

THE NO-HAIR CONJECTURE IN 2D DILATON SUPERGRAVITY

J. GAMBOA[†] and Y. GEORGELIN

*Division de Physique Théorique, Institut de Physique Nucleaire[‡],
F-91406 Cedex, Orsay, France*

Abstract. We study two dimensional dilaton gravity and supergravity following hamiltonian methods. Firstly, we consider the structure of constraints of 2D dilaton gravity and then the 2D dilaton supergravity theory is obtained taking the square root of the bosonic constraints. We integrate exactly the equations of motion in both cases and we show that the solutions of the equation of motion of 2D dilaton supergravity differs from the solutions of 2D dilaton gravity only by boundary conditions on the fermionic variables, i.e. the black holes of 2D dilaton supergravity theory are exactly the same black holes of 2D bosonic dilaton gravity modulo supersymmetry transformations. This result is the bidimensional analogue of the no-hair theorem for supergravity.

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[†] Address after September 1, 1993: Fachbereich 7 Physik, Universität Siegen, Germany.

[‡] Unite de Recherche des Universités Paris 11 et Paris 6 associée au CNRS.

I. Introduction

The quantization of the gravitational field is a problem plagued by technical and conceptual difficulties that has resisted a solution for many years [1]. However in spite of these difficulties some remarkable ideas have emerged which make one think that the quantized gravitational field necessarily involves new physics [2].

Among these ideas, the evaporation of black holes (BH) [3] is probably the most important result reached in the last twenty years and for which there is not a definitive explanation in terms of a true quantum theory of gravity.

Some time after the theoretical discovery of the evaporation of BH, Hawking [4] argued that the evaporation and creation of BH necessarily imply a radical change of the quantum mechanical laws and for these reasons he argued that pure states in quantum gravity could evolve to mixed states. Despite many efforts made in the last years, it has not been possible to prove this conjecture and for this reason many people think if simplifications are not introduced in the theory, probably the Hawking's conjecture never will be proved.

Recently Callan, Giddings, Harvey and Strominger (CGHS) proposed a simplified two dimensional gravity model coupled to a dilaton and conformal matter that has among its properties to be exactly soluble and to contain as a particular case the black hole solution [5]. The classical solubility of the model could be an indication that at the quantum level the solution of some old may be solved in terms of a completely quantized gravity theory.

However, in spite of the intense research in this area [6], there still remain open several classical problems that are important to solve in order to see if the CGHS model keeps some properties of four dimensional gravity. Among these properties, a no-hair theorem for $2D$ dilaton gravity has been proved in [7] but a similar result for $2D$ dilaton supergravity theory, to our knowledge, does not exist.

The purpose of the present research is to construct a dilaton supergravity model directly from the constraints of $2D$ dilaton gravity and to investigate the no-hair conjecture for the supergravity case. Our model of $2D$ supergravity is constructed from an action originally proposed by Russo and Tseytlin in [8] which is classically equivalent to the

original CGHS action. Our main result will be that the solutions of the equation of motion of $2D$ dilaton supergravity coupled to superconformal matter are exactly the $2D$ dilaton gravity coupled to conformal matter solutions modulo boundary conditions on the fermionic variables.

The paper is organized as follows: In the next section we study $2D$ dilaton gravity without fixing the gauge and we show the equivalence between these constraints and the constraints of $2D$ dilaton gravity in the conformal gauge. In section 3, we construct $N = 1$, $2D$ dilaton supergravity using hamiltonian methods and compute completely the algebra of constraints. In section 4 we analyze the equations of motion and we give the general solution for $2D$ dilaton (super)gravity coupled to (super)conformal matter. We also prove here the no-hair theorem for $2D$ dilaton supergravity. Finally, in section 5 we give the conclusions.

2. 2D Dilaton Gravity: Hamiltonian Analysis

The model considered by CGHS is described by the following action

$$S = -\frac{1}{8} \int_{\mathcal{M}} d^2x \sqrt{-g} \left\{ e^{-2\varphi} \left[R + 4g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi + \mu^2 \right] + 4 \sum_{i=1}^N g^{\alpha\beta} \partial_\alpha f_i \partial_\beta f_i \right\}, \quad (2.1)$$

where \mathcal{M} is the manifold on which the theory is defined, φ is the dilaton field, μ^2 the cosmological constant and f_i are N scalar fields that represents the matter degrees of freedom. Here $g^{\alpha\beta}$ is the two dimensional metric tensor and R the corresponding scalar curvature.

