AREAEE / Rep 297

AREAEE / Rep. 297

ARAB REPUBLIC OF EGYPT
ATOMIC ENERGY ESTABLISHMENT
PLASMA PHYSICS AND ACCELERATOR DEPARTMENT

PLASMA FOCUS MATCHING CONDITIONS

BY

H.M SOLIMAN, M.M MASOUD AND
T.A. EL-KHALAFAWY

1988

NUCLEAR INFORMATION DEPARTMENT
ATOMIC ENERGY POST OFFICE
CAIRO, A.R.E.

We regret that some
of the pages in the
microfiche copy of
this report may not
be up to the proper
legibility standards,
even though the best
possible copy was
used for preparing
the master fiche

AREAEE/ReP.297

ARAB REPUBLIC OF EGYPT
ATOMIC ENERGY ESTABLISHMENT
PLASMA PHYSICS AND ACCELERATOR DEPARTMENT

PLASMA FOCUS MATCHING CONDITIONS

BY
H.M. SOLIMAN, M.M. MASOUD AND
T.A. EL-KHALAFAWY

1988
NUCLEAR INFORMATION DEPARTMENT
ATOMIC ENERGY POST OFFICE
CAITO,A.R.E.

CONTENTS

	Page
ABSTRACT..	i
INTRODUCTION......................................	1
DISCUSSION..	9
CONCLUSION..	11
REFERENCES..	12

ABSTRACT

A snow-plough and slug models have been used to obtain the optimum matching conditions of the plasma in the focus. The dimensions of the plasma focus device are, inner electrode radius = 2 cm, outer electrode radius = 5.5 cm, and its length = 8 cm. It was found that the maximum magnetic energy of 12.26 kJ has to be delivered to plasma focus whose density is $10^{19}/cm^{3}$ at focusing time of 2.55 μs and with total external inductance of 24.2 nH.

The same method is used to evaluate the optimum matching conditions for the previous coaxial discharge system which had inner electrode radius = 1.6 cm, outer electrode radius = 3.3 cm and its length = 31.5 cm. These conditions are charging voltage = 12 kV, capacity of the condenser bank = 430 μf, plasma focus density $= 10^{19}/cm^{3}$, focusing time = 8 μs and total external inductance = 60.32 nH

INTRODUCTION

The dense plasma focus produced by a hydromagnetic coaxial gun system is a region of extreme high temperature and particle density(1). Formation of plasma focus has distinct phases, first is the initial gas breakdown and formation of an axisymmetric current sheath at insulator surface (breech), second is the hydromagnetic acceleration of current sheath by ($J_r \times B_\theta$) force towards the open end (muzzle), where J_r is the radial plasma current and B_θ is the azimuthal magnetic field. This mechanism occurs in Mather type but does not occur in Filippov design(2). In the axial phase, the current sheath gains momentum as the tube discharge current to increase. Experimentally, the arrival of the current sheath at the end of the gun is adjusted to occur at the peak tube current time. This is controlled for particular electrode geometry, by adjusting the length of the central electrode (CE), the applied voltage, and the initial gas pressure (3). The third phase is the rapid collapse of the current sheath off the end of the coaxial electrodes resulting in the formation of a thin filament of extremely hot and dense plasma. The diameter of the filament has been estimated to be between 1 and 2 mm and it lasts from 100 to 200 ns (4). During the current collapse towards the axis of coaxial electrodes, part of the stored magnetic energy in the coaxial electrodes and external circuit is rapidly converted to plasma energy.

To optimize the conversion of stored magnetic energy to plasma energy during the plasma focus formation, one must usually maximize the magnetic energy stored behind the current sheath in the coaxial electrodes just before its collapse to the axis to form a point of plasma focus. Maximum conversion of electrical capacitor energy to inductive magnetic occurs when the discharge current has a maximum value

System Under Consideration.

A schematic diagram of a plasma focus device is shown in Fig. (1), it consists essentially of central electrode with a diameter 4 cm, surrounded by outer electrode (OE) which consists of eight parallel bars of 8 cm length, uniformly distributed over a diameter of 11 cm, each of them has a diameter 0.8 cm. The length of the coaxial electrodes is 8 cm.

