. There are two main classes of X-ray binaries depending on the mass of the secondary, high mass X-ray binaries with an early type supergiant as the secondary, and low mass X-ray binaries with a late-type dwarf as the secondary. These two classes behave in radically different ways.

The high-mass X-ray binaries are young systems, which are in general not driven by Roche-lobe overflow. The strong stellar wind from the secondary is sufficient. Probably there does not form any accretion disk in connection with wind-fed accretion, although some numerical simulations have yielded temporary disks. As the neutron star has got an extremely strong magnetic field, on the order of 10^8 T, the accreted matter is channelled to the magnetic poles of the neutron star. Thus the X-ray emission is only observed when a magnetic pole faces towards us, such an object is called an X-ray pulsar. Many high-mass X-ray binaries are observed as X-ray pulsars.

The low-mass X-ray binaries on the other hand are old systems fed by Roche-lobe overflow leading to the formation of accretion disks. Because neutron stars in these systems have much weaker magnetic field, X-ray pulsars are rare, but on the other hand many systems undergo X-ray bursts. The X-ray bursts are the result of unstable nuclear fusion in the accreted material. This instability can only occur in a weak magnetic field. The European X-ray satellite EXOSAT lead to considerable progress in the study of low-mass X-ray binaries in the 80s. The most notable discovery may be the quasi-periodic oscillations observed from the so-called Z-sources (van der Klis et al. 1985). At least some of these oscillations may be explained by the interaction between the weak magnetosphere of the neutron star and the accretion disk.

1.3 Protostellar objects

Also protostars appear to have accretion disks. This explains for instance why all the planets in our solar system have co-planar orbits. However the details of the physics of these disks are still not known, although some researchers have suggested that the variability in T Tauri-stars can be explained by disk instabilities similar to the ones in dwarf novae, but occuring on longer time scales because the protostellar disks are much larger.

1.4 Active galactic nuclei

Some galaxies have abnormally bright centers, and in some cases there are observed jets emanating from these regions. It is hard to find a mechanism to explain how a sufficient amount of energy can be released in a region as small as the galactic nucleus must be. The least controversial suggestion is that the source of energy is matter accreting onto a supermassive black hole in the center of the galaxy. However there are no direct observations of accretion disks in active galactic nuclei, and the spectral distribution of the emitted radiation do not fit well to the spectrum of an accretion disk. Thus the common belief says that there is a black hole and an accretion disk, but the radiation from the black hole is reprocessed by surrounding matter. It also appears likely that the jet is emanating from the accretion disk, and perhaps it is accelerated by the magnetic field of the accretion disk.

1.5 Units and spelling

Nowadays the internationally accepted system for units is SI, and it has also been accepted by the astronomical community (IAU XXth General Assembly, Baltimore 1988, Resolution A3: Improvement of Publications). However many astronomers, especially theorists, still use the old cgs-units. There are two major drawbacks with using cgs-units:

- cgs is not one consistent system. There are at least three different varities, depending on the way Maxwell's equations are treated. One of the most popular is the Gaussian system, where the electric charge is redefined in order to eliminate the need for the permittivity of free space, ϵ_0 and the permeability of free space, μ_0. Gaussian units are however less popular in magnetohydrodynamics, where electromagnetic waves do not occur, because of the approximations having been done, and thus there is no natural place for the speed of light.

- Most undergraduate education use SI-units, and the change to cgs-units can be uncomfortable for the beginning graduate student, as he has no feel for the magnitude of the units, and familiar equations assume new shapes.

- Personally I also find it ridiculous to measure stars in g and cm. It is bad enough to measure them in m and kg.

Because I am particularly stubborn, I still insist on using SI-units, and will use them throughout the thesis.

There are many things that can be confusing for the person entering the physics of accretion disks. The first thing is the spelling of disk, two alternatives are in common use, disc and disk. It becomes even more confusing as some author seem to change habit from paper to paper. Without pretending to have investigated a statistically significant nor bias-free sample, I have got the impression that disc is most popular in the British journal Monthly Notices of the Royal Astronomical Society, and should thus be the spelling used in British English. As I personally prefer to use British English I should thus use disc, but in the keywords for Astronomy & Astrophysics (they are the same for Monthly Notices and Astrophysical Journal) only the form disk is used, thus I have chosen to also use disk.

Warning! The following chapters of this thesis will be filled with equations. Thus readers, who are allergic to long mathematical expressions should skip to the acknowledgement, and rather read my popular-level presentation in Astronomisk Årsbok 1994 (eds. A. Larsson & J. Schildt), which is in Swedish.

2 Accretion disk theory

2.1 Hydrodynamic flows in a cylindrical geometry

The fluid dynamical equations for a non-rotating system can be written as

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0, \tag{1}$$

$$\rho\left(\frac{\partial\mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p - \rho\nabla\Phi + 2\rho\nu\left(\nabla^2\mathbf{v} + \frac{1}{3}\nabla(\nabla\cdot\mathbf{v})\right), \tag{2}$$

$$\rho T\left(\frac{\partial s}{\partial t} + \mathbf{v}\cdot\nabla s\right) = -\nabla\cdot\vec{q} + \nabla\cdot\vec{q_r} + \nu\rho\left(\frac{1}{2}S^2 - \frac{2}{3}(\nabla\cdot\mathbf{v})^2\right), \tag{3}$$

where ρ is the density, $\mathbf{v}$ the fluid velocity, p the pressure, Φ the gravitational potential, ν the kinematic viscosity, T the temperature, s the entropy, $\vec{q}$ and $\vec{q_r}$ the conductive and radiative heat fluxes, respectively, $S_{ij} = v_{i,j} - v_{j,i}$ the rate of strain tensor, and for simplicity we assume the dynamic viscosity $\rho\nu$ to be constant (e.g. Priest 1982).

