RESIDUAL MASS CONSIDERATIONS IN MODAL ANALYSIS OF LARGE DYNAMIC STRUCTURAL SYSTEMS

Jeffrey S. Shulman, James P. Day
ADVENT Engineering Services, Inc.
3 Crow Canyon Court
Suite 100
San Ramon, CA 94583

ABSTRACT

Industry guidelines have specified that the seismic evaluation of Moderate and High Hazard Department of Energy (DOE) facilities be accomplished by use of dynamic analysis. The recommended approach is elastic response spectrum dynamic analysis to evaluate the elastic system demand on facility components. The application of modal response spectrum analysis to the seismic evaluation of nuclear facility structures, systems and equipment involves approximations due to limitations on the number of modes typically addressed in the complete dynamic solution.

A simplified approach for achieving improved rigor in accounting for responses of the higher frequency modes in a modal response spectrum analysis is demonstrated.

INTRODUCTION

UCRL-15910 [1] specifies that the seismic evaluation of Moderate and High Hazard DOE facilities be accomplished by use of dynamic analysis. It recommends that elastic response spectrum dynamic analysis be used to evaluate the elastic system demand on facility components. In the mode-by-mode dynamic analysis of seismic response, only those limited number of modes with frequencies below some cutoff frequency are typically addressed in the dynamic solution. Since its initial issue, Section 3.7.2 "Seismic System Analysis" of NUREG-0800 [2] has required that sufficient modes be included in a dynamic response analysis to ensure that the inclusion of additional modes does not result in more than a 10 percent increase in response. Various methods, which have been applied over the years with the intent of meeting or exceeding this convergence requirement, include:

- Trial-and-error procedures wherein the total response is computed for increasing cutoff frequencies until convergence is obtained.
- A total modal mass procedure in which the cutoff frequency is determined so that the total modal mass considered in the response calculations is at least 90% of the total system mass.
- A high cutoff frequency procedure wherein an arbitrary, high frequency (e.g. 100 Hz), well beyond the lowest frequency at which the response spectrum returns approximately to the zero-period acceleration (ZPA frequency, typically 33 Hz as for Regulatory Guide 1.60 [3] design response spectra), is selected.

- More recently, pseudo-static analysis procedures wherein response to the inertial forces associated with mass not participating in the (flexible) modes below the cutoff frequency are combined with the modal dynamic results for all modes through the cutoff frequency.

The trial-and-error procedures are inefficient, time-consuming and accepting of something (although quantifiable) less than a complete solution. Additionally, depending upon the mode number at which "satisfactory" convergence is determined, such procedures introduce potential inaccuracies associated with the higher mode predictions of the eigenvalue/eigenvector solution for systems with discretized inertia representations (e.g. mass lumping).

Procedures which address the dynamic solution through a cutoff frequency determined such that the total modal mass considered in the response calculation is above some fraction of the total system mass are also inefficient, time-consuming and accepting of something less than a complete solution. These procedures also introduce potential inaccuracies associated with the higher mode predictions of the eigenvalue/eigenvector solution. More significantly, perhaps, such procedures address only total modal mass (a scalar) as a measure of convergence and are silent on the quality of the modal mass distribution. Therefore, they do not allow for a true assessment of the convergence of the response determination.

Procedures which are based on the dynamic solution through an arbitrary, high cutoff frequency, well beyond the ZPA frequency, are also inefficient, time-consuming and accepting of something less than a complete solution. They also introduce potential inaccuracies associated with the higher mode predictions of the eigenvalue/eigenvector solution. Such procedures are not specifically responsive to a quantified convergence criteria.

With Revision 2 to Section 3.7.2 "Seismic System Analysis" [2] and its Appendix A "Acceptable Methods to Account for High-Frequency Modes," the Standard Review Plan (SRP) for the first time addressed concern for significant response which may be associated with modes having frequencies above the ZPA frequencies and proposed pseudo-static analysis procedures to address these contributions. Parallel considerations are manifested in the guidelines of UCRL-15910 [1] via its reference to ASCE Standard ASCE 4-86 [4]. ASCE 4-86 [5] actually goes beyond the SRP guidelines in specifically including the so-called "residual mass" associated with all higher frequencies above the cutoff frequency.