An insulator ring is located between the CE and OE across which the initial breakdown occurs.
Development of plasma focus device depends on : first, the axial acceleration of the current sheath in which snowplough model has been used and second, the slug model which has been used to study the radial collapse of the current sheath.

A) <u>Axial Acceleration</u>

The snow-plough model assumes that all the gas swept up by the current sheath propagates with the same velocity of the current sheath. In order to calculate the current sheath velocity, we must know the values of ambient gas pressure, geometry of the electrodes and the discharge current. Essentially, the model (5) assumes that the force accelerating the total mass of gas swept up by the current sheath equals to electro-magnetic driving force of the current sheath, the momentum balance equation in mks units is:

$$\frac{d}{dt}\left[(\rho A z)\frac{dz}{dt}\right] = \frac{B^2}{2\mu} A = \int_a^b \frac{\mu^2 I^2}{2\mu\,(2\pi r)^2}\, 2\pi r\, dr = \int_a^b \frac{\mu I^2}{4\pi r}\, dr \qquad (1)$$

where B is the magnetic induction associated with the discharge current I, ρ is the gas density, a and b are the radii of the inner and outer electrodes respectively and A is the inter electrode cross - sectional area.
Assuming $I = I_o \sin wt$ where I_o is the peak current, w is the angular frequency and boundary condition is z=0 at t = 0, integrating equation (1) gives

$$\dot{z} = k\left(t - \frac{\sin 2\omega t}{2\omega}\right) \Big/ \left(t^2 - \frac{\cos 2\omega t}{2\omega^2} - \frac{1}{2\omega^2}\right)^{\frac{1}{2}} \qquad (2)$$

where

$$K = \sqrt{\frac{\mu}{\rho}}\; \frac{I_o\left[\ln(b/a)\right]^{\frac{1}{2}}}{2\pi\left[(b^2 - a^2)\right]^{\frac{1}{2}}}$$

B) Radial Collapse

Plasma focus occurs due to the radial collapse of current sheath to the axis of the coaxial electrodes at muzzle and conversion of stored magnetic energy to plasma energy in the focus. The radial and axial dimensional r,z convergence is due to the (JxB) pinch force. In the radial collapse a slug of plasma is contained in the region between the shock front and the magnetic piston, the front of the slug is the shock front and the back of the slug is thec current sheath (or magnetic piston). This slug is obtained as a result of gas encountered by the shock front which moves from r_o to r_s, Fig.(2) shows the formation of radial collapse at the open end of the coaxial electrodes. Velocity of shock front[6], V_s

$$V_s = \frac{dr_s}{dt} = -\left[\frac{\mu(\gamma+1)}{\rho}\right]^{\frac{1}{2}} \times \frac{I}{4\pi r_p} \qquad (3)$$

where γ is the specific heat ratio of plasma = 0.67 for hydrogen gas.
Also the speed of the free axial front of the plasma focus[6]

$$\frac{dz_f}{dt} = -\frac{dr_s}{dt} \qquad (4)$$

where z_f is the length of the plasma focus.

Optimum Condition of the System

Optimization of the plasma focus formation depends upon the magnetic energy stored in and outside the coaxial electrode and upon the rate and manner in which it is converted to plasma energy. Consider that the maximum energy stored magnetically in coaxial electrodes and external circuit can be delivered to the plasma at the time of plasma focus formation, the period of discharge,

$$\tau = 2\pi\sqrt{LC} \qquad (5)$$

$t = \frac{\tau}{4}$ is the optimum focusing time, which corresponds to maximum discharge current i.e a maximum stored magnetic energy delivered to plasma.

The magnetic energy stored in the coaxial electrodes [7]

$$U = \frac{1}{2} L I^2 = 10^{-9} z \ln\left(\frac{b}{a}\right) I^2 \text{ Joule} \qquad (6)$$

Where Z is the length of coaxial electrodes, a & b are the radii of the inner and outer electrode respectively and I is the discharge current.
At a maximum discharge current, i.e a maximum stored energy, $\frac{dI}{dt} = 0$, the voltage is related to current by
$V = IR + \frac{d}{dt}(LI)$, by neglecting R, then

$$V = IL' = 2 \times 10^{-9} I \ln\left(\frac{b}{a}\right) \dot{z} \qquad (7)$$

At a maximum discharge current wt $\frac{\pi}{2}$ then $I \simeq I_o \sin\frac{\pi}{2}$,

