DYNAMICS OF FAST IONS IN TOKAMAKS

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by

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Abstract

Fast ions play a prominent role in the heating of tokamak plasmas by, e.g., neutral-beam injection, ion-cyclotron-resonance heating, and alpha-particle heating. In this thesis, a number of physical and mathematical problems concerning the dynamics of fast ions in tokamaks are addressed.

First, the motion under adiabatic perturbations is studied. The frequencies of instabilities excited in tokamaks sometimes vary slowly with time. The existence of an adiabatic invariant of particle motion in such circumstances is shown to lead to a rapid convection of particles in the radial direction. Generalized adiabatic invariants are constructed for systems where the slowly varying parameter is subjected to small, but rapidly varying, fluctuations.

Second, the onset of stochastic motion under resonant perturbations is considered. It is shown that the finite width of fast-ion drift orbits significantly affects the threshold for stochastic motion caused by magnetic field ripple or ion-cyclotron-resonance heating. Finite-orbit-width effects are also shown to reduce the strength of resonant interaction between alpha particles and internal kink modes.

Third, the diffusive motion in the stochastic regime is analysed mathematically. Monte Carlo operators for the motion on long time-scales are constructed, and the validity of the quasilinear diffusion coefficient is examined.

Finally, the effects of close ion collisions are investigated. It is demonstrated that close encounters with fast ions produce a high-energy tail in the distribution functions of impurity ions, and that close collisions between fusion-generated alpha particles give rise to a population of such particles with energies extending up to twice the birth energy.

The thesis comprises the following papers:

A. *Particle dynamics in chirped-frequency fluctuations*, C.T. Hsu, C.Z. Cheng, P. Helander, D.J. Sigmar, and R. White, Physical Review Letters **72**, 2503 (1994).

B. *Generalized adiabatic invariants in one-dimensional Hamiltonian systems*, P. Helander, M. Lisak, and V.E. Semenov, Physical Review Letters **68**, 3659 (1992).

C. *Finite orbit width effects on stochastic ripple diffusion*, L.-G. Eriksson and P. Helander, Nuclear Fusion **33**, 767 (1993).

D. *The stochastic nature of ion-cyclotron-resonance wave-particle interaction in tokamaks*, P. Helander and M. Lisak, Physics of Fluids **B4**, 1927 (1992).

E. *Effects of trapped alpha particles on internal kink modes in tokamaks*, P. Helander and M. Lisak, Journal of Plasma Physics **47**, 281 (1992).

F. *Monte Carlo operators for orbit-averaged Fokker-Planck equations*, L.-G. Eriksson and P. Helander, Physics of Plasmas **1**, 308 (1994).

G. *Simulation of nonquasilinear diffusion*, P. Helander and L. Kjellberg, Physics of Plasmas **1**, 210 (1994).

H. *Formation of hot ion populations in fusion plasmas by close collisions with fast particles*, P. Helander, M. Lisak, and D.D. Ryutov, Plasma Physics and Controlled Fusion **35**, 363 (1993).

1. INTRODUCTION

1.1 Fast ions in tokamak plasmas

Four and a half decades after the invention of the tokamak by Andrei Sakharov and Igor Tamm [1], tritium experiments have now been carried out under conditions approaching those required for thermonuclear ignition in two of the world's largest tokamaks, JET at Culham and TFTR at Princeton [2,3]. In deuterium-tritium fusion reactions, energetic alpha particles are created which contribute considerably to the heating of the plasma, and in a future fusion reactor most of the plasma heating will be accomplished in this way. In the largest present-day tokamaks, most of the heating power comes from neutral-beam injection (NBI) and ion-cyclotron-resonance heating of minority ions (ICRH).

Common to all these methods of heating is that energy is transferred to the plasma by a small population of high-energy ions. Their speed is typically much higher than that of thermal ions in the background plasma, and the energy of the fast-ion population constitutes a fairly large fraction of that stored in the entire plasma. For this reason, the behaviour of the fast ions is important for the overall plasma dynamics. Several issues are of importance:

(i) The *distribution* of the fast ions determines the heating profile, which in turn affects the transport and the temperature and density profiles of the plasma. The fast-ion distribution is also of importance for the stability of the plasma. Both the spatial distribution and the velocity distribution are of significance.

(ii) The *transport and the resulting losses* of fast ions are crucial to the energy balance. If the transport is too fast, the plasma may not be able to reach or sustain ignition; if it is too slow, helium ash may accumulate and quench the burning plasma.

