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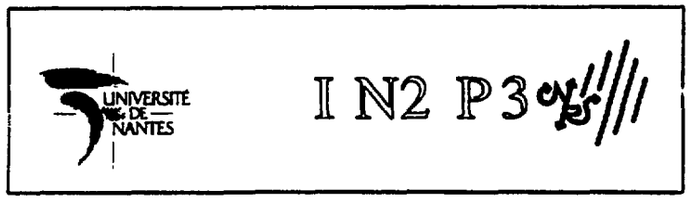

From Stochastic phase space evolution to Brownian motion in collective space

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Rapport Interne LPN - 93-08

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Abstract :
Within the framework of stochastic transport equations in phase space, we study the dynamics of fluctuations on collective variables in homogeneous fermion systems. The transport coefficients are formally deduced in the relaxation time approximation and a general method to compute dynamically the dispersions of collective observables is proposed as a set of coupled equations : respectively, the BUU/Landau-Vlasov equation for the average phase space trajectories and the equations for the averages and dispersions of the observables. Independently, we derive the general covariance matrix of phase space fluctuations and then by projection, the dispersion on collective variables at equilibrium. Detailed numerical applications of the formalism are given ; they show that dynamics of fluctuations can be extracted from noisy numerical simulations and that the leading parameter for collective fluctuations is the excitation energy whatever is its degree of thermalization.

1. Introduction

The time evolution of complex systems, as colliding heavy ions, is governed by a competition between the mean field and many-body correlations. During the last ten years, much progress has been done with semi-classical models like Landau-Vlasov [1] and Boltzmann - Uehling - Uhlenbeck (BUU) [2] (for a review see for example ref. 3). Such models provide only the average trajectory followed by the system in the phase space. These models have shown their good predictive power for the collective observables and their ability to link the experimental results in heavy ion reactions to fundamental properties of nuclear matter and nuclear effective interactions[4]. However, the situation is less satisfactory that it seems ; the availability of 4π detectors with high granularity has given access to an event-by-event analysis of nuclear reactions and then to the dispersion of the collective variables around their average values ; when the beam energy increases there

are clear evidences -disappearance of the fusion/fission or fusion/evaporation process, intermediate mass fragment emissions- that the nuclear system enters critical regimes where applications based on average one-body theories are meaningless.

These features have recently pushed forward attempts to treat the dynamics of fluctuations ; all different approaches are linked to the so-called Boltzmann-Langevin approach which has been pioneered out by Bixon and Zwanzig [5] for the hydrodynamical fluctuations of classical systems. The fluctuating part of the system dynamics arises from the loss of information occurring when one restricts its description to the one-body level. To a given one-body density distribution corresponds an infinite set of N-body distributions which will evolve differently under the high-order correlated contributions of the particle interactions.

As shown, by Bixon and Zwanzig [5], and then by Ayik and Grégoire [6], the equation of motion for the one-body density is a generalized Langevin equation, the average of which is the Boltzmann (or its quantum extensions) equation. The projected-out terms act as a random force whose time and space correlations can be connected with the properties of the collision kernel.

In ref. 7), Randrup and Remaud have developed a transport theory for treating the stochastic one-body dynamics via a Fokker-Planck equation. This formalism, conceptually close to Ayik and Grégoire's, has been shown to be amenable to detailed numerical simulations[8] in the case of two-dimension systems.

One of the main practical difficulty of these theories remains their application to the dynamics of actual systems. The Landau-Vlasov and BUU models applied to finite, quantum systems as nuclei give rise to specific methods based on the discretization of the one-body phase space in randomly distributed cells : the pseudo-particles. As can be shown with a high degree of accuracy [9], the average nuclear dynamics can be described through the quasi-classical dynamics of the pseudo-particles, provided that proper smearings of the mean field and scalings of nucleon-nucleon cross sections are performed. The pseudo-particle method introduces a specific numerical and unphysical noise which depends on the number of pseudo-particles treated in the simulation; thus, the fluctuations of the physical system cannot be directly extracted from the simulation [7,8].

This paper aims at showing that the dispersion on the collective variables can be actually computed from the BUU or Landau-Vlasov simulations provided that they yield the correct average transport coefficients in phase space such as relaxation times and diffusion coefficients. Some recently published papers [8,10] compute numerically the covariance matrix of the whole phase space, we notice that this procedure used for two-dimension systems is difficult to extend to realistic situations. The previous publications also suffer for a lack of unambiguous

comparisons with analytical predictions which could probe their numerical validity. This paper has then two purposes : to derive an effective method to track the dispersion of collective variables in numerical simulations and to check its validity by comparison with analytical values of thermal fluctuations at equilibrium. The last objective is based on Van Kampen's approach, whose pioneering book [11] provides the bases in the case of classical gases. We shall make an extension to fermion systems and then shall devote some pages to a general survey of the theory in order to provide the reader with a self-sustaining presentation.

Our study deals with the statistical fluctuations in Fermi systems and then is strongly correlated with the well documented Landau theory of Fermi Liquids [12]. However, we consider systems with high excitation energies, where the elementary excitations cannot be described as quasi-particle distributions close to the Fermi energy, we then explicitely treat excitations in real particle ensembles and expect to recover the results of the Landau theory[13,14] at the limiting case of systems close to equilibrium and at low temperature. At variance with Pethick and Ravenhall [14], we do not consider systems unstable to density fluctuations and then we always assume uniform spatial distributions. Our approach is closer to the one of H.Hofmann et al. [13] who treat statistical fluctuations in Fermi systems built on the Landau kinetic equation. We start from the Boltzmann equation and treat explicitely the full collision term to be free from low temperature limitations and equilibrium assumptions.

Our paper is organized as follows. Sect. 2 is a review of the stochastic phase space dynamics according to the Boltzmann-Langevin approach and allows to settle the notations which shall be further used in the paper ; sect. 3 presents the method to derive the Brownian motion in the collective space induced by the phase space fluctuations. This leads to a general equation for the collective observable dispersions. However, the transport coefficients of this equation, although formally derived, are difficult to be analytically expressed, and then, in the general case, equilibrium dispersions are unknown. In sect. 4, we give the general covariance matrix for phase space fluctuations in homogeneous systems and its projections on collective space which are the equilibrium solutions of sect. 3. In sect. 5, we show how to implement the formalism of sect. 3 in pseudo-particle models and to study the fluctuations dynamics on specific collective variables. By comparison, with the analytical results of sect. 4, we check the whole consistency of the approach and the possibility to get observable dispersions out of numerical models.

2. Stochastic phase space evolution

In this section we shortly review the essential concepts and formulae underlying the current theories of Brownian motion in phase space [6,7,11] and outline the frame within which collective observables develop in time. One considers a noise

source that kicks the one-body distribution $\hat{f}(\mathbf{r},\mathbf{p},t)$ and forces it to fluctuate around an average BUU kinetic evolution $f(\mathbf{r},\mathbf{p},t)$. The final result can be formalized in terms of a stochastic kinetic equation similar to the Boltzmann-Langevin one anticipated by Bixon and Zwanzig [5]. It has been constructed resorting to the standard truncation of the hierarchy of reduced s-body distributions ($1 \leq s \leq N$) or Green's functions [15] up to the two-body level including three-body Pauli correlations, and reads ,

$$\frac{\partial \hat{f}}{\partial t}(\mathbf{r},\mathbf{p},t) = -(\mathbf{v}.\nabla_{\mathbf{r}} + \mathbf{F}.\nabla_{\mathbf{p}}\hat{f}(\mathbf{r},\mathbf{p},t)) + K_f(\mathbf{r},\mathbf{p},t) + \delta K_f(\mathbf{r},\mathbf{p},t); \tag{2.1}$$

where $\mathbf{F}$ is the total force including the spatial gradient of the mean field and K_f is the BUU collision kernel,

$$K_f(\mathbf{r}_1,\mathbf{p}_1,t) = \int d^3p_2 d^3p_3 d^3p_4 W_{1234}(\bar{f}_1\bar{f}_2 f_3 f_4 - f_1 f_2 \bar{f}_3 \bar{f}_4). \tag{2.2}$$

