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# NEUTRINO OSCILLATIONS IN STRONG MAGNETIC FIELDS

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ABSTRACT

Neutrino conversion processes between two neutrino species and the corresponding oscillations induced by strong magnetic fields are considered. The value of the critical strength of magnetic field $B_{cr}$ as a function of characteristics of neutrinos in vacuum ($\Delta m_\nu^2$, mixing angle $\theta$), effective particle density of matter $n_{eff}$, neutrino (transition) magnetic moment $\tilde{\mu}$ and energy $E$ is introduced. It is shown that the neutrino conversion and oscillations effects induced by magnetic fields $B \geq B_{cr}$ are important and may result in the depletion of the initial type of $\nu$'s in the bunch. A possible increase of these effects in the case when neutrinos pass through a sudden decrease of density of matter ("cross-boundary effect") and applications to neutrinos from neutron stars and supernova are discussed.

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The basic idea of the vacuum neutrino conversion and oscillations effects [1] has found its applications in a considerable amount of studies of various astrophysical, cosmological and laboratory environments. The subsequent investigations in this field are strongly stimulated first of all by a possible solution of the solar neutrino puzzle on the bases of the matter and magnetic field enhancement of spin and flavour neutrino conversion [2-6] (see [7-11] for a review). Another important motivation for consideration of neutrino conversion and oscillations is based on the common belief that these effects may be involved in the processes of supernova bursts and cooling of neutron stars (see, for example, [13-16] and references therein). It must be mentioned here that in most of the performed studies of neutrino conversion and oscillations between two species in magnetized matter, the considered strengths of magnetic field is of the order of $B \leq 10^5\ G$ that is quite adequate to the solar neutrino problem. There are also studies and discussions of the neutrino resonant conversion for the case of the shell of a supernova accounting for much stronger magnetic fields (see, for example, [6, 12]). However, in some recent studies in this field the possible influence of strong magnetic fields on neutrino conversion and oscillations was not considered at all (see, for example, [13, 14, 15, 16]).

The magnetic fields of the order of $B \sim 10^{12} - 10^{14}\ G$ that are believed to exist in neutron stars at different stages of evolution could not only induce new particle interaction phenomena [17, 18] that may play a visible role in energy losses (see also [19]), but may influence the neutrino conversion and oscillations processes as well. In this paper supposing that neutrinos have non- vanishing magnetic or/and flavour transition moments we consider the magnetic field induced effects of neutrino spin and/or spin-flavour conversion and oscillations between different neutrino species. We focus on the particular case of neutrino conversion and oscillations effects induced by strong magnetic fields in the presence of matter, also accounting for mixing of neutrinos in vacuum. On the bases of general analysis of the problem we discuss consequences of these effects for neutrinos produced within the interior and emitted from a neutron star ( including the possible amplification of oscillations that can appear as the "cross-boundary effect" [20, 21]), and also get constraints on the value of the neutrino magnetic moment from the consideration of the reheating of a supernova.

For simplicity we restrict our consideration to the case of two neutrino flavours, $\nu_e$ and $\nu_\mu$. In vacuum the flavour eigenstates $\nu_e$ and $\nu_\mu$ can be expressed in terms of the mass eigenstates $\nu_1$ and $\nu_2$:

$$\begin{aligned} \nu_e &= \nu_1 \cos\theta + \nu_2 \sin\theta, \\ \nu_\mu &= -\nu_1 \sin\theta + \nu_2 \cos\theta, \end{aligned} \tag{1}$$

where $\theta$ denotes the vacuum mixing angle.

In the general case when neutrinos pass through the magnetized matter (composed of electrons, protons and neutrons) the evolution of neutrinos is described by the following Schrödinger-type equation (see, for example, [7])

$$i\frac{d}{dt}\nu(t) = H\nu(t). \tag{2}$$

The Hamiltonian $H$ is the sum of the three terms

$$H = H_V + H_{int} + H_F, \tag{3}$$

where $H_V$ contains a contribution from a vacuum mass matrix, $H_{int}$ contains a contribution from neutrino interactions with matter and $H_F$ contains a contribution from interactions with the magnetic field. If for the case of Dirac neutrinos one uses the bases in which neutrinos have a definite projection along the direction of propagation

$$\nu = (\nu_{e_L}, \nu_{\mu_L}, \nu_{e_R}, \nu_{\mu_R}),$$

then the Hamiltonian is given by

$$H^D = \begin{pmatrix} -\frac{\Delta m^2_\nu}{4E}c + V_{\nu_e} & \frac{\Delta m^2_\nu}{4E}s & \mu_{ee}B & \mu_{e\mu}B \\ \frac{\Delta m^2_\nu}{4E}s & \frac{\Delta m^2_\nu}{4E}c + V_{\nu_\mu} & \mu_{\mu e}B & \mu_{\mu\mu}B \\ \mu_{ee}B & \mu_{\mu e}B & -\frac{\Delta m^2_\nu}{4E} & 0 \\ \mu_{e\mu}B & \mu_{\mu\mu}B & 0 & \frac{\Delta m^2_\nu}{4E} \end{pmatrix}. \tag{4}$$