In order to perform the hamiltonian formulation of (2.1), it is convenient to transform this action to the form action proposed by Russo and Tseytlin [8] by means the transformations

$$\phi = \frac{1}{4} e^{-2\varphi}, \quad (2.2)$$

$$h_{\alpha\beta} = \frac{1}{4} e^{-2\omega} g_{\alpha\beta}, \quad (2.3)$$

where

$$\omega = \frac{1}{2} (\ln \phi - \phi). \quad (2.4)$$

Then, the action (2.1) becomes

$$S = S_1 + S_2, \quad (2.5)$$

where

$$S_1 = -\frac{1}{2} \int d^2x \sqrt{-h} \left[h^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + R\phi + \frac{1}{4} \mu^2 e^\phi \right], \quad (2.6)$$

$$S_2 = \int d^2x \sum_{i=1}^N \sqrt{-h} h^{\alpha\beta} \partial_\alpha f_i \partial_\beta f_i, \quad (2.7)$$

being R the scalar curvature computed with the metric $h_{\alpha\beta}$.

The form (2.5) is more convenient because it separately represents the pure gravity (S_1) and matter (S_2) sectors. Formally the matter sector is mathematically equivalent to a

string, the hamiltonian formulation of which is known [9]. On other hand, the hamiltonian formulation of pure 2D gravity has been also considered [10-12] and, as a consequence, the constraints associated with (2.5) are

$$\begin{aligned}\mathcal{H}_\perp &= \frac{1}{2}[\phi'^2 - 4(h_{11}\pi^{11})^2 - 4(h_{11}\pi^{11})P_\phi - \frac{h'_{11}}{h_{11}}\phi' + 2\phi'' + \frac{1}{4}\mu^2 h_{11} e^\phi] + \sum_{i=1}^N \frac{1}{2}(P_i^2 + f_i'^2), \\ \mathcal{H}_1 &= P_\phi \phi' - 2h_{11}\pi'^{11} - \pi^{11}h'_{11} + \sum_{i=1}^N P_i f_i',\end{aligned}\quad (2.8)$$

where $P_{\alpha\beta} = \frac{\delta L}{\delta h^{\alpha\beta}}$, $P_\phi = \frac{\delta L}{\delta \phi}$ and $P_i = \frac{\delta L}{\delta f_i}$ are the canonical momenta associated with $h^{\alpha\beta}$, ϕ and f_i respectively.

Geometrically, the constraints (2.8) are the generators of temporal and spatial deformations and contain all the dynamics of the theory. Nevertheless, their structure is still complicated and for this reason it is convenient to transform these quantities making a change of variables in order to simplify their structure.

Following [10,12] our first canonical transformation is

$$\pi = \pi^{11}h_{11}, \quad \chi = \ln h_{11}, \quad (2.9)$$

where π and χ are canonical variables that satisfy

$$\begin{aligned}[\chi(x), \chi(x')] &= 0 = [\pi(x), \pi(x')], \\ [\chi(x), \pi(x')] &= \delta(x - x').\end{aligned}\quad (2.10)$$

Using (2.9), the constraints (2.8) becomes

$$\begin{aligned}\mathcal{H}_\perp &= \frac{1}{2}\left[\phi'^2 - 4\pi^2 - 4\pi P_\phi - \chi'\phi' + 2\phi'' + 4\mu^2 e^{\chi+\phi}\right] + \sum_{i=1}^N \frac{1}{2}(P_i^2 + f_i'^2) \\ \mathcal{H}_1 &= P_\phi \phi' + \pi\chi' - 2\pi' + \sum_i P_i f_i'.\end{aligned}\quad (2.11)$$

Our second canonical change of variables consists in diagonalizing the constraints (2.11); thus we propose

$$\begin{aligned}\psi &= \phi - \frac{1}{2}\chi, & b &= P_\phi \\ \chi &= \chi, & P &= \pi + \frac{1}{2}P_\phi.\end{aligned}\quad (2.12)$$

This transformation is also canonical because the quantities (ψ, b) and (P, χ) satisfy

$$\begin{aligned} [\psi(x), \psi(x')] &= 0 = [b(x), b(x')], \\ [\psi(x), b(x')] &= \delta(x - x') = [\chi(x), P(x')]. \end{aligned} \quad (2.13)$$

In consequence, the constraints (2.11) can be written in the following form

$$\begin{aligned} \mathcal{H}_\perp &= \frac{1}{2} \left[\psi'^2 + b^2 - 4P^2 - \frac{1}{4} \chi'^2 + 2\psi'' + \chi'' + \frac{1}{4} \mu^2 e^{\psi + \frac{3}{2}x} \right] + \sum_{i=1}^N \frac{1}{2} (P_i^2 + f_i'^2), \\ \mathcal{H}_1 &= b\psi' + P\chi' - 2P' + b' + \sum_{i=1}^N P_i f_i', \end{aligned} \quad (2.14)$$

Explicit calculation shows that the algebra of constraints is

$$\begin{aligned} [\mathcal{H}_\perp(x), \mathcal{H}_\perp(x')] &= (\mathcal{H}_\perp(x) + \mathcal{H}_\perp(x')) \delta'(x - x'), \\ [\mathcal{H}_\perp(x), \mathcal{H}_1(x')] &= (\mathcal{H}_\perp(x) + \mathcal{H}_\perp(x')) \delta(x - x'), \\ [\mathcal{H}_1(x), \mathcal{H}_1(x')] &= (\mathcal{H}_1(x) + \mathcal{H}_1(x')) \delta(x - x'), \end{aligned} \quad (2.15)$$

that is, the usual diffeomorphism algebra.

