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QUANTUM EFFECTS INDUCED BY A GAP IN THE SPECTRUM OF ATOM–BATH COUPLING CONSTANTS: "FREEZING" OF ATOMIC DECAY AND MONOCHROMATIC COLLECTIVE RADIATION

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ABSTRACT

A specific kind of inhibition of atomic decay ("freezing" of decay) and intense monochromatic collective radiation are predicted for a single two-level atom and for a system of atoms interacting with the field bath having the gap in the spectrum of coupling constants.

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1. INTRODUCTION.

In 1987 Yablonovitch predicted [1] and later together with co-authors experimentally proved [2,3] the existence of specific media with a gap in the total photon density of states over a narrow frequency range (photonic band gap). These media are lossless periodic dielectric microstructures (photonic crystals). An example of such photonic crystal is a slab of dielectric material in which a specific pattern of criss-crossed cylindric holes is drilled. This pattern of holes produces a fully three-dimensional periodic fcc structure [3]. According to Yablonovich, at a refractive index n=3.6, normalized hole diameter d=0.469a, where a is the lattice period, and a 78% volume fraction is removed a full gap is opened up in the microwave region and its width can reach about 19% of the center frequency. For this frequency region the propagating electromagnetic modes were absent in all directions. The cause of this phenomenon is the interplay between the coherent scattering due to the periodic structure and the scattering due to individual voids [4,5]. This dielectric structure could exhibit potentially new physical phenomena, for example, suppression of the dipole-dipole interaction between two atoms [6], inhibition of the single-photon spontaneous emission and formation of the quantum bound state of the photon in the vicinity of the atom [7].

In this paper, which is a continuation of our work [8], we demonstrate that the above mentioned effects can be produced not only by creating a photonic band gap but also by creating a gap in the spectrum of constants of the interaction of radiation with media. It should be emphasized that a gap in the spectrum of coupling constants may occur without deforming the total photon density of states to the extent that a photonic band gap opens up. However, for a sufficiently dense frequency spectrum a photonic band gap and gap in the spectrum of interaction constants seem to be equivalent. We have shown that the inhibition of atomic decay due to a gap in the spectrum of interaction constants occurs in a specific way. It is characterized by a complete termination of the decay after the atom has lost part of its energy. That is why we will refer to this kind of inhibition as the "freezing of decay". The energy lost by the atom and absorbed by bath excites a monochromatic radiation at a frequency which coincides with that of the gap-center. This radiation has a collective nature because in effect it is a specific distribution of excitations between the bath modes perturbed by the interaction with the atom. This collective radiation is a sub-Poissonian in statistics.

The outline of this paper is as follows. In Sec.2 we provide a mathematical treatment of effects caused by the gap in the spectrum of coupling constants. The treatment is carried out by introduction of the collective atom-field operators and the Hamiltonian diagonalization in terms of these operators. Sec.3 is devoted to a single-atom interaction with a gap-having bath. Here the exact results concerning the "freezing" of the decay and monochromatic collective radiation are obtained. In Sec.4 we extend the results obtained in Sec.2 to the system of two-level atoms interacting with the bath. In this Section the sub-Poissonian statistics of monochromatic collective radiation is also discussed.

2. GENERAL EQUATIONS.

Let us consider the standard atoms-field interaction Dicke-type Hamiltonian in the dipole and the rotating wave approximations

$$H = \hbar\omega_0\sum_{i=1}^{M}(\sigma_z)_i/2 + \hbar\sum_{j=1}^{N}\omega_j b_j^+ b_j + \hbar\sum_{i,j}(g_i\sigma_j^+ b_i \exp(i\phi_j) + h.c.), \quad (1)$$

where the radiation mode labelled by j has frequency ω_j, b_j^+ and b_j are the creation and annihilation operators respectively; $(\sigma_z)_i$, σ_i^+, σ_i^- are the pseudospin operators corresponding to the i-th atom and ω_0 is the frequency of the atomic transition.

To simplify the further discussion let us choose a real set of g_i and take into account any possible differences in the spacing of atoms by means of the phase exponentials $\exp(i\phi_i)$. Since the "rotating-wave" approximation is made in the Hamiltonian (1), there is an important integral of motion

$$I = \hbar\omega_0\left[\sum_{i=1}^{M}(\sigma_z)_i/2 + \sum_{j=1}^{N}b_j^+ b_j\right] . \quad (2)$$

We omit it from the Hamiltonian (1) and introduce an effective Hamiltonian

$$H_{eff} = \hbar\sum_{j=1}^{N}\Delta_j b_j^+ b_j + \hbar\sum_{j=1}^{N}(g_j S^+ b_j + h.c.), \quad (3)$$

where $\Delta_j=\omega_j-\omega_0$ and $S^+=\sum_{i=1}^{M}\sigma_i^+\exp(+i\phi_i)$. We shall return to the original picture given by Hamiltonian (1) when the final results are to be obtained. Collective atomic operators S^- and S^+ satisfy the commutation relation of the pseudospin operators

$$[S^+,S^-] = S_z,$$

where $S_z = \sum_{i=1}^{N} (\sigma_z)_i$.

