CN9401699

CNIC-00797
IAE-0127

中国核科技报告
CHINA NUCLEAR SCIENCE & TECHNOLOGY REPORT

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EMITTANCE GROWTH CAUSED BY NONUNIFORM CHARGE DISTRIBUTION OF BUNCHED BEAM IN LINAC

原子能出版社

中国核情报中心
China Nuclear Information Centre

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摘　要

束流发射度的明显增长与束团中电荷密度分布的均匀化密切相关，是空间电荷自场能转换成粒子横向动能的结果。作者推得了在直线加速器屏蔽筒中的几种常见的非均匀密度分布的有限长空间电荷束团的非线性自场能，并给出了它们与均匀分布束团之间的剩余场能以及由此引起的直线加速器中非均匀密度分布的电荷束团发射度增长的数值计算结果图表。

EMITTANCE GROWTH CAUSED BY NONUNIFORM CHARGE DISTRIBUTION OF BUNCHED BEAM IN LINAC

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ABSTRACT

The nonlinear space charge effect of bunched beam in linac is one of the important reasons that induces the emittance growth because of the conversion of the field energy to kinetic energy. The authors have worked out the internal field energies associated with some nonuniform space charge distributions of a bunched beam, such as Gaussian distribution, waterbag distribution and parabolic distribution. And the emittance growths caused by these nonuniformities are obtained.

INTRODUCTION

In high current beam for Free Electron Laser (FEL) and linear accelerator for high-energy physics, induction linac for heavy ion fusion, microwave devices and other applications, the space charge effect is no longer small compared with the applied force. Recent theoretical work and particle simulation studies on transport of intense beams in linear channels have identified nonuniform charge distribution as a major cause of emittance growth[1~5]. According to the theory, a nonuniform beam has higher field energy than the equivalent beam, and the particle distribution will become uniform. The difference in potential energy is converted to kinetic energy (and hence emittance growth) as the distribution tends to become more homogeneous. Furthermore, a general relationship between the possible emittance growth and excess energy has been given in Ref. [5].

However, we should point out that the above results concerning the calculations of space charge field energy are based on either a charge bunch in free space or a continuous beam in shelter wall. They do not really meet the situations of a space charge bunch in a linac or other microwave devices. Therefore, it is necessary to derive the formulae for calculating the nonlinear field energy of a space charge bunch with some common nonuniform distributions in a linac and according to the obtained formulae to discuss the emittance growth caused by the nonuniformities.

1 FORMULAE FOR CALCULATING THE NONLINEAR FIELD ENERGY OF A SPACE CHARGE BUNCH IN LINAC

We use a cylinder model of space charge to present a space charge bunch in linac. In the waveguide of linac, set the cylindrical space charge q in the position as shown in Fig. 1.

The potentials induced by some common distributions, according to Ref [6], are listed as follows:

For Kapchinskij-Vladimirskij (KV) distribution

$$\rho_{kv} = \frac{q}{\pi b^2 L} = \rho_0 \tag{1}$$

we have the potential as

$$\phi_{kv} = \frac{2qa}{\pi\epsilon_0 bL}\sum_{l=1}^{\infty}\frac{J_1(k_l b)J_0(k_l r)}{(k_l a)^3 J_1^2(k_l a)}(1 - e^{-k_l L/2}\cosh k_l z), (|z| \leqslant \frac{L}{2}) \tag{2}$$

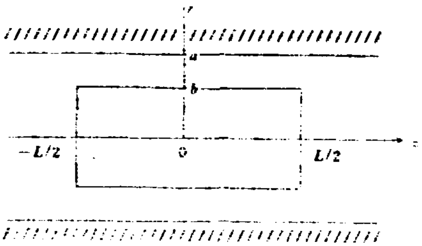

Fig. 1 Cylinder model of space charge

where a is the radius of the waveguide. b and L are the radius and length of the cylinder. respectively, $J_i(k_l x)$ is the Bessel function and k_l satisfies the equation: $J_0(k_l a) = 0$.