$I = I_o$

From equation (2) and (7) (take wt $\frac{\pi}{2}$), then

$$\dot{z} = \left[\frac{V}{2 \times 10^{-9} \times 2\pi \left(\ln b/a\right)^{\frac{1}{2}}}\right] \left(\frac{\mu}{\rho}\right)^{\frac{1}{4}} \frac{1}{(b^2 - a^2)^{\frac{1}{4}}} \qquad (8)$$

The magnetic energy (7) U_ρ = 12.53 ZVa $\sqrt{\rho}$ (9)
Suppose the gas density ρ in the focus region
= 1.67 x 10^{-5} gm/cm^3
i.e n = 10^{19} /cm^3 for hydrogen gas.
a = 2 cm, and Z = 8 cm.
Substitute these quantities in eqn (9), to obtain the optimum values of the applied voltage and the capacity of the condenser bank. Figs.(3) a,b,c show the relation between the capacity of the condenser bank and the gas density at different values of charging voltage, the gas density and the charginge voltage at different values of capacity of condenser bank and gas density versus capacity of condenser bank at different charging voltages respectively. The optimum values of the applied voltage and cappacity of condenser bank are V_{ch} = 15 kV and C = 109 µf (14 condensers each of them has a capacity = 7.71µF and maximum charging voltage = 18 kV). The bank energy = 12.26 k. Joule Substitute these quantities in eqn. (8) to obtain the time at which the sheath arrives the muzzle.

$t_a \simeq 1.06\ \mu s$ (10)

According to slug model, the time of radial collapse of current sheath can be obtained.(6)
In the radial phase, the inductance

$$L = \frac{\mu}{2\pi}\left(\ln(b/a)\right) z_o + \frac{\mu}{2\pi}\left(\ln(b/r_p)\right) z_p \qquad (11)$$

where z_o is the position of sheath at muzzle, then,

$$\frac{d}{dt}\left[(L_o + L)I\right] + IR = V - \frac{\int I\,dt}{C} \qquad (12)$$

take $\frac{dI}{dt}$ = 0 and neglect R

then eqn. (11) becomes

$$I\frac{\mu}{2\pi}\left[\left(\ln b/r_p\right)\frac{dz_f}{dt} - \frac{z_f}{r_p}\frac{dr_p}{dt}\right] = V - \frac{It}{c} \qquad (13)$$

from the axial acceleration (snow-plough model). the disc-charge current

$$I = \frac{\sqrt{2\pi V}\,\left(b^2 - a^2\right)^{\frac{1}{4}}}{\left(2\times10^{-9}\right)^{\frac{1}{2}}\left[\ln(b/a)\right]^{3/4}\left(\mu/\rho\right)^{\frac{1}{4}}} \qquad (14)$$

The rate of change of magnetic piston radius with time (6)

$$\frac{dr_p}{dt} = \frac{\frac{2}{(\gamma+1)}\frac{r_s}{r_p}\frac{dr_s}{dt} - \frac{r_p}{\gamma I}\left(1-\frac{r_s^2}{r_p^2}\right)\frac{dI}{dt} - \frac{1}{\gamma+1}\frac{r_p}{z_f}\left(1-\frac{r_s^2}{r_p^2}\right)\frac{dz_f}{dt}}{(\gamma-1)/\gamma + \left(\frac{1}{\gamma}\right) r_s^2/r_p^2}$$

Substitute eqn. (3, (4), (14), (15), in eqn. (13) and consider $\frac{dI}{dt} = 0$, V = 15 kV, a = 2 cm, z = 8 cm, $\rho = 1.67 \times 10^{-5}$ gm/cm^3 b = 5.5 cm, r_p = 1.5cm, $r_s \simeq 0.4$ cm, z_f = 0.3 cm and $\gamma = 0.67$ then the time of radial collapse of current sheath $t_r \simeq 1.487$ µs