W_{1234} is the microscopic transition rate for the collision vertex $(\mathbf{p}_1\mathbf{p}_2) \rightarrow (\mathbf{p}_3\mathbf{p}_4)$ and $\bar{f} = 1-f$. One easily recognizes the usual gain and loss terms of the Boltzmann equation, that we explicitely define for a further use :

$$K_f(\mathbf{r}_1,\mathbf{p}_1,t) = W^+\bar{f}_1 - W^- f_1 = -\frac{f_1 - f_1^{(0)}}{\tau_f}; \tag{2.2 \textit{bis}}$$

$\tau_f = 1/(W^+ + W^-)$ is the momentum-dependent relaxation time which governs the approach towards the instantaneous, momentum-dependent equilibrium [7] $f^{(0)} = W^+ * \tau_f$.

The fluctuating collision term in (2.1) is related to the propagation of initial two-body correlations, and can be expressed as,

$$\delta K_f(\mathbf{r}_1,\mathbf{p}_1,t) = \int d^3r_2 d^3p_2 \{V(1,2), \mathcal{U}_{12}(t) f_2^{(c)}(1,2,0)\}, \tag{2.3}$$

where V(1,2) is the effective interaction between particles 1 and 2, $\{\}$ denotes a Poisson bracket, $\mathcal{U}_{12}(t)$ is the two-particle propagator in phase space and $f_2^{(c)}$ (1,2,0) is the irreducible two-body distribution,

$$f_2^{(c)}(1,2,t) = f_2(\mathbf{r}_1,\mathbf{p}_1,\mathbf{r}_2,\mathbf{p}_2,t) - f(\mathbf{r}_1,\mathbf{p}_1,t)f(\mathbf{r}_2,\mathbf{p}_2,t) \tag{2.4}$$

at $t = 0$.

It is worthwhile noticing at this point that the procedure invoked to derive (2.1) with the fluctuating contribution (2.3) is identical to the construction of either the classical or the quantal Langevin equation [11] . In accordance with the usual treatments, one assumes a statistical ensemble of initial configurations that we propagate in time. The ensemble of the fluctuating part of the collision term vanishes, i.e., $< \delta K_f >= 0$, (hereafter, we use the symbol $< >$ to denote ensemble averages) and the average of (2.1) reduces to the BUU equation.

It is possible to compute the correlation of the noise,

$$\sigma^2_{K_f}(\mathbf{r},\mathbf{p},t\ ;\ \mathbf{r}',\mathbf{p}',t') =< \delta K_f(\mathbf{r},\mathbf{p},t)\delta K_f(\mathbf{r}',\mathbf{p}',t') >; \tag{2.5}$$

which reads,

$$\sigma^2_{K_f}(\mathbf{r},\mathbf{p},t\ ;\ \mathbf{r}',\mathbf{p}',t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')2D_f(\mathbf{p},\mathbf{p}'). \tag{2.6}$$

The diffusion matrix can be expressed from the microscopic transition rate[6] of (2.2) :

$$\begin{aligned} 2D_f(\mathbf{p},\mathbf{p}') = \delta(\mathbf{p}-\mathbf{p}') \int\int\int d^3p_2 d^3p_3 d^3p_4 W_{1234}(f_1 f_2 \bar{f}_3 \bar{f}_4 + \bar{f}_1 \bar{f}_2 f_3 f_4) \\ + \int\int d^3p_3 d^3p_4 W_{11'34}(f_1 f_{1'} \bar{f}_3 \bar{f}_4 + \bar{f}_1 \bar{f}_{1'} f_3 f_4) \\ - \int\int d^3p_2 d^3p_4 W_{121'4}(f_1 f_2 \bar{f}_{1'} \bar{f}_4 + \bar{f}_1 \bar{f}_2 f_{1'} f_4) \\ - \int\int d^3p_2 d^3p_3 W_{121'3}(f_1 f_2 \bar{f}_{1'} \bar{f}_3 + \bar{f}_1 \bar{f}_2 f_{1'} f_3). \end{aligned} \tag{2.7}$$

Furthermore, in the regime where fluctuations are limited to small amplitude excursions $\delta f = \hat{f} - f$, one can compute the evolution rule for the covariance of the distribution function,

$$\sigma_f^2(\mathbf{r},\ \mathbf{p},\ t, \mathbf{r}', \mathbf{p}', t) = < \delta f(\mathbf{r},\ \mathbf{p},\ t) \delta f(\mathbf{r}',\ \mathbf{p}',\ t) >; \tag{2.8}$$

which reads, in self-defined abridged notation,

$$\frac{d}{dt}\sigma_f^2(11't) = -(\mathbf{v}.\nabla_{\mathbf{r}} + \mathbf{F}.\nabla_{\mathbf{p}} + \mathbf{v}'.\nabla r' + \mathbf{F}'.\nabla p')\sigma_f^2(11't)$$

$$+ \int\int\int d^3p_2 d^3p_3 d^3p_4$$

$$\{W_{1234}[\sigma_f^2(1'3)(\bar{f}_1\bar{f}_2 f_4 + f_1 f_2 \bar{f}_4) + \sigma_f^2(1'4)(\bar{f}_1\bar{f}_2 f_3 + f_1 f_2 \bar{f}_3)$$

$$-\sigma_f^2(1'1)(\bar{f}_2 f_3 f_4 + f_2 \bar{f}_3 \bar{f}_4) - \sigma_f^2(1'2)(\bar{f}_1 f_3 f_4 + f_1 \bar{f}_3 \bar{f}_4)]$$

$$+(\textit{same with } 1 \iff 1')\}$$

$$+2D_f(1,1',t). \tag{2.9}$$

This equation is similar (with a different definition of the D-coefficient) to eq.(44) of ref.13 but does not rely on any close-to-equilibrium assumption. A complementary viewpoint to deal with the dynamics of phase space fluctuations has been put forward by Randrup and Remaud[7] ; based on a Fokker-Planck approach which is more amenable to numerical applications, it yields essentially the same results. This equation is very complex and has only been solved numerically in two-dimension infinite systems [7,8,10].

3. Brownian motion in collective space

In this section, we shall analyze the features of the stochastic motion that the previously discussed phase space dynamics assigns to one-body macroscopic

observables. If Q(p) is an arbitrary one-body operator and $\hat{q}(t)$ its average with respect to the stochastic one body distribution $\hat{f}(\mathbf{r},\mathbf{p},t)$, the BUU-Langevin equation (2.1) induces a classical Langevin equation with multiplicative noise,

$$\dot{\hat{q}}(t) = v_Q + K_Q + \delta K_Q(t). \tag{3.1}$$

In eq. (3.1) we have a conservative velocity v_Q related to the free-flow in phase space, which vanishes for spatially homogeneous systems and a dissipative velocity

$$K_Q = \frac{1}{N}\int\int d^3r d^3p K_f(\mathbf{r},\mathbf{p},t)Q(\mathbf{p}), \tag{3.2}$$

that can be more symmetrically cast as

$$K_Q = -\frac{1}{N}\int d^3r \int\int\int\int d^3p_1 d^3p_2 d^3p_3 d^3p_4 W_{1234}\frac{\Delta Q}{2} f_1 f_2 \bar{f}_3 \bar{f}_4, \tag{3.3}$$

where

$$\Delta Q = Q(\mathbf{p}_1) + Q(\mathbf{p}_2) - Q(\mathbf{p}_3) - Q(\mathbf{p}_4); \tag{3.4}$$