Further we use the diagonalization method which has been described in [8,9]. For this purpose it is necessary to represent the effective Hamiltonian (3) as a quadratic form

$$H = h\mathbf{F}^+(\Omega + G)\mathbf{F} \quad (4)$$

of vectors F^+ consisting of pseudo spin operators S^+ and b_i^+:

$$\mathbf{F}^+ = (S^+, b_1^+, b_2^+, \ldots\ldots. b_N^+)$$

and $\mathbf{F}$ as its Hermitian conjugate. In Eq.(4) Ω is the diagonal matrix with the elements

$$\Omega_{ij} = \Delta_i, (i=1,2\ldots.N), \ \Delta_0 \equiv 0,$$

and G is the symmetric matrix

$$G_{ij} = \delta_{i0} g_j + \delta_{0j} g_i, \ g_0 \equiv 0.$$

The matrix $\Omega + G$ can be diagonalized using the unitary transformation

$$U(\Omega + G) = \Lambda U, \quad (5)$$

where Λ is the diagonal matrix with elements $\lambda_{ij} = \lambda_i$, $(i=0,1\ldots.N)$, determined by the following characteristic equation:

$$\lambda_i = \sum_{j=1}^{N} g_j^2/(\lambda_i - \Delta_j). \quad (6)$$

The relationship between the elements U_{ij} and U_{i0} is

$$U_{ij} = g_j U_{i0}/(\lambda_i - \Delta_j). \quad (7)$$

This equation together with the unitary condition $(U^+U = 1)$

$$\sum_{i=0}^{N} U_{ij} U_{ik} = \delta_{jk} \quad (8)$$

uniquely defines the elements U_{i0} and, consequently, the matrix U_{ij}:

$$|U_{i0}|^2 = \left[1 + \sum_{k=1}^{N} g_k^2/(\lambda_i - \Delta_k)^2 \right]^{-1}. \quad (9)$$

According to Eq.(8) the following normalization condition takes place

$$\sum_{i=0}^{N} |U_{i0}|^2 = 1.$$

Using the obtained transformation matrix U_{ij} one can introduce new collective operators $\mathbf{C} = \mathbf{UF}$ which are a superposition of the field and atoms operators

$$C_i = U_{i0} S^- + \sum_{j=1}^{N} U_{ij} b_j = S^- \cos\vartheta_i + B_i \sin\vartheta_i, \quad (10)$$

where $\cos\vartheta_i = U_{i0}$, and B_i is the field part of the collective operator C_i. This part is the superposition of the field modes

$$B_i = \sum_{k=1}^{N} v_{ik} b_k,$$

where

$$v_{ij} = g_k \left[(\lambda_i - \Delta_k) \left(\sum_{m=1}^{N} g_m^2/(\lambda_i - \Delta_m)^2 \right)^{1/2} \right]^{-1}. \quad (11)$$

In terms of the operators C_i the effective Hamiltonian (3) is diagonally, i.e.

$$H_{eff} = \hbar\sum_{i=0}^{N}\lambda_i C_i^{+}C_i. \qquad (12)$$

From this Hamiltonian the following Heisenberg equations for C_i can be obtained

$$dC_i/dt + i\lambda_i C_i = iU_{i0}\sum_{j=0}^{N}\lambda_j U_{j0}\{S_z + 1\}C_j. \qquad (13)$$

Now let the frequency spectrum of an atom-field interaction has a *parabolic* gap near the frequency ω_g. Generally speaking ω_g does not coincide with any of the frequencies ω_i

$$g_i^2 = (\omega_g - \omega_i)^2\bar{g}_i^2. \qquad (14)$$

Here $\bar{g}_i^2$ is a smooth and gapless spectrum (for example, the Gaussian one which we chose for numerical calculations). It can be shown that $\omega_g-\omega_0$ is a solution of the characteristic equation (6) under the following conditions

$$\omega_g = \langle\omega\rangle + G(\omega_0 - \omega_i), \qquad (15)$$

where

$$\langle\omega\rangle = \sum_{k=1}^{N}\omega_k\bar{g}_k^2/\sum_{k=1}^{N}\bar{g}_k^2$$

is the average frequency of the bath-mode frequency distribution and

$$G = (1 + \sum_{i=1}^{N}\bar{g}_i^2)^{-1}. \qquad (16)$$

It is of great importance that under the conditions (14) and (15) the value of $U_{g0}^2 = G$ corresponding to the eigenvalue $\omega_g - \omega_0$ turns outto exceed by far other coefficients U_{i0}^2. This interesting fact is illustrated by Fig.1, where the distribution of U_{i0}^2 for the smooth and gapless g_i^2 spectrum is shown, and by Fig.2, where the distribution of U_{i0}^2 for the spectrum of g_i^2 with a gap is demonstrated.

As can be seen below, the large value of U_{g0}^2 distinguishes the collective mode C_g from other collective modes and gives rise to a number of new physical effects which will be discussed below.

4. SINGLE ATOM.

When the general multiatomic problem is reduced to the interaction of a single atom with an electromagnetic field there is a possibility to give an explicit description of some new physical effects. If initially an atom is excited and the bath is in the vacuum state, the single-atom version of equations (13) takes the simple form

$$dC_i/dt + i\lambda_i C_i = 0 . \qquad (17)$$

Substituting the solution of the equation (17)

$$C_i(t)=C_i(0)\exp(-i\lambda_i t). \qquad (18)$$

into the equation (10) we obtain

$$\sigma^-(t) = \sum_{i=0}U_{i0}C_i(t) = U(t)\sigma^-(0) + \sum_{i,j=1}U_{i0}U_{ji}b_j(0)\exp(-i\lambda_i t), \qquad (19)$$

where

$$U(t) = \sum_{i=0}^{N}U_{i0}^2\exp(-i\lambda_i t) . \qquad (20)$$

Using the equations (18)-(20) we are going to discuss below two phenomena resulting from a gap in the spectrum of coupling constants.

4.1 "Freezing" of decay.

If there is no gap in the coupling constants spectrum (Fig.1a), all the coefficients U^2_{i0} are comparable (Fig.1b). In this case due to the differences of temporal phase shifts ($\lambda_i t$ in Eq.(20)) the amplitude U(t) will decay approaching zero value at $t \to \infty$ in the limit of the dense spectrum where $|\omega_i - \omega_{i+1}| \Rightarrow 0$. The decay rate can be estimated as halfwidth of the bath-atom coupling constants frequency spectrum $|g_i(\omega)|^2$. If the frequency spectrum is discrete (i.e. $|\omega_i - \omega_{i+1}|$ is finite), there occur "revivals" in time dependence of U(t) (see Fig.3 as well as [8]).

