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Jishnu Dey

Mira Dey

Carlos F. de Araujo Jr.

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Lauro Tomio

INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

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Jishnu Dey[1], Mira Dey[2]
International Centre for Theoretical Physics, Trieste, Italy,

Carlos F. de Araujo Jr.
Instituto de Física Teórica - UNESP, 01405-900, São Paulo, SP, Brazil

and

Lauro Tomio
International Centre for Theoretical Physics, Trieste, Italy,
and
Instituto de Física Teórica - UNESP, 01405-900, São Paulo, SP, Brazil.

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[1]Permanent address: 1/10 Prince Golam Md. Road, Calcutta 700 026, India.
[2]On leave of absence from: Department of Physics, Lady Brabourne College, Calcutta 700 017, India.

ABSTRACT

Sharp transitions are perhaps absent in QCD, so that one looks for physical quantities which may reflect the phase change. One such quantity is the sound velocity which was shown in lattice theory to become zero at the transition point for pure glue. We show that even in a simple bag model the sound velocity goes to zero at temperature $T = T_v \neq 0$ and that the numerical value of this T_v depends on the nature of the meson. The average thermal energy of mesons go linearly with T near T_v, with much smaller slope for the pion. The $T_v - s$ can be connected with the Boltzmann temperatures obtained from transverse momentum spectrum of these mesons in heavy ion collision at mid-rapidity. It would be interesting to check the presence of different $T_v - s$ in present day finite T lattice theory.

1. Introduction

One of the first papers to talk about velocity of sound, u_s, in the context of quarks was Bhaduri et al.[1]. The work was devoted mostly to study of compressibility at finite density in hadrons, nuclear matter and quark matter.

In a later paper, Gavai and Gocksch [2] calculated u_s using Monte Carlo methods on the lattice for pure glue, clearly underlying its importance : in order to study the plasma produced in nuclear collision or early universe a theoretical model is needed and one usually employs relativistic hydrodynamics for this purpose. The hydrodynamic equations do involve parameters that are *a priori* unknown and which contain the memory of the underlying theory (QCD), and one such parameter is the velocity of sound in the quark gluon fluid. Here one is thinking of transition from colour singlet hadron phase to the deconfined phase which is highly non-perturbative in nature and to deal with such phenomena non-perturbative methods are called for. They find that the velocity increases with T and then goes down and ultimately crosses zero to become imaginary. Subsequently using the density of states method Huang et al.[3] also found the same behaviour for u_s in lattice.

The present-day lattice calculations, with light quarks at finite T, indicate that the phase transition may be second order or weak first order, and that the meson masses increase linearly with T near the critical temperature, T_c. The rise of the pion mass with T is much weaker than that of the ρ in lattice calculation [4] However, sound velocity has not been computed on the lattice in presence of quarks. Even if there is no clear cut first order phase transition, it is possible that the velocity of sound still goes to zero at a particular temperature, being a general feature of the transition. Zero sound velocity means decoupling of the pressure of the system from density or from the volume. Thus, $u_s \to 0$ may be a good signal even if there is no sharp phase transition in QCD as claimed in [5]. In this paper we show that indeed the sound velocity crosses zero at a particular temperature in a simple model.

In a previous paper [6] we have calculated the average thermal energy in the pion and ρ channels using a simple static bag immersed in a heat bath. We imposed the

Bekenstein entropy bound (deduced originally for black holes) between the energy, entropy and the radius of the hadron. It is found that the average thermal mass in the pion channel increases as $\sim \pi T$ while that of ρ meson goes as $\sim 2\pi T$ and one can estimate the temperature upto which the bound is validated. This bound, although generally true and proved for scalar and electromagnetic fields is rather exotic. So instead, we calculate the sound velocity in the same model.

$$\left(\frac{u_s}{c}\right)^2 = \left(\frac{\partial P}{\partial \epsilon}\right)_S = \frac{\gamma K}{\epsilon} \tag{1}$$

Here K is the isothermal compressibility, γ the ratio of 2 specific heats, C_p and C_V, and ϵ is the energy density.

Our model predicts the linear rise of thermal energy with T and the crossing of zero of u_s. The T_v at which the sound velocity crosses zero is also different for π and ρ. As a hot system (for example in heavy ion collision or early Universe) cools, the mesons can form at T_v, if we make the assumption that there is phase transition at this point.