From eqn (10) (16)the focusing time = $t_a + t_r$ = 1.06 + 1.487 = 2.547 µs

The time of peak current (focusing time) $t = \frac{2\pi\sqrt{LC}}{4}$

$4t = 2\pi\sqrt{LC}$

$L = L_{switches} + L_{capacitor\ bank} + L_{cables}$

$L = 24.2$ nH

$L_{(5\ switches)} = \frac{25}{5} = 5$ nH

$L_{(14\ condensers)} = \frac{160}{14} = 11.43$ nH (from certificate), then

$L_{cables} = 7.77$ nH

60 cables, each of them has a length 1.25 mt (type RG 11/U, L = 0.377 μH/mt)

Applied the above calculations to obtain the optimum conditions to operate a plasma focus device, which has the following dimensions:

a = 1.6 cm, b = 3.3 cm and z = 31.5 cm

For $n = 10^{19}$ / cm^3 for hydrogen gas

then $\rho = 1.67 \times 10^{-5}$ gm/cm^3

c = 430 μf (- 56 condensers, each of them has a capacity 7.71 μf)

$V_{ch} = 12$ kV

magnetic energy ≃ 30.968 kJ

t_a =3μs.

t_r =5μs.

focusing time = 8 μs

total external inductance $L = \frac{(4t)^2}{4\pi^2 C} = 60.32$ nH

L_5 switches $= \frac{25}{5} = 5$ nH

L_{56} condensers $= \frac{160}{56} \simeq 3$ nH, then

$L_{cables} = 52.32$ nH

~ 25 cables, each of them has a length $\simeq$ 3.5mt
(type RG 11/U, L = 0.377 μH/mt.)
(8)

In the experiment of plasma focus , the operating parameters were: b = 2.5 cm, a = 1.7 cm, z = 59 cm, capacity of condenser bank = 15 μf, charging voltage = 16 kV and parasitic inductance = 1×10^{-7} μH.

From these parameters, the estimated optimum value of focusing time $\simeq$ 2 μs. and the plasma sheath arrives the muzzle at $t_a = 2.36$ μs then $t_{focus} < t_a$, this experiment is not matched.

DISCUSSION

Development of plasma focus device required that a maximum value of stored magnetic energy has to be delivered to plasma during the plasma focus formation, using a snowplough model (axial phase) and slug model (radial phase) to obtain the optimum conditions for operation of plasma focus device. Parameters of optimization plasma focus device are listed in table (1) and (2).

Table (1)

Inner electrode radius	2 cm
outer electrode radius	5.5 cm
length of coaxial electrodes	8 cm
gas density	1.67×10^{-5} gm/cm^{3}
bank energy	12.26 kJ
charging voltage	15 kV
capacity of condenser bank	109 μf
total external inductance	24.2 nH
focusing time	2.55 μ.s

Table (2)

Inner electrode radius	1.6 cm
outer electrode radius	3.3 cm
length of coaxial electrodes	31.5 cm
gas density	1.67×10^{-5} gm/cm^3
bank energy	30.968 kJ
charging voltage	12 kV
capacity of condenser bank	430 µf
total external inductance	60.32 nH
focusing time	8 µs

CONCLUSION

The plasma focus device with dimension a = 2 cm, b = 5.5 cm and Z = 8 cm is matched with the electrical system, when the plasma focus with density $10^{19}/cm^{3}$ formed at a time of maximum value of discharge current. This corresponding to a maximum magnetic energy of 12.26 kJ, and magnetic stored energy will be delivered to the plasma at a focusing time = 2.55 μs. where the total external inductance has to be 24.2 nH.

To match the old plasma device with dimension a = 1.6cm b = 3.3 cm and Z=31.5 cm, where plasma focus density $\simeq 10^{19}/cm^{3}$, the maximum stored energy delivered to plasma during a plasma focus formation has to be 30.968 kJ, for total external inductance = 60.32 nH and the focusing time is around 8 μs.

A plasma focus device [8], which operates at charging voltage 16 kV, capacity of condenser bank = 15 μf and total external inductance = 10^{-7} μH, is not matched because the focusing time is less than the time of arrival of current sheath to muzzle.

REFERENCES

1) Mather, J.W. (1965) Phys. fluids Suppl. 8, S 366.

2) Mather, J.W. (1972), plasma physics vol. 9 P. 187.

3) Bostick, W.H., Grunberger, L and Prior W, (1968) Phys. Soc. 13 (2), 1544

4) Patou, A. Simonmet, A. and Watteau, J.P. (1969) Le Journal de physique, 29,973.