The fluctuating velocity of the multiplicative type,

$$\delta K_Q(t) = \frac{1}{N}\int\int d^3r d^3p \delta K_f(\mathbf{r},\mathbf{p},t)Q(\mathbf{p}), \tag{3.5}$$

has a vanishing ensemble average and a correlation given by :

$$\sigma^2_{K_Q}(t,t') = < \delta K_Q(t)\delta K_Q(t') > = 2D_Q(t)\delta(t-t'). \tag{3.6}$$

The time-dependent diffusion parameter is

$$D_Q(t) = \frac{1}{N^2}\int d^3r \int\int d^3p d^3p' Q(\mathbf{p})Q(\mathbf{p}')D_f(\mathbf{p},\mathbf{p}'). \tag{3.7}$$

Introducing the phase space diffusion matrix (2.7) into (3.7) it is straightforward to see that :

$$D_Q = \frac{1}{N^2}\int d^3r \int\int\int\int d^3p_1 d^3p_2 d^3p_3 d^3p_4 W_{1234} f_1 f_2 \bar{f}_3 \bar{f}_4 (\frac{\Delta Q}{2})^2. \qquad (3.8)$$

Due to the definition of ΔQ(3.4), the diffusion exactly vanishes for the observables which are invariant during the collision process (e.g. particle number, linear momentum, and energy). The simplest scheme making room to an analytical solution for (3.1) can be set out of with particular assumptions. First, we consider that the dissipative velocity (3.2) or (3.3) can be represented in a relaxation time approximation,

$$K_Q(t) \sim -\frac{q(t) - q^{(0)}}{\tau_f}; \qquad (3.9)$$

with $q^{(0)}$ the asymptotic equilibrium value of the ensemble averaged $q = < \hat{q} >$ and τ_f an effective dissipative parameter which characterizes the global phase space relaxation and then is independent on Q. The Landau theory predicts that this parameter at the Fermi level is proportional to T^{-2} at the low temperature limit (see for example, eq.77 of ref. 14). In fig.1, we compare the results of the Landau theory with the exact integral of the microscopic transition rates, at the Fermi level (see (2.2) and (2.2bis)) ; the curve for the Landau result has been normalized at T=5 MeV to scale the scattering probabilities used in the two calculations. One sees clearly the known [16] progressive departure from the T^{-2} law as the temperature gets larger and comparable to the Fermi energy (about 25 MeV in the particular case).

As it is well known, one can derive the evolution law for the fluctuation of q in the statistical ensemble,

$$\sigma_Q^2(t) =< [\delta q(t)]^2 >; \qquad (3.10)$$

where $\delta q(t) = \hat{q}(t) - q(t)$. It reads

$$\frac{d}{dt}\sigma_Q^2 = -\frac{2}{\tau_Q}\sigma_Q^2(t) + 2D_Q(t), \qquad (3.11)$$

where τ_Q is equal to the global relaxation time of (3.9). When D_Q is constant and takes its asymptotic equilibrium value $D_Q^{(0)}$, we find for $t >> \tau_Q$, the standard result of the Brownian motion theory :

$$\sigma_Q^{2(0)} = D_Q^{(0)} \tau_Q. \tag{3.12}$$

The above equations are closely related with the standard theory of the Brownian motion ; they provide us with a practical method to compute the dispersion on collective variables : the diffusion coefficient and characteristic time for each collective variable can be extracted from (3.3), (3.8) and (3.9) since they closely resemble the U-U collision term which is routinely computed in simulations with test particle models (see sect. 5).

4. The equilibrium covariance of the one-body stochastic distributions

In the preceding chapter we have shown that the procedure of contracting the full phase space upon either one or a numerable set of collective one-body observables naturally maps the stochastic noise as well as the respective diffusion matrices. The covariance σ_f^2 of the distribution function (see 2.9) is a very complex mathematical object. In the general case, it depends on at least seven variables (time and two momentum vectors in homogeneous systems) and then is out of reach of the most powerful computers. We have shown that out of the critical regimes, it is not an essential ingredient to establish the collective diffusive-dissipative path. However, it remains essential to study the equilibrium fluctuations.

It is easy to show that for any observables $Q(\mathbf{p})$, one has :

$$\sigma_Q^2(t) = \frac{1}{N^2} d^3r \int\int d^3p d^3p' Q(\mathbf{p}) Q(\mathbf{p}') \sigma_f^2(\mathbf{r}, \mathbf{p}, t, \mathbf{r}, \mathbf{p}', t). \tag{4.1}$$

Then the equilibrium dispersion $\sigma_{Q_i}^{2(0)}$ for each observable can be computed out of an integration if $\sigma_f^{2(0)}$ is known. This can be done by resort to Van Kampen's method[11] that we shall extend to the case of fermion systems ; this method consists of a projection of the covariance matrix on the eigenfunctions of the linearized Boltzmann operator. Eq.(2.9) reads in a matrix representation :

$$\frac{d}{dt}\sigma_f^2 = A\sigma_f^2 + \sigma_f^2 \tilde{A} + 2D_f, \tag{4.2}$$

where the correlation function has to be understood as the matrix element of σ_f^2 in the phase space representation. The operator A is the linearized Boltzmann operator, plus the streaming term and $\bar{A}$ its transposed expression. In a spatial homogenous system, i.e $\mathbf{F} = 0$ in (2.1), its elements are :

$$< \mathbf{r}_1, \mathbf{p}_1 | A | \mathbf{r}_2, \mathbf{p}_2 >= \delta(\mathbf{r}_1 - \mathbf{r}_2) \{ \delta(\mathbf{p}_1 - \mathbf{p}_2) \frac{\mathbf{p}_1}{m} \frac{\partial}{\partial \mathbf{r}_1}$$

$$+2 \int d^3p_3 d^3p_4 (f_1 f_3 \bar{f}_4 + \bar{f}_1 \bar{f}_3 f_4) W_{1234}$$

$$- \int d^3p_3 d^3p_4 (\bar{f}_1 f_3 f_4 + f_1 \bar{f}_3 \bar{f}_4) W_{1234}$$

$$-\delta(\mathbf{p}_1 - \mathbf{p}_2) \int d^3p_3 d^3p_4 d^3p_5 (f_3 \bar{f}_4 \bar{f}_5 + \bar{f}_3 f_4 f_5) W_{1345} \}. \tag{4.3}$$

At equilibrium, the diagonal part of the covariance matrix is well-known since it gives the standard expression for the fluctuations of the occupation probability ; we then formally separate the diagonal and the off-diagonal parts of the covariance matrix :

$$< \mathbf{r}_1, \mathbf{p}_1 | \sigma_f^{2(0)} | \mathbf{r}_2, \mathbf{p}_2 >=< \mathbf{r}_1, \mathbf{p}_1 | \sigma_{fD}^{2(0)} | \mathbf{r}_2, \mathbf{p}_2 > + < \mathbf{r}_1, \mathbf{p}_2 | \sigma_{fOD}^{2(0)} | \mathbf{r}_2, \mathbf{p}_2 >, \tag{4.4}$$

with

$$\sigma_f^{2(0)}(\mathbf{r}_1, \mathbf{p}_1; \mathbf{r}_2, \mathbf{p}_2) = f^{(0)}(\mathbf{r}_1, \mathbf{p}_1) \bar{f}^{(0)}(\mathbf{r}_1, \mathbf{p}_1) \delta(\mathbf{r}_1 - \mathbf{r}_2) \delta(\mathbf{p}_1 - \mathbf{p}_2). \tag{4.5}$$

The index 0 refers to the equilibrium value ; for fermion systems $f^{(0)}$ is then the Fermi-Dirac distribution. Inserting this expression in (4.2) in the stationnary case (the l.h.s. term vanishes), one gets for the equilibrium off-diagonal part :

$$A \sigma_{fOD}^{2(0)} + \sigma_{fOD}^{2(0)} \bar{A} = 0, \tag{4.6}$$

where the inhomogenous term cancels out, in agreement with eq. 26 of ref. 10). The product of matrix A with the diagonal part of $\sigma_{fD}^{2(0)}$ exactly cancels the whole diffusion matrix (2.8).