2. The Model

The simplest model of relativistic quarks is to confine them in the so - called MIT bag[7]. Other models like the soliton bag (see for example [8] or the model of Krein et al. [9]) reproduce the characteristics of the MIT bag model. The bag model wavefunction does not obey chiral symmetry but in thermodynamics detailed knowledge of wavefunction is not relevant. Perhaps it is for this reason that the thermodynamics built on the bag model has been so successful [10][11][12], and be matched onto the string models [13]. We must emphasize that the bag model is sometimes believed to fail to fit the hadron excitation, but this is not so. It has been shown that the Laplace transform of the bag partition function reproduces the observed meson spectrum [11] and also baryon spectrum [12].

There are many models which invoke pion and sigma mesons as fundamental fields to reproduce the chiral symmetry of the QCD Lagrangian. But these models do not have confinement. The Nambu Jona-Lasinio (NJL) model also suffers from the same defect, although it *has* been used for finite T. Soliton bag models do not have chiral symmetry.

To our knowledge the only model which has explicit chiral symmetry and confinement is the ref. [9]. As the authors point out, the solution of this model is "very MIT-like in character". It would be very interesting to extend the model directly to finite T, but this is not straight forward.

It is assumed that the quarks are confined in the bag because of a pressure difference B, between the perturbative vacuum inside the bag and the QCD vacuum outside. The wavefunctions go to zero on the bag boundary. A spherical bag is assumed in most cases and then the wavefunctions are just spherical Bessel functions. Of course there is the large and the small component of the wave functions and the phenomenology therefore is richer compared to that of the non-relativistic case. One can write the energy of the hadron in the model as

$$M = nq/R + BV + Z/R \tag{2}$$

where the first term is the quark kinetic energy (for baryon $n = 3$, and $n = 2$ for meson). For equilibrium one gets

$$M = 4BV \tag{3}$$

and the pressure is 1/3 of the energy density, as for radiation. To obtain q it is enough to use the boundary conditions at a radius R. BV is the volume energy needed to dig a hole in the QCD vacuum. The third term contains centre of mass correction factor, the instanton contribution (also one-gluon-exchange) and also has a Casimir energy term, which must be present as soon as we talk of two vacua. The instanton contribution dominates [14], [15]. Thus the $N - \Delta$ or the $\rho - \pi$ mass difference comes from this term [15]. So one might call our model an improved bag model with instanton induced interaction. Again, it must be emphasized, that the model fits the ground state hadron spectrum very well. The instanton induced interquark (quark-antiquark) potential is roughly independent of T as has been observed in lattice calculations [16]. This has been observed not only by us [6] but also by [17].

The states of the bag, labelled q, in units of $1/R$, are given in Table 1, along with the corresponding degeneracies, dq. We have listed $q < 10$. The sum over states converge with this set in all cases. The number of quarks(antiquarks) are $N_+(N_-)$ as follows:

$$N_\pm = \sum_i dq_i f_\pm, \tag{4}$$

where

$$f_\pm = 1/(1 + \exp\frac{q_i \pm \mu}{RT}), \tag{5}$$

and the chemical potential is μ/R. The total number of quarks (antiquarks) $N_+ - N_-$ is fixed to be 1 for each flavour for the quark and -1 for the antiquark, and the energy and the entropy are summed over flavours. For pions, since we do not distinguish between u and d quarks, it is sufficient to multiply the energy and entropy by factors of 2. Notice that since the mesons have a definite spin, we do not use spin degeneracy of the quarks. Thus if the u-quark is up and the d-antiquark is down we may be considering the π^+, we have the same energy and entropy if the spins were the other way round or if it was the π^-. The energy is given by

$$E_\pm = \sum_i (q_i/R) dq_i f_\pm, \tag{6}$$

and the corresponding entropy is:

$$S_\pm = \sum_i [\frac{q_i \pm \mu}{RT} f_\pm - ln(1 - f_\pm)] dq_i \tag{7}$$

The μ is taken to satisfy eq.(4) and then the radius is found iteratively till it satisfies

$$E \equiv Z/R + \sum_{flavours} (E_+ + E_-) = 3BV. \tag{8}$$

We take the bag pressure B to be $(220\ MeV)^4$ and find $Z = -3.927$ for π and -2.528 for ρ meson. The results are given in Table 2, in th range of T 110 - 160 MeV. We find the self-consistent pion masses to be 248 - 491 MeV. These masses are close to the value found at these temperatures in the lattice calculations namely $\sim \pi T$. The ρ-masses in this temperature range are about 0.956 to 1.154 GeV, close to $2\pi T$ and about twice the pion mass. We do not report K meson masses in this paper since there are as yet no lattice calculations for them, and they are not expected to be qualitatively different compared to ρ meson.