For waterbag (wb) distribution

$$\rho_{wb} = \frac{2q}{\pi b^2 L}\left(1 - \frac{r^2}{b^2}\right) \tag{3}$$

we have the potential as

$$\phi_{wb} = \frac{8qa^2}{\pi\varepsilon_0 b^2 L}\sum_{l=1}^{\infty}\frac{J_2(k_l b)J_0(k_l r)}{(k_l a)^4 J_1^2(k_l a)}\left(1 - e^{-k_l L/2}\cosh k_l z\right), \left(|z| \leqslant \frac{L}{2}\right) \tag{4}$$

For parabolic (pa) distribution

$$\rho_{pa} = \frac{3q}{\pi b^2 L}\left(1 - \frac{r^2}{b^2}\right)^2 \tag{5}$$

we have the potential as

$$\phi_{pa} = \frac{48qa^3}{\pi\varepsilon_0 b^3 L}\sum_{l=1}^{\infty}\frac{J_3(k_l b)J_0(k_l r)}{(k_l a)^5 J_1^2(k_l a)}\left(1 - e^{-k_l L/2}\cosh k_l z\right), \left(|z| \leqslant \frac{L}{2}\right) \tag{6}$$

For Gaussian (ga) distribution

$$\rho_{ga} = \frac{q}{2\pi\sigma^2 L}e^{-r^2/2\sigma^2} \tag{7}$$

we have the potential as

$$\phi_{ga} = \frac{q}{\pi\varepsilon_0 L}\sum_{l=1}^{\infty}\frac{J_0(k_l r)}{(k_l a)^2 J_1^2(k_l a)}e^{-k_l^2\sigma^2/2}\left(1 - e^{-k_l L/2}\cosh k_l z\right), \left(|z| \leqslant \frac{L}{2}\right) \tag{8}$$

The total self-electric energy of the cylinder space charge bunch can be obtained using the equation

$$W = \frac{1}{2}\int \rho\phi \mathrm{d}\tau \tag{9}$$

Substituting the formulae (1) to (8) into the Eq. (9) we get the field energy as follows:

$$W_{kv} = \frac{2q^2a^2}{\pi\epsilon_0 Lb^2}\sum_{l=1}^{\infty}\frac{J_1^2(k_1b)}{(k_la)^4J_1^2(k_la)}(1 - \frac{\sinh k_lL/2}{k_lL/2}\,\mathrm{e}^{-k_lL/2}) \tag{10}$$

$$W_{wb} = \frac{32q^2a^4}{\pi\epsilon_0 Lb^4}\sum_{l=1}^{\infty}\frac{J_2^2(k_1b)}{(k_la)^5J_1^2(k_la)}(1 - \frac{\sinh k_lL/2}{k_lL/2}\,\mathrm{e}^{-k_lL/2}) \tag{11}$$

$$W_{pa} = \frac{1152q^2a^5}{\pi\epsilon_0 Lb^6}\sum_{l=1}^{\infty}\frac{J_3^2(k_1b)}{(k_la)^8J_1^2(k_la)}(1 - \frac{\sinh k_lL/2}{k_lL/2}\,\mathrm{e}^{-k_lL/2}) \tag{12}$$

$$W_{ga} = \frac{q^2}{2\pi\epsilon_0 L}\sum_{l=1}^{\infty}\frac{\mathrm{e}^{-k_l^2L^2}}{(k_la)^2J_1^2(k_la)}(1 - \frac{\sinh k_lL/2}{k_lL/2}\,\mathrm{e}^{-k_lL/2}) \tag{13}$$

Now, let $W_0 = W_{kv}$, and

$$U_n = W_n - W_0 \tag{14}$$

where subscript n may be wb, pa or ga, respectively. U_n is the difference in potential energy between nonuniform distribution and the equivalent K-V distribution. This additional field energy is converted into particle kinetic energy as the distribution tends to become more homogeneous. And hence it leads to the emittance growth.