When the spectrum of $|g_i(\omega)|^2$ has a gap (Fig.2a), the atom decays within some initial time (Fig.4). But as it follows from equation (20), when the time exceeds the boundary value (denoted by T_{st}) the character of the evolution is completely changed:

$$U(t)\Big|_{t>>T_{st}} \approx U^2_{g0}\exp(-i\omega_g t) = G\exp(-i\omega_g t). \qquad (21)$$

The atom is prevented from losing all its energy and the upper level population is held at a stationary "frozen" value.

The nature of this phenomenon can be understood from the analysis of the bath correlation function

$$K(t) = \sum_{i=1}^{N} g_i^2 \exp(-i\omega_i t). \qquad (22)$$

Due to the existence of a gap in the coupling constants spectrum the real part of the bath correlation function has a negative peak

(Fig.5). The negative peak in the real part of the bath correlation function is believed to be a specific feature of feedback systems [1]. Thus it can be stated that a kind of feedback organized by the gap is established between the atom and the bath after time T_{st}. This feedback holds the upper level population of atom at a stationary "frozen" value.

Stabilization time T_{st} is estimated as the position of the negative peak in K(t). This can be proved by substitution the equation (14) into the equation (22). The stabilization time T_{st} depends in fact on the gap width rather than on the values of g_i^2.

4.2 The monochromatic collective radiation.

Time-dependent bath mode operators can be expressed using the equations (10) and (19)

$$b_i(t) = V_{i0}(t)\sigma^-(0) + \sum_{k=1} V_{ik}(t) b_k(0), \qquad (23)$$

where

$$V_{ij}(t) = \sum_{l=1} U_{il} U_{lj} \exp(-i\lambda_l t) . \qquad (24)$$

If initially an atom is excited and the field is in the vacuum state, then, according to the equation (23), all but one normally ordered correlation function are zero. Due to the presence of only one photon in the whole system the nonzero correlation function is factorized

$$\langle b_i^+(t) b_j(\tau)\rangle = V^*_{i0}(t) V_{j0}(\tau) .$$

In the case when there is no gap in the spectrum of coupling constants, the amplitudes of oscillations at each frequency λ_i

of the spectrum are comparable, as it follows from the equation (24). It is the presence of a gap that makes the amplitude of oscillations at the gap-center frequency much larger then the amplitudes of other oscillations ($U^2_{g0} >> U^2_{ij}$). After a sufficiently long period of time ($t >> T_{st}$) the dynamics of each bath mode acquires the monochromatic character

$$V_{i0}(t)\Big|_{t >>T_{st}} \approx U_{gi}U_{g0}\exp(-i\omega_g t) = \bar{g}_i G \exp(-i\omega_g t)$$

and

$$<b_i^+(t)b_i(\tau)>\Big|_{t,\tau >>T_{st}} \approx \bar{g}_i^2 G^2 \exp(i\omega_g(t-\tau)) .$$

In other words, due to the gap in the spectrum of interaction constants the monochromatic radiation is generated. Obviously, it is a laser effect resulting from the coherent interaction of an atom with the bath modes. To stress the nature of this effect we propose to use a term "monochromatic collective radiation". This collective radiation takes the largest part of energy lost by an atom during the evolution to the stationary "frozen" state.

The uniqueness of oscillations at the gap-center frequency can be also described in terms of collective operators. Time-dependent bath operators can be expressed as a linear superposition of the collective operators

$$b_i(t) = \sum_{j=0} U_{ij} C_j(0)\exp(-i\lambda_j t) .$$

As is seen from this equation, the unique nature of oscillations at the gap-center frequency is the consequence of the peculiarity of the collective mode described by collective operator C_g. Due to the gap the collective mode C_g, which oscillates at the gap-center frequency, is excited stronger than the others

$$<C_g^+C_g> >> <C_i^+C_i> .$$

Using the language of collective modes it is possible to unify the description of both described phenomena. Actually, the peculiarity of collective mode C_g underlies both the "freezing" of decay and the monochromatic collective radiation. This possibility is important for further discussion because it permits to test the appearance of the effects in more complicated problems than the single-atom and single-excitation problem discussed above. For example, in Appendix we discuss the case when there are many photons initially in the system, i.e. the bath initially excited. According to the equations (A.4) and (A.5), the initial excitation can eqilize the intensities of collective modes. If the initial number of photons in the bath modes is so large that for collective mode C_i

$$<\sigma^+(0)\sigma^-(0)>G \approx \sum_{j=1} U^2_{ij} <b_j^+(0)b_j(0)>,$$

the intensity of this collective mode is comparable with that of the collective mode C_g. Therefore, the described effects can't be observed: oscillations of the bath modes become nonmonochromatic and the upper level population of an atom can not bestationary any more.

5. MULTIATOMIC SYSTEM.

Applying the above method to the case of a group of atoms interacting with the bath, one can demonstrate that the "freezing" of decay and the monochromatic collective radiation can occur in this case too.

Let us start from the simplest exactly solvable model of a single initially excited atom. The other atoms are supposed to be initially in the ground state and the bath is in the vacuum state. The equations (13) can be rewritten in the general form

$$dC_i/dt + i\lambda_i C_i + i(M-1)U_{i0}\sum_{j=0}^{N}\lambda_j U_{j0}C_j =$$

$$= 2iU_{i0}\sum_{j=0}^{N}\lambda_j U_{j0}\theta C_j, \qquad (25)$$

where $\theta = \sum_{k=1}^{M}\sigma_k^{+}\sigma_k^{-}$. For the case when only one atom is initially excited the right-hand part of the equations (25) can be dropped to form the following set of equations

$$dC_i/dt + i\lambda_i C_i + i(M-1)U_{i0}\sum_{j=0}^{N}\lambda_j U_{j0}C_j = 0$$

with the solution

$$C_i(t) = \sum_{j=0}^{N}B_{ij}\exp(-i(x_j + \omega_0)t)C_j(0), \qquad (26)$$

where the eigenvalues x_i are given by the following characteristic equation

$$\sum_{k=0}^{N}\lambda_k U_{k0}/(\lambda_k - x_i) = (M-1)^{-1}, \quad (i=0,1,2....N), \qquad (27)$$

and

$$B_{ij} = \frac{(\omega_g - \omega_0 - x_i)U_{j0}}{(\lambda_i - x_j)U_{g0}} L_i, \qquad (28)$$

where

$$L_i = \left[\sum_{l\neq g}\left(\frac{(\omega_g - \omega_0 - x_j)U_{jo}}{(\lambda_l - x_j)U_{g0}}\right)^2 + 1\right]^{-1/2}.$$

