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ABSTRACT

We propose and solve approximately a phenomenological model for Anderson localization of the macroscopic wavefunction for an inhomogeneous superconductor quench-disordered on the mesoscale of the order of the coherence length ξ_0. Our treatment is based on the non-linear Schrödinger equation resulting from the Ginzburg-Landau free-energy functional having a spatially random coefficient representing spatial disorder of the pairing interaction. Linearization of the equation, valid close to the critical temperature T_c, or to the upper critical field H_{c_2} (T_c) maps it to the Anderson localization problem with T_c identified with the mobility edge. For the highly anisotropic high-T_c materials and thin ($2D$) films in the quantum Hall geometry, we predict windows of re-entrant superconductivity centered at integrally spaced temperature values. Our model treatment also provides a possible explanation for the critical current $J_{c\perp}$ becoming non-zero on cooling before $J_{c\parallel}$ does in some high-T_c superconductors.

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Inasmuch as a superconductor is well described by a complex order–parameter doubling as the macroscopic wavefunction having a long–range phase coherence, the question of its Anderson localization *per se* due to spatial inhomogeneity on the mesoscale becomes an apt one. Here, we are *not* concerned with the usual dirty superconductors with atomic scale, microscopic disorder [1–6] parametrized by $(1/k_F\,\ell_e)$ and tunable by resistivity that, in the weak–scattering limit ($k_F\,\ell \gg 1$), can be disposed of by the well–known Anderson Impurity Theorem [1], namely that one can re–define the one–electron bases as the eigenstates of the impurity Hamiltonian out of which to construct the Cooper pairs. We are then left with a minor parametric effect on T_c, e.g. due to the smearing out of the density of states. Weak–localization (WL) effects close to the mobility edge ($k_F\,\ell \gtrsim 1$) can also be incorporated [4] through a renormalization (reduction) of the scale–dependent diffusion constant that in turn enhances the instantaneous Coulomb pseudopotential over the retarded pairing interaction, thereby disfavouring superconductivity. (By the same token, however, in the case of a superconductor with an unretarded attraction (of electronic origin, say), the WL should effectively enhance the pairing interaction and, thereby, favour superconductivity.) There is yet another subtle interplay of microscopic disorder and interaction, namely the Al'tshuler–Aronov [5] singularity (the soft Coulomb gap) in the density of states at the Fermi–level that depresses T_c. Strong localization ($k_F\,\ell \lessapprox 1$) will, on the other hand, suppress the long–range superconducting order altogether, except possibly for the delocalization by a strong pairing potential giving an (Anderson) Insulator–to–superconductor transition. Here the superconducting coherence length ξ_0 (as $T = OK$) remains finite below the mobility edge – a quantum critical transition [7,8]. Such microscopically disordered superconductors, thin films in particular have been studied extensively in the recent years [9]. In the opposite, macroscopic, limit of the length scale of disorder, we have the case for a classical percolation in an effectively random mixture of the normal and the superconducting components. Weak superconductivity in granular systems with random Josephson couplings belongs here [9,10].

In this work, however, we are concerned with the intermediate case of an inhomogeneous superconductor disordered on a mesoscale [9,11,12] $L_0 >$ coherence length $\xi_0 \gg \lambda_F$ (Fermi–wavelength). (By the length scale of disorder here we mean the length L_0 over which the relevant quantity (the potential, say) is self–correlated.) This is the case, for example, when particles of one phase are included in the matrix of another phase, as in a nanophase composite, or when the system has extended defects like dislocations. Mesoscale inhomogeneities are ubiquitous in the high–T_c materials with weak inter–layer tunneling even when they are clean on the atomic scale. We will consider here a general model of such an inhomogeneous superconductor [11,12] with $L_0 \gtrsim \xi_0$, so that it is possible to define meaningfully the pairing interaction *locally* ($< \xi_0$) that varies randomly on the mesoscale L_0 about a configurationally defined mean value. It is then possible to describe the superconductor phenomenologically by an appropriately coarse-grained order parameter $\psi(x)$. We will treat the Anderson localization of this $\psi(x)$, rather than that of the underlying one-electron wavefunctions, and discuss its implications for superconductivity. It is important to realize that the usual localization at the level of one-electron wavefunctions in a microscopically disordered normal metal is hardly reflected in its thermodynamics – a consequence of the well known theorem due to Wegner [13] ensuring smoothness of the density of states. However, localization of $\psi(x)$ and the resulting gradients due to the relevant mesoscale disorder can off-set the condensation energy of a superconductor, and hence affect its thermodynamics appreciably – close to the critical temperature.

