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Abstract

Former studies have shown that correlation methods can be used for determination of various two-component flow parameters, among these the *correlation length*. In cases where the flow can be described as a mixture, in which the minority component forms spatially limited *perturbations* within the majority component, this parameter gives a good indication of the maximum extension of these perturbations. In the former studies, spherical symmetry of the perturbations has been assumed, and the correlation length has been measured in the direction of the flow (axially) only. However, if the flow structure is anisotropic, the correlation length will be different in different directions. In the present study, the method has been developed further, allowing also measurements *perpendicular* to the flow direction (radially).

The measurements were carried out using laser beams and the two-component flows consisted of either glass beads and air or air and water. In order to make local measurements of both the axial and radial correlation length simultaneously, it is necessary to use 3 laser beams and to form the *triple* cross-covariance. This led to some unforeseen complications, due to the character of this function.

The experimental results are generally positive and size determinations with an accuracy of better than 10% have been achieved in most cases. Less accurate results appeared only for difficult conditions (symmetrical signals), when 3 beams were used.

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1 Introduction

A former study by the author [1], has shown that a setup with two crossing, axially shifted laser beams can be used for measurements of local flow parameters in a two-component flow, with excellent results. The measurements have been based on the crossed beam correlation technique, introduced by Fisher and Krause in 1967 [2] for local, non-intrusive measurements on turbulent flow. A similar concept was suggested by I. Pázsit in 1985 [3], using X-rays for two-phase flow measurements. With this method, 3 local two-component flow parameters can be determined simultaneously: (1) Perturbation velocity, (2) mean density and (3) axial correlation length.

The last parameter is, in a random flow, equal to the maximum extension of the perturbations (minor flow component) along the flow direction (axially). In the study mentioned, we have assumed spherically symmetrical (isotropic) perturbations, in which case it is only necessary to measure the correlation length in one direction. This assumption is, however, far from always valid in reality, where some kind of anisotropy often can be expected, as, for example, slugs in a water-vapour flow. This means that measurements of the correlation length in several directions may be required. Generally though, some symmetry is present and, with a suitable choice of geometry, correlation length measurements in, at the most, three directions should normally be sufficient. For instance, there is often a symmetry line along the flow direction (as in the slug flow case), with cylindrical symmetry around this line. In such a case, measurements are needed in two directions only - radially and axially.

It has been suggested that, utilizing the soft X-ray radiation generated by the plasma, the same crossed beam correlation technique as for two-phase flow could be used for determination of local parameters of plasma density fluctuations in tokamak fusion experiments [4]. At JET (Joint European Torus), where such experiments are conducted, the existence of density perturbations have been observed using microwave reflectometry [5]. The observed perturbations usually have had extensions that were some 50-100 times larger axially (toroidally) than radially (poloidally), making it clear that both axial and radial correlation length measurements would be required.

To be able to determine radial correlation lengths, some further development of the crossed beam correlation technique was found to be necessary. One possibility was to apply a third beam, crossing one of the original beams in the same axial plane and being parallel to the other. By gradually increasing the radial distance between the two parallel beams, the local radial correlation length is given by the distance where the triple cross-covariance becomes zero. This possibility has been examined and a detailed description of the theory and experiments is given in this report.

2 Theory

We assume a two-component flow with a mixing ratio sufficiently far from 0.5 that the minority component can be considered as forming density perturbations propagating with the flow. We also assume that the majority component is transparent to light and the minority component is either opaque to the light or has a different refractive index for the wavelength(s) in use. If we then apply a collimated light beam that enters the flow on one side and is monitored by a small detector on the other side, aligned with the light beam, a binary signal (light or no light) is obtained, as the passing perturbations either cut off or scatter away the light from the detector. It should be mentioned that, for flow mixing ratios around 0.5, the situation becomes much more complicated, but such measurements are outside the scope of the present work.