For a rather large number of atoms (M >>1) the right-hand part of equation (27) is very small. Thus the eigenvalues x_i are practically independent of M. As it follows from the inequality

$$U_{g0}^2 >> U_{i0}^2,$$

there is eigenvalue x_g which is very close to the difference $\omega_g - \omega_0$:

$$|x_g - \omega_g + \omega_0| << |x_i - \lambda_j|,$$

Therefore, in accordance with the equation (28) the inequality

$$1 > B_{gg} = |U_{g0}| >> |B_{ij}| \qquad (29)$$

is satisfied and the uniqueness of the collective mode $C_g(t)$ is manifested for a multiatomic system in the same way as for a single-atom one. As a consequence, the decay of an initially excited atom will be "frozen" at the value of a higher level population which is equal to G similar to the single-atom case. The collective monochromatic radiation will arise at the gap center frequency practically with the same intensity.

In Appendix we have investigated the case of an initial multi-atom excitation and the influence of the initial field excitation considering the multi-atomic effects by taking into account the right-hand of Eq.(25) perturbative manner. The results of Appendix are as follows: if the number of initially excited atoms is small in comparison with their total number, and the initial excitation of bath is so small that the following condition

$$<\theta(0)>G \gg \sum_{j=1} U_{ij}^2 <b_j^+(0) b_j(0)> ,$$

is satisfied for all i, the decay of initially excited atoms will be "frozen" and collective radiation will arise similarly to the single-atom case. It should be noted that the discussed case of monochromatic collective radiation is an example of laser generation without inversion, because the number of initially excited atoms may be arbitrarily small in comparison with the total number of atoms.

To describe the statistical properties of the monochromatic collective radiation, it is useful to introduce the characteristic operator

$$P(z) = \int d\xi \exp(-i\xi z) <\exp(i\xi \hat{N})>,$$

where $\hat{N} = \sum_{i=1}^{N} b_i^+ b_i$ is the operator of the total number of excitations. From integral (2) we have

$$\hat{N} = I - \sum_{i=1}^{M} \sigma_i^+ \sigma_i^- ,$$

and

$$<I(t)> = <I(0)> = <\sum_{i=1}^{M} \sigma_i^+(0)\sigma_i^-(0) + \sum_{i=1}^{N} b_i^+(0) b_i(0)>. \quad (30)$$

If we assume the initial state of the field to be vacuum and the number of excited atoms to be equal to V (their total number is M) then using the equation (30) we find

$$<\exp(i\xi \hat{N})> = \exp(i\xi V) \prod_{i=1}^{M} \{1 + (\exp(-i\xi) - 1) <\sigma_i^+(t)\sigma_i^-(t)>\}. \quad (31)$$

According to the equations (B.7) and (B.8) one can write for all initially excited atoms

$$<\sigma_i^+(t)\sigma_i^-(t)>\Big|_{t \gg T_{st}} \approx G^2 <\sigma_i^+(0)\sigma_i^-(0)> . \quad (32)$$

Due to the gap the upper-state population of initially excited atoms is "frozen" similarly to the single-atom case. An energy is not redistributed among all atoms contrary to the case when the gap is absent (see, for example [10]). In other words, one can say that the interaction of atoms between themselves through the radiation bath is suppressed. As follows from the equations (31) and (32)

$$P(z) = \sum_{k=0}^{V} C_V^k (1 - G^2)^{2V-2k} G^{2k} \delta(z - k). \quad (33)$$

The photon number distribution described by the equation (33) is a binomial one. It gives the average number of photons

$$<z> = V(1 - G^2)$$

and the variance

$$D(z) = VG^2(1 - G^2) = G^2 <z>. \quad (34)$$

The variance given by the equation (34) is smaller than the Poissonian one. However from the fact that the value of G is close to one it follows that the radiation antibunching of photons is slight.

6. CONCLUSIONS.

In this work we have demonstrated the existence of two new physical effects due to the placing of atomic system in a medium which rearranges the atom-bath interaction in such a way that a gap appears in the atom-bath interaction spectrum. We have shown that in this case the special kind of inhibition of atomic decay (which we named "freezing" of decay") and connected with it the effect of monochromatic collective radiation take place. Unlike the usual laser radiation this collective radiation has the sub-Poissonian statistics. We have also investigated the influence of the initial bath excitation on the described effects. A rather strong initial excitation of the bath can make the effects of the "freezing" of decay and the monochromatic collective radiation unobservable.

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APPENDIX A. SINGLE-ATOM PERTURBATION THEORY.

Unfortunately a complete analitical solution of Eq.(13) is unknown. However the perturbation theory can be used for description of the system evolution (under the condition discussed below) and for illustration of the unique role of the C_g mode. This theory is based on the inequality

$$1 > |U_{g0}| >> |U_{i0}|. \quad (A.1)$$

Thus, all of U_{i0} for i≠g would be treated as small parameters.

Let us introduce the "interaction picture"

$$\tilde{C}(t) = C(t)\exp(-i(\omega_g - \omega_0)t)$$

and write the following equations

$$d\tilde{C}_i/dt + i\varepsilon_i\tilde{C}_i = 2iU_{i0}\sum_{j=0}^{N}\varepsilon_j U_{j0}\sigma^+\sigma^-\tilde{C}_j, \quad (A.2)$$

$$d(\sigma^+\sigma^-\tilde{C}_i)/dt + i\varepsilon_i\sigma^+\sigma^-\tilde{C}_i = 2iU_{i0}\sum_{j=0}^{N}\varepsilon_j U_{j0}\sigma^+\sigma^-\tilde{C}_j -$$

$$- i\sum_{j=0}^{N}\varepsilon_j U_{j0}(\sigma^+\tilde{C}_j\tilde{C}_i - \tilde{C}_j^+\sigma^-\tilde{C}_i), \quad (A.3)$$

where $\varepsilon_i = \lambda_i - (\omega_g - \omega_0)$.