(iii) The *coupling* between the fast ion-population and the bulk plasma is of significance for stability. The fast ions may stabilize modes otherwise unstable, and give rise to new instabilities. Both types of behaviour have been observed in experiments.

The present thesis addresses a number of physical and mathematical questions related to these issues. Since the fast ions rarely collide with other particles, many aspects of the physics of fast ions can be understood by considering the influence of various perturbations on their collisionless trajectories. This is the basic philosophy of the thesis and, accordingly, the single-particle dynamics of fast ions is examined in detail.

1.2 The orbits

A charged particle moving in a toroidally symmetric magnetic field of, e.g., a tokamak has three constants of motion: the energy $E = mv^2/2$, the magnetic moment $\mu = mv_\perp^2/2B$, and the canonical momentum conjugate to the toroidal angle φ, $p_\varphi = \partial L/\partial\dot{\varphi}$, where L denotes the Lagrangian. The conservation of canonical momentum and energy is exact, and follows, respectively, from axisymmetry and the fact that the tokamak magnetic field is constant in time. The magnetic moment is an adiabatic invariant, which is conserved if the magnetic field varies only slightly over the length scale of the Larmor radius.

From the three invariants of motion, the shape of particle orbits can be deduced. There are two main types of orbits. The particles either circulate freely around the torus, or they are trapped on the outside, being unable to penetrate the stronger magnetic field on the inside of the torus, as in Fig.1a.

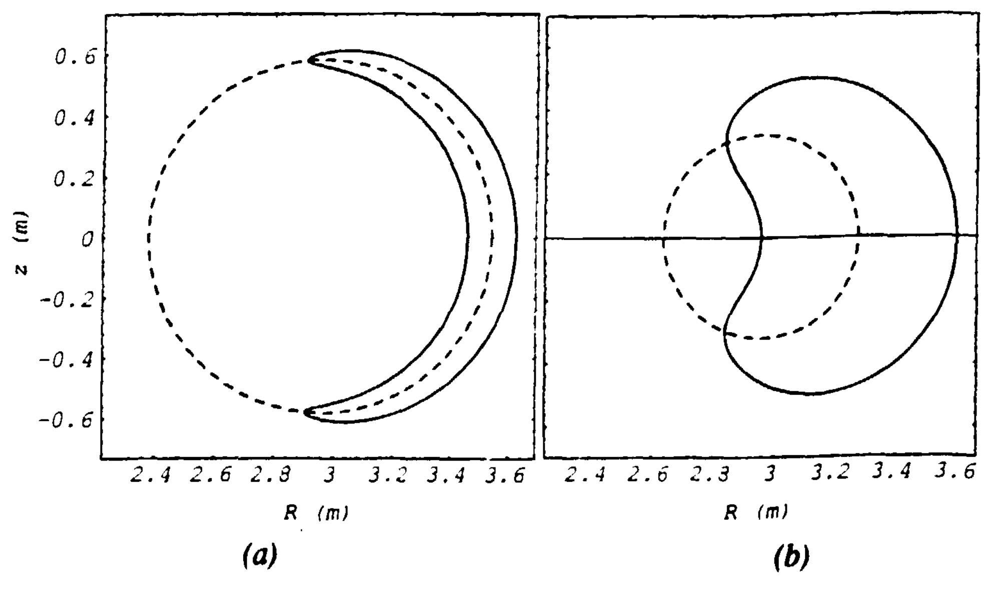

Fig. 1: Banana (a) and potato (b) orbits of trapped particles in a tokamak.

The trajectories of trapped particles are usually referred to as banana orbits. In the analysis of problems where orbit topology is important, it is frequently assumed that the banana width is small in comparison with the distance to the magnetic axis. Since the banana width is proportional to the velocity, this is not true for sufficiently fast particles. The orbits of some of these particles look more like potatoes, cf Fig.1b. This figure shows the guiding-centre trajectory a 3.5 MeV alpha particle in JET. The orbit width is about 65 cm, quite comparable to the minor radius of the machine. For highly energetic particles, finite-orbit-width effects can be assumed to be of significance, and should be taken into account.