If we have a radially homogeneous flow with isotropic (spherical) perturbations, it is in principle possible to determine the maximum extension of the perturbations, given by the correlation length, from one such signal ($i(t)$). This is done by forming the autocovariance:

$$C(\tau) = \langle \delta i(t) \delta i(t+\tau) \rangle \tag{1}$$

from the signal and determining the time for which this function becomes zero (the cut-off time, τ_c in Figure 1). This is called the correlation time, which (for randomly distributed perturbations) is equal to the maximum passage time of the perturbations. To determine the correlation length, we simply multiply τ_c with the perturbation velocity. If this velocity is not known, we could use a modified (and well-known) method: A second beam is applied, parallel to the first, but at a different axial level. If the cross-covariance:

$$C(\tau) = \langle \delta i_1(t) \delta i_2(t+\tau) \rangle \tag{2}$$

is formed from the detected signals from these two beams, a peak is expected to appear at a time, τ_0, which is equal to the average time it takes for the perturbations to pass the distance between the beams. A condition for this, is that we have a transport of the flow components in the axial direction only and that the flow properties do not change significantly between the beams. In fact, if the properties do not change at all, the resulting cross-covariance should be exactly equal to the autocovariance from one beam, but with a time displacement equal to the transit time (τ_0) as shown in Figure 1. The perturbation velocity is finally calculated by simply dividing the axial distance between the beams with the transit time.

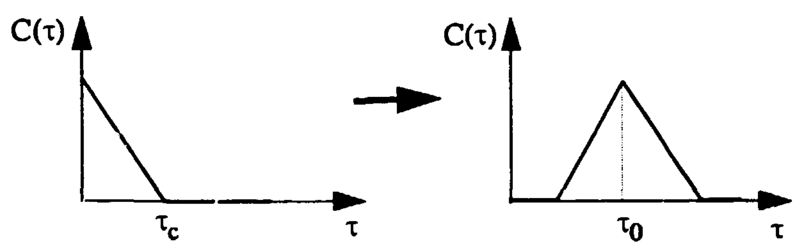

Figure 1: Autocovariance from one beam and cross-covariance from two, parallel and axially shifted beams.

If the flow is not homogeneous, for instance, if we have a radial distribution of the perturbation sizes and/or velocity across the flow, the above method will not give sufficient information, only a weighted average along the beam. Then a third method [3] can be applied: We rotate one beam so that it becomes perpendicular to the other one (see Figure 2). When forming the cross-covariance from these two signals, we expect to get information only from a small (control) volume around the crossing point of the two beams [1]. This assumption is the essential basis of the crossed beam correlation technique.

All the above applies only to cases where the perturbations are isotropic. If we have a flow

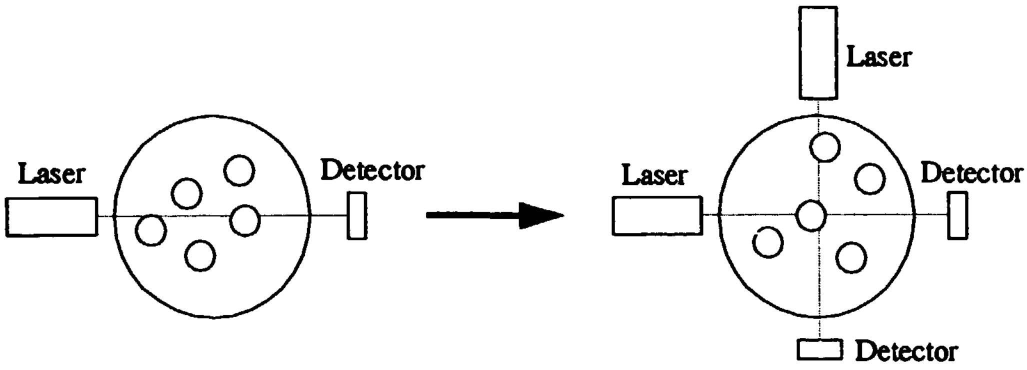

Figure 2: Geometries for axial correlation length determination.

where the perturbations have different extensions in different directions, the situation becomes more complicated. A thorough and general description of this case is given in [4]. The aim of the present work has been to experimentally investigate a method capable of measuring the axial and the radial correlation lengths in a flow where the perturbations are cylindrically symmetric around the flow direction axis.

We start, again, by assuming a radially homogeneous flow. As above, we then apply two, parallel beams - from this setup, the axial correlation length can be determined. If we then gradually increase the *radial* distance between the beams (from zero), the cross-covariance peak height will decrease and the radial correlation length is now given by the radial inter-beam distance for which the cross-covariance becomes zero everywhere.