This homogeneous equation may have non trivially null solutions if σ^2 belongs to the kernel of the operator A. This implies that σ^2 can be expanded on the basis of the eigenstates of the collision operator which have vanishing eigenvalues, those eigenstates being built from the well-known five invariants of the Boltzmann collision operator, respectively the identity operator (which insures conservation of the total particle number), the three components of the linear momentum and the kinetic energy. The eigenstates of the linearized quantum Boltzmann operator with vanishing eigenvalues are then linear functions of this invariants.

Since the streaming term acts only on r dependent functions, they are also eigenfunctions of the operator A. According to (4.6), the off-diagonal part of the covariance matrix then reads :

$$\sigma_{fOD}^2(\mathbf{r}_1,\mathbf{p}_1;\mathbf{r}_2,\mathbf{p}_2) = \sum_{i,j=0}^{4} \theta_{ij}\psi_i(\mathbf{r}_1,\mathbf{p}_1)\psi_j(\mathbf{r}_2,\mathbf{p}_2); \tag{4.7}$$

where

$$\psi_0(\mathbf{r},\mathbf{p}) = f^{(0)}(\mathbf{r},\mathbf{p})\bar{f}^{(0)}(\mathbf{r},\mathbf{p}) \qquad \text{and} \qquad \psi_i(\mathbf{r},\mathbf{p}) = q_i(\mathbf{p})\psi_0(\mathbf{r},\mathbf{p}). \tag{4.8}$$

The q_i functions are the Boltzmann invariants respectively $1, p_x, p_y, p_z$ and $\mathbf{p}^2$ for i = 0,...,4. To compute the θ_{ij} coefficients we use the fact that the fluctuations exactly vanish for the constants of motion of the system. These constraints yield a linear system of equations, the matrix B of which reads :

$$B = \begin{pmatrix} B_{00} & 0 & 0 & 0 & 3mT \\ 0 & mT & 0 & 0 & 0 \\ 0 & 0 & mT & 0 & 0 \\ 0 & 0 & 0 & mT & 0 \\ 3mT & 0 & 0 & 0 & 10m^2TE_T \end{pmatrix} \tag{4.9}$$

The only non-analytical coefficient is B_{00}, the integral of ψ_0 over the whole phase space; the other terms are expressible as functions of the temperature T

and of the total kinetic energy E_T. By inversion of the non-singular matrix B, one gets the equilibrium covariance matrix :

$$\sigma_f^{2(0)}(\mathbf{r}_1,\mathbf{p}_1;\mathbf{r}_2,\mathbf{p}_2) = f_1^{(0)}\bar{f}_1^{(0)}\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{p}_1-\mathbf{p}_2)$$

$$+f_1^{(0)}\bar{f}_1^{(0)}f_2^{(0)}\bar{f}_2^{(0)}(\theta_{00}+\theta_{04}(\mathbf{p}_1{}^2+\mathbf{p}_2{}^2)+\theta_{44}\mathbf{p}_1{}^2\mathbf{p}_2{}^2+\theta_{11}\mathbf{p}_1\mathbf{p}_2), \qquad (4.10)$$

where :

$$\theta_{00} = \frac{-B_{44}}{B_{00}B_{44}-B_{04}^2} \; ; \quad \theta_{04} = \frac{B_{04}}{B_{00}B_{44}-B_{04}^2}$$
$$\theta_{44} = \frac{-B_{00}}{B_{00}B_{44}-B_{04}^2} \; ; \quad \theta_{11} = -\frac{3}{B_{04}} = -\frac{1}{mT} \qquad (4.11)$$

A lenghty but straightforward computation shows that the covariance matrix (4.10) is the equilibrium solution of the general evolution equation (2.9) for the homogeneous systems. One should notice that setting $\bar{f} = 1, 1-f, 1+f$ respectively, the covariance (4.10) is valid for classical, fermion and boson systems at equilibrium, with an appropriate redefinition of the matrix B.

Equation (4.10) exhibits strong departure from the usual result of statistical physics which reduces to the diagonal part ; the off-diagonal terms of (4.10) are also missing in H.Hofmann et al. [13]. In the systems that we consider, the number of particles is exactly conserved (as well as the linear momentum and the energy), then any transient local fluctuation in the phase space density has to be counterbalanced by another fluctuation of opposite sign elsewhere in the system ; thus this microcanonical treatment induces strong correlations and is at the origin of these off-diagonal terms of the covariance matrix. Indeed, (4.10) projected on the number, linear momentum and energy operators exactly vanishes. These conservation laws for the covariance matrix imply that its zeroth and first moments exactly vanish and then if one would perform an angular moment expansion as in ref.13), one would also find that the diffusion coefficients D_{kl} (see their eq. 51), linearly linked to the covariance matrix, exactly vanish for k,l=0,1.

According to (4.1), the equilibrium dispersion of any one-body observable for thermalized homogenous systems is then amenable to direct computation. For instance, let us consider the quadrupole moment of the velocity distribution, which is the leading quantity to analyze the relaxation of systems toward their equilibrium :

$$Q = 2p_z^2 - p_x^2 - p_y^2; \tag{4.12}$$

by direct use of (4.12), the dispersion simply reads :

$$\sigma_Q^{2(0)} = 8m^2 T E_T, \tag{4.13}$$

where E_T is the total kinetic energy, then the dispersion of the quadrupole moment grows linearly with the temperature at low excitation energy. At very high excitation energies, it grows proportionally to T^3. Another quantity of interest in the context of heavy ion reactions is the transverse momentum gained by particles, which can be connected to the equation of state of nuclear matter. The transverse momentum dispersion has a more complex expression, but it can be reduced for the cases of low and high temperatures. In the general case, it reads :

$$\sigma_{P_\perp}^{2(0)} = 2mT + (\frac{3\pi mT}{4p_F^3})^2 \{\theta_{00} I_1^2 + 16m^2 \theta_{44} I_2^2 + 8m\theta_{11} I_1 I_2\}, \tag{4.15}$$

I_1 and I_2 being the Fermi integrals :

$$I_1 = \int_0^\infty d\epsilon f(\epsilon) \qquad \text{and} \qquad I_2 = \int_0^\infty d\epsilon \epsilon f(\epsilon). \tag{4.15}$$

Another simple expression is found for the dispersion of the observable $F = P_z \ \mathrm{sign}(P_z)$ which is connected to the flow :

$$\sigma_F^{2(0)} = mT. \tag{4.16}$$

We can notice that all these expressions depend exclusively on the kinetic energy or on the temperature which are strongly related at equilibrium . They formally include the diagonal and off-diagonal contributions of the covarinace matrix. The off-diagonal terms cancels out for the quadrupole moment (4.13) and the flow observable (4.16) whereas they contribute for the transverse momentum dispersion.