For an accretion disk we will assume that the gravitational potential is given entirely by a central gravitating star, $\Phi = -\frac{GM}{r}$, where G is the gravitational constant and r the distance from the star. In order to reformulate the equations we will use cylindrical coordinates (ϖ, ϕ, z) and also assume axisymmetry and a stationary state.

$$\frac{1}{\varpi}\frac{\partial}{\partial\varpi}(\varpi\rho v_\varpi) + \frac{\partial}{\partial z}(\rho v_z) = 0, \tag{4}$$

$$\rho\left(v_\varpi\frac{\partial v_\varpi}{\partial\varpi} + v_z\frac{\partial v_\varpi}{\partial z}\right) = -\frac{\partial p}{\partial\varpi} - \frac{GM\rho\varpi}{(\varpi^2+z^2)^{3/2}} + F_{\nu,\varpi} \tag{5}$$

$$\rho\left(v_\varpi\frac{\partial l}{\partial\varpi} - \frac{v_\varpi l}{\varpi} + v_z\frac{\partial l}{\partial z}\right) = \varpi F_{\nu,\phi} \tag{6}$$

$$\rho\left(v_\varpi\frac{\partial v_z}{\partial\varpi} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \frac{GM\rho z}{(\varpi^2+z^2)^{3/2}} + F_{\nu,z} \tag{7}$$

$$\rho T\left(v_\varpi\frac{\partial s}{\partial\varpi} + v_z\frac{\partial s}{\partial z}\right) = -\frac{1}{\varpi}\frac{\partial}{\partial\varpi}\left[\varpi\left(q_\varpi + q_{r,\varpi}\right)\right] - \frac{\partial}{\partial z}(q_z + q_{r,z}) + \nu\rho\left(\frac{1}{2}S^2 - \frac{2}{3}(\nabla\cdot\mathbf{v})^2\right), \tag{8}$$

where $\mathbf{F}_\nu$ is the viscous force. We have expressed the angular velocity in terms of the specific angular momentum $l = v_\phi\varpi$, and multiplied the azimuthal component of the equation of motion with ϖ.

2.2 Steady accretion disks

It is difficult to find a self-consistent solution to the whole set of hydrodynamic equations above. Most research has therefore concentrated on finding different classes of restricted solutions to the complete set of equations (see e.g. Frank et. al. 1992. Ch. 10), or simplify the problem by first integrating the equations in the vertical direction and then consider geometrically thin disks with a parameterized description of the viscous stress. As the restrictions usually include neglecting meridional circulation and accretion it is clear that they do not provide a truthful description of real accretion disks.

2.2.1 Thick accretion disks

The interest for thick accretion disks dwindled in the mid 80s, as it was demonstrated that they are unstable to non-axisymmetric perturbations (e.g. Papaloizou & Pringle 1984). Nevertheless *they posses some unique properties that cannot occur in thin accretion disks.* **Two important** points should be mentioned here. First of all it is possible for a thick accretion disk to emit more energy than given by the Eddington limit (the luminosity at which a spherical star would blow up because of its radiation pressure). Secondly it turns out that if we prescribe the angular momentum distribution on the surface of the disk, we are able to calculate the luminosity without any further knowledge of the interior of the accretion disk (Abramowicz et al. 1980).

2.2.2 Slim accretion disks

The problem is considerably simplified by integrating the hydrodynamic equations in the vertical direction. which leads to the so-called slim disk model (Abramowicz et al. 1988). The resulting equations are similar to the thin disk equations discussed below, but take into account deviations from Keplerian rotation, the radial pressure gradient, the radial advection of heat, and the inertial term $v_r \partial v_r / \partial r$ in the equation of motion. Thus they are able to truthfully model the transsonic flow in the inner part of the accretion disk.

The model has mainly been applied on black hole accretion. In this context there appears an interesting feature, the innermost part of the disk will be cooled by the flow through the inner cusp of the accretion disk. This stabilises the disk against some of the instabilities that occur for the standard thin accretion disk.

2.2.3 Thin accretion disks

It is possible to go from the slim disk model to the standard thin disk model (Shakura & Sunyaev 1973), by neglecting the radial derivatives compared to other terms, and imposing a Keplerian rotation, however a more heuristic derivation is more enlightening physically. Let us start with considering a thin cylindrical shell of thickness dr situated at the radius r in the accretion disk. Matter is flowing into this shell at the rate $\dot{M}$, and is simultaneously leaving the shell with the velocity $-v_r$ (the velocity is taken positive in the outwards direction), thus the total amount of matter lost per time unit is $(2\pi r)(2H)(-v_r)\rho$, where H is the half-thickness of the disk. Conservation of mass requires

$$\dot{M} = -2\pi r v_r \sigma, \tag{9}$$

where $\sigma = 2H\rho$ is the mass per area unit.

In the same way we can find the inflow of angular momentum. The angular momentum in the shell is $r^2\Omega 2\pi r\sigma$, and this is advected inwards with the velocity $-v_r$. To conserve angular momentum a torque, $\tau(r)$, must act on the shell

$$r^2\Omega 2\pi r\sigma(-v_r) = \tau(r). \tag{10}$$

Currently there is no satisfactory theoretical understanding of the torque, however we will present a phenomenological model below. This limits the usefulness of accretion disk theory, and τ can be said to be the holy grail of accretion disk theory.

As matter is falling inwards it loses potential energy. half of this is transformed to kinetic energy and the other half gets dissipated. The dissipation leads to heating of the disk. which finally radiates away the energy. The dissipated potential energy in the shell is

$$\frac{1}{2}\frac{GM\dot{M}}{r^2}\mathrm{d}r. \tag{11}$$

We will assume that the disk is optically thick. so that it radiates like a black body, and that all the energy is radiated locally. i e there is no radial transport of thermal energy within the disk. With these assumptions the radiated energy is

$$2(2\pi r)\,\mathrm{d}r\frac{aT^4}{3}, \tag{12}$$

where a is the radiation constant. Thus we get for the temperature distribution in the disk

$$T = \left(\frac{3G}{8\pi a}\frac{M\dot{M}}{r}\right)^{1/4}, \tag{13}$$

which is independent of the torque τ. as long as the disk is optically thick.