The dynamical as well as the statistical mechanical variation of ψ in space-time is described by the phenomenological Ginzburg-Landau free-energy functional $F[\psi]$, given as [12,14]

$$F[\psi] = \int f(\psi)\, \underset{\sim}{d},$$

with

$$f(\psi) = \alpha|\psi|^2 + \frac{\beta_0}{2}\, |\psi|^4 + \gamma_0|\nabla\psi|^2, \tag{1}$$

where $\alpha = \alpha_0(T) + \alpha_d(x)$ with $\alpha_d(x)$ random. The latter represents disorder in the pairing interaction (or, what is physically the same as the spatial randomness of the locally defined critical temperature). The parameter β_0 measures the repulsive self-coupling and γ_0 the stiffness to distortion. Both of these may reasonably be assumed independent of x. Here ψ is taken to be normalized such that $|\psi(x)|^2 = n_s$, where n_s is the superfluid electron-pair density that includes the condensate density (n_0 in the zero-momentum condensate) and also "above-the-condensate" fraction. Thus $n_s \to 0$ as $T \to T_c$ and $2n_s \to n$ (the total electron number density) as $T \to 0$. Also, in general $n_s \gg n_0$ but roughly proportional to it as $T \to T_c$.

The stationary macroscopic wavefunction then obeys the cubic non-linear Schrödinger equation obtained variationally as

$$-\gamma_0\nabla^2\psi + \beta_0|\psi|^2\psi + \alpha(x)\psi = 0\ . \tag{2}$$

Equation (2) is to be viewed as the *dynamical* equation of motion for the meanfield ψ where the time-dependent unitary phase factor $e^{i2\mu t/\hbar}$ has already been absorbed in the basic Hamiltonian (as $H - \mu N$) underlying the GL free-energy functional [14]. This allows us to set $\frac{\partial\psi}{\partial t} = 0$ on the RHS of Eq.(2) in the absence of chemical potential gradients. Indeed, Eq.(2) directly leads to the Hamiltonian equations of motion for the phase and the superfluid-electron number (the well-known Josephson equations) for the lumped case of coupled (weak-linked) superconductors, valid at all temperatures, and vice-versa [14,15]. Thus, the free-energy functional $F[\psi]$ acts as the effective Hamiltonian. Analogous situation obtains for the superfluid He II described by the Bernoulli equation for the Hartree field, the Gross–Pitaevskii equation, if one ignores 'the – above–the – condensate' part of the superfluid fraction [14].

Close to the critical temperature T_c, yet to be determined, $|\psi|$ is small and, therefore, we need to consider only the linear equation, which may be re–written as

$$-\gamma_0^2\nabla^2\psi + \alpha_1(x)\psi = -\alpha_0(T)\psi \ . \tag{3}$$

Equation (3) now describes precisely the Anderson localization problem (as quite distinct from the percolation problem) with $\alpha_1(x)$ as the random 'potential' and $-\alpha_0(T)$ as the 'energy' eigenvalue, tunable by varying the parameter T (the temperature). Recalling that $\alpha_0(T) < 0$ for $T < T_{c_0}$ (the critical temperature for the un–disordered system), it is clear that as we lower the temperature T, we will cross the 'mobility edge' at a certain $T = T_c$ that separates the extended states for $T < T_c$ from the localized states for $T > T_c$. We emphasize once more that the terms 'localized' and 'extended' here do not refer to the underlying one–electron states, but to the condensate wavefunction. This transition is readily identified as the normal (localized)–state to the superconducting (delocalized)–state transition in a bulk superconductor. We know on general thermodynamic grounds that the long–range coherence of a superconductor demands extended–ψ states: Localized states are essentially finite–sized systems subject to large thermal fluctuations. The T_c (the 'mobility edge') will, of course, depend on disorder, and in general decrease monotonically with increasing disorder (corresponding to the 'mobility edge' moving into the 'extended' band).

Thus T_c degrades with disorder. However, the transition remains sharp in the sense of not being merely a cross–over, because in three dimensions ($d = 3$) the localization length $L_c(T) \sim (T_c - T)^{-\nu}$ with $\nu \approx 1$ (and, in any case, $\nu \geq 2/3$) while the meanfield superconducting correlation length $\xi(T) \sim (T_c - T)^{-1/2}$ implying $L_c(T)/\xi(T) \to \infty$ as $T \to T_c$. This has an interesting consequence for the layered, highly anisotropic high–T_c superconductors. As is well known, despite spatial anisotropy, the mobility edge is unique, and the localization lengths, though anisotropic with $L_{c\perp}$ (perpendicular to the ab–planes) $< L_{c\|}$ (parallel to the ab–planes) because of weak–interplanar tunneling matrix elements, both diverge with the same exponent $\nu \simeq 1$ as T_c is approached from above ($T > T_c$). Now, given that $\xi_\perp(T) \ll \xi(T)_\|$ and that both diverge as we approach criticality (T_c) with an index $1/2 < \nu \simeq 1$, and hence slower than $L_{c\perp}$ and $L_{c\|}$, we expect the situation for $T \gtrsim T_c$ where $L_{c\perp} > \xi_\perp(T)$ but $L_{c\|} < \xi_\|(T)$. This would show up as a non–zero transport critical current $J_{c\perp}(T) \neq 0$ along the c–axis while $J_{c\|}(T) = 0$ for a certain temperature range $T - T_c \gtrsim 0$, as recently reported [16]. This is essentially a precursor effect – true bulk superconductivity occurs only for $T < T_c$.