5) Chow, S.P., Lee, S. and Tan, B.C. (1972) J. Plasma Physics Vol. 8, Part 1, PP. 21-31.

6) Lee. S, (1984) J. Fiz. Mal. 5, 135.

7) John Marshall, (1975) Proceedings of the high beta workshop. Los Alamas 87545

8) Burkhardt, L.C. and Lovberg, R.H. (1962) the Phys. of fluids Vol. 5 no. 3 P. 341.

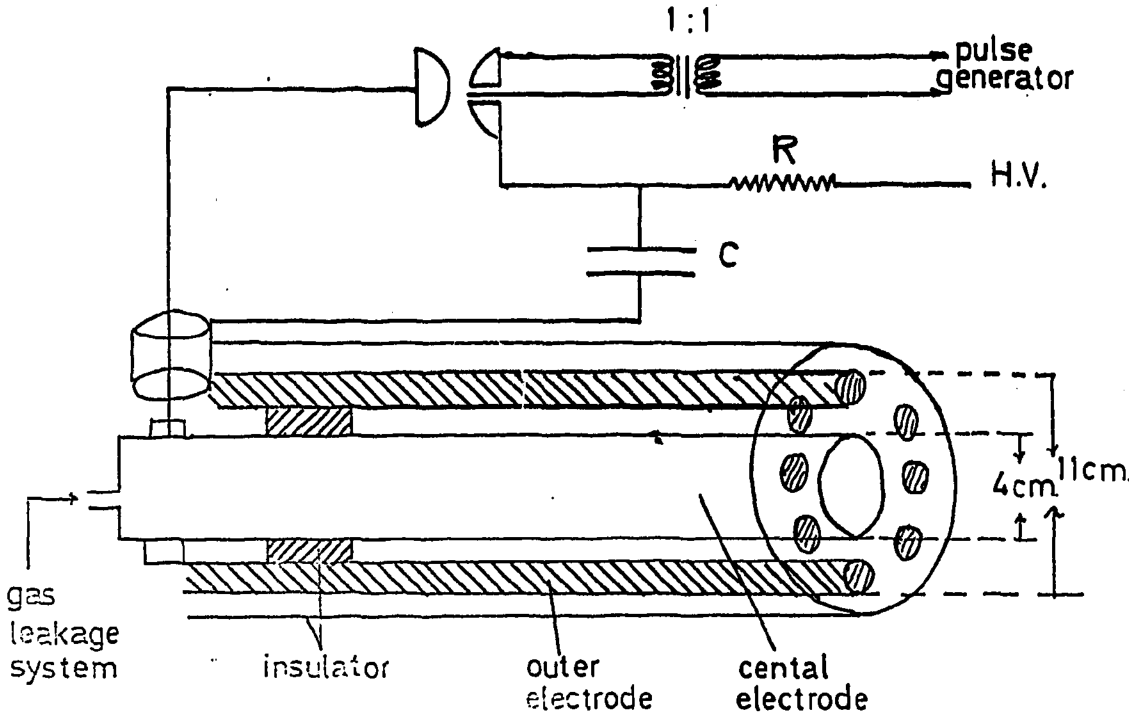

Figure (1) Schematic diagram of plasma focus device.

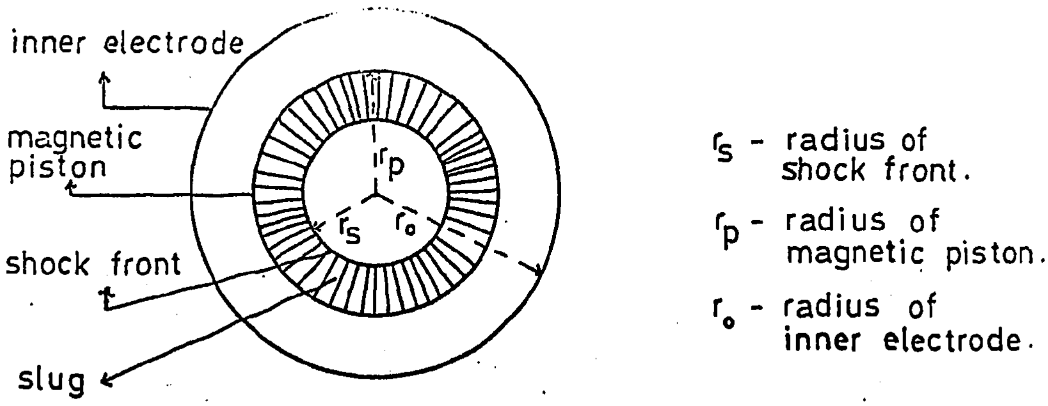

Figure (2) Cross-section of the radial compression .