Here $\Delta m^2_\nu = m^2_2 - m^2_1$ , $c \equiv \cos 2\theta$, $s \equiv \sin 2\theta$, $E$ is the energy of relativistic neutrinos, $B$ is the transverse magnetic field, $\mu_{ii}$ and $\mu_{ij}$ are the neutrino and the flavour transition magnetic moments. It is supposed that the magnetic field does not rotate along the path of neutrinos (see, for example, [22]). The Hamiltonian (4) corresponds to the case of sterile neutrinos $\nu_{e_R}$ and $\nu_{\mu_R}$. When $\nu_e$ or $\nu_\mu$ pass through matter (composed of electrons, protons and neutrons) the energies of the particles for a given momentum get additional Wolfenstein terms $V_{\nu_l}$:

$$V_{\nu_e} = \sqrt{2}G_F\Big(n_e - \frac{1}{2}n_n\Big), \ V_{\nu_\mu} = -\frac{1}{\sqrt{2}}G_F n_n, \tag{5}$$

where $n_e$ and $n_n$ are the electron and neutron number densities.

For the two Majorana neutrinos in the bases written as

$$\nu = (\nu_e, \nu_\mu, \bar{\nu}_e, \bar{\nu}_\mu)$$

in the corresponding Hamiltonian

$$H^M = \begin{pmatrix} -\frac{\Delta m^2_\nu}{4E}c + V_{\nu_e} & \frac{\Delta m^2_\nu}{4E}s & 0 & \mu B \\ \frac{\Delta m^2_\nu}{4E}s & \frac{\Delta m^2_\nu}{4E}c + V_{\nu_\mu} & -\mu B & 0 \\ 0 & -\mu B & -\frac{\Delta m^2_\nu}{4E}c - V_{\nu_e} & \frac{\Delta m^2_\nu}{4E}s \\ \mu B & 0 & \frac{\Delta m^2_\nu}{4E}s & \frac{\Delta m^2_\nu}{4E}c - V_{\nu_\mu} \end{pmatrix} \tag{6}$$

$\mu$ denotes the flavour transition magnetic moment, the only non- vanishing one in this case. Contrary to the case of Dirac neutrinos, here right- handed antineutrinos are not supposed to be not sterile.

Using these Hamiltonians we can consider different neutrino conversion processes $\nu_i \to \nu_j$ and the corresponding neutrino oscillations $\nu_i \leftrightarrow \nu_j$, such as

$$\nu_{e_L} \to \nu_{e_R}, \ \nu_{e_L} \to \nu_{\mu_R}, \ \nu_{e_L} \to \bar{\nu}_{\mu_R}. \tag{7}$$

Accounting for these processes one can readily obtain the probabilities of finding a neutrino of the type $j$ ($\nu_j$) after the initial neutrino $\nu_i$ travels a distance $x$ in magnetized matter:

$$P(\nu_i \to \nu_j) = \sin^2 2\theta_{eff} \sin^2\Big(\frac{\pi x}{L_{eff}}\Big), i \neq j, \tag{8}$$

while the survival probabilities are

$$P(\nu_i \to \nu_i) = 1 - P(\nu_i \to \nu_j), \tag{9}$$

where the effective mixing angle $\theta_{eff}$ and effective oscillation length $L_{eff}$ are given by

$$\tan 2\theta_{eff} = \frac{2\tilde{\mu}B}{\frac{-\Delta m_\nu^2}{2E}A + \sqrt{2}G_F n_{eff}}, \tag{10}$$

$$L_{eff} = 2\pi\Big[\Big(\frac{\Delta m_\nu^2}{2E}A - \sqrt{2}G_F n_{eff}\Big)^2 + (2\tilde{\mu}B)^2\Big]^{-1/2}. \tag{11}$$