The canonical hamiltonian for this theory is

$$H_c = \int dx [N^\perp \mathcal{H}_\perp + N^1 \mathcal{H}_1], \quad (2.16)$$

where N^\perp and N^1 are the Lagrange multipliers associated with the first class constraints \mathcal{H}_\perp and \mathcal{H}_1 respectively.

The expression (2.16) is the starting point for computing the invariant ADM mass and this (in the context of $2D$ dilaton gravity) has been carried out by Bilal and Kogan [13] and de Alwis in [14].

In analogy with string theory, we can redefine the constraints (2.14) in the following form

$$\begin{aligned} L_\pm = \mathcal{H}_\perp \pm \mathcal{H}_1 &= \frac{1}{2} (h_\pm^2 \pm 2h'_\pm - J_\mp^2 \mp 2J'_\mp + \frac{1}{4} \mu^2 e^{\psi + \frac{3}{2}x}) \\ &+ \sum_{i=1}^N \frac{1}{2} Q_{\pm, i}^2, \end{aligned} \quad (2.17)$$

where h_a , J_a and $Q_{\pm,i}$ are defined as

$$\begin{aligned} h_a &= b + a\psi', \\ J_a &= 2P + \frac{a}{2}\chi', \\ Q_{i,a} &= P_i + af'_i, \quad (a = \pm 1) \end{aligned} \tag{2.18}$$

and satisfy the algebra

$$\begin{aligned} [h_a(x), h_b(x')] &= 2a\delta_{ab}\delta'(x - x'), \\ [J_a(x), J_b(x')] &= 2a\delta_{ab}\delta'(x - x'), \\ [Q_{a,i}(x), Q_{b,j}(x')] &= 2a\delta_{ab}\delta_{ij}\delta'(x - x'). \end{aligned} \tag{2.19}$$

Using (2.19) the algebra of constraints is now

$$[L_a(x), L_b(x')] = 2a\delta_{ab}(L_a(x) + L_a(x'))\delta'(x - x'). \tag{2.21}$$

In the compact case, of course, (2.20) define a classical Virasoro algebra with generators L_n . In the non-compact case, however, the situation is more involved because the Fourier expansion, generally speaking, is not defined. Even so, some progress has been reported recently [15].

3. 2D Dilaton Supergravity from 2D Dilaton Gravity

In this section we derive 2D dilaton gravity directly from the results obtained in the previous section.

The procedure we follow here is the square root method proposed originally by Dirac in 1928 [16]. His central idea was to construct a fermionic operator \hat{S} such that its square root gives the Klein-Gordon equation. The classical version of this procedure and their relation with supersymmetry is due to Berezin and Marinov [17] and their procedure can be summarized by the following scheme

Operator \rightarrow *Constraint*

(Anti)Commutator \rightarrow *(Anti)Symmetric Poisson Bracket*

In the context of four dimensional supergravity theory, this procedure was used in [18] and is reviewed in [19] for the spinning string case and in [20] for the supergravity case.

In order to construct the fermionic constraint, let us start with the following ansatz [21]

$$\begin{aligned} \mathcal{S}_a = & \alpha \Gamma_a h_a + \beta \Gamma'_a + \gamma \Theta_{-a} J_{-a} + \delta \Theta'_{-a} + (\omega \Theta_a + \kappa \Gamma_{-a}) e^{\frac{1}{2}\psi + \frac{3}{2}\xi} \\ & + \rho \sum_{i=1}^N \Xi_{i,a} Q_{i,a}, \text{ (no summing } a) \end{aligned} \quad (3.1)$$

where $\alpha, \beta, \gamma, \delta, \omega, \kappa$ and ρ are unknown coefficients that will be fixed at the end of the calculation. The spinors Γ_a, Θ_a and $\Xi_{i,a}$ are real fermionic variables that, by definition, satisfy the following Clifford algebra

$$\begin{aligned} \{\Gamma_a(x), \Gamma_b(x')\} &= i\delta_{ab}\delta(x-x'), \\ \{\Theta_a(x), \Theta_b(x')\} &= -i\delta_{ab}\delta(x-x'), \\ \{\Xi_{i,a}(x), \Xi_{j,b}\} &= i\delta_{ab}\delta_{ij}\delta(x-x'). \end{aligned} \quad (3.2)$$

Following the square root method, the aim now is to compute the symmetric Poisson bracket

$$\{\mathcal{S}_a(x), \mathcal{S}_b(x')\} = i\delta_{ab}\bar{L}_a(x)\delta(x-x'), \quad (3.3)$$

in order to find the fermionic corrections introduced in the bosonic constraints $L_a(x)$.