A theoretically rigorous combination of the high-frequency modes having frequencies beyond the ZPA frequency should be by the Algebraic Sum Method [2]. The application of the Square-Root-of-the-Sum-of-the-Squares (SRSS) combination to such high frequency modes is highly inaccurate and could present significant unconservatisms. The SRSS method assumes random phasing of modal responses at the time of peak response. While this assumption has been shown to be adequate throughout the majority of the frequency range for earthquake-type responses, higher frequency modes are all nearly in-phase and therefore should be combined by Algebraic Summation. At these frequencies, the seismic input motion does not contain significant energy content and the structure simply responds to the inertial forces from the peak zero period acceleration in a pseudo-static fashion. The phasing of the maximum response from modes at these high frequencies will be essentially deterministic and in accordance with this pseudo-static response to the peak ZPA. Consequently, theory would suggest that for all procedures discussed, the Algebraic Sum Method should be used to combine all modes with frequencies above the ZPA frequency. This is, for the most part, an academic issue where incomplete superposition of the modes already corrupts what is essentially a series solution.

The basic approach commonly recommended for consideration of these high frequency modal responses is to perform the modal extraction up to some predetermined cutoff frequency (preferably the ZPA frequency). Standard, yet still somewhat controversial, industry techniques are applied for combining (flexible) modes up to and including the cutoff frequency. The combined effects of the flexible modes and the effects of residual mass not participating in the modes extracted are superimposed by SRSS to predict the total response. The normally-recommended procedures for addressing the effects of residual mass prescribe the development of a residual mass matrix for use in pseudo-static analyses for accelerations corresponding to the spectral accelerations at the cutoff frequency (preferably the ZPA frequency). Both the SRP [2] as well as the ASCE 4-86 [4] introduce the concept of fractional mass (for each dynamic degree of freedom) not included in the modes summed through the cutoff frequency mass, thereby associated with the higher modes. The approach requires some mode-by-mode bookkeeping which presently is not available in many of the computer programs currently used to perform seismic analysis. In addition to the bookkeeping, the static analysis used to account for the unincluded fractional mass requires the application of nodal forces representing the corresponding pseudo-static inertial forces. The bookkeeping and analysis input becomes more involved with the existence of off-diagonal terms in the dynamic system mass matrix, introduced by consistent-mass formulations or reduction techniques such as Guyan reduction. Consequently, in spite of very clear regulatory guidance regarding the proper consideration of higher-mode response, a large sector of the industry still proceeds with awkward and/or approximate alternatives to what is an elegantly simple and exact method.

THEORETICAL BACKGROUND

A much simplified approach to addressing the residual mass contributions of the high-frequency modes was initially presented by Powell [6]. This approach circumvents the need for customized bookkeeping and relies on a straightforward body-force analysis to account for the "higher frequency mass." It requires no more than the most basic postprocessing capability offered by virtually all general purpose finite element programs.

Consider the following expression for the inertia loads developed in a "rigid" system:

$$\{F_R\}_i = [M]\,\{S_{ao}\}_i \qquad (1)$$

where $\{F_R\}_i$ is the vector of loads; $\{R_R\}_i$ is the vector of responses (e.g. displacement, stress, acceleration); [M] is the system mass matrix and $\{S_{ao}\}$ is an acceleration vector having constant values S_{ao} for each dynamic degree of freedom in the ith direction of input motion and zero values for all other dynamic degrees of freedom. Fundamental to the approach is the understanding that for such a rigid system, a constant acceleration (S_{ao}) response spectra analysis, with algebraic summation of all the modal responses, renders identical results, expressed as:

$$\{F_R\}_i = \sum_{n=1}^{N} \{F_n\}_i \qquad (2)$$

$$\left(\{R_R\}_i = \sum_{n=1}^{N} \{R_n\}_i\right) \qquad (3)$$

N =	total number of dynamic degrees of freedom of dynamic system
$\{F_n\}_i$ =	load vector in mode n due to acceleration S_{ao} in the ith direction of input.
$\{R_n\}_i$=	response vector in mode n due to acceleration S_{ao} in the ith direction of input.