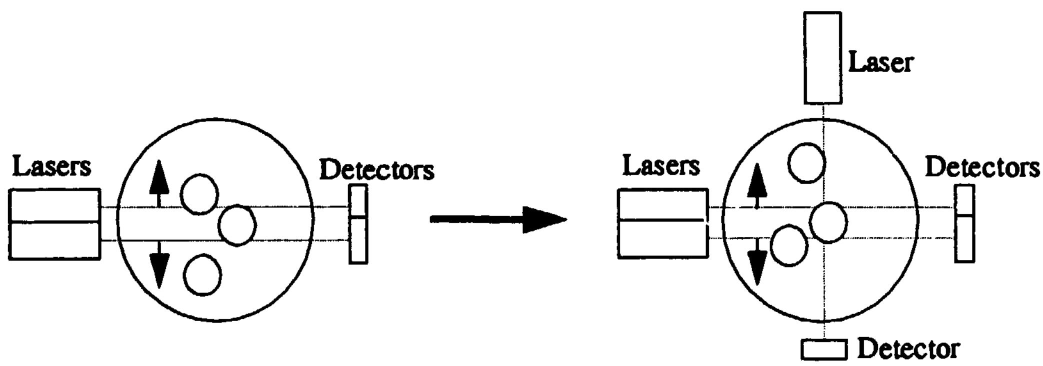

Figure 3: Geometries for axial and radial correlation length determination.

In the case of a radially inhomogeneous flow, we can use the same trick as above; using crossed beams. In this case, however, we need three beams, where the first two are crossing in the same axial plane and the third beam is parallel to either one of the other two, but lying in another axial plane (downstream the flow). Then we form the triple covariance from the three detected signals:

$$C(\tau) = \langle \delta i_1(t) \delta i_2(t) \delta i_3(t+\tau) \rangle \tag{3}$$

At first thought, one would expect this function to yield the same information as for 2, parallel beams, but now for a small control volume around the crossing point. Unfortunately, this is not true in all cases, as will be explained.

While the cross-covariance of 2 signals always takes a positive value when the signals are correlated, the triple cross-covariance can take both positive and negative values, as it is an odd (third) order moment. This means that the symmetry properties of the 3 signals will also affect the result (compare with the third moment or *skewness* for one signal). For perfectly symmetrical signals, the triple cross-covariance will be 0 everywhere, regardless of if the signals are correlated or not. By symmetrical, we here mean that the signals have symmetrical amplitude probability distributions. Thus, it is only for correlated *and* asymmetrical signals, that a peak (positive or negative) in the triple cross-covariance can be expected. To date, no solution to this complication has been found. However, the method has still been able to produce useful results, in cases where the signals *are* asymmetrical, as will be presented in the following.

The first two described geometries (Figure 2) have been thoroughly studied earlier [1]. In this report, the results from an experimental study of the last two geometries (shown in Figure 3) will be presented.

3 Experimental setup

Three different types of two-component flow were created during the described experiments. These were as follows:

1. Falling glass beads in a single, vertical plexiglass tube. In this case, the perturbation velocity, mean density and perturbation diameter were all constant over a cross-section of the tube, at any given axial level (except near the walls). The perturbations (glass beads) were spherically symmetrical and two different diameters were available.

2. Falling glass beads in dual plexiglass tubes. In this setup, a stepwisely changing radial distribution of the correlation length and the velocity was created by using two concentric tubes, into which glass beads of different size, starting from different heights, were dropped.

3. Water-air flow. In this case, we tried to imitate a two-phase water flow in a narrow channel by circulating water in a closed loop, through a vertical plexiglass tube, where air was introduced through narrow holes at the bottom of the tube. The water and air flows could be regulated separately, to create different flow regimes.

For the measurements, either two or three red He-Ne lasers and photodiode detectors were used. The lasers were mounted so that two of the beams were crossing in the same axial plane - these two could be moved sideways using micrometer screws. The third laser could be moved vertically and was mounted so that its beam was above the other two and parallel to one of them. The geometry of this arrangement is shown in Figure 4. In the setup with falling glass beads, the flow direction was actually opposite, so in this case we had to calculate the cross-covariance with negative τ-values instead.

Unfortunately, the laser beams are not infinitely thin, as was assumed in the theory above - they have a diameter of approximately 0.5 mm. This and the fact that they have to be almost completely cut off in order to give a pulse in the detector, leads to an overall lower than

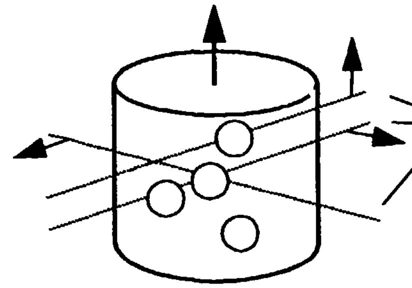

Figure 4: Geometrical arrangement of laser beams.

expected measured correlation length. This deviation can be up to 1 mm, depending also on how well the beams are centred onto the detectors.