A straightforward calculation gives for \bar{L}_a

$$\begin{aligned} \bar{L}_a = & \alpha^2 h_a^2 + \alpha\beta h'_a - 2i\alpha^2 a \Gamma_a \Gamma'_a - \gamma^2 J_{-a}^2 \\ & - \gamma\delta J'_{-a} + 2i\gamma^2 a \Theta_{-a} \Theta'_{-a} + (\kappa^2 - \omega^2) e^{\psi + \frac{3}{2}\xi} \\ & + \rho^2 \sum_{i=1}^N (Q_{a,i}^2 - 2ia \Xi_{a,i} \Xi'_{a,i}). \end{aligned} \quad (3.4)$$

The coefficients that appear in (3.4) can be explicitly evaluated in the limit

$$(\Gamma_a, \Theta_a, \Xi_{i,a}) \rightarrow 0.$$

In fact, after comparing with (2.19) we find¹

$$\alpha = \frac{1}{\beta} = \frac{1}{\sqrt{2}}, \quad \gamma = \frac{1}{\delta} = \frac{1}{\sqrt{2}}, \quad \rho = \frac{1}{\sqrt{2}}, \quad (3.5)$$

$$\kappa^2 - \omega^2 = \frac{\mu^2}{8}, \quad (3.6)$$

of course, the next step is to verify if the constraints \tilde{L}_a and S_a satisfy a closed superalgebra.

Computing this algebra, we find

$$\begin{aligned} \{S_a(x), S_b(x')\} &= i\delta_{ab}\tilde{L}_a(x)\delta(x-x'), \\ [\tilde{L}_a(x), S_b(x')] &= \delta_{ab}(S_a(x) + 2S_a(x'))\delta'(x-x'), \\ [\tilde{L}_a(x), \tilde{L}_b(x')] &= \delta_{ab}(\tilde{L}_a(x) + \tilde{L}_a(x'))\delta'(x-x'). \end{aligned} \quad (3.7)$$

The superalgebra (3.7) deserves two comments: 1) As in 2D dilaton gravity, there are no $\delta'''(x-x')$ terms present in the superalgebra and the total central charge for this model is zero; 2) Our model depends on an arbitrary constant κ (or *omega*) and, as a consequence, in this dilaton 2D supergravity theory the cosmological constant can be positive or negative. It is interesting to note that a negative cosmological constant is mandatory in a theory with only scalar field, such as a pure Liouville theory.

The constraints (\tilde{L}_a, S_a) are first class and the canonical hamiltonian

$$H = \int dx (N^a \tilde{L}_a + i\lambda^a S_a), \quad (3.8)$$

vanishes due to the general covariance.

It is easy to see that the action

$$S = \int d^2x \left(b\dot{\psi} + P\dot{\chi} + P_i \dot{f}_i - \frac{i}{2}\Gamma_a \dot{\Gamma}_a - \frac{i}{2}\Theta_a \dot{\Theta}_a - \frac{i}{2}\Xi_{a,i} \dot{\Xi}_{a,i} - N^a \tilde{L}_a - i\lambda^a S_a \right), \quad (3.9)$$

(sum in i and a)

is invariant under reparametrization and local supersymmetry, generated by \tilde{L}_a and S_a respectively.

¹ The reader might note here that we have chosen the plus sign in front of the coefficients. This is not a loss of generality, because the same situation occurs, for instance, when we construct the spinning particle from the spinless particle.

4. Equations of Motion and Their Solutions for 2D Dilaton (Super)Gravity

In this section we will analyze the equation of motion and their solution for 2D dilaton (super)gravity coupled to conformal matter. This a subtle problem that was originally studied in the context of 4D general relativity by Dirac [22], de Witt [23] and more recently by Regge and Teitelboim [24]. As was emphasized by these authors the hamiltonian of 4D general relativity exhibits some peculiarities that are not present in other theories. In fact, it can be shown that if we want to reproduce correctly the Einstein field equations, it is mandatory to add to the hamiltonian of general relativity a surface term that asymptotically define the energy and the momentum of the gravitational field. When this observation is done, one has a well defined variational principle.

At the quantum level, this observation is very important because defining the surface term is equivalent to defining a positive ADM mass and, in consequence, to having a ground state for quantum gravity.