In the vertical direction the pressure gradient and gravitational force of the central body balances each other. As an order of magnitude estimate this can be written

$$\frac{p}{H} = \frac{GM\rho}{r^2}\frac{H}{r}. \tag{14}$$

Defining the isothermal sound speed $c_s^2 = p/\rho$ and the Keplerian velocity $v_{\mathrm{Kepl}}^2 = GM/r$, Eq. (14) can be rewritten as

$$\frac{c_s}{v_{\mathrm{Kepl}}} = \frac{H}{r}. \tag{15}$$

As the disk is optically thick and it can be shown that convection is unimportant (Shakura et al. 1978), we can use the diffusion approximation for treating the vertical energy transport

$$F(z) = -\frac{16\sigma_{\mathrm{SB}}T^3}{3\kappa_{\mathrm{R}}\rho}\frac{\partial T}{\partial z}, \tag{16}$$

where σ_{SB} is the Stefan-Boltzmann constant.

Finally to get a complete theory for accretion disks we need a description of the torque. It is most natural to assume that it is due to viscous friction. However it can easily be shown that molecular viscosity is insufficient (e.g. Frank et al. 1992), but as the accretion disk has an extremely high Reynolds number, one may expect turbulence to appear. Shakura & Sunyaev (1973) suggested a simple phenomenological model based upon this assumption. To construct a viscosity coefficient we need a length scale, l, and a velocity, v. It appears unlikely that the turbulent eddies can be larger than the thickness of the disk. or that the turbulent velocity can exceed the sound speed. Thus we write $\nu_t = \alpha_{\mathrm{SS}} H c_s$, where α_{SS} is a free parameter that should be smaller than unity. We can then express the torque as

$$\tau(r) = -\nu\sigma\frac{\mathrm{d}\Omega}{\mathrm{d}r}. \tag{17}$$

All that is missing now is an equation of state and a description of the opacity. As these relationships are practically identical to what is used in stellar structure calculations, I will not present them here, and neither will I present the solutions to the equations, as they are easily accessible in the literature (e.g. Frank et al. 1992).

2.2.4 Variations on a given theme

The thin disk model can be modified in a few ways without breaking the assumption of a geometrically thin disk. First of all, as the viscosity prescription is given it is proportional to the pressure in the disk, however it is not clear whether one should consider the total pressure or the gas pressure. This is of significance for the stability properties of the accretion disk (Sakimoto & Coroniti 1981).

Another variation is to make the disk optically thin in the vertical direction. The cooling of the disk becomes less efficient then, so the disk has to be hotter in order to emit the same amount of energy. If the disk becomes sufficiently hot the Coulomb coupling between electrons and protons will be to weak to keep them at the same temperature. Because the electrons lose energy more readily the electron temperature will be much lower ($\sim 10^9$ K) than the ion temperature (up to $\sim 10^{11}$ K) (e.g. Shapiro et al. 1976). The high temperatures have lead to speculations that electron-positron pairs will be produced in the disk (e.g. Kusunose & Mineshige 1991, Björnsson & Svensson 1991).

2.3 Instabilities

I have now presented the main theoretical tools for finding equilibrium solutions to acc⋯ion disks, but so far we have not considered the stability of the resulting solutions. This requires including the time derivatives in the equations (Lightman 1974a). From a local analysis it is possible to find two unstable modes for a thin accretion disk, a viscous and a thermal one (Shakura & Sunyaev 1976).

The time evolution of the surface density can be described by a diffusion equation. If the pressure is given by radiation pressure and the disk opacity by electron scattering the diffusion coefficient will be negative. The result is a secular instability leading to clumping in the disk. This is the viscous instability.

The thermal instability is caused by a feedback mechanism between the temperature dependences of the emissivity and viscosity. The instability appears when the disk is cooled by bremsstrahlung radiation (Pringle et al. 1973), the radiated power per unit volume $\propto \rho^2 T^{1/2}$. A pressure supported disk in this scenario will get $H \propto T^{1/2}$ and $\rho \propto T^{-1/2}$. Thus we see that the radiated power decreases when the temperature increases, which leads to a further increase in temperature and so on.

For slim disks there appear two additional acoustic modes (Wallinder 1991), but then the viscous and thermal instabilities can be stabilised by the radial advection.

The meaning of the instabilities is somewhat unclear. Some authors consider the instabilities to be indications of problems in the theory (e.g. Pringle 1981), whereas other authors identify the instabilities with physical processes leading to observable time variability in accreting systems (e.g. Wallinder 1991).

2.4 Numerical simulations

As should be clear from the foregoing presentation, there are effects that usually are not taken into account in the standard model, because it is difficult to treat them analytically. However such effects can in general be treated numerically. The first to perform numerical simulations of accretion disks were Hawley et al. (1984a,b), although the main emphasis in their early papers was not so much on the astrophysical results as on the testing and calibration of the numerical method. The work has been continued later by Hawley (e.g. 1990, 1991) investigating the nonlinear evolution of instabilities in thick accretion disks and tori. Similar work has also been carried out by Eggum et al. (1985, 1988) for disks accreting close to the Eddington limit.

Most of these simulations have been limited to geometrically thick disks, as it is more difficult to accomodate for thin disks, where the vertical length scale is much smaller than the radial extent. Another way to think of the difficulties is to remember that the flow is a combination of a supersonic azimuthal flow and a subsonic radial inflow. A possible way round this limitation is to carry out the simulations within the thin disk formalism, thus only evolving quantities integrated over the vertical coordinate. Apart from allowing the study of thinner disks it also saves computational time as it decreases the number of dimensions with one. Such models have been used for simulating e.g. dwarf nova outbursts (e.g. Meyer & Meyer-Hofmeister 1984, Papaloizou et al. 1983, Mineshige & Osaki 1983, Cannizzo et al. 1982).

A different approach to numerically modelling thin accretion disks without the aim of reproducing the observed behaviour of accretion disks, is to cut out a small slab, usually of the size of a few vertical scale heights, from the accretion disk. Even though this approach is only remotely connected to observational properties it may yield knowledge of the basic physics of accretion disks. A typical example is the program started by Kley, Lin and Papaloizou employing a two-dimensional code simulating the behaviour in a meridional cut (Kley & Lin 1992, Kley et al. 1993a,b). They use the numerical code to investigate in detail hydrodynamical phenomena like meridional circulation, viscous oscillations and angular momentum transport through convection. Their work demonstrates the strength of combining numerical modelling with a thorough knowledge of hydrodynamics. Two dimensional models have also been used by Ryu & Goodman (1994) to investigate the consequences of tidal waves on the local hydrodynamics of the accretion disk, and Hawley & Balbus (1991, 1992) to investigate the nonlinear evolution of magnetic shearing instabilities, which is something I will come back to later.