Therefore, for a ZPA cutoff frequency corresponding to mode M (M≤N) of the system, the contribution of the higher frequency modes (beyond the cutoff frequency) can be determined as:

$$\{F\}_i = \{F_R\}_i - \sum_{n=1}^{M} \{F_n\}_i \qquad (4)$$

$$\left(\{R\}_i = \{R_R\}_i - \sum_{n=1}^{M} \{R_n\}_i\right) \qquad (5)$$

Where:

$$\sum_{n=1}^{M} \{F_n\}_i \text{ and } \sum_{n=1}^{M} \{R_n\}_i \qquad (6)$$

are algebraic summations operating on the "flexible" modes up to and including the cutoff frequency for spectral input characterized by a constant spectral acceleration equal to the ZPA in the ith direction of seismic excitation.

APPLICATION

The issue of potential dynamic coupling between structural systems has received much attention in nuclear facility seismic design. Nuclear facility structures and components have been approximated by mathematical models to permit analysis of responses to seismic excitation. Historically, the large number of degrees of freedom that would be necessary and the possible ill-conditioning of the resulting stiffness matrix associated with a combined, single mathematical model of several connected components encouraged the use of separate component models for analysis. Thus, one usually finds one or more so-called primary structural models, each of which supports one or more secondary models. Different models of the same structure would be required for different purposes. For example, models used to generate the seismic excitation input for subsequent separate analyses of secondary components (systems) may not be suitable for the detailed localized analysis of the primary structure (system). ASCE 4-86 [4] as well as SRP 3.7.2 [2] provides some guidelines relating to dynamic coupling criteria, albeit primarily in applications where the secondary system(s) are connected to the primary system at a single point. The criteria operate on parameters such as secondary system to primary system frequency and mass ratios. ASCE 4-86 [4] goes on to state that for multipoint attachment of a secondary system to the primary system, the stiffness of the subsystem may restrict movement of the primary system. Therefore, in addition to mass ratio considerations, the relative stiffness of the subsystem to structure should be investigated. The Standard admits that, at present, no criteria are available to determine when coupling is required for this situation.

In addition to the computational benefits that can be gained by the optimum treatment of dynamic coupling in the seismic analysis of nuclear facility components (systems), it is particularly beneficial in evolving design situations, wherein the individual components may each be undergoing design iterations pursuant to satisfying their own or mutual design/performance requirements. The design of the three major structural components of the SP-100 Ground Engineering System (GES) Test Site, i.e. the Nuclear Test Assembly (NAT), the vacuum vessel and the vacuum vessel support structure, was a classic example of this interdependence. Design responsibilities for these three major components rested with different organizations. Westinghouse Hanford Company (WHC), with design responsibility for the vacuum vessel, identified the need for an analysis strategy which:

- Clearly defined the interfaces between the three design organizations.
- Minimized organizational interfaces while, at the same time, optimized efficiency in interactive analyses accompanying design evolution.
- Provided justifiable simplifications in the ongoing seismic analysis activities.

The development of such a strategy is very much dependent on the structural dynamic (mass and stiffness) characteristics of the individual components, in an absolute as well as in a relative sense. Such a strategy must account for the other constraints which may be, if only implicitly, affecting these characteristics. Constraints designed to completely eliminate dynamic coupling as a parameter in the design evolution often result in greater burdens on the design process, including impact on more important component performance goals and material costs.

A detailed description of the vacuum vessel was included in a recent paper presented at the 1991 Pressure Vessels and Piping Conference [7]. A schematic representation of the system consisting of the three major structural components, identifying their interfaces, is shown in Figure 1.