Neglecting the second and higher degrees of U_{i0} terms and returning to the original picture given by Hamiltonian (1) we obtain

$$C_g(t) \approx C_g(0)\exp(-i\omega_g t) -$$
$$-2U_{g0}\sum_{j=0}^{N}U_{j0}\sigma^+(0)\sigma^-(0)C_j(0)\exp(-i(\lambda_j+\omega_0)t), \quad (A.4)$$

$$C_i(t) \approx C_i(0)\exp(-i(\lambda_i + \omega_0)t), \quad (\lambda_i \neq \omega_g - \omega_0). \tag{A.5}$$

According to Eq.(A.4) and (A.5) the behavior of all the collective modes except C_g resemble the bosonic one. It can be also seen from the commutation relations for the collective modes

$$[C_i, C_j^+] = \delta_{ij} - 2U_{i0}U_{j0}\sigma^+\sigma^-. \tag{A.6}$$

If all indices i and j in (A.6) are different from g then the second term in right-hand parts of Eq.(A.6) are small and the commutation relations of these collective modes resemble the bosonic ones. Because of a large value of U_{g0} an atom-field interaction nonlinearity (the second term in the right part of Eq.(A.4)) is significant for the mode C_g only.

APPENDIX B. MULTIATOMIC PERTURBATION THEORY.

It is possible to develop a perturbation theory using the solution of Eq.(25) with the right-hand side omitted. If this solution is denoted by $C_i^0(t)$ then

$$C_i(t) = C_i^0(t) + \delta C_i(t), \tag{B.1}$$

where the nonlinear perturbation term $\delta C_i(t)$ is the main subject of this Appendix consideration.

Equations for θC_i operators have the form

$$d\theta C_i/dt + i\lambda_i\theta C_i + i(M-1)U_{i0}\sum_{j=0}^{N}\lambda_j U_{j0}\theta C_j =$$

$$= iU_{i0}\sum_{j=0}^{N}\lambda_j U_{j0}\theta^2 C_j + i\sum_{j=0}^{N}\lambda_j U_{j0}(C_j^+ S^- C_i - S^+ C_i C_j), \tag{B.2}$$

where $\theta = \sum_{i=1}^{M}\sigma_i^+\sigma_i^-$. The linear part of this equation resembles the Eq.(25). Therefore θC_i can be presented as

$$\theta C_i(t) = [\theta C_i(t)]^0 + \delta[\theta C_i(t)], \tag{B.3}$$

where $[\theta C_i(t)]^0$ denotes the solution of the linear part of the equation (B.3). Substituting the Eq.(B.3) in Eq.(25) and using the unitary transformation

$$\delta C_i(t) = \sum_{j=0}^{N} B_{ij}\exp(-ix_j t)\delta\tilde{C}_j(t)$$

the following formulae can be obtained

$$d\delta\tilde{C}_i/dt = 2if_i(t)\sum_{j=0}^{N}\lambda_j U_{j0}\theta C_j, \tag{B.4}$$

where

$$f_i(t) = \sum_{j=0}^{N} B_{ij}U_{j0}\exp(ix_j t) \tag{B.5}$$

Thus

$$f_g(t) = \sum_{j=0}^{N} U_{j0}^2\exp(ix_j t)\Big|_{t \gg T_{st}} \approx G\exp(ix_g t).$$

In accordance with the inequality (29)the values $B_{ij}U_{j0}$ (for all $i \neq g$) are comparable and therefore

$$|f_i(t)|\Big|_{t \gg T_d} \Rightarrow 0.$$

Here T_d is the characteristic decay time estimated as an inverse width of the bath frequency spectrum. Thus

$$d\delta C_i(t)/dt\Big|_{t\,\gg\,T_d} \to 0,$$

i.e. all the collective modes except mode C_g have been stabilized and their evolution has acquired a bosonic character for the time $T_d \ll T_{st}$. As one can see below this is true within the accuracy of the consideration.

Let us substitute Eq.(B.3) into Eq.(B.2) and make the unitary transformation

$$\delta[\theta C_i(t)] = \sum_{j=0}^{N} B_{ij}\exp(-ix_j t)\delta[\theta\tilde{C}_j(t)].$$

Within the required accuracy (up to the second degree of U_{10}) the following equation is obtained(here and after $t \gg T_{st}$):

$$d\delta C_g/dt \approx 2if_g(t)\Big[\sum_{j=0}^{N}\lambda_j U_{j0}[\theta C_j]^0 +$$

$$+ \Big(\sum_{k=0}^{N}\lambda_k U_{k0}B_{kg}\Big)\delta[\theta\tilde{C}_g]\exp(-ix_g t)\Big]. \qquad (B.6)$$

In the second term of the right-hand part of the Eq.(B.6) the $\sum_{k=0}^{N}\lambda_k U_{k0}B_{kg}$ sum is estimated as $1/(M-1)$. If a number of atoms is sufficiently large ($M \gg 1$) this term can be dropped under the condition

$$\langle\theta\rangle/M \ll 1.$$

Consequently one of the conditions of the developed theory applicability is obtained. It is a small number of initially excited atoms compared with the total number of atoms.