The quantity U_n can be used to measure the nonuniformity of density distribution of a space charge bunch. Using formulae (10) to (13), we get U_n/W_0 for some common distributions as follows:

$$\frac{U_{wb}}{W_0} = 16(\frac{a}{b})^2\left[\sum_{l=1}^{\infty}\frac{J_2^2(k_lb)P_{K_lL}}{(k_la)^5J_1^2(k_la)}\Big/\sum_{l=1}^{\infty}\frac{J_1^2(k_lb)P_{K_lL}}{(k_la)^4J_1^2(k_la)}\right] - 1 \tag{15}$$

$$\frac{U_{pa}}{W_0} = 576(\frac{a}{b})^4\left[\sum_{l=1}^{\infty}\frac{J_3^2(k_lb)P_{K_lL}}{(k_la)^8J_1^2(k_la)}\Big/\sum_{l=1}^{\infty}\frac{J_1^2(k_lb)P_{K_lL}}{(k_la)^4J_1^2(k_la)}\right] - 1 \tag{16}$$

$$\frac{U_{ga}}{W_0} = \frac{1}{4}(\frac{b}{a})^2\left[\sum_{l=1}^{\infty}\frac{\mathrm{e}^{-k_l^2a^2}P_{K_lL}}{(k_la)^2J_1^2(k_la)}\Big/\sum_{l=1}^{\infty}\frac{J_1^2(k_lb)P_{K_lL}}{(k_la)^4J_1^2(k_la)}\right] - 1 \tag{17}$$

where

$$P_{t,L} = \left(1 - \frac{\sinh k_t L/2}{k_t L/2} e^{-k_t L/2} \right) \tag{18}$$

2 NUMERICAL RESULTS

Using the Eqs. (15) to (17), we calculated the U_n/W_0 numerically. From the numerical results of U_n/W_0 for some common distributions we find that U_n/W_0 is depending on the bunch length L and the ratio b/a of the beam and wall radius. When L is constant, the values of U_n/W_0 will increase as the ratio b/a increases. The curves of U_n/W_0 versus the ratio b/a of the beam and wall radius for some common distributions, i. e. , waterbag (wb) distribution, parabolic (pa) distribution and Gaussian (ga) distribution, are shown in Fig. 2 .

Furthermore, in Gaussian distribution, the values of U_n/W_0 vary with not only b/a , but also the ratio a/b of the truncated distance and beam radius. The U_{ga}/W_0 will increase as the ratio a/b decreases. Fig. 3 shows the curves of U_{ga}/W_0 versus the ratio b/a of the beam and wall radius for different ratio a/b of the truncated distance and beam radius in Gaussian distribution. It is interesting that the difference in self-electric field energy of Gaussian distribution and [illegible] distribution is very slight when $a/b = 0.5$.

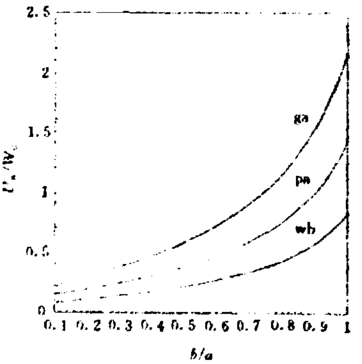

Fig. 2 Curves of U_n/W_0 vs b/a for some common distributions, $a/b = 0.3$ for Gaussian distribution.

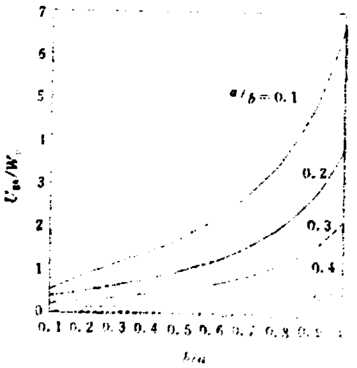

Fig. 3 Curves of U_{ga}/W_0 vs b/a for different a/b in Gaussian destribution.