For different neutrino conversion processes (7) $\tilde{\mu}$, $A$ and $n_{eff}$ are equal to

$$\tilde{\mu} = \begin{cases} \mu_{ee} & for\ \nu_{e_L} \to \nu_{e_R} \\ \mu_{e\mu} & for\ \nu_{e_L} \to \nu_{\mu_R} \\ \mu & for\ \nu_{e_L} \to \bar{\nu}_{\mu_R} \end{cases}, \tag{12}$$

$$A = \begin{cases} \frac{1}{2}(\cos 2\theta - 1) & for\ \nu_{e_L} \to \nu_{e_R} \\ \frac{1}{2}(\cos 2\theta + 1) & for\ \nu_{e_L} \to \nu_{\mu_R} \\ \cos 2\theta & for\ \nu_{e_L} \to \bar{\nu}_{\mu_R} \end{cases}, \tag{13}$$

$$n_{eff} = \begin{cases} n_e - n_n & for\ \nu_{e_L} \to \bar{\nu}_{\mu_R} \\ n_e - \frac{1}{2}n_n & for\ \nu_{e_L} \to \nu_{e_R,\mu_R} \end{cases} \tag{14}$$

The probability (10) may have a considerable value (the neutrino conversion processes and oscillations become important) if the following two conditions are valid:

1) the "amplitude of oscillations" $\sin^2 2\theta_{eff}$ is far from zero (or $\sin^2 2\theta_{eff} \sim 1$), and

2) the length $x$ of the neutrinos path in the medium must be greater than the effective oscillation length $L_{eff}$ ($x \sim$ or $> \frac{L_{eff}}{2}$).

The condition 1) is realized if $\tan 2\theta_{eff} \geq 1$, then from (10) it follows that at least one of the following two relations must be satisfied

$$\frac{\Delta m_\nu^2}{2E}A - \sqrt{2}G_F n_{eff} = 0, (\tilde{\mu}B \neq 0) \tag{15a}$$

$$2\tilde{\mu}B \geq \Big|\frac{\Delta m_\nu^2}{2E}A - \sqrt{2}G_F n_{eff}\Big|. \tag{15b}$$

Using the definitions (see, for example, in [7]) of the oscillation length in vacuum

$$L_V = \frac{2\pi}{\Delta E} = \frac{4\pi p}{m_2^2 - m_1^2}, (m_2 > m_1), \tag{16}$$

interaction oscillation length

$$L_{int} = \frac{2\pi}{\sqrt{2}G_F n_{eff}}, \tag{17}$$

and oscillation length in strong magnetic field

$$L_F = \frac{\pi}{\tilde{\mu}B}, \tag{18}$$

we can re-write the relations $(15a, b)$ as follows:

$$\frac{A}{L_V} - \frac{1}{L_{int}} = 0, (L_F < \infty), \tag{19a}$$

$$\frac{1}{L_F} \geq \left|\frac{A}{L_V} - \frac{1}{L_{int}}\right|. \qquad (19b)$$

From (15a) it follows that the neutrino conversion and oscillations can be induced by magnetic field of any strength if the resonant condition [5, 6] similar to the MSW one [2, 3] is realized (although for small values of $\tilde{\mu}B$ the effective oscillation length (11) would be large). However, from (15b) one can see that even if the resonant condition (15a) is not valid, the neutrinos conversion and oscillations effects would appear for strong enough magnetic fields. This situation has no anology with similar effects in matter when the only possibility (in the case of a small vacuum mixing angle) to get a substantial increase of neutrino conversion may be realized in the MSW resonant region.

Let us consider the relation (15b) and suppose that the right-hand side is not equal to zero. In the case of exact equality from (15b) we determine the critical strength of magnetic field [20]

$$B_{cr} = \left|\frac{1}{2\tilde{\mu}}\Big(\frac{\Delta m_\nu^2 A}{2E} - \sqrt{2}G_F n_{eff}\Big)\right| \qquad (20)$$

that constrain the range ($B \geq B_{cr}$) of field strengths for which the value of $\sin^2 2\theta_{eff}$ is not small (i.e., at least is not less than $\frac{1}{2}$) for all possible values of the right-hand side term in (15b).

It is also possible to express $B_{cr}$ in a more convenient numerical estimation form:

$$B_{cr} \approx 43\frac{\mu_B}{\tilde{\mu}}\left| - \Big(2.5\frac{n_{eff}}{10^{31}cm^{-3}}\Big) + A\Big(\frac{\Delta m_\nu^2}{eV^2}\Big)\Big(\frac{MeV}{E_\nu}\Big)\right| \; [Gauss]. \qquad (21)$$

For the case of strong magnetic fields ($B > B_{cr}$), $\sin^2 2\theta_{eff} \approx 1$, we find that for large enough lengths of a neutrino $\nu_i$ pass given by $x \approx L_{eff}\frac{k}{2}, k = 1, 2, ...$ in the magnetized medium characterized by $n_{eff}$ the probability (8) of conversion process $\nu_i \to \nu_j$ can reach the value of the order of $P(\nu_i \to \nu_j) \sim 1$.