After returning to the original picture given by the Hamiltonian (1) and ignoring of negligible terms the final result is

$$C_g(t) \approx |U_{g0}|\exp(-i\omega_g t)C_g(0) + \sum_{x_i \neq x_g} B_{gi}\exp(-i(x_i + \omega_0)t)C_i(0) +$$

$$+ |U_{g0}|\exp(-i\omega_g t)\delta\tilde{C}_g(t), \qquad (B.7)$$

where

$$\delta\tilde{C}_g(t) \approx 2\sum_{j=0}^{N} d_j\{1 - \exp(-i(x_j - \omega_g + \omega_0)t)\}[\theta C_j(0)]$$

and

$$d_j = U_{j0}\sum_{k=0}^{N}\lambda_k U_{k0}L_k/(\lambda_k - x_j),$$

$$C_i(t) \approx \sum_{j=0}^{N} B_{ij}\exp(-i(x_j + \omega_0)t)C_j(0). \qquad (B.8)$$

REFERENCES

1. E.Yablonovitch, Phys.Rev.Lett. **58**, 2059 (1987).
2. E.Yablonovitch and T.Gmitter, Phys.Rev.Lett. **63**, 1950 (1989).
3. E.Yablonovitch, J.Phys.:Condens. Matter **5**, 2443 (1993).
4. K.M.Leung and Y.F.Lin, Phys.Rev. B **41**, 10188(1990).
5. S.John and R.Rangarajan, Phys.Rev. B **38**, 10101(1988).
6. G.Kurizki and A.F.Genak, Phys.Rev.Lett. **61**, 2269(1988).
7. Z.Zang, S.Satpathy, Phys.Rev.Lett. **65**, 2650 (1990).
8. S.Kilin and D.Mogilevtsev, Laser Phys. **2**, 153 (1992).
9. R.Loudon "The Quantum Theory of Light", Clarendon Press, Oxford, 1973, chapter 8.
10. G.S.Agarval "Quantum statistical theories of spontaneous emission and their relation to other approaches", Springer-Verlag, Berlin, 1974.

FIGURE CAPTIONS

Fig.1 An example of smooth (Gaussian) spectrum of g^2_{Uk} (a) and corresponding distribution of U^2_{10} (b). The frequency spectrum is equidistant Gaussian with the distance Δ between nearest frequencies . The model calculation is made for the number of field oscillators $N = 41$. All λ_i are the roots of the characteristic equations (3).

Fig.2 An example of spectrum of g^2_{Gk} with the gap (a) and corresponding distribution U^2_{10} (b). The quantities g^2_{Gk} are chosen in such a way that $\bar{g}^2_{Gk}$ spectrum is the Gaussian one with characteristics similar to Fig.1 spectrum and

$$\sum_{k=1}^{N} g^2_{Uk} = \sum_{k=1}^{N} g^2_{Gk}.$$

Fig.3 A typical temporal dependence of U(t) for the smooth (Gaussian) g^2_{Uk} spectrum from Fig.1. Stage I corresponds to motion within time interval $[0,T_d]$. Stage II corresponds to motion within time interval $[T_d, 2T_d]$.
Dimensionless time $T = 3t\sigma/2\pi$.

Fig.4 The same as in Fig.3 for the spectrum of g^2_{Gk}(Fig.2) with the gap; $T = 3t\sigma/2\pi$.

Fig.5 The real part of the bath correlation function K(t) for the spectrum of g^2_{Gk} (Fig.2) with the gap; $T = 3t\sigma/2\pi$.

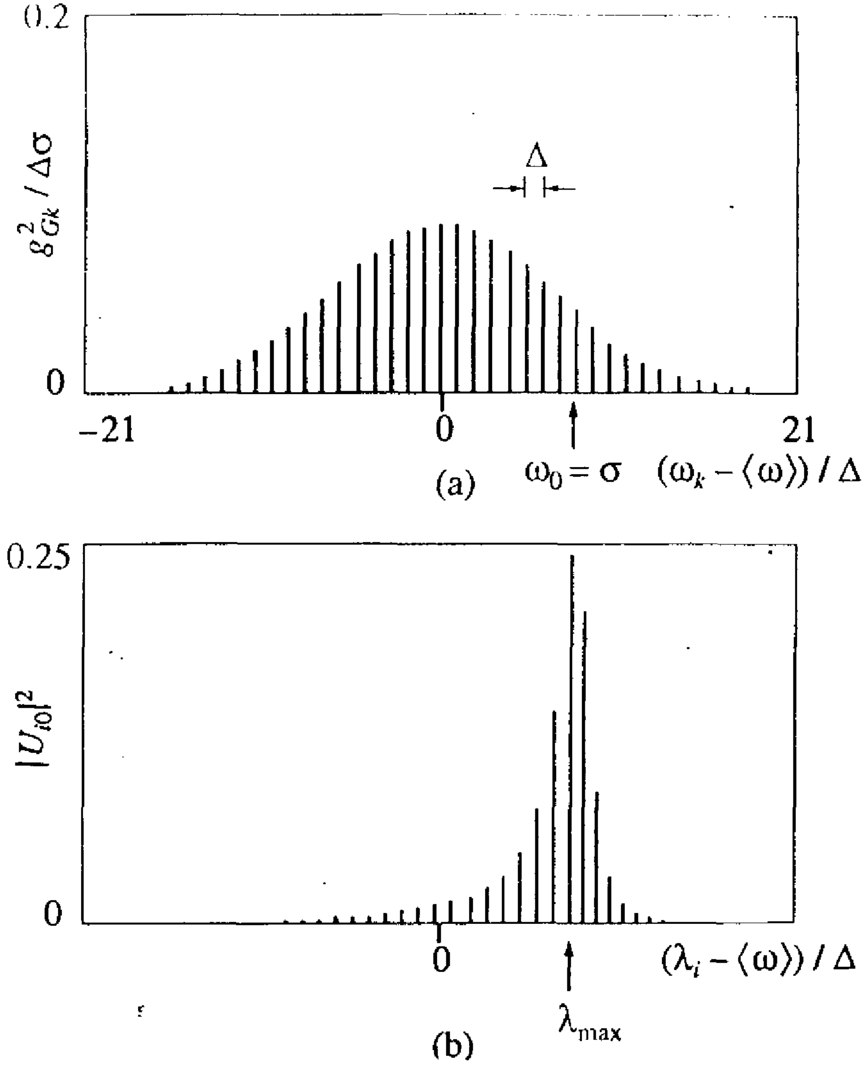

0.2
$g_{Gk}^2 / \Delta\sigma$
Δ
0
−21
0
21
$\omega_0 = \sigma$
$(\omega_k - \langle\omega\rangle) / \Delta$
(a)
0.25
$|U_{i0}|^2$
0
0
$(\lambda_i - \langle\omega\rangle) / \Delta$
λ_{max}
(b)