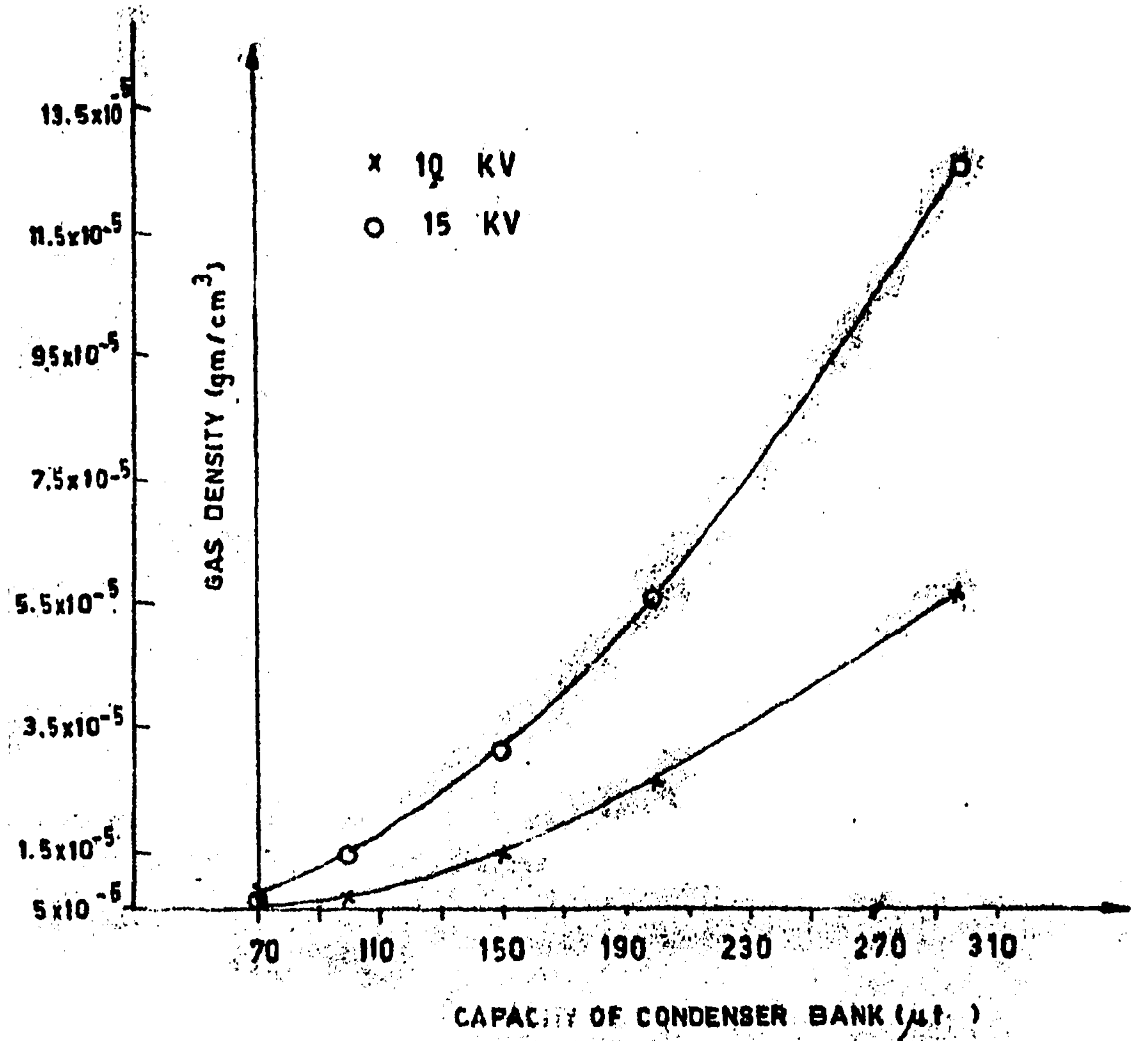

FIG (3a) Variation of Capacity of Condenser Bank with Gas Density at U_{ch} = 10 KV & 15KV