2. ADIABATIC PERTURBATIONS

If the magnetic field of the tokamak is perturbed, the constants of motion are generally not conserved. If the perturbation is time-dependent, the conservation of energy is broken. The adiabatic invariance of the magnetic moment is violated by sufficiently small-scale or fast perturbations, and p_φ fails to be conserved if axisymmetry is broken. In these cases, the equations of motion are usually not integrable, and the exact particle motion becomes difficult to predict. However, if the perturbation is slow in comparison with the typical time-scales of particle-motion, new constants of motion, so-called adiabatic invariants, arise.

2.1 Adiabatic invariants

The concept of adiabatic invariance was introduced by A. Einstein in 1911, see e.g. Ref. [4], and has proved very useful in many areas of theoretical physics. Adiabatic invariants formed the basis of the old quantum physics, and are of fundamental importance in the theory of particle motion in slowly varying electromagnetic fields. Over the years, many works devoted to the study of adiabatic invariants have accumulated. An account of the early history of the problem can be found in Ref. [5]; more recent studies are listed and reviewed in Refs [6] and [7].

In a one-dimensional Hamiltonian system $H(q,p;\lambda(t))$ with a slowly varying parameter $\lambda(t)$, it can be shown that the action integral, defined by

$$I(H,\lambda) = \frac{1}{2\pi} \int p(q,H,\lambda)\, dq \qquad (1)$$

is an adiabatic invariant [4]-[8]. (For early proofs of this classical result, see Ehrenfest [9] and Burgers [10].) This means that if the system executes finite oscillations with some period T during which λ changes only little (of the order ε), then I remains approximately constant (to all orders ε^n) over time intervals much longer than T. Of course, this is a very common situation in physics, which is the reason for the widespread applicability of adiabatic invariants.

Adiabatic invariants also exist in systems with many degrees of freedom and for systems where the parameter λ is a slowly varying function of the space coordinates [7]. Well-known examples of this are the magnetic moment and the longitudinal invariant of charged-particle motion in a magnetic field.

2.2 Particle dynamics in a slowly varying wave

It is sometimes observed that the frequency of instabilities appearing in tokamaks varies in time. For instance, the so-called fishbone instability results in sudden bursts of magnetohydrodynamic oscillations. During each burst, the frequency of the oscillations decreases markedly [11], as illustrated in Fig. 2. Fishbone activity is often observed to lead to severe losses of fast ions. In the PBX tokamak, where fishbones were first observed, up to 40% of the NBI heating power was lost due to the ejection of fast ions by the instability [12].

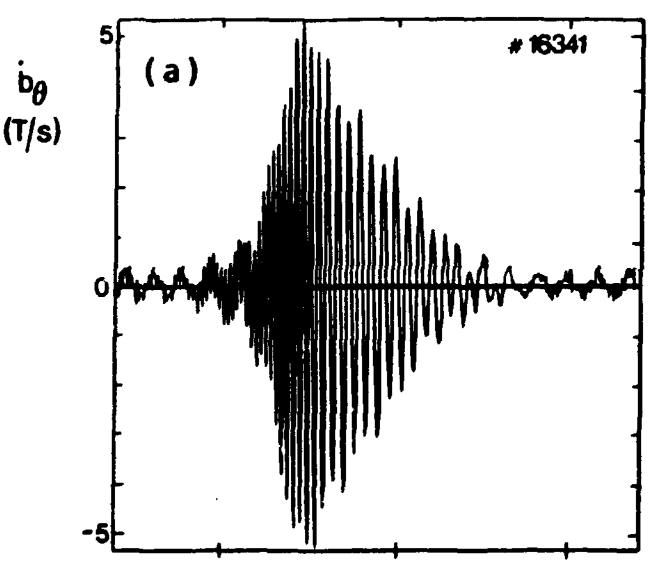

Fig. 2. A burst of the fishbone instability. From Ref. [11].

The motion of particles in a tokamak magnetic field perturbed by instabilities have been studied extensively [13-15], but generally the frequency has been taken to be independent of time. As pointed out in paper A, a time-dependent frequency may lead to essentially different particle dynamics. This is most easily understood in terms of adiabatic invariants.