5. Simulation and results

The preceding chapters establish the general method that we propose to compute dynamically the fluctuations of fermion systems. We present here a numerical check of its validity, we restrict our analysis to infinite excited systems for which we have analytical solutions to compare with. In this case, the problem reduces to the solution of a coupled set of equations :

$$\frac{\partial f}{\partial t}(\mathbf{p},t) = K_f(\mathbf{p},t), \tag{5.1}$$

$$\dot{q}(t) = K_Q = -\frac{q(t) - q^{(0)}}{\tau_Q}, \tag{5.2}$$

$$\frac{d}{dt}\sigma_Q^2 = -\frac{2}{\tau_Q}\sigma_Q^2(t) + 2D_Q(t), \tag{5.3}$$

where one recognizes respectively the BUU/Landau-Vlasov equation for the average phase space trajectory (average of 2.1), the equations for the average value of Q (average of 3.1) and of its dispersion (3.11). The link between them is given by the microscopic structure of K_f (2.2), K_Q (3.3) and D_Q (3.8). A ten-year old experience of several groups has shown how to compute K_f by the microscopic account of the test-particle collisions[3] ; it is straightforward to include simulations the balance, for each microscopic allowed collisions, of the operators ΔQ (3.4) and $(\Delta Q)^2/2$ which contribute to τ_Q and D_Q. Due to the graining of the phase space, these quantities are blurred by a numerical noise which can be smeared out by an appropriate averaging over the time.

The initial distribution in momentum space is chosen to reproduce, as closely as possible, the situation of two colliding nuclei. We take in the center of mass frame, two shifted Fermi spheres, respectively centered at $(0,0,-\Delta k/2)$ and $(0,0,\Delta k/2)$ where the z axis corresponds to the beam direction. For the study of the flow, we introduce an angle between the beam direction and the symmetry axis of the two Fermi spheres. This is an artificial way to create a flow in infinite nuclear matter where it does not appear naturally. While in real nuclear collisions the flow is created from a vanishing initial flow state, here we follow the inverse process ; this provides us with a cheap method to estimate the dispersion on this variable.

We have performed several simulation with different values of the initial shift Δk, corresponding to different excitation energies of the system. The mean dis-

tribution function is displayed in fig.2, at different time steps, as a function of the particle energy. As already found with such simulations [9], there is an obvious relaxation towards equilibrium. We notice numerical fluctuations introduced by the coarse graining of the system coordinates and the discrete character of the collision simulation. However, as can be seen on bottom figures, the asymptotic configurations are the theoretically expected Fermi-Dirac distributions.

Let us focus now on the relaxation in collective space. First we consider the relaxation of the quadrupole moment operator (4.13). The exponential decay of this observable is obvious as seen in fig.3a, which allows a rather precise estimate of the relaxation time τ_Q. Even if the systems are highly excited, the relaxation can be characterized by a fairly constant time parameter, we may use this feature to compute τ_Q with a global exponential fit instead of extracting it from (5.2). A similar behaviour is observed for the flow observable F in fig.3b. Whereas the transverse momentum on fig.3c shows the behaviour of collective variable which does not vanish at equilibrium but for which a relaxation time can nevertheless be deduced.

The relaxation times of the quadrupole moment and of the flow extracted from the numerical simulations are plotted in fig.4 as functions of the final temperature. Results at very low temperature are not shown since the simulations become prohibitively long.

We can notice that even if the structures of these two operators are completely different, their relaxation times are nearly the same, which a-posteriori justifies the hypothesis (3.9) of the global relaxation approximation. The relaxation time for the collective variables incorporates relaxation properties in the whole phase space ; however in fig.4 we show that the values extracted from the numerical simulations fit nicely the relaxation time τ_f analytically computed at the Fermi level (see (2.2) and (2.2bis)).

We then compute as a function of time the diffusion coefficients for two collective variables (fig.5). As already noticed, some care should be taken in presence of the numerical fluctuations. Due to the structure of the collision term, the numerical noise on the gain and loss coefficients tends to cancel out in the mean quantities ; for the diffusion process, we shall average the computation of the instantaneous diffusion coefficient on several time steps provided that this averaging time is kept much smaller that the global relaxation time. Here, this condition is met since the averaging time has been taken equal to 2.5 10^{-23} sec, it could have been taken larger for the less excited systems.

Even if some fluctuations remain, one clearly sees that taking apart the first time steps of very excited systems (Δk = 0.5 and 1.3); the computed diffusion

coefficient is remarkably constant in time. We show in fig.6, the temperature dependence of the asymptotic diffusion coefficients for the quadrupole and transverse momentum observables. So we confirm for the whole phase space diffusion matrix, the conclusion of ref.7) concerning its diagonal part : even if the system is very far from its equilibrium, the fluctuations of the system keep close track of their instantaneous equilibrium value. The fact that the collective relaxation time and the diffusion coefficient are almost independent on the level of equilibration of the system is important, since it states that the leading parameter for fluctuations dynamics is the excitation energy regardless this energy is thermally equilibrated or not.

The validity of the relaxation time approximation being verified, we can calculate the expected equilibrium dispersions according to the fluctuation dissipation theorem (see sect. 4) using the values of τ_Q and D_Q extracted from simulations. In fig.7, we present a comparison between the analytical results of (4.12) and (4.16) and the results of the simulations. The equilibrium dispersions are reproduced with a margin of error less than 15%, which could be reduced by an increase of the number of test-particles associated with each nucleon. This agreement indicates that the test-particle simulations can be used to compute dispersions of collective variables with a negligible increase of computing and then opens a new field of applications of transport models for heavy ion collisions

6. Conclusion

We have presented a formal and numerical scheme to approach the dynamics of fluctuations for collective variables as arising from the stochastic character of the phase space evolution[6]. We have voluntarily limited ourselves to the linear regime where the concept of constant relaxation time for the whole system is meaningful. At variance with other works, we consider that the above fluctuations may serve at best as seeds for the density fluctuations expected in critical systems. We then do not intend to apply this formalism to non-linear processes as multifragmentation and droplet or bubble formations. However, we have to notice that the limitation to the linear regime are less severe than it seems ; since, in the actual (fermion) systems that we have studied, the linear regime applies successfully for the fluctuations even at high excitation energies.

We have presented a consistent brownian motion approach to the dynamics of collective variables in the linear regime, which allows a direct link of the transport coefficients in collective space to the microscopic two-body correlations induced by the residual interaction on the phase space distributions. We have used this microscopic link in the simulations.

The solution of the Brownian equation is generally not available, even in the

thermal equilibrium case. We then have presented the independent derivation of the equilibrium fluctuations, from the general invariant properties of the collision kernel of the Boltzmann equation. This method initiated by Van Kampen for classical system, provides a simple (and novel) expression for the covariance matrix of phase space fluctuations which can be easily projected on each collective variable, we have used the resulting analytical dispersions to check the validity of the numerical simulations.

Finally, we have demonstrated that the method is efficient by simulations based on pseudo-particle models which are numerical implementations of the Brownian approach. These simulations show first that the inherent numerical fluctuations of the calculations do not prevent the extraction of transport coefficients. They show too that the numerical simulations provide the correct approach of the fluctuations towards their equilibrium.

The fact that pseudo-particle models -here used in infinite matter- have been successfully applied to finite systems, paves the way to actual dynamical studies of fluctuations in systems like interacting heavy ions. As a supplementary conclusion which may help this application to real systems whose equilibrium is difficult to ascertain, we notice that the excitation energy (thermally equilibrated or not) is the leading parameter for the fluctuations.

Acknowledgment : The authors thank J. Randrup and P. Chomaz for very fruitful discussions. One of us (E. S. H.) also wishes to ackowledge warm hospitality during her stay at the Laboratoire de Physique Nucléaire , Université de Nantes, and the CNRS od France for financial support.