3.1 Data processing

The detected signals are digitised through an ADC at a rate of up to 50 kHz. A PC is used for controlling the measurements and storing the data. After digitising, the time signals are converted into true binary form by a simple discrimination routine, in which all values above a selectable threshold are set to one and all values below are set to zero. This process also reduces the amount of data significantly. After this process, the cross-covariance is calculated from the two or three signals, for a time displacement (τ) between 0 and T, where T is the selectable maximum time shift.

To take advantage of the fact that the signal values are either one or zero, the cross-covariance is formed by first calculating the mean values and the cross-*correlation* function(s) from the discriminated signals and then, for two signals, simply subtracting the product of the two mean values. For three signals, the following relation is used:

$$\begin{aligned}\langle \delta i_1(t)\,\delta i_2(t)\,\delta i_3(t+\tau)\rangle &= \langle i_1(t)\,i_2(t)\,i_3(t+\tau)\rangle + \\ &+ 2\langle i_1(t)\rangle\langle i_2(t)\rangle\langle i_3(t)\rangle - \langle i_1(t)\,i_2(t)\rangle\langle i_3(t)\rangle - \\ &- \langle i_1(t)\,i_3(t+\tau)\rangle\langle i_2(t)\rangle - \langle i_1(t)\rangle\langle i_2(t)\,i_3(t+\tau)\rangle\end{aligned} \qquad (4)$$

3.2 Error estimation

There are 3 main error sources in the determination of the correlation length from the current measurements:

1. Statistical uncertainties in the determination of the peak position and the cut-off time in the plotted cross-covariance.

2. The finite sampling interval, giving rise to a maximum error of $v \cdot t_s$, where v is the perturbation velocity and t_s is the sampling interval.

3. The inaccuracy of the beam positioning together with the errors due to the finite beam

diameter, as described above.

These 3 factors together will determine the overall inaccuracy. No great effort has been made to determine a precise estimate of this inaccuracy, especially not for cases where the statistical errors become important. We can nevertheless conclude that there is a systematical error of roughly - 0.5 mm due to the beam factor and a random error of approximately ± 0.5 mm due to the other factors, in cases when the statistical errors are not dominant.

4 Results

As described above, three different setups were used for the experiments, for different purposes. In the first case, the assumption of radially homogeneous flow was fulfilled and only two beams were used. In the other two cases, a radial distribution was present and therefore three beams were used in these cases.

4.1 Falling beads in single tube

The first experiments were made with only two, parallel laser beams, crossing a single plexiglass tube. This was done in order to find out if the described method for determination of the radial correlation length (by varying the radial beam separation), would give results that were consistent with the results formerly obtained for the axial correlation length. The theoretical correlation length is, in this case, the same everywhere in the tube, given by the diameter of the beads. The velocity is also constant across the tube (at a given axial level).

Bead diameter	3 or 4 mm
Height of fall	27 cm
Radial beam position	tube center
Axial beam separation	20 mm
Radial beam separation	0, 0.5, 1, 1.5, 2, 2.5 3, 3.5* and 4* mm
Sampling frequency	10 kHz/channel
Measurement time	16 s

Table 1: Experimental parameters for single tube measurements. The values marked with an asterisk apply for the measurements on 4 mm beads only.

Two series of measurements were carried out, using two different bead diameters (3 and 4 mm). The experimental parameters are shown in Table 1. The resulting cross-covariances are shown in Figure 5 and Figure 6. The axial beam separation was 20 mm and the peaks are found at 8.67 ms, giving a velocity of 2.31 m/s. From this, and the correlation times of 1.2 and 1.6 ms, axial correlation lengths of 2.8 and 3.7 mm respectively are finally obtained.

The cross-covariance peak values are plotted against the radial beam separation in Figure 7. As can be seen, the resulting plots are very similar to the original cross-covariances (at zero radial beam separation) and they also yield very similar correlation lengths: 2.9 and 3.5 mm respectively.