First, consider the one-dimensional motion of a particle in a slowly accelerating (or decelerating) wave. If the velocity of the wave changes sufficiently slowly, an adiabatic invariant (1) exists. This implies that a particle which is initially trapped in a potential well of the wave remains trapped at all times, approximately satisfying the Landau resonance condition $\omega=kv$, where ω is the angular frequency, k is the wave number, and v is the velocity of the particle

In three dimensions, the picture is similar. A particle initially resonant with a weakly time-dependent wave stays in resonance. In a tokamak, the unperturbed guiding-centre motion of, e.g., a fast ion is characterized by two frequencies, ω_θ, and ω_ζ, corresponding to the poloidal and toroidal motion, respectively. Resonant interaction with a wave takes place when $\omega=n\omega_\zeta+m\omega_\theta$, where m and n are integers. If this condition is satisfied at any instant of time, the orbit adjusts itself so as to remain in resonance even if the wave frequency changes. If ω decreases, this usually means that the orbit is shifted radially outwards. As pointed out in paper A, this is a highly efficient mechanism of expelling fast ions from the centre of the plasma, and may contribute to the understanding of, e.g., fast-ion losses during fishbone activity. It should be noted that this radial convection of resonant particles takes place no matter how small the wave amplitude is. On the other hand, the number of resonant particles is proportional to the square root of the wave amplitude.

2.3 Generalized invariants

The usual theory of adiabatic invariants applies only to strictly Hamiltonian systems where the slowly varying parameter λ changes independently of the oscillations of the system. In paper B, a generalization of the classical invariant (1) was found. In order for the adiabatic invariance of the action integral I to hold, it is essential that not only λ, but also its time derivative $\dot{\lambda}$ should change slowly. If λ varies little over one period of oscillation of the system, but is subjected to small but rapid oscillations (so that $\dot{\lambda}$ varies significantly), the conservation of I breaks down. However, if the time variation of the parameter λ is a function of the form

$$\dot{\lambda} = f(H,\lambda)\ \varphi(q,p),$$

then, as shown in paper B, the quantity

$$J(H,\lambda) = \frac{1}{2\pi} \int \psi(q,p)\ dq,$$

where $\varphi = \partial\psi/\partial p$, is adiabatically invariant. This quantity J, which represents a generalization of the ordinary adiabatic invariant can, hopefully, be useful for the investigation of a variety of physical problems.

3. RESONANT PERTURBATIONS, CHAOS, AND THE EFFECTS OF FINITE ORBIT WIDTH

As we have seen, even a very small perturbation may dramatically affect the motion of a particle if the interaction is resonant. If the perturbing field is strong enough, chaotic motion may result [6,16]. This is true for all Hamiltonian systems and, in particular, for particle motion in a tokamak, in which case it may have both desirable and undesirablr consequences. It is therefore of considerable interest to determine the critical perturbation strength required for the onset of chaos.

3.1 Stochastic ripple diffusion

Because of the finite number of toroidal field coils (e.g., 32 in JET), the magnetic field in a tokamak is not perfectly axisymmetric; it is slightly rippled. For this reason, the conservation of the canonical momentum is not exact. The magnetic ripple mainly affects banana orbits near the turning points, where the orbits, because of the ripple, are displaced somewhat in the vertical direction. If these displacements are random, i.e. if the displacements at successive turning points are uncorrelated, the particle motion becomes random, and diffusion sets in [17]. This is the case if the amplitude of the ripple is large enough, so that the dynamical system describing the influence of the ripple on the orbits exhibits stochastic behaviour. In order to avoid ripple diffusion, which can lead to very rapid losses of, e.g., fusion-generated alpha particles, the number of coils generating the toroidal field must be large enough, so that the ripple amplitude is kept low. This is a major constraint on the design of next-generation tokamaks.

In the past few years, experiments have been carried out on the TFTR and JET tokamaks to investigate stochastic ripple diffusion [18,19], and a comparison with theory has been attempted. However, the width of the fast-ion orbits is very large in these machines, but the standard theory of ripple diffusion [17,20] only applies in the zero-banana-width limit. Paper C attempts to remedy this. When the theory is extended to account for large orbit width, the calculated threshold value in ripple amplitude for the onset of stochastic ripple diffusion changes considerably. Depending on the orbit, the threshold is raised or lowered by up to a few orders of magnitude. The changes are most pronounced for particles with their turning points on the inside of the torus, and may be attributed to details in the orbit topology.

