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PROOFS OF CONTRACTED LENGTH NONCOVARIANCE

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Introduction. Relativity theory has ascertained that a material rod scale is physically not a spatial thing but a space-time configuration. In other words, a time component is added to three space ones. In the mathematical language this means that the scale must be described by a space-like 4-vector. The generally accepted definition of moving scale length gives a recipe of obtaining corresponding four numbers in each reference system. If the Lorentz-invariance condition is fulfilled, these four values must represent the same 4-vector. Or otherwise, the interval corresponding to the contracted length must be a Lorentz-invariant one. At first the answer to the question on contracted length covariance was obtained just by comparing such intervals in two moving reference systems [1,2]*.

Remind the following before turning to the consideration of other proofs.

The interval is a four-dimensional quantity defined by two point events and an analog of three-dimensional distance between two points. Or as one says, the metric of Minkowski's (four-dimensional) space is defined by the interval squared

$$-s^2 = c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2, \qquad (1)$$

dependending on the coordinate difference of these events. The interval is the main invariant of relativity theory, and so it is also named the fundamental invariant (see, e.g., [4]). Clocks and scales are material representatives of the interval.

Remind that the interval is a quantity which does not change (it remains an invariable one) when transiting from one inertial reference system to another one. Since this transition is related to changing motion velocity, the interval invariance must mean its independence (constancy) of velocity.

Contracted length. Let us consider the traditiional definition of moving scale length l_c from the viewpoint of the foregoing. Let for simplicity the scale be oriented and move along the x-axis of a S-system. In the framework of this

*Although the own statement of the problem is in essence main here as the interval writing down taking the contraction formula into account already speaks of noncovariance of the traditional definition. On the other hand, all the foregoing here can be considered as the mathematical formulation of the former physical conclusion (see, e. g., [3]) that the generally accepted definition of moving scale length contradicts the principle of relativity.

definition it is characterized by two simultaneous events at its ends or a four-component quantity

$$l_c^n = (0, \Delta x, 0, 0) = (0, l_c, 0, 0). \qquad (2)$$

Therefore the space-like interval squared answering this moving scale takes the form

$$s_c^2 = \Delta x^2 = l_c^2. \qquad (3)$$

As known, a direct consequence of the simultaneity demand of endmarks $\Delta t = 0$ (simultaneity of this pair of events) is the contraction formula

$$l_c = l^*(1 - v^2/c^2)^{1/2}. \qquad (4)$$

Here l^* is the scale length at rest (proper length) and v is its velocity (the velocity of the S^*-system relative to S).

Based on (4), it follows that the interval s_c depends evidently on the motion velocity

$$s_c = l^*(1 - v^2/c^2)^{1/2}. \qquad (5)$$

As noted above, such a dependence means that the traditional definition does not satisfy the Lorentz-invariance condition of interval. Besides, it is evident that the dependence of the scale length l_c on velocity must also mean with necessity the v dependence of Δt [5].

Now we want to pay attention to one, apparently very early indication of violation of interval invariance, which remained absolutely unnoticed. The question is the known «Lectures on physical foundations of relativity theory (1933—1934)» by L.Mandel'shtam [6]. We read there: «...if two events lie in that in the system S^* where the scale is at rest two flashes are made at the scale ends simultaneously, then $\Delta t^* = 0$ and $s_c^2 = l_*^{2*}$ for these two events in this system. Therefore the scale length measured in the rest system defines a space-like interval. Thus, the resting clock measures a time-like interval, and the resting scale measures a space-like interval». In order to make sure of noncovariance of the generally accepted definition, it was enough to compare the above interval with the corresponding value (3) in the moving system. On the other hand, one can come to the last expression supposing $v \to 0$ in (5).

*In our designations.

As far as one can judge, the very first indirect evidence of contracted length noncovariance (exactly, contracted volume) was obtained by M.Laue as early as 1911 [7]. He used the expression

$$G^i = \int T^{ik} dV_k \tag{6}$$

for the calculation of the electromagnetic field energy and momentum G^i of a moving charge. Here T^{ik} is the energy-momentum tensor of the electromagnetic field. dV_k is the four-dimensional quantity that has only one time component in accordance with the generally accepted definition (see, e.g., [8]). As the Lorentz covariance of T^{ik} is beyond doubt, the noncovariance G^{i*} on the left-hand side (6) must mean the noncovariance of dV_k and, consequently, of contracted length.

Relativistic length. In the framework of the concept of relativistic (radar) length (see, e.g., [8]) instead of (2) we have

$$l_r^i = (vl_r/c,\ l_r,\ 0,\ 0), \tag{7}$$

whence taking the elongation formula

$$l_r = l^*(1 - v^2/c^2)^{-1/2} \tag{8}$$

into account interval constancy $s_r = l^*$ (and its coincidence with the above magnitude for the resting scale) is obvious. In other words, the four-component quantity l_r^i (in contrast to l_c^u) is a 4-vector.

Conclusion. Certainly the considered proofs of contracted length noncovariance cannot be regarded as absolutely independent. Rather one should tell about different modifications of this procedure although the proof related to ascertainment of interval inconstancy (its dependence on velocity) looks the most convincing one. However, the last indirect proof based on the noncovariance of the electromagnetic field energy and momentum of a moving charge scarcely gives rise to doubt.

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Доказательства нековариантности сокращенной длины

Обсуждаются различные доказательства нековариантности сокращенной длины. Наиболее убедительным из них представляется способ, основанный на установлении непостоянства (зависимости от скорости) интервала, отвечающего сокращенной длине. Подчеркивается, что известная нековариантность энергии и импульса электромагнитного поля движущегося заряда (*«проблема 4/3»*) является прямым следствием нековариантности сокращенной длины.

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Strel'tsov V.N. D2-94-179

Proofs of Contracted Length Noncovariance

Different proofs of contracted length noncovariance are discussed. The way based on the establishment of interval inconstancy (dependence on velocity) seems to be the most convincing one. It is stressed that the known noncovariance of the electromagnetic field energy and momentum of a moving charge (the «problem 4/3») is a direct consequence of contracted length noncovariance.

The investigation has been performed at the Laboratory of High Energies, JINR.

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