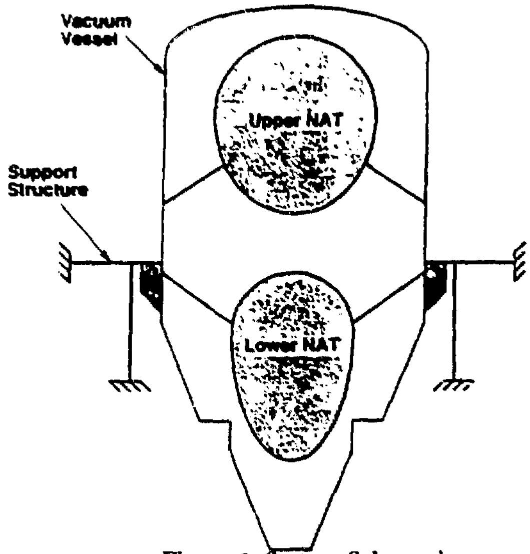

Figure 1. System Schematic

Seismic studies were performed to address the potential for decoupling of the upper and lower NAT internals from the vacuum vessel/vessel support structure. The demonstration of such decoupled behavior would:

- Allow for simplified consideration of the NAT in the seismic analysis of the vacuum vessel. Therefore, seismic response of the vessel would be invariant under those NAT modifications which left the NAT mass and "static interaction" stiffness essentially unchanged.

- Allow for seismic analysis of the NAT for seismic input that would be invariant under those NAT modifications which left the NAT mass and "static interaction" stiffness essentially unchanged. This could be a significant advantage in analysis if the NAT analyst chooses to analyze a "fixed-base" NAT model using amplified response spectra computed by the vessel analyst at the NAT support locations. Alternatively, the NAT analyst could choose to use a coupled NAT/vacuum vessel/vacuum vessel support structure model together with the response spectra specified at the base of the support structure. In any case, design iterations on the NAT, which do not significantly effect its mass, could proceed without concern for the effect of these iterations on the seismic design basis for the NAT.

Neither the NAT internals nor the supported vacuum vessel have mass which can be considered insignificant relative to the other. Consequently, investigation of the viability of decoupling a subsystem from its primary system required demonstration that a simplified primary system model, reduced to the extent that the secondary system was represented only in terms of its mass and its potential for static (stiffness) interaction with the primary system, could accurately predict amplified seismic response at the attachment points for the secondary system.

Seismic response spectra analyses were performed utilizing SSE response spectra defined at the base of the vessel support structure as shown in Figures 2 and 3 for the horizontal and vertical directions respectively.

Two analyses were performed using response spectrum modal analysis applied to the combined model of the NAT, vacuum vessel and vacuum vessel support structure. In the first analysis (Model 1), master (or dynamic) degrees of freedom were selected throughout the combined model including appropriate locations internal to the NAT assembly. Consequently, this first analysis would explicitly account for the structural dynamics of the NAT and therefore address potential dynamic interactions between the NAT and the vacuum vessel/vacuum vessel support structure system. In the second analysis (Model 2), master degrees of freedom were selected throughout the combined model excluding locations internal to the NAT assembly. This second reduced analysis essentially addressed a simplified primary system (vessel and vacuum vessel support structure) which neglects dynamic modes associated with the NAT internals but which considers the mass and static stiffness of the NAT. In both analyses, master degrees of freedom were retained for all degrees of freedom corresponding to the locations of attachment and translational directions of restraint provided by the vacuum vessel in supporting the upper and lower NAT internals.

The methodology employed in determining the degree of NAT coupling is based on evaluating the difference in the resulting maximum absolute accelerations as predicted by Models 1 and 2 at and in the direction of vacuum vessel support for the upper and lower NAT internals. In both analyses, all modes with frequencies below 30 Hz were extracted. For each such flexible mode, designated by mode number i, a reduced eigenvector, $\{\phi_i\}$, characterizing displacements for the master degrees of freedom is computed. For each mode, a mode coefficient, A_i is computed as follows:

$$A_i = S_i \Gamma_i / \Omega_i^2 \qquad (7)$$

Where S_i = spectral acceleration at mode i
Γ_i = participation factor for mode i
Ω_i = natural circular frequency of mode i

The participation factor, Γ_i, is defined as:

$$\Gamma_i = \{\phi_i\}^T [M] \{D\} \qquad (8)$$

Where $[M]$ = reduced mass matrix
$\{D\}$ = a unit vector describing the spectral excitation direction