3.2 Ion-cyclotron-resonance heating

A phenomenon somewhat similar to ripple diffusion occurs when an externally generated RF wave is present in the plasma. The field is then not constant in time, and axisymmetry is broken. As a result, none of the three invariants are conserved, and diffusion takes place in both coordinate- and velocity-space. The former leads to radial transport, and the latter to heating of the particles interacting with the wave. During ICRH in large tokamaks such as JET, minority ions are heated to energies of several MeV in this way. The heating mechanism relies on resonant wave-particle interaction at the ion cyclotron frequency. Every time an ion passes through a region in the plasma where the wave frequency equals the cyclotron frequency, energy is transferred from the wave to the particle, or vice versa. In order for the ions to gain energy on the average, the interaction must be random, i.e. it must be decorrelated between successive passages through resonance. At low temperatures, such decorrelation occurs as a result of collisions with the background plasma particles. At high temperatures, decorrelation must be accomplished by other mechanisms, so that a random walk in velocity space may take place.

In paper D, the collisionless interaction between fast minority ions and a wave in the ion-cyclotron range of frequencies is studied. In particular, the threshold in wave amplitude required for the interaction to be random is calculated and compared to experimental values. Earlier analyses [21-23] of this problem neglect a number of important effects, including those of finite Larmor radius (FLR) and finite banana width (FBW). The results of these studies indicate that the wave amplitude required for the randomization of fast-particle orbits is several magnitudes larger than that actually used in present-day tokamaks. As found in paper D, however, because of FLR and FBW the required amplitude is reduced by one to two orders of magnitude for highly energetic ions. The reason for this is that if the width of the orbits is assumed to be small, the particles are constrained to move along the magnetic field lines and become very difficult to perturb. In other words, the orbits are artificially deprived of a degree of freedom in their response to external perturbations. When this freedom is restored, the orbits are more easily perturbed, and become stochastic at lower wave amplitudes.

Even when FLR and FBW effects are taken into account, the wave amplitude required for randomization of particle motion is still somewhat too large to account for the heating actually observed for most particles. This is, however, easily explained by the existence of a *spectrum* of waves in ICRH experiments. The calculations in paper D were carried out for the case of a monochromatic wave field. If several waves are present, the threshold for stochastic motion can be expected to be considerably lower.

3.3 Finite-orbit-width effects on the interaction with internal kink modes

Even if a population of fast particles is small, it may, because of its high energy content, significantly influence the properties of the bulk plasma. In particular, instabilities in the plasma may be stabilized or destabilized because of the presence of hot ions. At present, the most frequently discussed tokamak instabilities influenced by fast ions are toroidal Alfvén eigenmodes (TAE) and the internal kink mode.

Toroidal Alfvén eigenmodes can be destabilized by alpha particles with a parallel velocity close to the Alfvén velocity (or one third of it) [24,25]. The instability is global in nature, and poses a potential threat to the confinement of alpha particles, since their orbits become stochastic at very low TAE amplitudes [14]. The ultimate importance of these modes remains uncertain. The threshold for TAE destabilization has been much debated [26,27], and in the very recent TFTR tritium experiments, TAE modes have not been observed so far [3]. A possible explanation for this is the stabilizing influence of finite-orbit-width effects, which should be significant since alpha particles in TFTR move along very wide drift orbits.

Internal kink modes are believed to be responsible for fishbone- and sawtooth-oscillations. As mentioned above, fishbones are sometimes observed to severely deteriorate the confinement of fast ions. In JET experiments, however, fishbones do not seem to have any serious impact on fast-ion confinement or energy confinement [11]. On the other hand, sawtooth oscillations, which have plagued tokamak experiments for two decades now, result in a large-scale redistribution of the plasma, causing a sudden drop in the central temperature and density, and a rapid expulsion of fast ions from the centre of the plasma [28,29]. Experimentally, it has been shown that both these instabilities are intimately connected with the presence of fast ions. Fishbone oscillations have been observed only during NBI and ICRH, never in purely Ohmic discharges, and sawteeth are apparently sometimes suppressed during high-power auxiliary heating [30].

Theoretical considerations [31,32] suggest that fishbones and sawteeth are intrinsically related phenomena, corresponding to different branches of the internal kink mode. The mode structure is localized near the centre of the plasma, where the safety factor q is less than unity. Depending on the pressure, a population of hot ions may either stabilize or destabilize the mode, and in this way both fishbone excitation and sawtooth suppression can be explained. More specifically, the fast-ion poloidal beta-value $\beta_{p,\alpha}$, a quantity which depends on the poloidal magnetic field and the pressure profile of the hot ions inside the q=1 surface, determines whether or not the mode is stable. When $\beta_{p,\alpha}$ lies between two certain values depending on plasma parameters, $\beta^{(1)}_{p,\alpha}<\beta_{p,\alpha}<\beta^{(2)}_{p,\alpha}$, stabilization occurs; below $\beta^{(1)}_{p,\alpha}$, low-frequency fishbones (and sometimes sawteeth) are unstable, and above $\beta^{(2)}_{p,\alpha}$ a high-frequency branch of the mode, not present when fast ions are absent, is excited.