The analogue of this problem for 2D dilaton gravity has been considered by Park and Strominger [25] and more recently by Bilal and Kogan [13] and de Alwis [14]. In particular in these last references, the authors use the Regge-Teitelboim method to obtain an explicit formula for the ADM mass independent of the time and manifestly positive. Our aim in this section is to use the Regge-Teitelboim method to obtain the equations of motion, and we will also write an explicit expression for the surface term, although as we comment below, we will not impose boundary conditions on the fields.

Let us start considering the bosonic dilaton gravity. Following the reference [24], the variation of the hamiltonian

$$\delta H = \int dx \left(\frac{\delta H}{\delta \psi} \delta \psi + \frac{\delta H}{\delta b} \delta b + \frac{\delta H}{\delta \chi} \delta \chi + \frac{\delta H}{\delta P} \delta P + \frac{\delta H}{\delta f_i} \delta f_i + \frac{\delta H}{\delta P_i} \delta P_i \right), \quad (4.1)$$

contains a surface term \mathcal{D} that does not allow one to obtain correctly the equation of motion. For this reason, it is necessary to redefine the hamiltonian in the form

$$H \rightarrow H - \mathcal{D}, \quad (4.2)$$

in order to cancel the undesired term that appears in (4.1).

By explicit calculation of (4.1), it is found that the surface term is

$$\mathcal{D} = \sum_{a=\pm 1} \left[aN^a \delta h_a + a(N^a h_a - aN'^a) \delta \psi - aN^a \delta J_{-a} - \frac{a}{2} (-N^a J_{-a} + aN'^a) \delta \chi \right]_{z=-\infty}^{z=+\infty}, \quad (4.3)$$

where we have assumed that the matter fields f_i vanishes at $x = \pm\infty$.

Inserting (4.3) in (4.2) we obtain a well defined variational principle and the equation of motion obtained are

$$\begin{aligned} \dot{\psi} &= \sum_{a=\pm 1} (N^a h_a - aN'^a), \\ \dot{b} &= - \sum_{a=\pm 1} \left[-a(N^a h_a - aN'^a)' + \frac{1}{8} \mu^2 N^a e^{\psi + \frac{3}{2}x} \right], \\ \dot{\chi} &= \sum_{a=\pm 1} \left[2(-N^a J_{-a} + aN'^a) \right], \\ \dot{P} &= - \sum_{a=\pm 1} \left[\frac{a}{2} (-N^a J_{-a} + aN'^a)' + \frac{3}{16} \mu^2 N^a e^{\psi + \frac{3}{2}x} \right], \\ \dot{f}_i &= \sum_{a=\pm 1} Q_{i,a}, \\ \dot{P}_i &= \sum_{a=\pm 1} \left[aN^a Q_{i,a} \right]'. \end{aligned} \quad (4.4)$$

These equations are complicated to solve in an arbitrary gauge and, for this reason, is convenient to fix the gauge; let us take the proper-time gauge

$$N_+ = 1 = N_- \quad (4.5)$$

The previous equations of motion becomes

$$\begin{aligned} \dot{\psi} - 2b &= 0, \\ \dot{b} - 2\psi'' + \frac{1}{4} \mu^2 e^{\psi + \frac{3}{2}x} &= 0, \\ \dot{\chi} + 8P &= 0, \\ \dot{P} + \frac{1}{2} \chi'' + \frac{3}{8} \mu^2 e^{\psi + \frac{3}{2}x} &= 0, \\ \dot{f}_i - 2P_i &= 0, \\ \dot{P}_i - 2f_i'' &= 0, \end{aligned} \quad (4.6)$$

and after eliminating b, P and P_i , we obtain

$$\begin{aligned}\bar{\psi} - 4\psi'' &= 2\frac{\mu^2}{2}e^{\psi+\frac{3}{2}x}, \\ \bar{\chi} - 4\chi'' &= -3\mu^2 e^{\psi+\frac{3}{2}x}, \\ \bar{f}_i - 4f_i'' &= 0.\end{aligned}\tag{4.7}$$

It is convenient to write these equations in terms of light light cone coordinates $x_{\pm} = \tau \pm \frac{1}{2}x$. Using this parametrization (4.7) takes the form²

$$\begin{aligned}4\partial_+\partial_-\psi &= \frac{\mu^2}{2}e^{\psi+\frac{3}{2}x}, \\ 4\partial_+\partial_-\chi &= -3\mu^2 e^{\psi+\frac{3}{2}x}, \\ 4\partial_+\partial_-\bar{f}_i &= 0,\end{aligned}\tag{4.8}$$

from these equations we see that the matter fields are trivially integrable and their solution can be expressed in terms of two chiral functions $f_i = f_-(x_-) + f_+(x_+)$, which may be determined imposing appropriate boundary conditions for the matter fields. The first couple of equations, by other hand, gives the following relation between ψ and χ

$$\psi + \frac{1}{6}\chi = \gamma(g_+(x_+) + g_-(x_-)),\tag{4.9}$$

where γ is a constant and $g_{\pm}(x_{\pm})$ are two chiral functions that depends, in this case, on the coordinate system.