3 EMITTANCE GROWTH

A general relationship between the possible emittance growth and the values U_n/W_0 is derived by M. Reiser in Ref. [5]. In linear approximation, the formula for the ratio ϵ_f/ϵ_i of final and initial beam emittance can be written as[5]

$$\frac{\epsilon_f}{\epsilon_i} = \left[1+\frac{U_n}{2W_0}\left[\frac{k_0^2}{k_i^2}-1\right]\right]^{1/2} \tag{19}$$

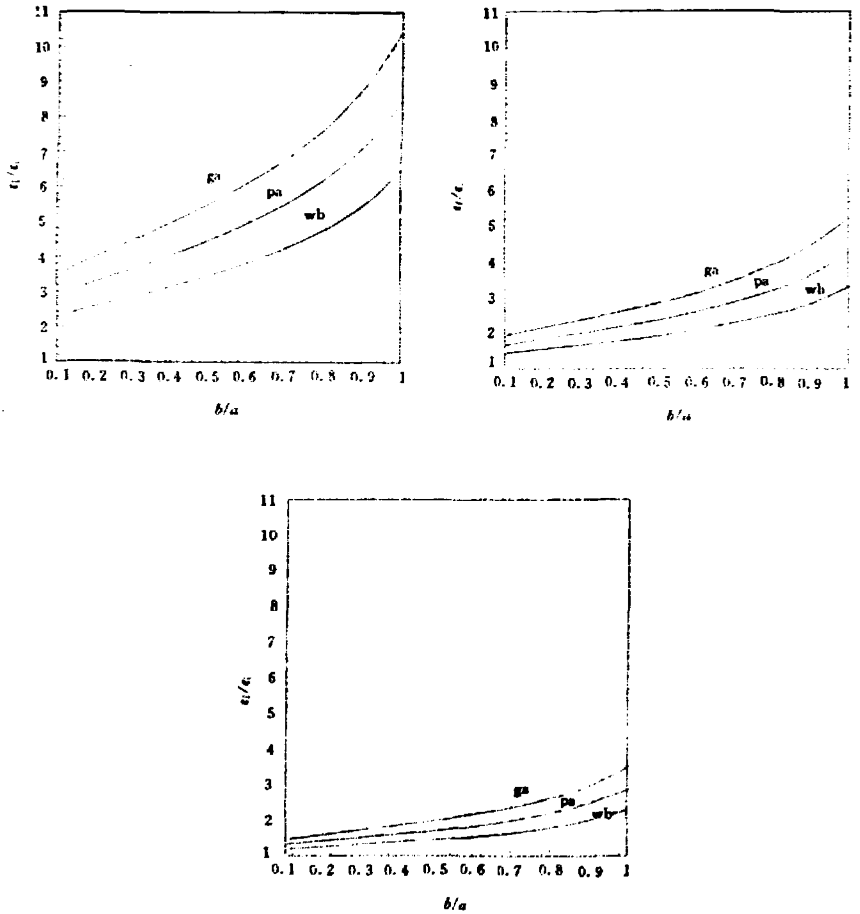

Fig. 4 Emittance growth ϵ_f/ϵ_i vs b/a for some common distributions, (a) $k_i/k_0=0.1$, (b) $k_i/k_0=0.2$, (c) $k_i/k_0=0.3$.

where k_0 is external focusing constant, k_i is initial focusing with self fields. The presence of self field of the beam will reduce the net focusing force acting on the particle, therefore, the constant k_i is always less than k_0.

Substituting the values U_s/W_n into Eq. (19) and taking the ratio $a/b = 0.3$ in Gaussian distribution, we calculated the ϵ_f/ϵ_i for $k_i/k_0 = 0.1, 0.2$ and 0.3. The emittance growth ϵ_f/ϵ_i versus b/a for some common distributions are plotted in Fig. 4.

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（北京2108信箱）
中国核科技报告编辑部排版
核科学技术情报研究所印刷
☆
开本787×1092 1/16・印张1/2・字数6千字
1993年9月北京第一版・1993年月9北京第一次印刷
ISBN 7-5022-1094-6
TL・668

CHINA NUCLEAR SCIENCE & TECHNOLOGY REPORT

ISBN 7-5022-1094-6
TL·668

P.O.Box 2103
Beijing, China

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