All of the numerical work above is based upon a common conceptual frame, in which the accretion disk is considered to be a steady or almost steady object like a star. These methods are most closely related to the numerical simulations of convection in late-type stars (e.g. Nordlund et al. 1992). However it is equally possible to think of the accretion disk as an example of a hydrodynamic flow pattern from the secondary on to the accreting object, and treat it with methods more closely related to the simulations of flow in turbines or around aircraft wings. A severe complication is that the size of the primary object is in general so much smaller than the size of binary that it is not possible to resolve the flow close to the primary even with an inhomogeneous mesh. Nevertheless attempts at simulating both accretion through Roche-lobe overflow and wind accretion has been published (e.g. Blondin et al. 1990; Sawada et al. 1986, Sawada & Matsuda 1992).

A popular method for calculating the flow is smooth particle hydrodynamics, where the fluid motion is approximated by a number of particles moving around in the gravitational potential. The particles are allowed to interact in a way that approximates a streaming fluid. A significant advantage with this method is that it does not require a grid, however instead the resolution of the method is limited by the number of particles used. The method has been reviewed by Monaghan (1992). For accretion the method has been used by several groups both for Roche-lobe overflow driven accretion in cataclysmic variables (e.g. Lanzafame et al. 1992, 1993), and wind-driven accretion (Theuns & Jorissen 1993).

3 Magnetohydrodynamics

It is straightforward to generalise the hydrodynamic equations to the case of an electrically conducting plasma coupled to a magnetic field. We need to add the Lorentz force, $\mathbf{j}\times\mathbf{B}$, where $\mathbf{B}$ is the magnetic field, $\mathbf{j} = \nabla \times \mathbf{B}/\mu_0$ the current, and μ_0 the permeability of free space, to the momentum equation, and the Joule heating j^2/σ, where σ is the conductivity, to the energy equation. A new equation describing the time evolution of the magnetic field, the induction equation, also turns up

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B}), \tag{18}$$

where η is the magnetic diffusivity.

Naturally it is not easier to find a self-consistent solution for a magnetic accretion disk, but as for a star the effects of the magnetic field can be of interest even if it does not change the basic structure of the disk or star. In this section I will present some types of magnetohydrodynamic modelling, which can be of use for addressing some problems.

3.1 Magnetic flux tubes

It has been found in numerical simulations that the magnetic field tend to form coherent structures (e.g. Brandenburg 1994), typically flux tubes. Thus an obvious way to simplify the problem is to consider the motion of the magnetic flux tubes in an ambient medium. This approach was pioneered by Spruit (1981a,b) for a thin flux tube, which means that we only consider quantities averaged over the cross section of the tube.

First of all because a magnetic field is divergence-free, the magnetic flux along the tube is conserved

$$BA = \text{const}, \tag{19}$$

where B is the magnetic field along the flux tube and A the cross section of the flux tube. For simplicity we will assume that the magnetic flux tube is not twisted, so that B is the total magnetic field. The differential form of the flux conservation is

$$\frac{\mathrm{d}B}{B} + \frac{\mathrm{d}A}{A} = 0. \tag{20}$$

In the same spirit the conservation of mass can be expressed as

$$\frac{\mathrm{d}\rho}{\rho} + \frac{\mathrm{d}A}{A} + \frac{\mathrm{d}l}{l} = 0, \tag{21}$$

where l denotes the length of the flux tube, when the plasma is frozen into the magnetic field. It is reasonable to also assume that the flux tube is in pressure equilibrium with its surroundings

$$\frac{\rho_e k T_e}{\mu_e m_H} + \frac{1}{3} a T_e = \frac{\rho_i k T_i}{\mu_i m_H} + \frac{1}{3} a T_i + \frac{B^2}{2\mu_0}, \tag{22}$$

where subscripts e and i denote quantities measured exterior to and interior to the flux tube, μ_e and μ_i are, respectively, the molecular weight outside and inside the flux tube. We need to complement this local description of the flux tube with an expression describing the exchange of heat between the flux tube and the surrounding medium. The simplest assumption is to consider the flux tube as being in thermal equilibrium with its environment, then $T_e = T_i$, and it is also reasonable to put $\mu_e = \mu_i$. Then we see that the radiation pressure drops out, and $\rho_i < \rho_e$. This introduces the important concept of magnetic buoyancy, a magnetic region subject only to the gravitational force will tend to rise.

We need to find an equation of motion for the flux tube too. The forces acting on the flux tube will be the buoyancy force $(\rho_i - \rho_e)\mathbf{g}$, a viscous force, and the Lorentz force. The Lorentz force can be split into two components, the magnetic pressure, which is included in the local pressure balance above, and tension. The tension force in its turn can be divided in two components, along and perpendicular to the field lines, (Priest 1982)

$$\frac{B}{\mu_0}\frac{\mathrm{d}}{\mathrm{d}s}(B\hat{\mathbf{s}}) = \frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{B^2}{2\mu_0}\right)\hat{\mathbf{s}} + \frac{B^2}{\mu_0}\frac{\hat{\mathbf{n}}}{R_c}, \tag{23}$$

where s is the path lenght along the flux tube, R_c the radius of curvature of the field lines, and $\hat{\mathbf{s}}$ and $\hat{\mathbf{n}}$ are the unit vectors along the flux tube and the principal normal to it, respectively. A more stringent discussion of the dynamics of flux tubes is given in Ferriz-Mas & Schüssler (1993).