The above picture of condensate–localization has an interesting consequence in the presence of an applied magnetic $\underset{\sim}{B}$ ($\lesssim B_{c_2\perp}(T)$ the upper critical field) perpendicular to a planar (quasi-$2d$) superconductor – a thin film, or a stack of HTSC bilayers with the c–axis normal to the film. Introducing the field through the usual local gauge transformation $\underset{\sim}{\nabla} \to \underset{\sim}{\nabla} - \frac{i2e}{\hbar c}\underset{\sim}{A}$ with $\nabla \times A = \underset{\sim}{B}$, we get from Eq.(3) for this quantum Hall geometry

$$-\gamma_0 \left(\underset{\sim}{\nabla} - \frac{i2e}{\hbar c}\underset{\sim}{A}\right)^2 \psi + \alpha_1(x)\psi = -\alpha_0(T)\psi, \tag{4}$$

where again linearization is valid as B is close to $B_{c_2}(T)$. For a uniform field, Eq.(4) just describes formally the 'Integral Quantum Hall Effect' in the presence of disorder, with an altered meaning though. It is well known from IQHE, that despite disorder and 2–dimensionality, the middle of each Landau sub–band contains an energy interval of extended states that eventually make the IQHE plateaus observable and understandable [17]. In the present case this translates into the prediction that as we lower the temperature T (analogue of energy parameter) below $T_c(B)$, we pass through temperature intervals of superconducting (extended) states separated by wide intervals of the normal (localized) states. This should give multiply–re–entrant–superconducting (MR) oscillations! For a weak mesoscale disorder we can readily estimate the temperature interval ΔT in terms of the GL parameters as $\left(\frac{2\pi B}{\phi_0}\right)\frac{\gamma_0}{\left(\frac{d\alpha_0}{dT}\right)_{T_c}} \simeq \left(\frac{2\pi B}{\phi_0}\right)\xi_{0\perp}^2 T_c = \left(\frac{BT_c}{B_{c2\perp}(0)}\right)$ which should be observable experimentally. This may provide a temperature–interval calibration of good precision.

The main purpose of this work is to point out the possibility and plausibility of reformulating the problem of inhomogeneous superconductor with mesoscale disorder directly as a localization problem for the macroscopic superconducting wavefunction within the GL phenomenology. Such a disordered superconductor may be microscopically in the clean limit and it is then not relevant to speak of localization of the one–electron states. However, one can take full advantage of the well established theory of Anderson localization, parametrized now with the GL parameters for an inhomogeneous superconductor. It is to be noted that in the present treatment of the effect of disorder on the macroscopic wavefunction ψ, we take the latter to be the coherent meanfield amplitude of the condensate, ignoring the thermal fluctuations that have relaxational dynamics making γ_0 complex [18]. This is the analogue of the inelastic scattering (dephasing) in the usual localization problem. One expects these fluctuations to be slow when not small as very close to T_c within the Ginzburg critical regime. Our phenomenological approach is somewhat complementary to that of Bulaevskii *et al.* [12], who used the replica trick to do statistical mechanics with the GL free–energy functional containing disorder in the pairing interaction (i.e. $\alpha(x)$). They obtain a phase of superconducting droplets that should correspond to our regime $T_c < T < T_{c_0}$ of localized superconducting order parameter. (In point of fact, the usual mapping of a superconductor onto an XY–model will, in the present case, give a random XY–model. Indeed, for $\alpha_0 = 0$ (i.e. $T = T_{c_0}$) a pure spin–glass model with $\alpha_1(x)$ playing the role of randomly signed local coupling results. However, for $T < T_{c_0}$, one has a spin–glass model with 'ferromagnetic' bias and the conventional LRO should develop at sufficiently low temperatures.

In conclusion, we have proposed Anderson–localization of the condensate wavefunction *per se* for a superconductor disordered on mesocales. Our treatment, based on the GL phenomenology predicts multiply–re–entrant superconducting (MRS) oscillations as the temperature is lowered through $T_c(B)$ for a quasi–$2d$ superconductor cooled in a perpendicular field B.

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