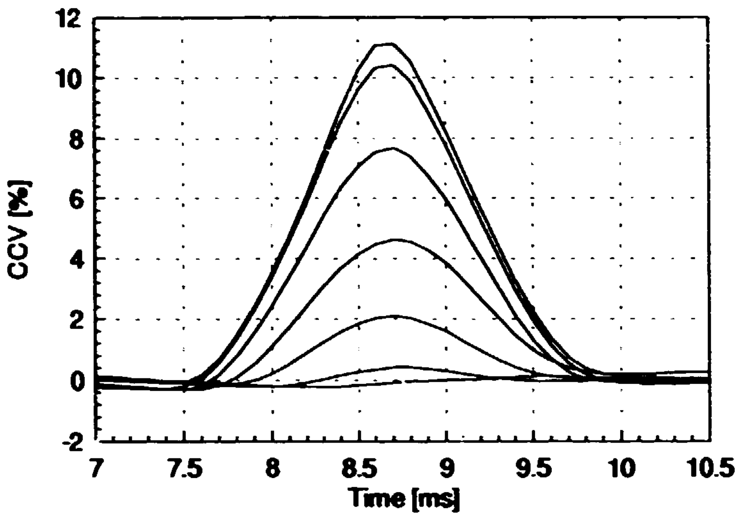

Figure 5: Cross-covariance for radially shifted, parallel beams and 3 mm glass beads.

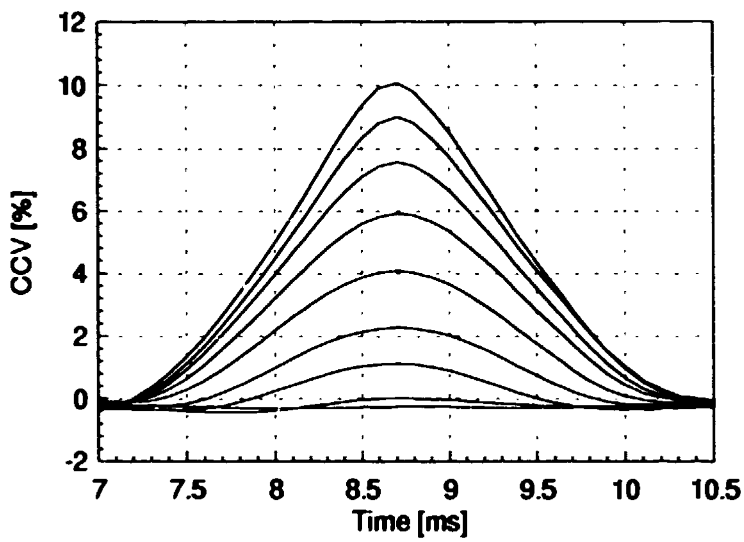

Figure 6: Cross-covariance for radially shifted, parallel beams and 4 mm glass beads.

4.2 Falling beads in dual tubes

In this case, the velocity and bead sizes were made different in the inner and the outer tube. We then used three laser beams to measure the local radial correlation length in each tube, in accordance with the theory above.

The velocity difference was achieved by dropping the beads from different heights (above the detection level) into each tube (20 cm in the inner and 36 cm in the outer tube). The diameter

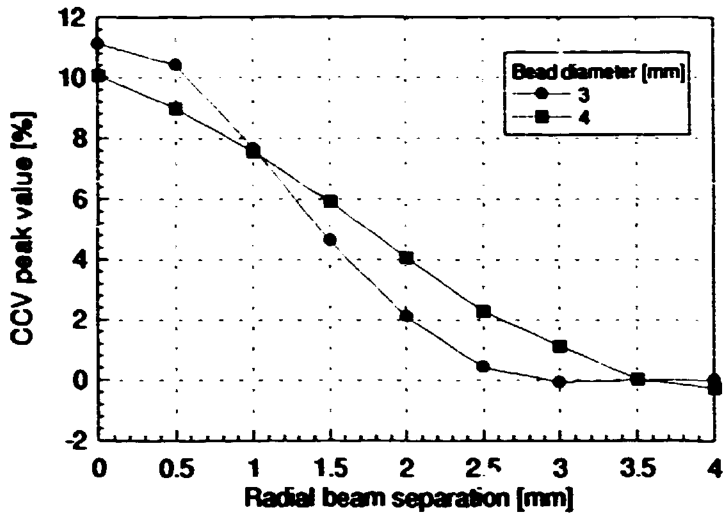

Figure 7: Radial cross-covariance for different bead sizes.