Fig.1

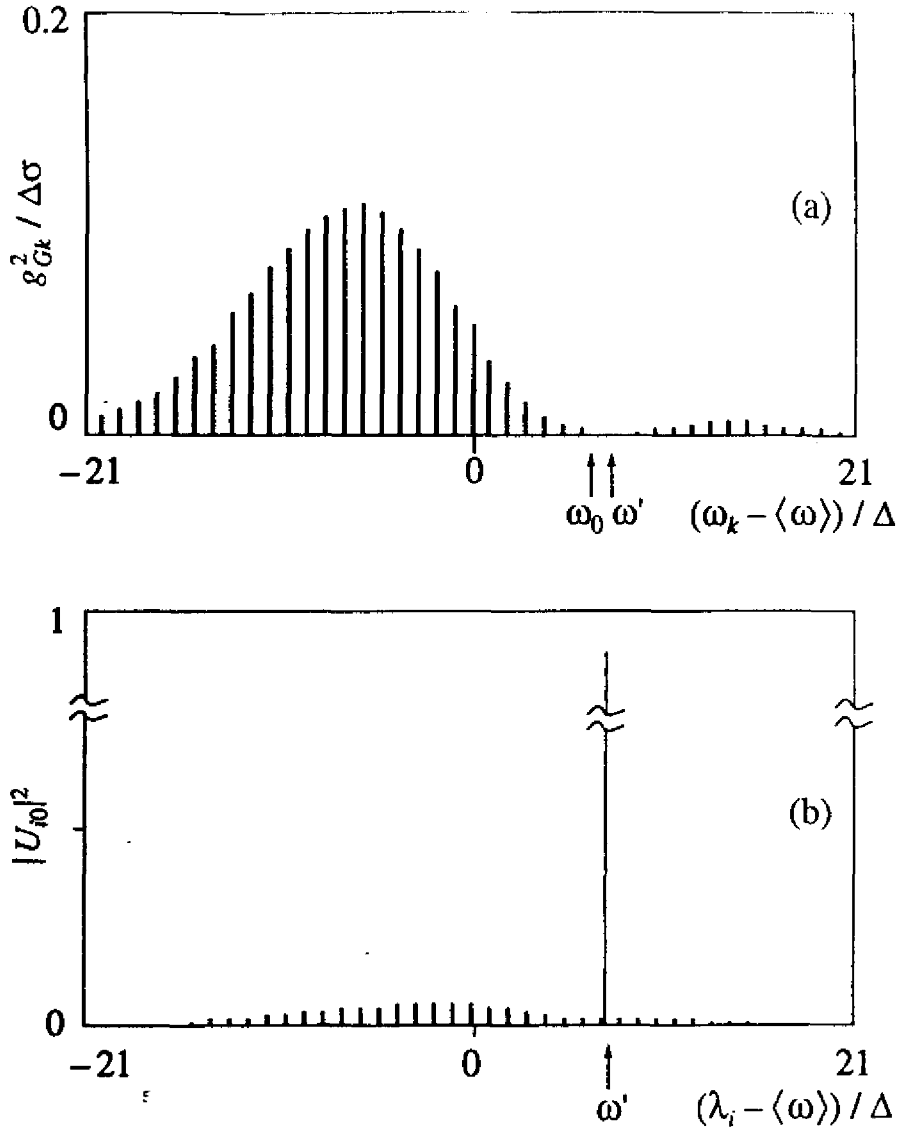
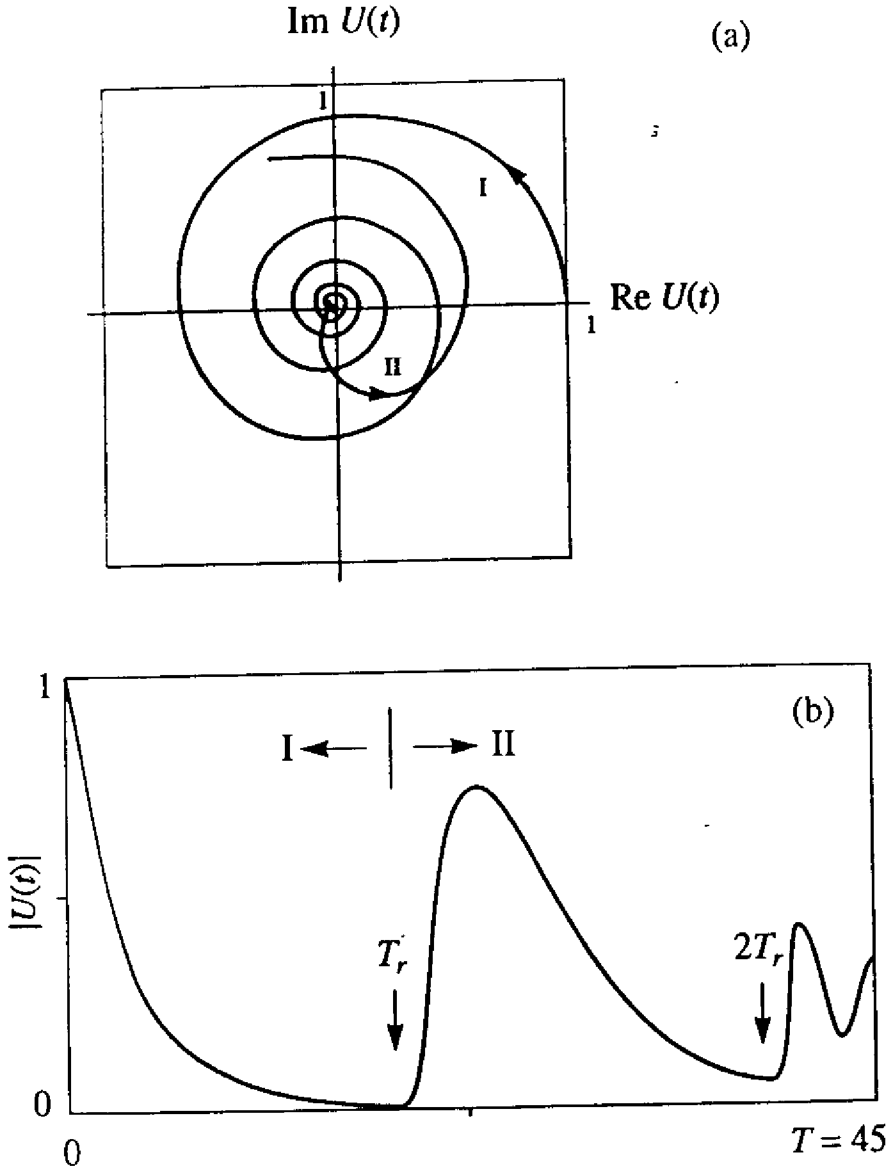
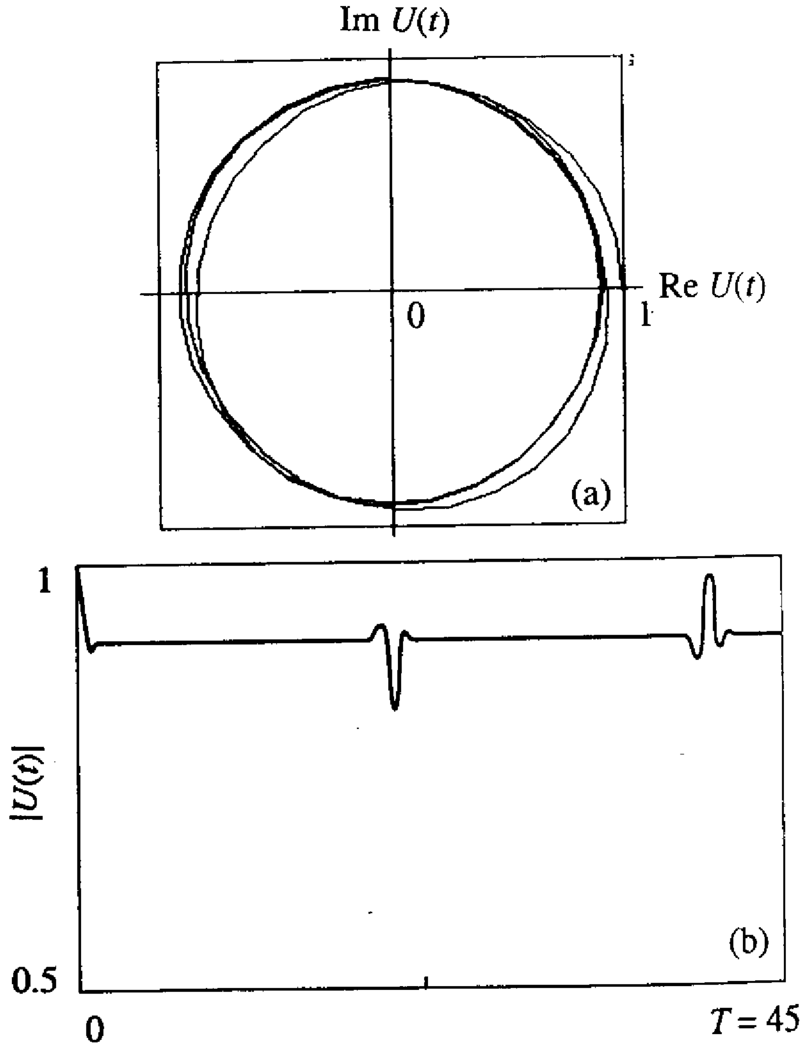