Using (4.9), for instance, one can bring the second equation in (4.8) in the form

$$\partial_+\partial_-\bar{\chi} = \mu^2 e^{\bar{\chi}+\gamma g(x)},\tag{4.10}$$

$$(\bar{\chi} = \frac{4}{3}\chi, g(x) = g_+(x_+) + g_-(x_-)).$$

In order to solve (4.10) one can take a coordinate system where $g_{\pm}(x_{\pm}) = 0$ ³ and the equation (4.10) becomes the Liouville equation, whose general solution is [27]

$$e^{\bar{\chi}} = \frac{8}{\mu^2} \frac{h'(x_+)k'(x_-)}{[h(x_+) - k(x_-)]^2},\tag{4.11}$$

² The first pair of equations is very similar to the Toda equation, v.i.z. $\partial_+\partial_-\Phi_i = e^{K_{ij}}\Phi_j$ where K is a Cartan matrix of some simple Lie algebra and Φ_i is a field with two components [26].

³ This choice corresponds to choose Kruskal-Szekeres coordinates, see e.g. [5] and de Alwis in [6]

where h and k are two functions that depend on x_- and x_+ respectively and that satisfy the restrictions

$$h' > 0, \quad k' > 0, \quad (4.12)$$

$$(h' = \frac{dh}{dx}).$$

In the same way, one can solve the equation of motion for ψ and by (4.9) ($g = 0$) obtaining

$$e^{-\tilde{\psi}} = \frac{8}{\mu^2} \frac{h'(x_+)k'(x_-)}{[h(x_+) - k(x_-)]^2}, \quad (4.13)$$

$$(\tilde{\psi} = 8\psi).$$

If we choose appropriately the functions $h(x_+)$ and $k(x_-)$ and set the matter fields to zero, the black hole solution discussed by CGHS in [5] is obtained.

Now, let us discuss the equations of motion and their solutions for 2D dilaton supergravity.

The variation of the hamiltonian in this case is

$$\begin{aligned} \delta H = \int dx \left[\frac{\delta H}{\delta \psi} \delta \psi + \frac{\delta H}{\delta b} \delta b + \frac{\delta H}{\delta \chi} \delta \chi + \frac{\delta H}{\delta P} \delta P + \frac{\delta H}{\delta f_i} \delta f_i \right. \\ \left. + \frac{\delta H}{\delta P_i} \delta P_i + \frac{\delta H}{\delta \Gamma} \delta \Gamma + \frac{\delta H}{\delta \Theta} \delta \Theta + \frac{\delta H}{\delta \Xi} \delta \Xi \right] + \mathcal{D}_s. \end{aligned} \quad (4.14)$$

Using (4.7) we obtain that the equations of motion

$$\begin{aligned} \dot{\psi} &= \sum_{a=\pm 1} \left[N^a h_a - a N'^a + \frac{i\lambda^a}{\sqrt{2}} \Gamma_a \right], \\ \dot{b} &= - \sum_{a=\pm 1} \left[-a(N^a h_a - a N'^a + \frac{i\lambda^a}{\sqrt{2}} \Gamma_a)' + \frac{1}{8} \mu^2 N^a e^{\psi + \frac{3}{2}x} + \frac{i\lambda^a}{2} (\omega \Theta_a + \kappa \Gamma_{-a}) e^{\frac{1}{2}\psi + \frac{3}{2}x} \right], \\ \dot{\chi} &= \sum_{a=\pm 1} 2 \left(-N^a J_{-a} + a N'^a + \frac{i\lambda^a}{\sqrt{2}} \Theta_{-a} \right), \\ \dot{P} &= - \sum_{a=\pm 1} \left[\frac{a}{2} (-N^a J_{-a} + a N'^a + \frac{i\lambda^a}{\sqrt{2}} \Theta_{-a})' + \frac{3}{16} \mu^2 N^a e^{\psi + \frac{3}{2}x} - \frac{3}{4} \lambda^a (\omega \Theta_a + \kappa \Gamma_{-a}) e^{\frac{1}{2}\psi + \frac{3}{2}x} \right], \\ \dot{\Gamma}_a &= i a N^a \Gamma_a' + i (a N^a \Gamma_a)' - \frac{i}{\sqrt{2}} \lambda^a h_a + i \sqrt{2} \lambda'^a + i \kappa \lambda^{-a} e^{\frac{1}{2}\psi + \frac{3}{2}x}, \\ \dot{\Theta}_{-a} &= -i a N^a \Theta_{-a}' - i (a N^a \Theta_{-a})' - \frac{i}{\sqrt{2}} \lambda^a J_{-a} + i \sqrt{2} \lambda'^a + i \omega \lambda^{-a} e^{\frac{1}{2}\psi + \frac{3}{2}x}, \end{aligned} \quad (4.15)$$