Bead diameter	3 mm (inner tube) and 4 mm (outer tube)
Falling distance	36 cm (inner tube) and 20 cm (outer tube)
Crossing point	0 or 18 mm
Axial beam separation	25 mm
Radial beam separation	0, 1, 1.5, 2, 2.5, 3, 3.5 and 4 mm
Sampling frequency	5 kHz/channel
Measurement time	16 s

Table 2: Experimental parameters for dual tube measurements.

of the falling beads was 3 mm in the inner tube and 4 mm in the outer tube. The experimental parameters are shown in Table 2 and the resulting triple cross-covariances are shown in Figure 8 and Figure 9.

The effects of much poorer statistics as compared to the former measurements (with two beams) are very obvious in these curves. Nevertheless, we can use the peak values to plot the radial cross-covariance, as shown in Figure 10.

The axial beam separation was 25 mm and the peaks are positioned at 9.45 and 12.8 ms in the inner and outer tube respectively, yielding velocities of 2.65 and 1.95 m/s. The correlation times are 1.1 and 1.8 ms, yielding axial correlation lengths of 2.9 and 3.5 mm in the inner and the outer tube respectively. The radial correlation lengths, obtained from the curves in Figure 10, are 3 and 4 mm respectively. It should be noted though, that these values are, statistically, highly uncertain.

In a case like this, where we have two discrete velocities and bead sizes, it is also interesting

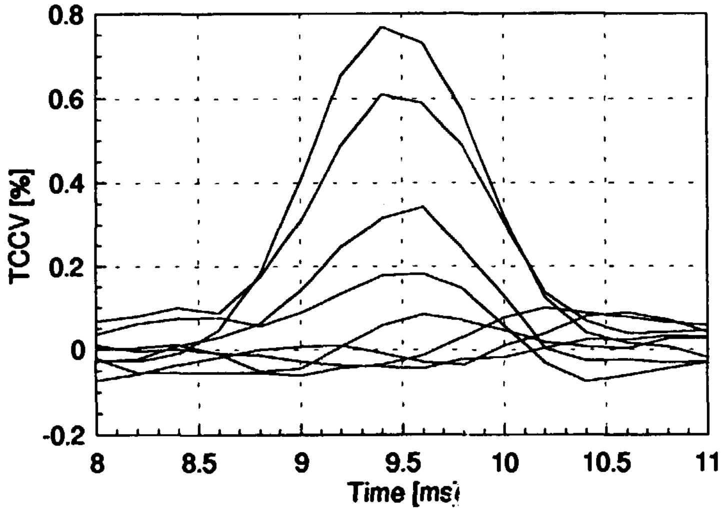

Figure 8: Triple cross-covariance for beams crossing in inner tube.

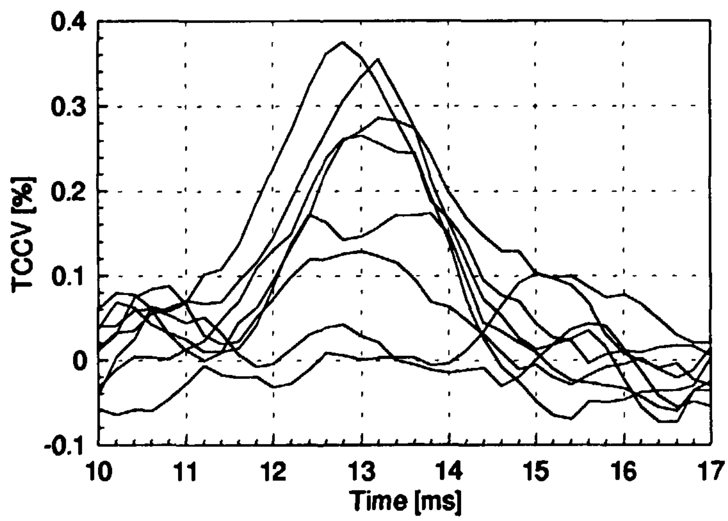

Figure 9: Triple cross-covariance for beams crossing in outer tube.

to look at the cross-covariance from the two parallel beams, passing the whole cross-section of the flow. In this case, the statistical basis is much better and, if the velocity difference is large enough, we still expect to be able to separate the two peaks. The result of such a measurement is shown in Figure 11.