Therefore, the initially emitted, for example, left-handed neutrino on the path lengths $x \geq \frac{L_{eff}}{2}$ can undergo convertion to the right-handed neutrino or to the right-handed antineutrino.

It is obvious that these oscillation processes take place only in the presence of strong magnetic fields $B \gg B_{cr}$, and the oscillation length $L_{eff}$, as it follows from (11), is $L_{eff} \approx L_F$. For $B \ll B_{cr}$ the influence of magnetic field is not important and oscillations (if they exist) are completely determined by the vacuum mixing angle and neutrino interaction with matter.

Now let us consider neutrinos that are produced in the interior of a neutron star where magnetic fields of the order of $10^{13}$ $G$ (or even a few orders of magnetude stronger) can exist (see, for example, [23, 24, 25]). For definiteness we suppose that initially $\nu_{eL}$'s are produced in the inner layers of the neutron star and shall take into account only that of the conversion processes (7), $\nu_{e_L} \to \nu_{e_R}$, that can be induced by the magnetic field on the neutrino pass from the centre to the surface of the neutron star.

In order to determine the scale of $B_{cr}$ on the bases of (20) and (21) we use the following values for characteristics of neutrinos and matter of the neutron star: $\mu \sim 10^{-10}\mu_B$, $\mu_B$ is the Bohr magneton, $n_{eff} \sim 10^{33}$ $cm^{-3}$, $\Delta m_\nu^2 \approx 10^{-4}$ $eV^2$, $\sin 2\theta = 0.1$ and $E_\nu \approx 20$ $MeV$. It follows that the main contribution is given by the "matter" term and for this case

$$B_{cr} = 1.11 \times 10^{14} \; G. \qquad (22)$$

Magnetic fields just of this order may exist on the surfaces of neutron stars [24, 25].

From (11) for effective oscillation length we get $L_{eff} \simeq 1\ cm$, that is much less than the characteristical scales of the neutron star structures (the thickness of the crust is, for instance, $L_{crust} \sim 0.1 R_{NS} \approx 1\ km$).

From these estimations we can conclude that for neutrinos passing from the inner layers to the surface the conversion and oscillations effects induced by the magnetic field can be important. However, if one is dealing not with a single neutrino but with a bunch of neutrinos that are emitted in different inner points of the neutron star then the average of the $x$ dependent term in formula (8) must be taken. Therefore the probability of $\nu_{eR}$'s appearing in the initial bunch of $\nu_{e_L}$'s is given by

$$\bar{P}(\nu_{e_R}) = \frac{1}{2}\sin^2 2\theta_{eff}. \tag{23}$$

It follows that the induced by strong magnetic field conversion and oscillations effects could yield in the approximate equal distribution of neutrinos between the two neutrino species ( $\sin^2 2\theta_{eff} \sim 1$ if $B \gg B_{cr}$); it also means that there would be a factor of two decrease in amount of initially emitted $\nu_{eL}$'s in the bunch.

Now let us consider the case of not too strong magnetic fields, viz., $B < B_{cr}$ along the whole neutrinos path inside the neutron star. If we exclude the possibility for the neutrinos to pass through the resonant conversion point [5, 6] determined by the Eq.(15$a$) we then get that the neutrino bunch after travelling through the neutron star will still be composed only of the left-handed neutrinos. However, when the bunch of neutrinos escapes from the neutron star it passes through a sudden change of density of matter and enters into the nearly empty space where $n_{eff} \to 0$. Effectively it may result that the neutrinos enter and pass through the region of strong field ($B > B_{cr}$) determined on the base of Eq.(15$b$). The neutrino conversion processes and oscillations may thus appear due to the "cross-boundary effect" (CBE) [20, 21].

To consider the CBE we suppose that the magnetic field on the surface of the neutron star is of the order of $B \sim B_0 = 10^{12}\ G$ and that the strength of the magnetic field decreases with the distance $r$ from the surface of the neutron star according to the law

$$B(r) = B_0\Big(\frac{r_0}{r}\Big)^3, \tag{24}$$

where $r_0$ is the radius of the neutron star.