The main advantage of the thin flux tube formalism is that it is computationally inexpensive. For a numerical computation the cost per time step will be proportional to the number of grid points along the flux tube, essentially inversely proportional to the grid spacing, which should be compared to a conventional three-dimensional simulations, where the cost per time step is inversely proportional to the cube of the grid spacing. Thus the method has been used for investigating the migration of the solar magnetic field (e.g. Moreno Insertis et al. 1992). An obvious limitation is that the formalism in itself does not account for the interaction between flux tubes, however this problem has also been investigated (Bogdan 1985). On the other hand the formalism loses some of its appeal if we include a large number of flux tubes and their mutual interactions, as the computational cost will increase at least in proportion to the number of flux tubes, and even faster if interactions between the flux tubes become important.

3.2 Mean-field dynamos

A completely different approach to magnetohydrodynamics appears in dynamo theory, the theory of generation of magnetic fields in a turbulent medium. The main advantage with mean-field dynamos is that they allow one to calculate the overall structure of the global field without any detailed knowledge of the small-scale turbulence. As it is beyond the capabilities of current computers to simultaneously resolve the local turbulence, and provide a faithful representation of the global magnetic field of an astrophysical object, we are forced to employ mean-field dynamos or some analogous model for describing the generation of the magnetic field.

The basis for the mean-field dynamo is to split the magnetic and velocity fields in mean and fluctuating parts

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{b}, \ \mathbf{v} = \mathbf{V}_0 + \mathbf{v}'. \tag{24}$$

Note that strictly speaking these means are to be taken as ensemble averages. Averaging the induction equation we get

$$\frac{\partial \mathbf{B}_0}{\partial t} = \nabla \times \left(\mathbf{V}_0 - \eta \nabla \times \mathbf{B}_0 + \overline{\mathbf{v}' \times \mathbf{b}}\right), \tag{25}$$

where $\overline{\mathbf{v}' \times \mathbf{b}}$ represents the turbulent electromotive force. In the simplest physically realistic case the electromotive force can be written

$$\overline{\mathbf{v}' \times \mathbf{b}} = \alpha \mathbf{B}_0 - \beta \nabla \times \mathbf{B}_0, \tag{26}$$

where α is a pseudo-scalar, and β a scalar (Krause & Rädler 1980). We note now that the diffusive term in Eq. (25) and the last term in Eq. (26) can be merged. The coefficient of this term will be denoted η_t, the turbulent diffusivity, and is in general much greater than η. Thus we get the archetypical equation describing a mean-field dynamo

$$\frac{\partial \mathbf{B}_0}{\partial t} = \nabla \times (\mathbf{V}_0 \times \mathbf{B}_0 + \alpha \mathbf{B} - \eta_t \nabla \times \mathbf{B}_0). \tag{27}$$

There are a number of technical complications, which we have skipped so far. First of all in general α and β should be second and third rank tensors, respectively. This makes the computations somewhat longer, but is not of any threat to the theory. A serious threat, however, is that the right-hand side of Eq. (26) assumes that the higher order statistical moments of the fluctuations are negligible, but this has never been proved. A third complication is that the calculated magnetic field is the ensemble average, whereas what in practice is observed is the magnetic field averaged over some small length scale. These averages will not in general coincide, albeit there are happy circumstances under which they do.

In the simplest case α and η_t are constants, independent of the magnetic field, but this is unrealistic. If the generated magnetic field becomes sufficiently strong, it will modify the turbulence, decreasing the α-effect. This renders the induction equation nonlinear, which can cause interesting and unexpected effects. During the last decade the interest in chaotic solutions appearing in nonlinear equations has grown. It has been demonstrated that chaotic solutions appear for reduced models, models consisting of a system of ordinary differential equations, of the solar dynamo (e.g. Jones et al. 1985), however no examples are known of two-dimensional stellar dynamos displaying chaos. Recently chaotic solutions of the induction equation have been found for Keplerian disks and tori (Brooke & Moss 1994; Torkelsson & Brandenburg 1994b,c). The accepted way for identifying a chaotic solution is to demonstrate that the largest Lyapunov exponent λ is positive. One calculates the Lyapunov exponent by taking the magnetic field $\mathbf{B}_0$ at an arbitrary time, and generates a new field by adding a small disturbance to the magnetic field. The time evolution for the two fields are then. If the system is chaotic they will diverge exponentially like $e^{\lambda t}$. It is possible to define a spectrum of Lyapunov exponents describing the growth rates in orthogonal directions of the solution space, but the higher ones are more difficult to calculate in practice.

Recently the mean-field dynamos have been challenged by Cattaneo & Vainshtein (1991), who find that even a weak magnetic field will affect and decrease the turbulent diffusivity. Because of this the complicated magnetic field on the small scales cannot be dissipated, and thus the dynamo will rapidly saturate at field strengths that are much smaller than we in reality observe on the Sun.

4 Magnetohydrodynamics of accretion disks

4.1 Magnetohydrodynamic instabilities

Naively one might find it natural that turbulence will appear in accretion disks, because they involve a strong shear flow, but matters are not that simple. The differentially rotating fluid in an accretion disk is analogous to a Couette flow with somewhat unusual boundary conditions. According to Rayleigh's criterion Couette flow is linearly stable against axisymmetric disturbances as long as the angular momentum increases outwards (e.g. Landau & Lifshitz 1987). An elementary calculation shows that for Keplerian rotation the angular momentum grows as the square root of the distance from the rotational axis. and thus the flow is linearly stable against the production of turbulence. This is a fundamental problem not only for dynamo action in accretion disks, but for accretion disk theory in general, as it implies that we do not know what is causing the turbulent viscosity, or the magnetic torque that may make the turbulent viscosity unneccesary.

An obvious suggestion for the source of turbulence is convection in the accretion disk, but it has been demonstrated that in most cases, with the likely exception of protoplanetary nebulae, convection is not important for the transport of heat (Shakura et al. 1978). However there is a loophole in this argument, that the convection is not important for the thermal transport, does not in itself imply that it is unimportant for the turbulent viscosity or the dynamo action. This topic may deserve some more investigation.

What options are then left for producing turbulence inside an accretion disk? Dubrulle (1992) suggested that even if the disk is linearly stable to hydrodynamic instabilities, it might be non-linearly unstable. In this way she seems to be able to produce some turbulent viscosity in an accretion disk, but not enough for explaining for instance the dwarf nova outbursts. On the other hand perhaps it can drive a dynamo exerting a much stronger magnetic torque, but this remains to be investigated.