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Figures Captions

Figure 1 : Relaxation time of a Fermi system ; the dotted line indicates the low temperature expansion used in the Landau theory, the black circles show the results arising from the exact computation of the collision integral.

Figure 2 : Phase space distribution functions as functions of the particle energy in units of Fermi energy. Situations at different time steps in 10^{-23} sec units are shown ; for an inital shift of $1.2k_F$ of the two Fermi spheres, on the left hand and of $0.3k_F$ on the right. The two bottom figures show in full lines the analytical Fermi distributions for systems with the same total excitation energies.

Figure 3.a : Quadrupole moment relaxation (4.12) for r different values of the initial shift Δk of the two initial fermi spheres.

Figure 3.b : Same as fig. 3.a for the flow observable.

Figure 3.c : Same as fig. 3.a for the transverse momentum.

Figure 4 : Relaxation times of the quadrupole moment (stars) and the flow observable (triangles) as functions of the final temperature of the system. The theoretical phase space relaxation time (dots) is shown again for comparison.

Figure 5.a : Quadrupole moment diffusion coefficient plotted versus time for different initial conditions.

Figure 5.b : Flow observable diffusion coefficient plotted versus time for different initial conditions.

Figure 6.a : Mean equilibrium quadrupole diffusion coefficient as a function of the final temperature.

Figure 6.b : Mean equilibrium flow observable diffusion coefficient as a function of the final temperature.

Figure 7.a: Comparison between the dispersions on the quadrupole moment extracted from the simulation and the exact analytical ones ; the results are plotted for different final temperature.

Figure 7.b : Comparison between the dispersions on the flow observable extracted from the simulation and the exact analytical ones.

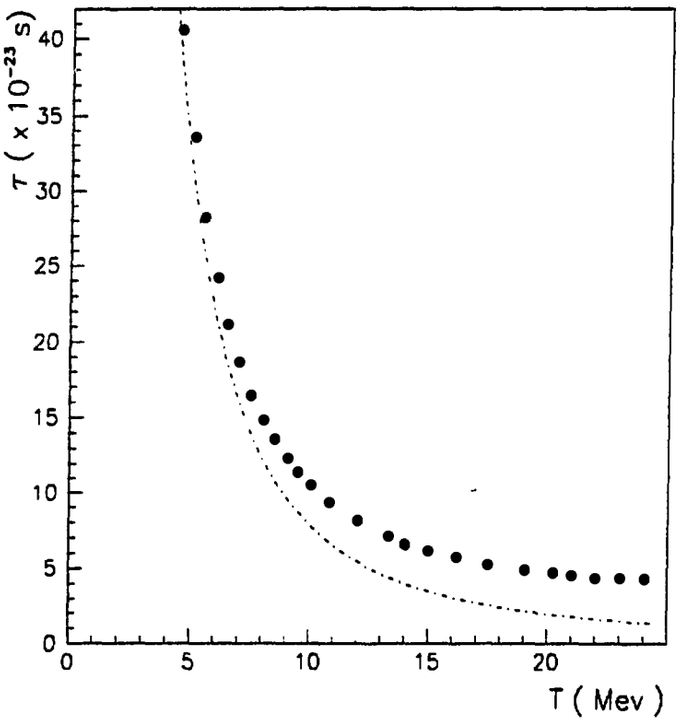

Figure 1

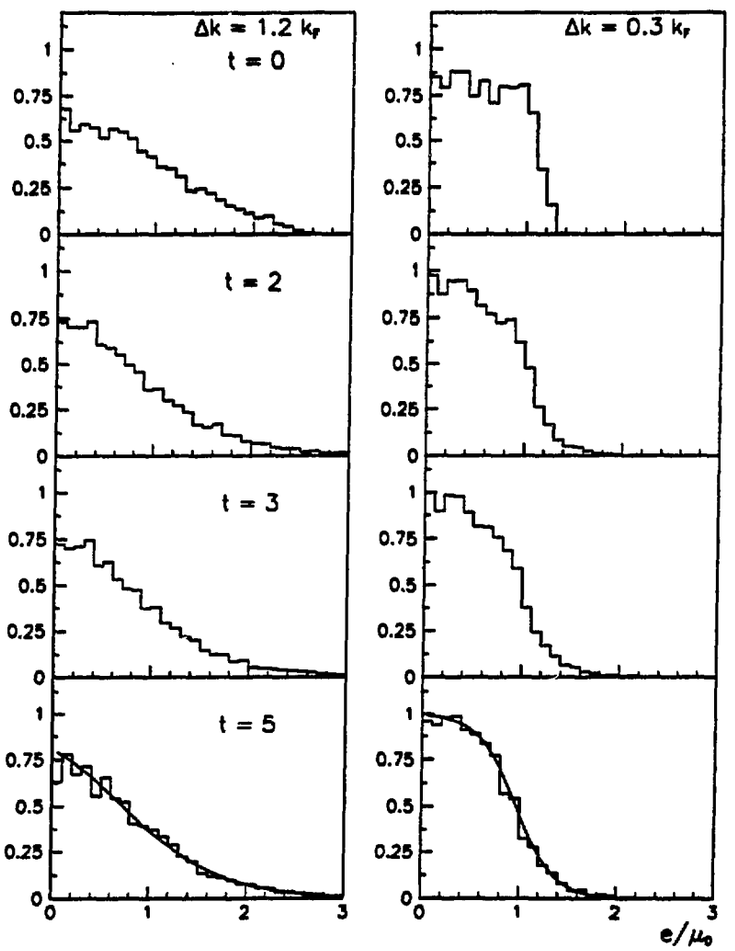

Figure 2

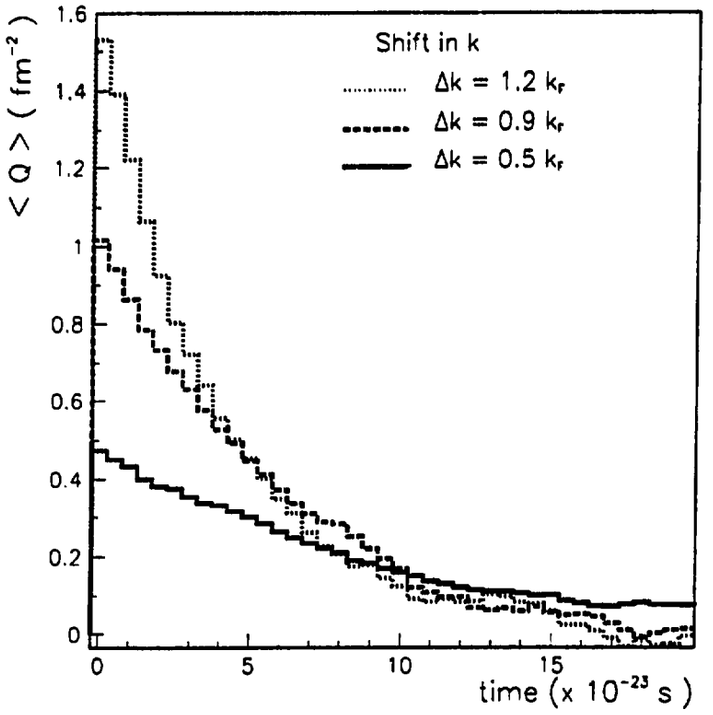
Shift in k
Δk = 1.2 k_F
Δk = 0.9 k_F
Δk = 0.5 k_F
< Q > (fm^-2)
time (x 10^-23 s)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0
5
10
15