$$\begin{aligned}
f_i &= \sum_{a=\pm a} (N^a Q_{a,i} - \frac{i\lambda^a}{\sqrt{2}} \Xi_{i,a}), \\
\dot{P}_i &= \sum_{a=\pm 1} [aN^a Q_{a,i} + ia\lambda^a \Xi_{a,i}]', \\
i\dot{\Xi}_{a,i} &= 2iaN^a \Xi'_{a,i} - \frac{i\lambda}{\sqrt{2}} Q_{i,a},
\end{aligned}$$

holds if in the canonical hamiltonian (3.8) the surface term \mathcal{D}_s is added, i.e. if we redefine H by

$$H_{\text{modified}} = H(3.8) - \mathcal{D}_s, \quad (4.16)$$

where the surface term is

$$\begin{aligned}
\mathcal{D}_s &= \sum_{a=\pm} \left[aN^a \delta h_a + a(N^a h_a - aN'^a + \frac{i\lambda^a}{\sqrt{2}} \Gamma_{-a}) \delta \psi - aN^a \delta J_{-a} \right. \\
&\quad - \frac{a}{2} (-N^a J_{-a} + aN'^a + \frac{i\lambda^a}{\sqrt{2}}) \delta \chi - iaN^a \delta \Gamma_a \Gamma_a - i\sqrt{2} \delta \Gamma_a \lambda^a \\
&\quad \left. + iN^a \delta \Theta_{-a} \Theta_{-a} + i\sqrt{2} \lambda^a \delta \Theta_{-a} \right]_{-\infty}^{+\infty}, \quad (4.17)
\end{aligned}$$

(we have supposed, such as in the bosonic case, that the matter fields vanish for $x = \pm\infty$).

By the same reasons given in the bosonic case, we have choose the proper time gauge

$$N_+ = 1 = N_-, \quad \lambda_+ = \xi = \lambda_-, \quad (4.18)$$

where ξ is a constant nilpotent spinor.

In this gauge the equations (4.15) take the following form

$$\begin{aligned}
\dot{\psi} &= 2b + \frac{i\xi}{\sqrt{2}} (\Gamma_+ + \Gamma_-), \\
\dot{b} &= 2\psi'' - \frac{i\xi}{\sqrt{2}} (\Gamma_+ - \Gamma_-)' + \frac{1}{4} \mu^2 e^{\psi + \frac{3}{2}\chi} + \frac{i\xi}{2} [\omega(\Theta_+ + \Theta_-) + \kappa(\Gamma_- + \Gamma_+)] e^{\frac{1}{2}\psi + \frac{3}{2}\chi}, \\
\dot{\chi} &= -8P + i\sqrt{2}\xi(\Theta_- + \Theta_+), \quad (4.19) \\
\dot{P} &= -\frac{1}{2}\chi'' + \frac{i\xi}{2\sqrt{2}} (\Theta_+ - \Theta_-)' - \frac{3}{8} \mu^2 e^{\psi + \frac{3}{2}\chi} - \frac{3i\xi}{4} [\omega(\Theta_+ + \Theta_-) + \kappa(\Gamma_+ + \Gamma_-)] e^{\frac{1}{2}\psi + \frac{3}{2}\chi}, \\
i\dot{\Gamma}_a &= 2ia\Gamma'_a - \frac{i}{\sqrt{2}} \xi h_a + i\kappa\xi e^{\frac{1}{2}\psi + \frac{3}{2}\chi}, \\
i\dot{\Theta}_{-a} &= -2ia\Theta'_{-a} - \frac{i}{\sqrt{2}} \xi J_{-a} + i\omega\xi e^{\frac{1}{2}\psi + \frac{3}{2}\chi},
\end{aligned}$$

$$\begin{aligned}
f_i &= 2P_i - \frac{i\xi}{\sqrt{2}}(\Xi_{i,+} + \Xi_{i,-}), \\
\dot{P}_i &= 2f_i' + i\xi(\Xi_{i,+} - \Xi_{i,-})', \\
i\dot{\Xi}_{a,i} &= 2ia\Xi_{a,i}' - \frac{i\xi}{\sqrt{2}}Q_{i,a}.
\end{aligned}$$