In future tokamak experiments, fusion-generated alpha particles are expected to play a role similar to that of fast neutral-beam-injected and RF-heated ions for the stability of fishbones and sawteeth [33]. However, alpha particles have very wide banana orbits, and FBW effects can be assumed to be of importance. In particular, if a large fraction of the alpha particles interacting with the instability have orbits extending outside the q=1 surface, the strength of the wave-particle interaction is reduced. This effect was pointed out in paper E, and was found to reduce the growth-rate in the unstable regions of the mode, and to increase the values of $\beta_{p,\alpha}^{(1)}$ and $\beta_{p,\alpha}^{(2)}$. Thus, the value of $\beta_{p,\alpha}$ required to attain the stable domain of operation increases, as well as that required to excite high-frequency fishbones. A similar conclusion was made in Ref. [34], where the large-orbit-width limit (potato limit) was investigated.

4. WELL DEVELOPED CHAOS AND QUASILINEAR DIFFUSION

In the previous section, the threshold for the onset of stochastic particle motion was discussed. If the threshold is exceeded, the constants of motion describing particle orbits change randomly on long time-scales. It is then difficult to follow the motion of an individual particle, but instead a statistical description becomes meaningful. The motion can be pictured as a random walk in the three-dimensional space of constants of motion, and the resulting diffusion tensor completely determines the evolution of a distribution of particles. Far above the stochasticity threshold, it is possible to derive a simple analytical expression for the diffusion tensor [35-37]. The basic assumption underlying the resulting so-called quasilinear diffusion is that the wave-particle interaction is randomized on very short time-scales; long-time correlations are neglected. For instance, in the case of stochastic ripple diffusion this means that the radial displacements of the orbit at successive turning points are completely uncorrelated.

4.1 Monte Carlo operators

A large tokamak experiment like JET produces huge amounts of experimental data. In order to take full advantage of this and make a detailed comparison with theory, it is necessary to develop accurate numerical models of the plasma. As for the fast ions, it is desirable to solve the kinetic equation for the distribution function. One way of doing this is to use Monte Carlo methods. In this approach, the kinetic equation is side-stepped, and instead one focuses on the individual particles. Each particle is followed along its unperturbed trajectory, and at regular time intervals the effects of collisions and wave-particle interaction are simulated by applying a Monte Carlo operator, which changes the constant of motions in an appropriate way. In other words, the Fokker-Planck equation is replaced by an equivalent Langevin equation, which is simply the equation of motion under random forces. The Langevin equation is then solved numerically by discretizing time. By following a sufficiently large ensemble of particles, accurate statistical information is obtained about the fast-ion population.

Mathematically, the central problem of the Monte Carlo method is to construct the required Monte Carlo operator. This problem is solved in paper F under the assumption that the wave-particle interaction is quasilinear. At each time step, the three constants of motion change in some random way. From the Fokker-Planck equation, it is possible to calculate the expectation values and the covariances of the changes. The Monte Carlo operator should faithfully reproduce these, and can be conveniently constructed by diagonalizing the covariance matrix. In the past, simulations of fast ions have been carried out by applying local Monte Carlo operators at a number of times each turn around the orbit [38,39]. This is quite inefficient since similar orbits have to be calculated over and over again. Paper F suggests a much faster way, viz., to solve the orbit-averaged Fokker-Planck equation directly with a Monte Carlo technique. The mathematical details are worked out in the paper. A computer code, FIDO, operating in this way has recently been constructed; preliminary results are presented in Ref. [40]. In principle, almost all single-particle phenomena associated with fast ions in tokamaks can be studied with such a code, e.g., ripple diffusion, wave-driven transport and heating, and neoclassical transport, at least as long as the interaction between particles and waves is quasilinear.