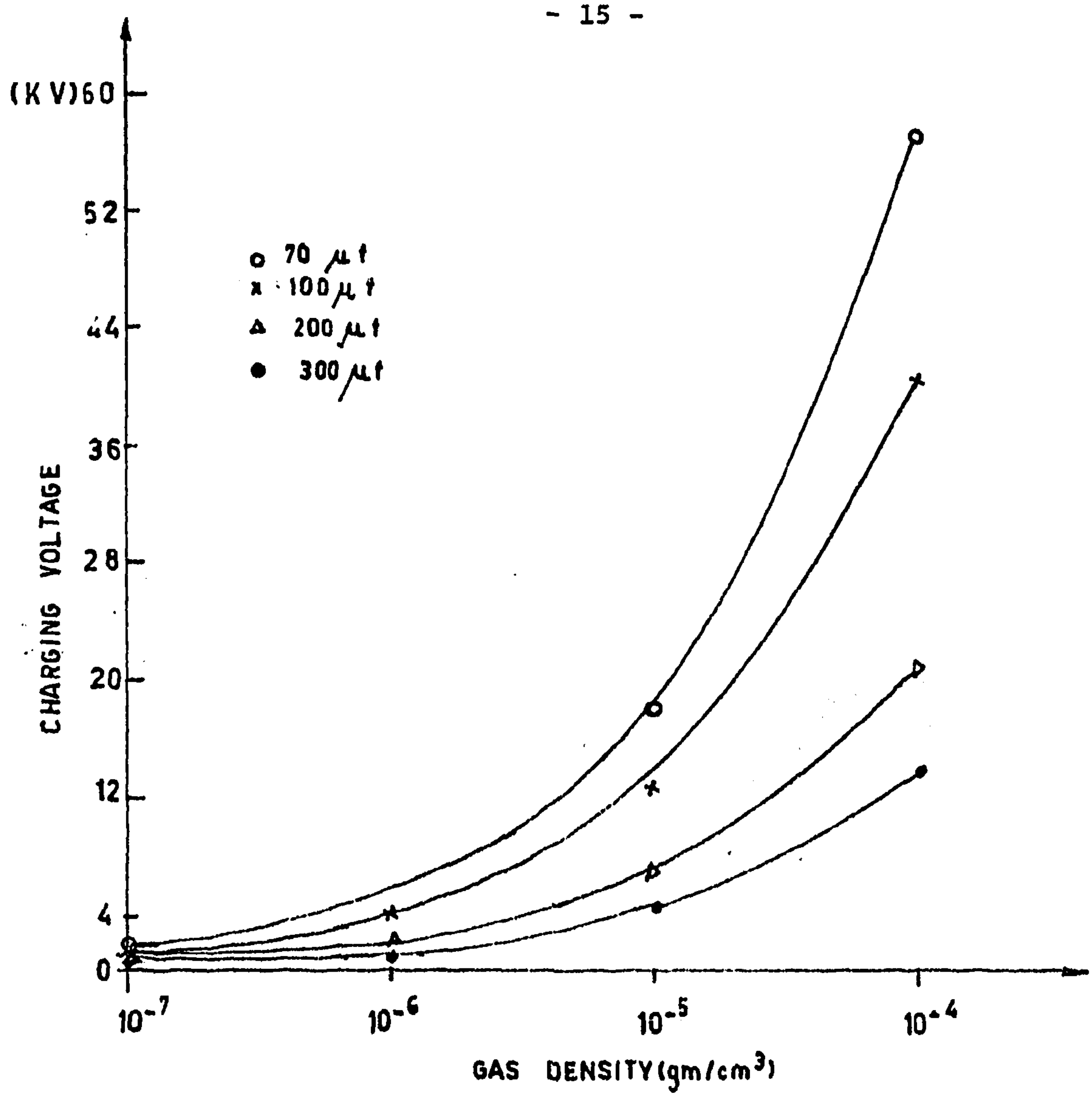

FIG.(3b) Variation of Gas Density with Charging Voltag at Different Values of Condenser Bank Capacity