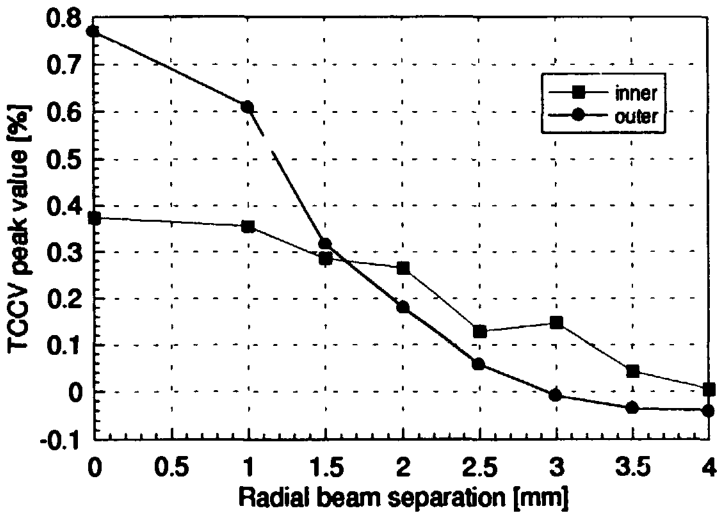

Figure 10: Radial cross-covariance in inner and outer tube.

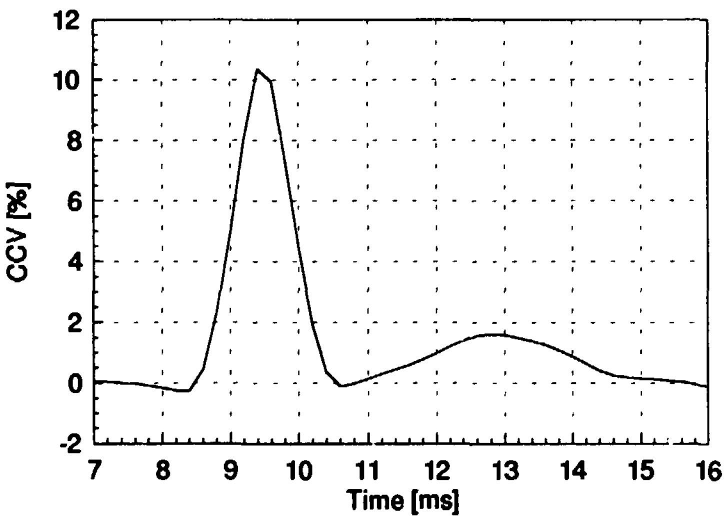

Figure 11: Cross-covariance from two, parallel beams.

4.3 Water-air flow

In a water-air flow, the shapes of the perturbations (bubbles) are not as well defined as in the glass bead flow. It is only in the case of small bubbles, that spherical symmetry can be expected. In all other cases, the perturbations will be either stretched out in the flow direction (e.g. slugs) or flattened (e.g. caps). It would of course be interesting to examine all of these different cases, but this was not done, due to two factors: First, the above described complication with symmetrical signals showed to be a major problem in these measurements, and second, when the air bubbles become very large, as in slug flow, the laser light begins to penetrate them rather than being scattered, giving rise to a "false" signal. These factors caused the water-air experiments to be rather unsuccessful, but the results from one measurement will nevertheless be presented, mainly to show an example where negative cross-covariance peaks are obtained.

Bubble size (visually estimated)	5 mm
Crossing point	0 mm
Axial beam separation	20 mm
Radial beam separation	0, 1, 2, 3, 4, 5 and 6 mm
Sampling frequency	2 kHz
Measurement time	82 s

Table 3: Experimental parameters for water-air flow measurements.

The experimental parameters are listed in Table 3 and the resulting cross-covariances are shown in Figure 12. From the peak values of these functions, a radial cross-covariance can be