Figure 3.a

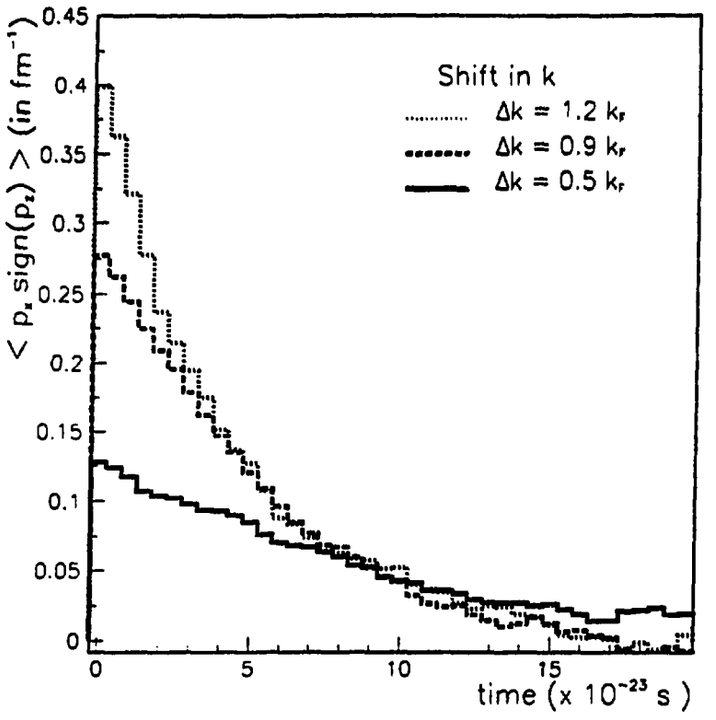
Shift in k
Δk = 1.2 k_r
Δk = 0.9 k_r
Δk = 0.5 k_r
< p_z sign(p_z) > (in fm^-1)
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
5
10
15
time (x 10^-23 s)

Figure 3.b

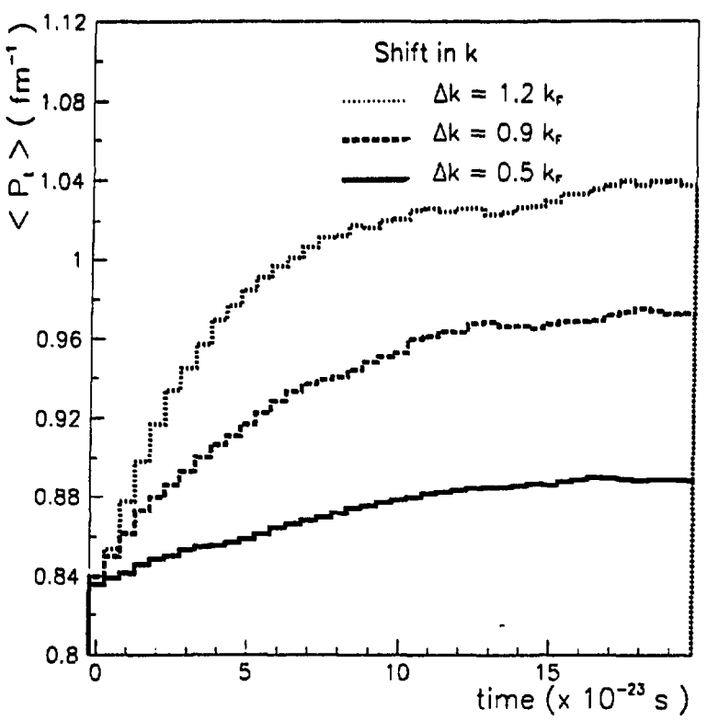
Shift in k
Δk = 1.2 k_F
Δk = 0.9 k_F
Δk = 0.5 k_F
< P_t > (fm^-1)
1.12
1.08
1.04
1
0.96
0.92
0.88
0.84
0.8
0
5
10
15
time (x 10^-23 s)

Figure 3.c

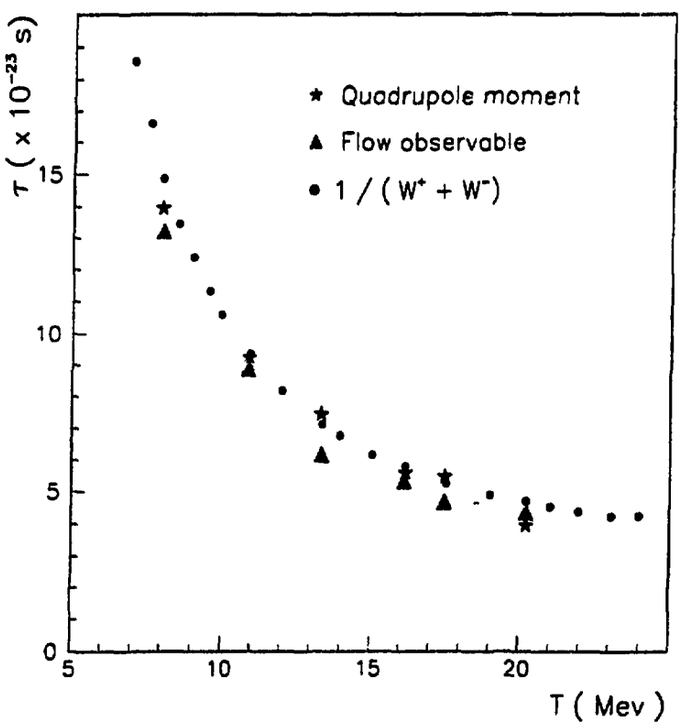
τ (x 10⁻²³ s)
★ Quadrupole moment
▲ Flow observable
● 1 / (W⁺ + W⁻)
0
5
10
15
5
10
15
20
T (Mev)

Figure 4

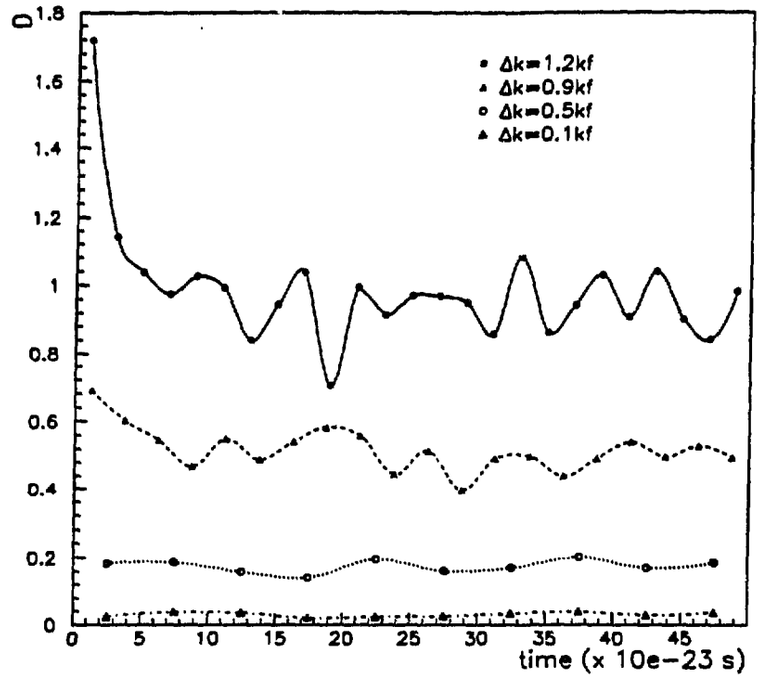
D
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
30
35
40
45
time (x 10e−23 s)
Δk=1.2kf
Δk=0.9kf
Δk=0.5kf
Δk=0.1kf