0.2
$g_{Gk}^2 / \Delta\sigma$
(a)
0
−21
0
21
ω_0 ω'
$(\omega_k - \langle\omega\rangle) / \Delta$
1
$|U_{i0}|^2$
(b)
0
−21
0
21
ω'
$(\lambda_i - \langle\omega\rangle) / \Delta$

Fig.2



Im U(t)
(a)
1
I
Re U(t)
1
II
1
(b)
I
II
|U(t)|
T_r
$2T_r$
0
0
T = 45

Fig.3



Im U(t)
Re U(t)
0
1
(a)
1
|U(t)|
(b)
0.5
0
T = 45

Fig.4

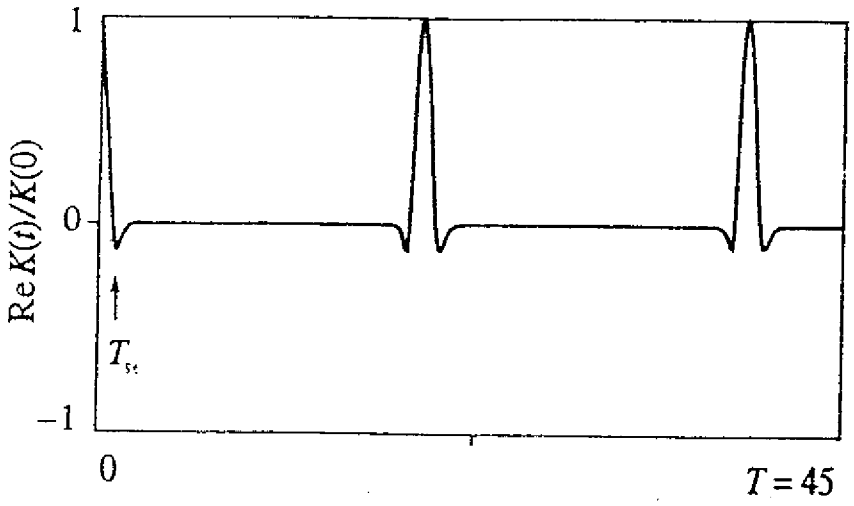


Fig. 5