The estimation for the critical field $B'_{cr}$ on the base of (20),(21) for the same values of $\mu$, $\Delta m_\nu^2$, $E_\nu$ and $\sin 2\theta$ (again for definiteness the conversion of the type $\nu_{eL} \leftrightarrow \nu_{eR}$ is considered) gives that

$$B'_{cr} = 5.4 \times 10^3\ G. \tag{25}$$

From (24) and (25) it follows that the magnetic field exceeds $B'_{cr}$ in regions characterized by

$$r \leq r_{cr} \approx 600 r_0. \tag{26}$$

. Therefore, along the distances of about $600r_0$ from the neutron star the magnetic field exceeds the critical field strength $B'_{cr}$. From the estimation for the effective oscillation length for the magnetic field at the surface of the neutron star

$$L_{eff}(B \sim B_0) = \frac{\pi}{\tilde{\mu}B_0} \simeq 10^2\frac{\mu_B}{\tilde{\mu}}\Big(\frac{1G}{B_0}\Big)[m] = 1\ m \tag{27}$$

it follows that the equal distribution of neutrinos between the two neutrino species ($\nu_{e_L}$ and $\nu_{e_R}$) appears after the neutrino bunch passes through a thin layer $\Delta x \gg 1\ m$ along which the decrease of the magnetic field is still negligible: $\Delta B(\Delta x) \ll B_0$.

So, in the case of "not too strong field" again as it was in the case of "strong field" after the neutrino bunch has passed a distance $L > L_{eff}$ from the neutron star the equal distribution of neutrinos among the two species $\nu_{e_L}$ and $\nu_{e_R}$ appears.

Consider the case when the neutrinos on their path inside the neutron star pass through the resonant region [5, 6]. In this region the condition of Eq.(15$a$) is valid. From (15 b) it follows that for any fixed strength of the magnetic field there is a layer ( between the two shells with radiuses $x_1$ and $x_2$) on the neutrino path to the surface of the neutron star where effectively the "strong field" case is realized. If the distance $x_2 - x_1$ is greater than the effective oscillation length $L_{eff} \sim L_F$ then after neutrinos pass through this *resonant region* again the equal neutrino distribution between the two neutrino species appears.

The effect discussed above of suppression of amount of electron neutrinos (or other active neutrinos) induced by strong magnetic fields may have sufficient consequences on the reheating phase of a Type II supernova [2] that can be used for getting constraints on the value $\tilde{\mu}B$. Let us suppose that the magnetic field induced neutrino oscillations do not destroy the proposed model [14] of about 60 % increase in the supernova explosion energy. If the magnetic field $B \sim 10^{14}\ G$ exists at the radius of $r_0 = 45\ km$ from the centre of the hot proto neutron star (the matter density in this region is $\rho \sim 10^{12}\ g/cm^3$) and decreases with distance according to (24) then on the distances $r \sim 160\ km$ from the centre the magnetic field is $\sim 0.6 \times 10^{13}\ G$. This field is of the order of the $B_{cr}$ determined by (20),(21) for the density $\rho \sim 6 \times 10^8\ g/cm^3$ and the magnetic moment $\tilde{\mu} \sim 10^{-10}\mu_B$. For this case the probability of finding, for example, sterile $\nu_{eR}$'s among the initially emitted $\nu_{eL}$'s is $\bar{P}_{\nu_{eL} \to \nu_{eR}} = 0.25$ (the effective length (11) for this effect is $L_{eff} \sim\ 10\ cm$). Therefore, in order to avoid the loss of a substantial amount of energy that will escape from the region behind the shock together with the sterile neutrinos, one has to constrain the magnetic moment on the level of $\tilde{\mu} \leq 10^{-11}\mu_B$.

We should like to point out the importance of the resonance enhancement [5, 6] of neutrino conversion and oscillations effect in magnetic fields that may substantially change the energetics of the shock and also give a stringent constraints on the value of $\tilde{\mu}B$.

It must be noted that with the appropriate choice of $n_{eff}$, $A$ and $\tilde{\mu}$ the conversion and oscillations processes between different neutrino species similar to (7) can be considered.

It is also interesting to consider the neutrino conversion and oscillations induced by the interstellar galactic magnetic fields that are of the order $B_G \sim 10^{-6}\ G$. The critical field estimated on the bases of Eqs.(20), (21) for ultra high energy neutrinos ($E \geq 10^{17}\ eV$) are $\leq 10^{-6}\ G$. Taking into account the estimation for the effective oscillation length $L_{eff}(B \sim B_G) = 10^{20}\ cm$, that is much less than the radius of the galaxy ($R_G \approx 3 \times 10^{22}\ cm$) we conclude that in this case the effect of neutrino conversion and oscillations in "strong magnetic field" can be presented.

The similar analysis on the bases of (15a,b) could be applied to the solar neutrinos. The estimations show that even if the strength of the field in some regions of the sun may exceed the critical value of the magnetic field, $B > B_{cr}$,

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