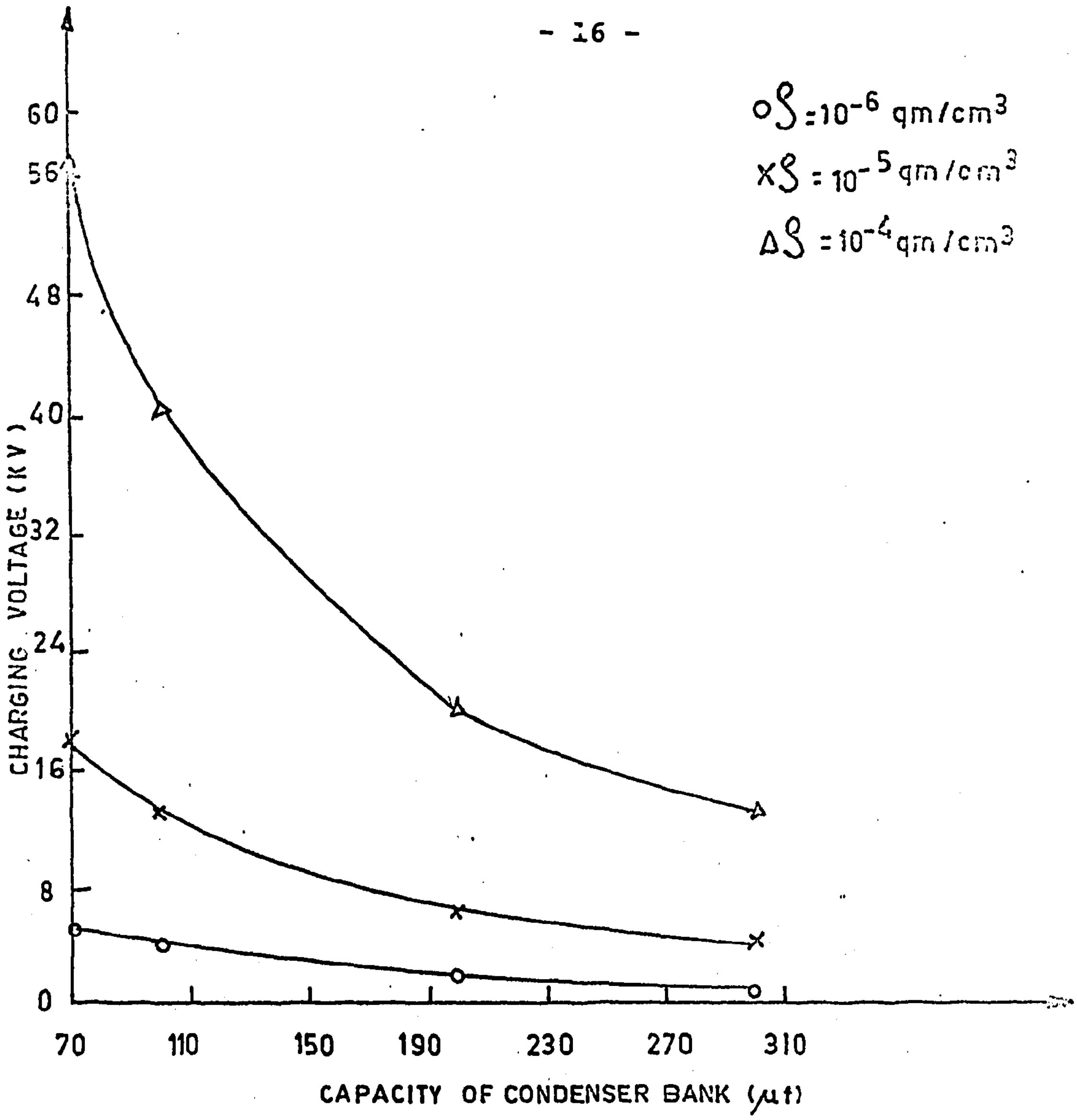

FIG.(3C) Capacity of Condenser Bank Versus Charging Voltage at different Values of Gas Density.