After the elimination the momenta b and P , these equations are

$$\begin{aligned}
\bar{\psi} - 4\psi'' &= \frac{i\xi}{\sqrt{2}}(\dot{\Gamma}_+ + \dot{\Gamma}_-) - i\sqrt{2}\xi(\Gamma_+ - \Gamma_-)' + \frac{\mu^2}{2}e^{\psi+\frac{3}{2}x} \\
&\quad + i\xi[\omega(\Theta_+ + \Theta_-) + \kappa(\Gamma_- + \Gamma_+)]e^{\frac{1}{2}\psi+\frac{3}{2}x}, \tag{4.19a}
\end{aligned}$$

$$\begin{aligned}
\bar{\chi} - 4\chi'' &= i\sqrt{2}\xi(\dot{\Theta}_- + \dot{\Theta}_+) - i2\sqrt{2}\xi(\Theta_+ - \Theta_-)' - 3\mu^2e^{\psi+\frac{3}{2}x} \\
&\quad + 6i\xi[\omega(\Theta_+ + \Theta_-) + \kappa(\Gamma_+ + \Gamma_-)]e^{\frac{1}{2}\psi+\frac{3}{2}x}, \tag{4.19b}
\end{aligned}$$

$$i\dot{\Gamma}_a - 2ia\Gamma_a' = -\frac{i}{\sqrt{2}}\xi h_a + i\kappa\xi e^{\frac{1}{2}\psi+\frac{3}{2}x}, \tag{4.19c}$$

$$i\dot{\Theta}_{-a} + 2ia\Theta_{-a}' = -\frac{i\xi}{\sqrt{2}}J_{-a} + i\omega\xi e^{\frac{1}{2}\psi+\frac{3}{2}x}. \tag{4.19d}$$

$$\bar{f}_i - 4f_i' = i\xi(\Xi_{i,+} - \Xi_{i,-})' - \frac{i\xi}{\sqrt{2}}(\dot{\Xi}_{i,+} + \dot{\Xi}_{i,-}), \tag{4.19e}$$

$$i\dot{\Xi}_{a,i} = 2ia\Xi_{a,i}' - \frac{i\xi}{\sqrt{2}}Q_{i,a}. \tag{4.19f}$$

These equations can be greatly simplified using the following observation: the set of equations (4.19) has the structure

$$DX = \xi(\text{something}), \tag{4.20}$$

where D is a linear operator like $D = a_2(x)\frac{d^2}{dx^2} + a_1(x)\frac{d}{dx} + a_0(x)$ and $X = (\psi, \chi, \Gamma, \Theta, \Xi)$. If we multiply (4.20) by ξ and use the fact that $\xi^2 = 0$, we obtain the equation

$$\xi DX = 0. \tag{4.21}$$

This result has the following implications: i) If the variables X are bosonic fields, (4.21) implies

$$DX = 0, \tag{4.22}$$

because ξ is a nonzero spinor; ii) If the variables X are fermionic fields, the complete expression (4.21) must be kept.

Multiplying (4.19) by ξ and using light cone like coordinates, we find that the equations (4.19a), (4.19b) and (4.19e) become

$$4\partial_+\partial_-\psi = \frac{\mu^2}{2}e^{\psi+\frac{3}{2}x}, \quad (4.23a)$$

$$4\partial_+\partial_-\chi = -3\mu^2e^{\psi+\frac{3}{2}x}, \quad (4.23b)$$

$$4\partial_+\partial_-\psi_i = 0, \quad (4.19c)$$

while (4.19c),(4.19d) and (4.19f) may be written in the form⁴

$$\xi\partial_\mp T_\pm = 0, \quad (4.24)$$

where T_\pm stands for Γ_\pm , Θ_\mp and Ξ_\pm .

The equations (4.23) are exactly the equations of motion for $2D$ dilaton gravity coupled to conformal matter while (4.24) appears as chirality conditions on the fermionic fields.

When the matter fields (f, Ξ) are put to zero, such in the bosonic case, we obtain the same black hole solution found in [5] plus the chirality condition (4.24) for Γ and Θ . Thus, we conclude that the super black holes are the ones of $2D$ dilaton gravity modulo chirality conditions on the fermionic fields. Exactly the same situation holds (although the derivation is more involved) in $4D$ supergravity [27].

⁴ Here we have used the identity

$$\partial_\tau \mp 2a\partial_x = (1 \pm a)\partial_- + (1 \mp a)\partial_+ = 2\delta_{\pm,a}\partial_- + 2\delta_{\mp,a}\partial_+.$$

5. Conclusions

In this paper we have studied $2D$ dilaton supergravity following canonical methods. We believe that this point of view is more convenient and permits one to derive the theory directly in analogy with string theory. Our procedure is also an alternative way to derive this theory and as a result, can be made free of the subtleties associated with supersymmetrization of Liouville-like theories.

We have also presented a simple derivation of a no-hair theorem that is the natural analogue of the four dimensional case.

Several problems, such as the explicit form of the ADM mass for $2D$ supergravity and the quantum corrections to effective action associated with the model presented here are, presently under research.

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