The absolute acceleration of the master degrees of freedom, in mode i, could be computed as:

$$\{a_i\} = \Omega_i^2 A_i \{\phi_i\} \qquad (9)$$

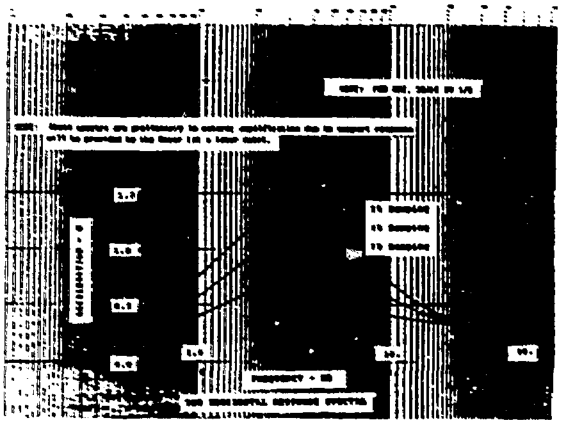

Figure 2. SSE Horizontal Response Spectra

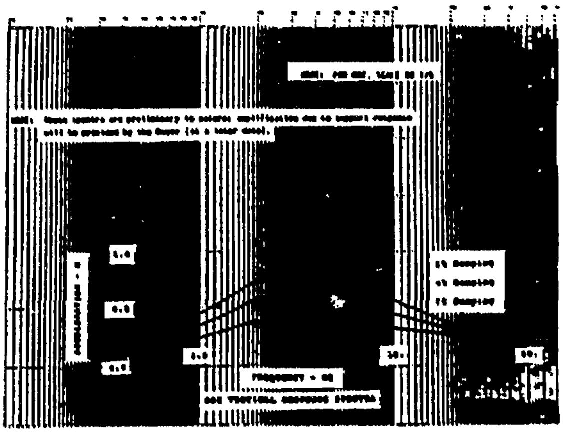

Figure 3. SSE Vertical Response Spectra

Guidance for combining modal responses to predict resulting maximum seismic responses is provided by ASCE 4-86 [4] as well as Regulatory Guide 1.92 [5]. Some debate regarding the combination of modal combinations persists, particularly in application to the flexible modes of the dynamic system, and has been well documented in Volume 4 of NUREG 1061 [6].

A frequent pitfall of comparisons between combined results of response spectrum analyses, performed on models with relatively minor differences, is that relatively small frequency shifts can dramatically affect the level of conservatism in the modal combination approach, particularly in addressing total absolute acceleration contributions of potentially closely-spaced, high frequency modes. In effect, the differences are associated more with unquantifiable dogma than true variations in dynamic response. While uncertainty regarding the conservatism of combined modal response spectrum analysis results is inevitable, the rigorous treatment of the high-frequency modes at least minimized "extraneous" potential inconsistencies.

The following sequence of analysis and postprocessing was performed for Model 1 and 2:

1) Three-dimensional response spectra modal analyses were performed addressing three directions of seismic excitation, for:

a. Design Basis Earthquake (DBE) spectra defined by Figures 2 and 3.

and b. spectra, in all three directions, defined by a constant spectral acceleration, for all frequencies corresponding to the ZPA of the DBE spectrum (a. above) for each direction, respectively.

The modal extractions and expansions were performed for all modes up and including 30 Hz (approximately the ZPA frequency).

2) Mode-by-mode absolute accelerations at the NAT attachment points' master degrees of freedom were computed for

a. Modal responses as per 1) a. and

b. Modal responses as per 1) b.

3) Mode-by-mode absolute accelerations at the NAT attachment points' master degrees of freedom were combined by

a. The simplified Gupta Method [4] for the responses of 2) a.

b. Algebraic Summation for the response of 2) b.

4) The results of 3)b. were subtracted from the acceleration values of 1)b., for each direction of seismic excitation.

5) The results of 4) were added to those of 3)a. and then combined across the three directions of seismic excitation.

The differences in the resultant accelerations, in each of three directions, at the NAT attachment points as predicted by Model 1 and Model 2 were compared to the appropriate ZPA. A small difference relative to ZPA was considered to be sufficient demonstration of decoupling.