Perhaps it is more interesting to note that some new instabilities appear when one includes the magnetic field in the investigation. In recent years much interest has concentrated on a shearing instability appearing in weakly magnetised disks (Balbus & Hawley 1991). Actually it was not a new instability that Balbus & Hawley discovered, it was discussed already by Velikhov (1959) and Chandrasekhar (1960, 1961), but Balbus & Hawley were the first ones to discuss it in the context of accretion disks. To understand the instability, think of a fluid element being displaced radially in the accretion disk. If it is displaced inwards, it will rotate too slowly because through the magnetic field it is still connected to the more slowly rotating matter at its initial position, and thus it will continue to fall inwards. In the opposite case the fluid element will rotate too rapidly and be accelerated outwards by the centrifugal force. The nonlinear evolution of the instability has been investigated numerically in two dimensions by Hawley & Balbus (1991,1992). This work has been critisized by Knobloch (1992) and Dubrulle & Knobloch (1993) on the grounds that a local analysis can produce spurious instabilities if applied carelessly.

Another instability of potential importance is due to the magnetic buoyancy, a magnetized region is less dense than its non-magnetic surroundings. According to Archimedes' principle this region will be buoyant and try to rise through a gravitational field. Whereas the Balbus-Hawley instability is most important for weakly magnetized accretion disks, the buoyancy instability is most important for strong magnetic fields. The buoyancy instability has been investigated in more detail by Sakimoto & Coroniti (1989, for later work within the thin flux tube formalism see Schramkowski & Achterberg 1993), describing the magnetic field as consisting of thin flux tubes. They found that for some disks the magnetic field in the disk may be sufficiently strong to be responsible for the friction. Some authors have argued that the buoyancy instability will lead to rapid expulsion of the magnetic flux (e.g Galeev et al. 1979). The magnetic field may then create a corona surrounding the accretion disk. Aly & Kuijpers (1990) and Heyvaerts & Priest (1989) have argued that the magnetic interaction between the disk and the corona may give rise to a strong torque acting on the disk. This can be a candidate for the transport of angular momentum.

Vishniac et al. (1990) suggested that dynamo action is driven by inertial waves in the accretion disk (see also Vishniac & Diamond 1992, Diamond et al. 1994). The idea of wave turbulence is not entirely new, it was suggested in Zeldovich et al. (1983) in the context of accretion disks. A major weakness with this model is that it requires a mechanism to excite the inertial waves, likely candidates are tidal waves excited by the secondary in a binary, or simply the impact of accreting matter at the outer edge of the disk.

Much of the work on magnetohydrodynamic instabilities has so far been done more or less analytically, but in order to understand the development of turbulence in accretion disks, it is necessary to understand the non-linear evolution of the instabilities. Even in fairly simplified cases this requires numerical calculations, and it appears likely that numerical simulations will become important in the future. Some work of this kind has already been done (e.g. Hawley & Balbus 1991, 1992, Brandenburg et al. 1994), but the models are still crude and there is reason to expect significant progress in the near future.

4.2 Dynamo action in accretion disks

Apart from being responsible for the angular momentum transport the turbulence could also give rise to a dynamo producing a large-scale magnetic field. The first one to present a detailed model of a mean-field dynamo for an accretion disk was Pudritz (1981a,b). He found an analytical solution to the relevant equations. Like in most later works the most easily excited solution was a steady quadrupolar one. The models were later extended by Stepinski & Levy (1988, 1990b) employing numerical methods for solving the linear induction equation as an eigenvalue problem. Because of their employing a thick disk model, they found that the most easily excited mode was an oscillating dipolar mode (Stepinski & Levy 1988), however after reformulating their problem in a cylindrical geometry, they found a steady quadrupolar mode for thinner disks (Stepinski & Levy 1990b). Later Campbell (1992) have solved the eigenvalue problem of the steady, axisymmetric magnetohydrodynamic equations including an α-effect for the dynamo action, and Camenzind (1991) and Torkelsson & Brandenburg (1994a,b,c) have employed time-stepping methods to simulate the time-evolution of the dynamo generated magnetic field, which esily allows for nonlinear effects in the dynamo.

Some researchers have used fundamentally different, but related methods for studying the generation of magnetic fields. Geertsema & Achterberg (1992) employed a shell model for describing the generation of magnetic fields in an accretion disk. Shell models describe the magnetohydrodynamic turbulence in the Fourier domain invoking a cascading mechanism for describing the transfer of the field between different scales. This investigation demonstrated a rapid production of a strong magnetic field.

A different approach to dynamo action in accretion disks has been presented by Tout & Pringle (1992). Building on previous investigations of instabilities in accretion disks (see above) they assume a number of processes being responsible for generating the different components of the magnetic field, the toroidal field is generated from the radial component by shear motion in the disk, a vertical field is generated from the toroidal one by a buoyancy instability, and finally the radial field is generated from the vertical by the Balbus-Hawley instability (Balbus & Hawley 1991). In a way this model is more closely related to a small scale direct simulation of the dynamo action, but it is less convincing than a direct simulation, and strictly speaking the resulting magnetic field is still a mean-field, albeit the averaging procedure is not well-defined.

One of the main aims of these dynamo models is to demonstrate that a magnetic torque is able to take care of the angular momentum transport in an accretion disk (Stepinski & Levy 1990a, Tout & Pringle 1992). This is not a new idea, already Eardley & Lightman (1975) suggested that the accretion was driven by magnetic viscosity, and it was suggested incidentally already by Shakura & Sunyaev (1973). However there is an important conceptual difference between these early works and mean-field dynamos. The mean-field dynamo builds up a large scale magnetic field trying to achieve a global redistribution of the angular momentum, whereas most other models assume a small scale field and a local viscosity. Therefore the angular momentum transport carried out by mean-field dynamos is more closely related to the torque that the jet exerts on the accretion disk in some models of magnetically accelerated jets (e.g. Blandford & Payne 1982).

4.3 Consequences of magnetic fields in accretion disks

For a moment let us pretend that there are no problems with the generation of magnetic fields in accretion disks. The next question then would be how the field affects our observations of accretion disks.