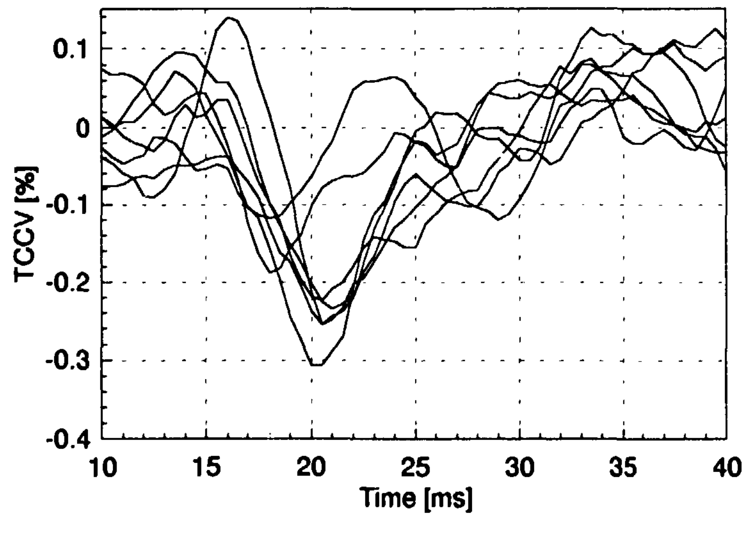

Figure 12: Triple cross-covariances in water-air flow experiment.

plotted, as shown in Figure 13 (absolute values). The bubble velocity can be calculated from the position of the peaks in Figure 12, giving a value of approximately 1 m/s, and from the correlation time, we end up at a maximum axial bubble extension of around 5 mm, in fair agreement with visual observations. Judging from Figure 13, a somewhat larger radial correlation length is obtained - such an observation would not be unreasonable, although no definite conclusions should be drawn, as the uncertainties are quite large in these measurements.

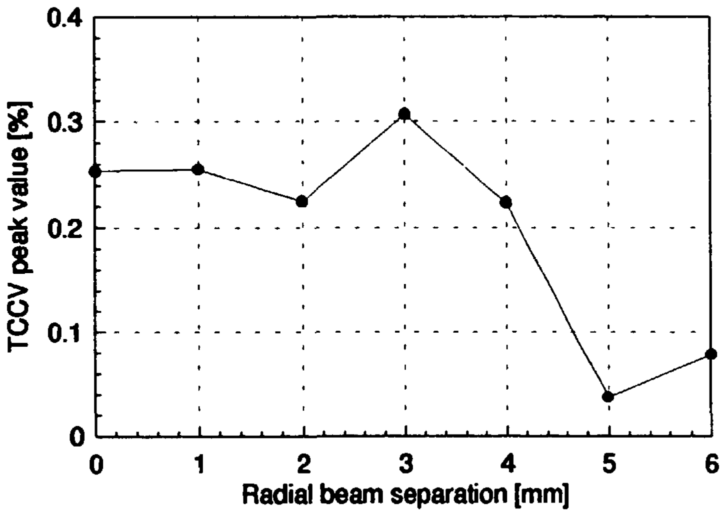

Figure 13: Absolute value of radial cross-covariance in water-air flow experiment.

5 Conclusions

The results show that determination of the (global) radial correlation length in a two-component flow, using two laser beams and the described method, is possible in the investigated cases and that this parameter can also be determined locally, by using three laser beams, in special cases. The method is expected to work equally well with other information carriers, such as X-rays, neutrons etc.

The accuracy of the described measurements is maybe not extremely good, but it should be noted that the method would probably not be used for very accurate size measurements, but rather for estimates of the order of magnitude of the perturbation extensions. In the experiments, we have been able to clearly distinguish between 3 and 4 mm glass beads; this accuracy should be sufficient in most cases.

Problems arising when applying the described methods are: (1) Practical problems, as, for example, how to position the three beams with the desired movement possibilities and (2) the rather extensive, time consuming calculations needed to get reasonably statistically reliable values. If this method should be applied in a monitoring system, radically different hardware would be required to get reasonable response times. Such a system would probably involve several, parallel beams for the radial correlation length determination and a number of delay channels for on-line cross-correlation calculations or a very powerful computer, capable of making such calculations in real-time.

References

[1] Thomson, O. "Laser measurements on two-component flow using the crossed beam cross-correlation method" CTH-RF-99 (1993).

[2] Fisher, M. J. & Krause, F. R. "The crossed-beam correlation technique" *J. Fluid Mech.* **28**:4 (1967) 705-717.

[3] Pázsit, I. "Two-phase flow identification by correlation techniques" *Ann. Nucl. Energy*, **13**:1 (1986), 37-41.

[4] Pázsit, I. "Density correlations in two-phase flow and fusion plasma transport" submitted for publication (1994).

[5] Costley A.E., Cripwell P. & Fukuda T. "Measurement of the toroidal motion of fine-scale density structures in JET plasmas" *1992 International conference on plasma physics*, Innsbruck, June/July 1992.