Table 1 presents the predictions of the resultant accelerations at the points of attachment of the NAT to the vacuum vessel for comparison. By comparing the acceleration resultants predicted by Model 1 and Model 2, the largest difference was seen to be 0.785 in/sec^2. This maximum difference is insignificant relative to the ZPA (0.88% of the y-direction ZPA, 0.63% of the x-direction and z-direction ZPA) indicating that the NAT dynamics,

TABLE 1
Resultant Nodal Accelerations
(SSE Response Spectra)
Units: in/sec^2

Location	Node	a_x			a_y			a_z		
		Model			Model			Model		
		1	2	1-2	1	2	1-2	1	2	1-2
Upper NAT Attach Points	2231	131.041	130.636	0.405	96.248	96.151	0.097	123.923	123.694	0.229
	2234	127.693	127.582	0.111	94.191	94.061	0.130	125.301	124.883	0.418
	2237	127.834	127.601	0.233	89.225	89.117	0.108	124.385	124.033	0.352
	2240	127.430	127.106	0.324	84.159	84.115	0.044	122.682	122.364	0.318
	2243	130.245	129.965	0.280	81.681	81.750	(0.069)	123.817	123.605	0.212
	2246	127.664	127.258	0.406	84.190	84.271	(0.081)	124.703	124.669	0.034
	2249	127.878	127.539	0.339	89.325	89.376	(0.051)	123.876	123.532	0.344
	2252	127.563	127.045	0.518	94.076	94.102	(0.026)	122.336	122.059	0.277
Lower NAT Attach Points	515	137.685	137.129	0.556	95.413	95.269	0.144	124.158	123.970	0.188
	516	141.948	141.163	0.785	89.169	89.105	0.064	124.233	124.023	0.210
	517	137.685	137.141	0.544	82.572	82.692	(0.120)	124.173	123.966	0.207
	518	142.040	141.279	0.761	89.247	89.262	(0.015)	124.198	123.998	0.200

which are considered only by Model 1, are inconsequential to the response of the vacuum vessel and to the forcing function delivered to the NAT through the NAT attachment points.

The demonstrated decoupling of the NAT from the vacuum vessel allowed for the following:

- In predicting seismic response of the vacuum vessel/vessel support structure, one need only account for the mass of the NAT and not its internal dynamics, i.e., Model 2 is an adequate predictor of vacuum vessel/vessel support response. The advantage gained is that changes to the NAT, other than to its mass, do not affect the vacuum vessel or vessel support seismic analyses.

- Seismic response of the NAT can be determined from a fixed-support NAT model, neglecting the vacuum vessel and vessel support structure, provided that seismic input is determined in an appropriate manner. This input would change only if the NAT mass changed appreciably, or possibly, if the vacuum vessel or vessel support structural dynamics changed.

CONCLUSIONS

A simplified approach to addressing the residual mass contributions of the high-frequency modes of a dynamic structural system has been demonstrated in a particular application regarding decoupling criteria. The method requires no more than the most basic postprocessing offered by virtually all commercially available general purpose finite element programs.

REFERENCES

[1] Kennedy, R.P., et al, "Design and Evaluation Guidelines for Department of Energy Facilities Subjected to Natural Phenomena Hazards," Report No. UCRL-15910, prepared for U.S. Department of Energy, June, 1990.

[2] U.S. Nuclear Regulatory Commission, "Standard Review Plan for the Review of Safety Analysis Reports for Nuclear Power Plants," NUREG-0800, Section 3.7.2, Rev. 2; Appendix A, Rev. 0; August, 1989.

[3] U.S. Nuclear Regulatory Commission Regulatory Guide 1.60, "Design Response Spectra for Nuclear Power Plants, December, 1973.

[4] ASCE Standard 4-86, "Seismic Analysis of Safety-Related Nuclear Structures," September, 1986.

[5] U.S. Nuclear Regulatory Commission Regulatory Guide 1.92, "Combining Modal Responses and Spatial Components in Seismic Response Analysis," February, 1976.

[6] U.S. Nuclear Regulatory Commission "Report of the U.S. Nuclear Regulatory Commission Piping Review Committee, Volume 4. Evaluation of Other Dynamic Loads and Load Combinations," December, 1984.

[7] M.S. Shurrab and H.H. Ziada, "Buckling Analysis of a Complex Vacuum Vessel Using the Finite-Element Method," PVP-Vol. 214, DOE Facilities Programs and Systems Interaction with Linear and Non-Linear Techniques, June, 1991.