The most well-known manifestation is in the acceleration of jets from accretion disks. There is a vast literature on this topic, and I have no intention of providing a complete list of it. For a recent review see for instance Blandford (1989). Most work on the acceleration of jets have assumed the existence of a magnetic field, without specifying its origin in detail. It appears that implicit in this assumption is the notion of magnetic flux being provided together with the accreting matter, and later in the disk only amplified through the differential rotation. This scenario has got an important problem. van Ballegooijen (1989) demonstrated that the magnetic flux will diffuse outwards faster than it is transported inwards by the accretion. There are some ways out of the obvious conclusion that the disk field will decay to zero. van Ballegooijen assumed that no magnetic flux is generated in the disk, thus his argument does not exclude fields generated by a dynamo in the disk. Spruit (1993) notes that van Ballegooijen assumes an isotropic diffusion, but in reality an accretion disk is dominated by motions in the $r-\phi$-plane, which suggests that the vertical diffusion is much weaker.

A new and exciting suggestion is the possibility of magnetic activity on accretion disks. Horne & Saar (1991) reported that the strength of the Balmer emission from accretion disks in cataclysmic variables is proportional to the Keplerian angular velocity. This bears some resemblance to stars, where some early investigations (e.g. Skumanich 1972) found that the strength of chromospheric emission lines like Ca H & K was proportional to the angular velocity of the star. The situation for stars have become somewhat more complicated and less clear since then (for a review see Hartmann & Noyes 1987), especially since theoretically one would expect the activity to depend upon the Rossby number. Nevertheless there is currently no other theory providing a satisfactory explanation for the variation in the strength of the Balmer emission lines.

Recently more circumstantial evidence for magnetic activity in accretion disks have been presented by Tout et al. (1993). Investigating archival IUE spectra of the novalike variable TT Ari, they find that it is bluer after its minimum in 1981 than before. If the magnetic viscosity model of Tout & Pringle (1992) describes the angular momentum transport in the accretion disk in the normal high state, one would expect energy to be released also in other forms than thermal radiation, thus decreasing the temperature of the inner part of the accretion disk. On the other hand a different, less efficient mechanism appears to be at work during the temporary low state, and it is not unreasonable to think that this mechanism only gives rise to thermal emission from the disk. Shortly after the return of the disk from the temporary low state, the dynamo has not recovered and thus the disk behaves as expected from the thin disk theory, implying that its inner part is hotter than usual and its emitted light bluer than usual.

As is evident from the description above of the manifestations of the magnetic field in accretion disks, most arguments at this stage involve a considerable amount of hand-waving. Significant progress in this area seems to require a close interaction between new observations, probably mainly time-resolved spectroscopy and spectropolarimetry, which to some extent can provide imaging of accretion disks, and improved theoretical work, combining detailed magnetohydrodynamic modelling and radiation transport.

5 Description of the papers included in this thesis

5.1 U. Torkelsson, Magnetic buoyancy in accretion disks

This paper is an extension of the investigation by Sakimoto & Coroniti (1989) of the effect of the buoyancy instability on the magnetic field in an accretion disk. The approach in itself is simple starting with a magnetic flux tube close to the mid-plane of the disk. This flux tube is always in pressure equilibrium with its surroundings, and initially, at least, in thermal equilibrium. Because the magnetic field contributes to the pressure the density is smaller inside the flux tube. The buoyancy forces the flux tube to move vertically. The model requires a description of the heat exchange between the flux tube and the rest of the accretion disk and knowledge of the vertical structure of the accretion disk. Sakimoto & Coroniti (1989) allowed for radiative exchange of heat, but I used two simple assumptions, the flux tube is always in thermal equilibrium with its surroundings, or it is moving adiabatically, which encloses the radiative heat exchange used by Sakimoto & Coroniti (1989). I assumed that the vertical structure of the accretion disk is either isothermal or adiabatic. If the flux tube is moving adiabatically, it will reach an equilibrium height, where both the density and the pressure is the same inside and outside the flux tube. When the accretion disk is adiabatic the flux tube will execute damped oscillations around this height. If the flux tube is in thermal equilibrium with its surroundings, it will not be able to reach an equilibrium position, but when it has reached higher regions with much smaller pressure, it will move very slowly, because the buoyancy force is weak and the viscous force can be significant on the expanded flux tube. Thus in both cases the magnetic field will tend to stay close to the surface of the accretion disk, which makes it interesting to investigate how a surrounding accretion disk corona affects the motion of the flux tube. Test calculations demonstrate that the corona tends to quench the buoyancy instability. However the presented model is still severely oversimplified in several respects, so it is not possible to draw any watertight conclusions from it.

5.2 U. Torkelsson & A. Brandenburg, Turbulent accretion disk dynamos

Using a time-stepping method originally developed for simulating stellar dynamos (Brandenburg et al. 1989) by Axel Brandenburg, David Moss & Ilkka Tuominen, I investigate the linear dynamo modes for a number of different disk models. The adaptation of the code to disk models was done by A. Brandenburg and myself, while the routine calculations afterwards were done by me. The first version of the paper was mainly written by me, and then it developed through discussions between me and Axel Brandenburg. The disk models were chosen to be similar to the ones used by Stepinski & Levy (1988, 1990b). The linear calculations in general reproduce the results of Stepinski & Levy, that is for the thick, flaring disk models of Stepinski & Levy (1988) the most easily excited mode is an oscillating dipolar, and for thinner disks it is a steady, quadrupolar for a positive α-effect. An important result is that the magnetic torque in the disk is mainly negative, especially in the innermost part of the accretion disk, thus demonstrating that it transports the angular momentum outwards (cf. Stepinski & Levy 1990a). We also included a few examples of nonlinear models, using both α-quenching and magnetic buoyancy as the nonlinearity. For α-quenching the solution is steady and almost quadrupolar for a positive α-effect, and chaotically varying for a negative α-effect. With magnetic buoyancy the nonlinear solution is steady, quadrupolar for a positive α-effect, and oscillating, dipolar for a negative α-effect. Another significant effect of the buoyancy is that more of the magnetic field will appear outside the accretion disk.