Figure 5.a

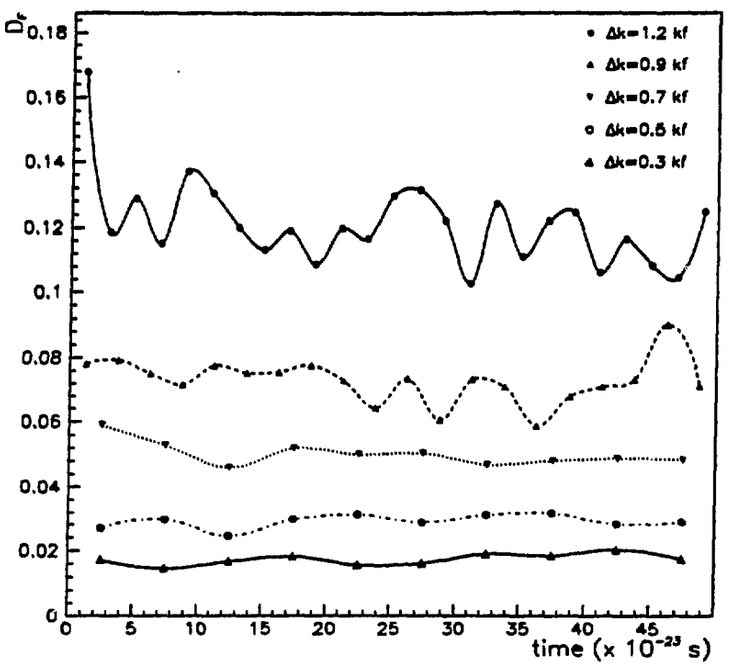
D_r
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
5
10
15
20
25
30
35
40
45
time (x 10^{-23} s)
Δk=1.2 kf
Δk=0.9 kf
Δk=0.7 kf
Δk=0.5 kf
Δk=0.3 kf

Figure 5.b

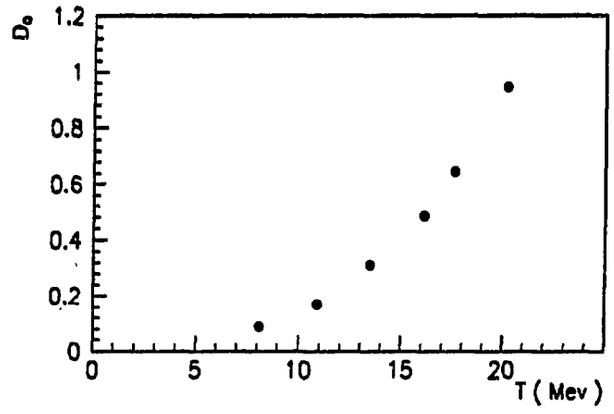

Figure 6.a

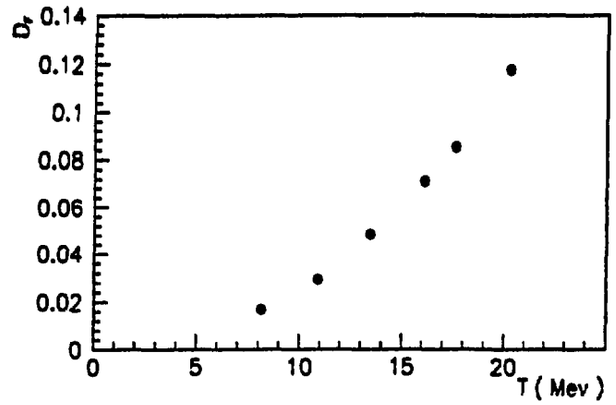

Figure 6.b

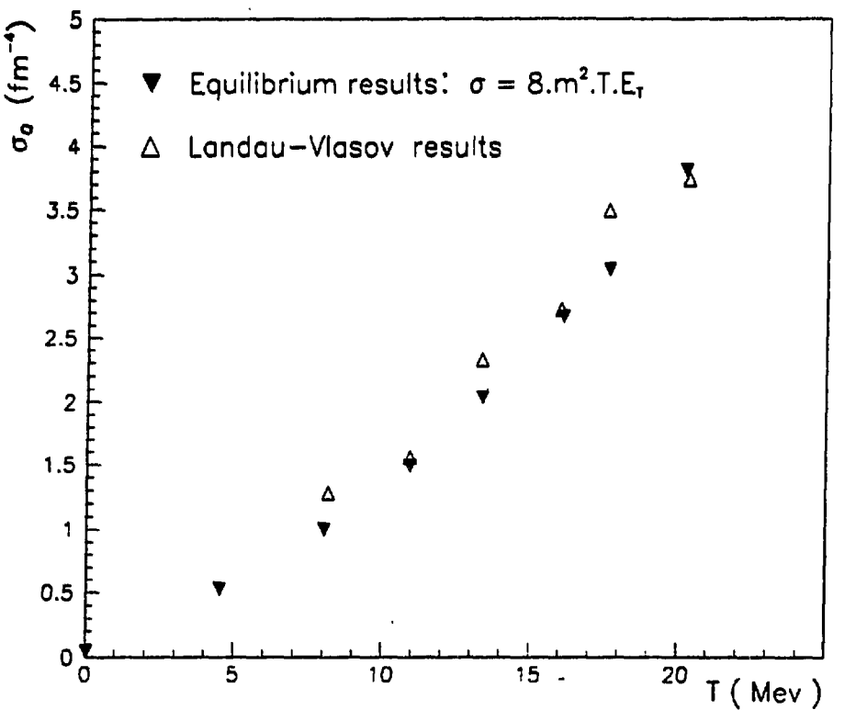
σ_o (fm^{-1})
Equilibrium results: $\sigma = 8.m^2.T.E_T$
Landau–Vlasov results
T (Mev)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
5
10
15
20

Figure 7.a

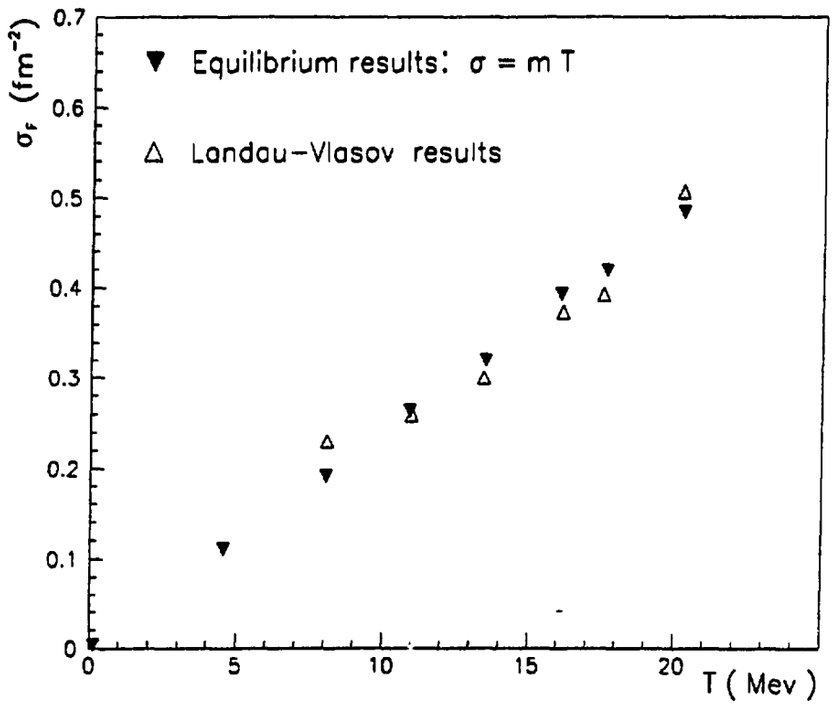
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
σ_f (fm^{-2})
Equilibrium results: $\sigma = m\,T$
Landau–Vlasov results
0
5
10
15
20
T (Mev)

Figure 7.b