4.2 Non-quasilinear diffusion

The validity of the quasilinear diffusion coefficient has been debated continuously ever since it was first derived [35,36]. In particular, the theory of stochasticity in Hamiltonian systems [6,16] has shed much light on the issue, and shows that the hypothesis of quasilinear diffusion of test particles in a prescribed wave field is justified if the threshold for chaotic motion is well exceeded. However, recent numerical simulations [41] surprisingly show that the margin has to be very large. If K is the ratio between the actual perturbation strength and that required for randomization of the particle motion, the quasilinear diffusion coefficient D_{QL} is correct within 10% only if $K>100$. For smaller values of K, the velocity diffusion coefficient D observed in the simulations deviates strongly from D_{QL}. The maximum deviation occurs when $K=17$; then $D/D_{QL}=2.2$. Apparently, the reason for the observed non-quasilinear diffusion is that the motion is not completely irregular. Traces of regular dynamics persist and give rise to correlation effects even far above the threshold for global chaos. This is of considerable theoretical as well as practical interest since, as shown in papers C and D, in experimentally relevant cases the parameter K may fall in the intermediate region where the diffusion is faster than quasilinear.

The simulations by Cary *et al.* [41] were carried out for the simple case of one-dimensional motion in a spectrum of waves with equal amplitudes and equal spacing with respect to the phase velocity. The equations of motion in the prescribed wave field were solved numerically for an ensemble of particles. Since this is a very time-consuming procedure, only a small ensemble of particles may be simulated, and the statistics becomes rather unsatisfactory. In order to improve the accuracy and to verify the somewhat controversial results of Ref. [41], the simple simulations presented in paper G were carried out. The exact equation of motion was replaced by an approximate mapping, which is very easily iterated numerically. Therefore a much larger ensemble of particles can be used, and the accuracy is improved significantly. The results obtained support those of Ref. [41].

Shortly after paper G was submitted for publication, Ref. [42] appeared in the same journal. A numerical algorithm different from that used by Cary *et al.* was used to integrate the equations of motion, and different results were obtained. It was claimed that the diffusion is initially non-quasilinear, as found by Cary *et al.*, but after some time settles down at the quasilinear rate. At most one of Refs [41,42] can be right. The simulations in paper G support Ref. [41]; no traces of any initially enhanced diffusion are seen. Fig. 4 illustrates this point. The observed velocity diffusion coefficient $D(t)=\langle[v(t)-v(0)]^2/2t\rangle$, where v is the velocity and t time, for a 100-step mapping is plotted vs time for K=17. The diffusion coefficient is seen to increase monotonically with time and to saturate at 2.2 D_{QL}. The number of particles in the ensemble is $2\cdot10^5$, and the time-scale of observation is slightly longer than that in Ref. [42].

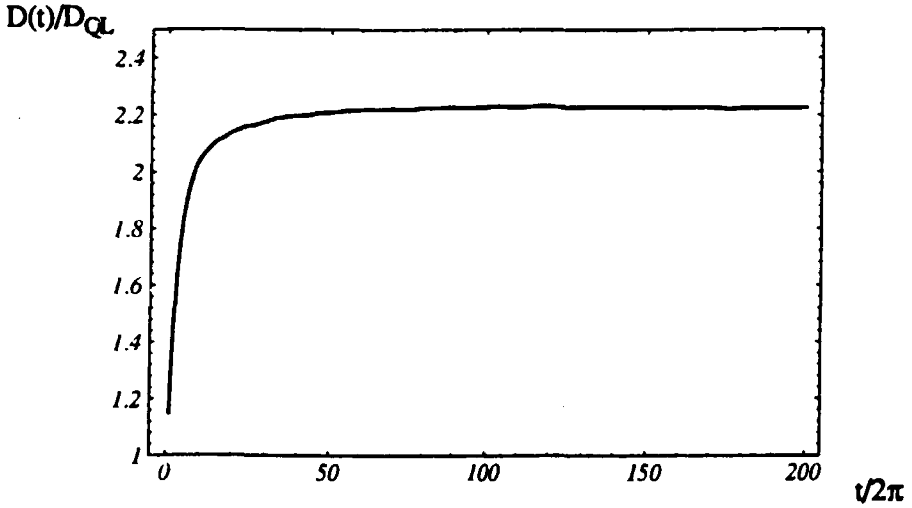

Fig. 4. Ratio of the numerically obtained diffusion coefficient to the quasilinear value vs time for an ensemble of $2 \cdot 10^5$ particles. K=17.