5.3 U. Torkelsson & A. Brandenburg, Chaos in accretion disk dynamos?

Continuing the work from the previous paper we started a systematic survey of the different types of solutions that turn up for accretion disk dynamos. First of all for positive C_αs it was found that the firstly excited steady, quadrupolar solution disappeared for sufficiently high dynamo numbers. After a gap, where no non-trivial solutions were found, there appeared oscillatory dipolar and quadrupolar solutions. Initially only the A0 solution was stable, but at higher dynamo numbers both solutions became stable. For $C_\alpha < 0$ there was a stable A0-solution, and an unstable S0, both oscillatory. The A0-solution passed through a bifurcation to a doubly periodic solution of mixed parity, and later also some more bifurcations through a triply periodic solution, back to a doubly periodic solution of even parity, and finally to a chaotically varying one. We proved its chaoticity by demonstrating that the first Lyapunov coefficient is positive. We also presented phase portraits and Poincaré maps of doubly periodic, triply periodic, and chaotic solutions. Like in the previous paper I carried out most of the routine computations and wrote practically everything of the first draft, and then the final version grow out of discussions between us, at which stage I managed to learn something on nonlinear dynamics and its applications in magnetohydrodynamics.

5.4 U. Torkelsson & A. Brandenburg, The many incarnations of accretion disk dynamos: mixed parities and chaos for large dynamo numbers

This paper sums up and extends our previous work on mean-field dynamos in accretion disks. The work was done more or less in the same way as in the earlier publications. By extending the previous bifurcation diagrams to higher dynamo numbers, we found chaotic solutions for both positive and negative C_αs. We proved that the positive solutions were chaotic by calculating the first Lyapunov exponent in the same way as in the previous paper. A surprising result in the view of Paper 3 is that at very high positive dynamo numbers the solution once more becomes steady and of almost positive parity, but with a small contribution from a dipolar solution, however this came not as a shock to us, because we had already found this solution in Paper 2. This time we also calculated a bifurcation diagram for magnetic buoyancy as the nonlinearity. Because of numerical problems we have not been able to continue the calculations to extremely high dynamo numbers in this case, but still we have found some interesting results. For positive dynamo numbers the first excited solution, the steady quadrupolar, now appears even below its critical dynamo number, and continues to exist in the gap that we found with α-quenching. Therefore the oscillatory dipolar solution is unstable, in accordance with the results of Krause & Meinel (1988) that only a solution bifurcating from the stable trivial solution is itself stable from the beginning. Another interesting result appears for negative dynamo numbers. The oscillatory dipolar and quadrupolar solutions coincide in the bifurcation diagram. The reason for this is clear from a plot of the magnetic field, the magnetic field is concentrated to two tori close to the upper and lower surface of the disk, and there is no magnetic field at all at the mid-plane of the disk. To go from the case of positive to negative parity can then be done by switching the sign of the magnetic field on only one side of the disk mid-plane.

5.5 Brandenburg, A., Nordlund, Å., Stein, R. F., Torkelsson, U., Simulating 3-D MHD Keplerian shear flows

This is the natural step to avoid the arbitrariness that is inherent to the mean-field dynamos. We simulate the three-dimensional magnetohydrodynamical flow in a small part of the accretion disk. For computational reasons, we were forced to assume stress-free and perfectly conducting boundaries in the radial direction and on the top and bottom of the computational domain, except for some cases where we assumed the flow to be symmetric around the mid-plane. In the azimuthal direction there are periodic boundary conditions. The initial flow inside the slab is Keplerian, and we only follow the deviations from Keplerian flow. The radial boundary conditions are a problem in the sense that they exclude accretion through the box, but this should only be minor complication, as the shear flow should in principle still be able to produce turbulence with realistic properties, at least in the middle of the box. A further limitation is that we use a Cartesian coordinate system, essentially excluding the effects of the curvature of the stream lines. This approach is allowed in the limit of a small box far from the gravitating object, but makes it impossible to investigate the inner parts of the accretion disk, or increase the size of the box to a significant fraction of the radial distance. In the future we plan to switch to cylindrical coordinates in a new, more sophisticated numerical code. This new code will also provide further advantages as it is conservative, which is not the case for the current code. An important point can be that we could not follow the evolution of the flow on a time scale as long as the thermal diffusion time scale. Thus it was not possible to investigate buoyancy instabilities, which to some extent are driven by thermal equilibration between magnetic and non-magnetic regions. Most of the calculations were done by Axel Brandenburg and Åke Nordlund with a numerical code that was originally written by Åke Nordlund and Robert Stein. Brandenburg also wrote much of the first version of the paper (note the spelling of disk), whereas I provided the accretion disk background, and later editing of the manuscript.

Acknowledgements

There is a number of organisations that I would like to thank for financial support, first of all the university of Lund for awarding me a predoctoral fellowship, but also Nordita; the International Astronomical Union: the Royal Physiographical Society, Lund; the WE-Heraeus-Stiftung, Germany. mainly for travel grants, and a number of institutes that I have visited during the course of this work for hospitality. Astrophysikalisches Institut, Potsdam; Observatory and Astrophysics Laboratory. Helsinki; Isaac Newton Institute, Cambridge, and High Altitude Observatory, Boulder, (my apologies to everyone that I may have left out by mistake). Most of the numerical calculations presented in this paper have been carried out on the Cray X-MP 4/16 (upgraded to Y-MP 4/64 during the course of this work) at the National Supercomputer Centre, Linköping.

There is of course also a number of persons I want to thank:

My parents for never interfering with my choice of career.

Ted Bjarme for careful reading of my attempts at popularising astronomy.

Glen & Tracy Saraduke for hospitality in Boulder.

David Moss for reading Paper 2 - 4, and providing linguistical advice on them.

The entire staff of Lund Observatory for a friendly atmosphere, and especially my supervisor Dainis Dravins.

Åke Nordlund for introducing me to 3D-simulations.

Finally Axel Brandenburg, who in practice acted as my advisor and guide through the jungle of dynamo theory.

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There are more things in heaven and earth, Horatio,
Than are dreamt of in your philosophy.
W. Shakespeare, *Hamlet*, Act 1, Scene 5