Once the self consistent solutions are given, as in Table 2, one can check the numbers in a pocket computer without difficulty. This is in sharp contrast to the gigaflop lattice calculations on super - computers operating continuously sometimes for 9 months. Without the support of these expensive calculations however the simple model would make no sense.

The world's highest energy gold beam (11.6 A GeV/c) was delivered to experiments at Brookhaven's Tandem AGS facilities on April 24, 1992. This has provided an unique opportunity, to achieve in a nuclear reaction, the highest baryon densities ever obtained in any experiment. Because of the large number of collisions involved in the Au+Au reaction, one might expect a complete thermalization of the system. The hope is that by using a "truly" heavy ion beam a "new" kind of physics such as the formation of dense nuclear matter can be achieved [18]. Temperatures $T_{HI} = 142 \pm 3\ MeV$ for the pions and $151 \pm 7\ MeV$ for the kaons [18] are found. These T_{HI} are found by fitting the number of particles at different energies to a Boltzmann form. Since heavy ion collision is a very complex many body phenomenon, one cannot rule out such descriptions. Experimentalists like Stachel [19] argue very convincingly that in heavy ion collisions particles come out only when a particular T_{HI} is reached during cooling. But lattice calculations are more clear-cut and according to us at the present moment *supplements* experiment.

We have also presented the values of the sound velocity for the pion and the ρ in Table 2 and in the figure. Denoting the temperature at which the sound velocity becomes zero by T_v we see that for pion $T_v \sim 133$ MeV, whereas for the ρ the T_v is about 149 MeV. One could try to vary the B value for a better fitting. There are convincing reasons for believing that the freeze-out temperature (where the pions come out) is about 150 MeV. For the ρ the Brookhaven detectors are not adequate. Measurement of the ρ require a much more sophisticated method, like dimuon and di-electron measurements [20]. But as we have mentioned before the ρ and the K meson should behave in a similar manner. It must be mentioned [21], that the temperatures of the pion, the kaon and the ρ meson are nearly the same in proton-nucleus and pp collisions in this mid-rapidity region. Thus

here indeed, for heavy systems colliding, one expects to see a genuine finite temperature effect. Generally this is explained by the hydrodynamic flow model [21]. It is interesting to see the T_v given by the simple bag model reproduces the order of magnitude of these different emission-temperatures. It would be nice to confirm these results from lattice calculations for the mesons, extraction from existing results are probably possible !

3. Discussions

It is satisfying that a simple model calculation gives both the lattice masses and the heavy ion distribution for the pions with very reasonable values for the bag constant. One can add Abelian gluons to the bag, but since gluon excited states appear above the quark states (lowest state : $2.74/R$ compared to $2.04/R$ for the quarks) their importance will be much less. The non-Abelian effects of the gluon is already contained in the model in the Z/R and BV factors. The difference in Z causes different equilibrium radii for π and ρ, resulting in different T_v-s.

Lattice results have suggested seemingly contradictory models for quark plasma, as discussed by DeTar et al. [22] While determination of quantities like energy density ϵ yield values consistent with a nearly free gas of quarks and gluons, measurement of screening parameters are consistent with confinement of colour singlets. They have conjectured a resolution of this *seeming* paradox which describes the quark plasma as an ensemble of colour singlet clusters of different sizes. Bulk thermodynamic quantities like ϵ would receive contributions from all clusters, whereas long range screening would be controlled by the lightest clusters. When sound velocity becomes zero, one expects different sizes. So although the bag model, being a one-phase model, cannot predict about quark plasma, one can see that when $u_s = 0$, the pressure has decoupled from the volume, the bulk modulus K being zero. At a given pressure the coexistence of several volumes is thus quite natural. At this juncture we remind ourselves of the liquid gas phase transition depicted by the simple Van der Waal gas where $K = 0$, but the discontinuity in volume makes the transition first order. In the present case the $K \rightarrow 0$ makes the $u_s \rightarrow 0$ showing the possibility of co-existence of different volumes but no further than that. We stress once more, u_s^2 becoming less than zero means the previous physical picture breaks down.

The model does not offer a physical picture of the new phase but at least the breakdown temperature is clear.