5. COLLISIONS

Collisions between fast ions and the bulk plasma particles are very rare since the collision frequency is proportional to v^{-3}, where v is the fast-ion velocity. Nevertheless, collisions are very important since they provide the means of transferring energy to the background plasma. Mathematically, they are described by the Boltzmann collision operator

$$C[f(v)] = \sum_i \int d^3v_i \int [f(v')\, f_i(v_i') - f(v)\, f_i(v_i)]\, |v - v_i|\, d\sigma, \qquad (2)$$

which includes contributions from collisions with all plasma constituents. In Eq. (2), $d\sigma$ represents the differential cross-section, and f_i is the distribution function of the i:th particle species in the plasma. The post-collision velocities v' and v_i' are related to the velocities v and v_i prior to the collision by the laws of energy and momentum conservation.

The collision operator (2) is too complicated for most practical purposes in plasma physics. The customary way of simplifying it is to expand for small |v'-v|. Such an expansion is permissible since collisions resulting in large deflections of the velocity vector v are very infrequent. This is a consequence of the long-range nature of Coulomb interaction, which ensures that close encounters between charged particles are rare. For fast ions, the dominant part of the resulting Fokker-Planck collision term is

$$C(f) = \frac{1}{\tau_s v^2} \frac{\partial}{\partial v} (v^3 f), \tag{3}$$

which describes the deceleration because of friction against the bulk plasma electrons. The Fokker-Planck expansion also gives terms describing scattering and diffusion in velocity space, but these terms are small for fast ions. The characteristic time of slowing-down τ_s is of the order of one second in a thermonuclear tokamak plasma. In a burning deuterium-tritium plasma, alpha particles are produced with a kinetic energy of 3.5 MeV in fusion reactions, and are gradually slowed down as described by Eq. (3). At high energies, a steady-state distribution results from the balance between alpha-particle production and slowing-down.

5.1 Close collisions

In the derivation of the Fokker-Planck collision term, it is assumed that the energy transfer in each collision is small, and the discrete nature of collisions is neglected. However, when fast ions are present in a plasma, large amounts of energy can be transferred in single, close collisions. In two recent articles [43,44], Ryutov has suggested that this phenomenon can be important in fusion devices. For example, in close collisions with fusion-generated alpha particles, thermal impurity ions can be accelerated to energies of several MeV. In a tandem-mirror plasma, this energy is sufficient for the impurities to penetrate the potential barriers at the ends of the machine; close collisions therefore serve as a purification mechanism of the plasma. In a tokamak, the high-energy tail in the impurity distribution created by close collisions with alpha particles can perhaps be useful for diagnostic purposes. The distribution of fast impurity ions can, in principle, be measured by detecting their Doppler-shifted line radiation, or by measuring the prompt losses of impurities striking the reactor wall. This distribution function would, it turn, yield information about the α-distribution. Furthermore, close collisions between the alpha particles themselves give rise to a tail in the α-distribution extending up to 7.0 MeV; the prompt losses of these 'superfast' alpha particles can also be measured. It should be pointed out, however, that all these measurements could be quite difficult to carry out because of the very small densities involved.

The slowing-down operator (3) fails to describe the kinetics of close collisions, since it neglects exactly these events. Instead, the full Boltzmann operator (2) must be used. On the other hand, the second term in the integrand of (2) can be neglected when calculating the distribution of very fast particles, since these particles are expected to be few. In paper H, this calculation is carried out with special attention paid to the effects of (i) nuclear elastic scattering (NES), (ii) anisotropy in the fast-particle velocity-distribution, and (iii) to α-α-collisions. The results obtained can be summarized as follows:

(i) Since the distance of closest approach in collisions between 3.5 MeV alpha particles and light ions (including other alpha particles) is smaller than the sum of their nuclear radii, the interaction is not purely electromagnetic in nature, and NES effects play a role. These effects are expected to increase the scattering cross-section, and can be accounted for in the kinetic calculation by an expansion of the NES differential cross-section.

(ii) If the fast-ion distribution is anisotropic in velocity space, which is the case for ions accelerated by ICRH or injected by NBI, a hot impurity population formed in close collisions with these particles 'inherits' this property. This can be utilized for determining the anisotropy of the fast-particle population indirectly by measuring that of sufficiently fast impurities.

(iii) Finally, the rate of formation and the velocity-distribution function of near 7.0 MeV alpha particles are calculated in paper H. This superfast α-population is formed in close collisions between fusion-generated 3.5 MeV alpha particles. NES effects are very important. In fact, the cross section is about 100 times larger than that suggested by Rutherford's formula for Coulomb collisions [8].

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