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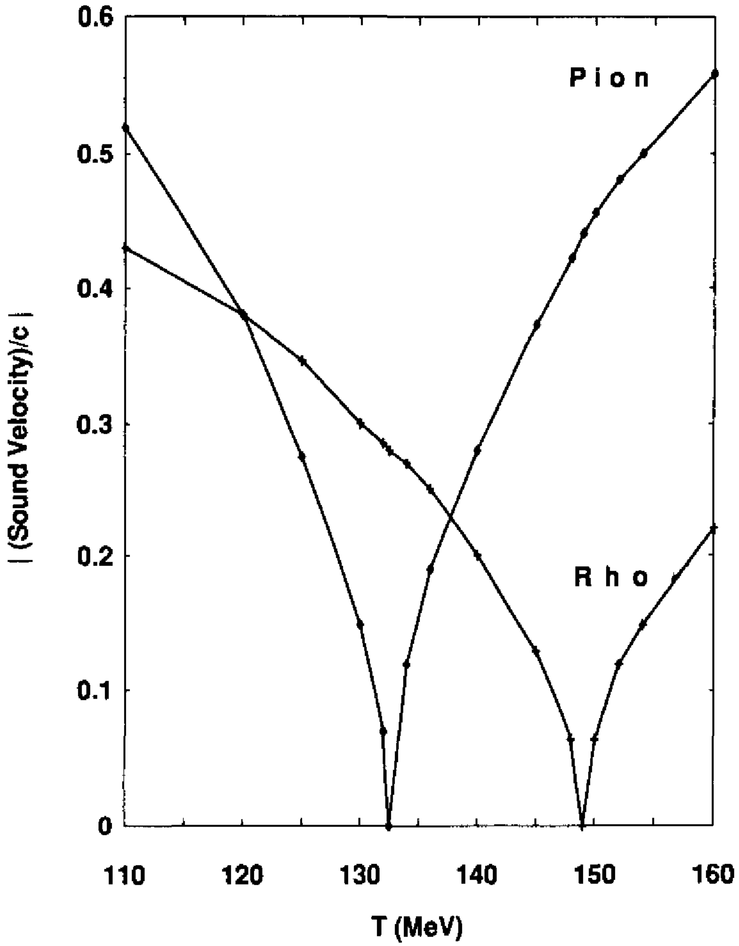
Table 1. Quark levels in the bag and corresponding degeneracies

$q =$	2.0428	3.8115	3.2039	5.1231	4.3273	6.3711	5.4295	5.3960
$dq =$	1	1	2	2	3	3	4	1
$q =$	7.0020	6.7778	8.4076	8.0596	9.3219	8.5776	7.5813	6.5179
$dq =$	1	2	2	3	4	1	4	5
$q =$	8.7657	7.5963	9.9312	8.6673	9.7323	9.7536		
$dq =$	5	6	6	7	8	3		

Table 2. Temperature, radius, μ in units of $1/R$, meson mass M, ratio M/T and u_s

Meson	T (MeV)	R (fm)	μ	M (MeV)	M/T	$(u_s/c)^2$
π	110	0.365	2.545	248	2.26	0.2704
	120	0.385	2.529	292	2.43	0.1444
	130	0.406	2.509	342	2.63	0.0225
	132	0.409	2.504	349	2.65	0.0049
	134	0.412	2.500	357	2.67	-0.0144
	136	0.416	2.495	368	2.70	-0.0361
	140	0.424	2.485	389	2.78	-0.0784
	150	0.441	2.458	438	2.92	-0.1936
	160	0.458	2.425	491	3.07	-0.3136
ρ	110	0.572	2.471	956	8.69	0.1849
	120	0.579	2.442	993	8.27	0.1444
	130	0.587	2.410	1033	7.95	0.0900
	140	0.594	2.374	1071	7.65	0.0400
	145	0.598	2.354	1092	7.53	0.0169
	148	0.600	2.342	1103	7.45	0.0049
	150	0.602	2.332	1114	7.43	-0.0049
	152	0.604	2.324	1126	7.40	-0.0144
	154	0.605	2.315	1131	7.34	-0.0225
	160	0.609	2.287	1154	7.21	-0.0484



Modulus of sound velocity for π and ρ meson. $\mid \frac{u_s}{c} \mid = 0$ at $T = 132.5$ MeV for π and 149 MeV for ρ meson, respectively.