CTH-IEFT/PP-1994-20

ISSN 0281-1308

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Petro P Sosenko

INSTITUTIONEN FÖR ELEKTROMAGNETISK FÄLTTEORI
CHALMERS TEKNISKA HÖGSKOLA

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Petro P. Sosenko

International Centre of Physics & M.M. Boholiubov Institute
for Theoretical Physics, 252130, Kyyiv, Ukraine;
LPMI, École Polytechnique, Palaiseau, France;
Institute for Electromagnetic Field Theory and Plasma Physics
Chalmers University of Technology, Göteborg, Sweden

Abstract

A quasiparticle concept [P.P. Sosenko, and V.K. Decyk. Physica Scripta 1993. **47**, 258] is generalized for the case of non-uniformly magnetized plasma. Exact and reduced continuity equations for the microscopic density in the quasiparticle phase space are derived, and the nature of quasiparticles is analyzed. The theory is developed for the general case of relativistic particles in electromagnetic fields, besides non-uniform but stationary magnetic fields. Effects of non-stationary magnetic fields are briefly investigated also.

1. Introduction

Experimental observations of space and fusion plasmas provide evidence of a complex plasma behaviour involving low-frequency (with respect to the cyclotron frequency) fluctuations and effects of nonlinear interaction between them, and necessitate fundamental knowledge of the latter. Progress in developing plasma diagnostics enables one to determine much more detailed plasma characteristics [1-3], while modern computers and numerical simulation techniques encompassing parallel computing make it possible to study plasmas for complicated realistic conditions [4-6].

When low-frequency plasma motion is considered the reduction of the original description employing explicitly the separation of time scales is facilitating both for analytical and computational studies. Even with the rapidly increasing power of computers, many important problems in plasma physics cannot be solved with conventional particle-in-cell simulation techniques, which model plasmas on the most basic level, using Maxwell's equations and Newton's law directly. Problems such as modeling a Tokamak for fusion energy research or modeling the magnetosphere in space plasma physics, have so large a range of space and time scales that the calculation is prohibitive on any forseeable computer. These so called Grand Challenge problems require a reduced description if they are to be solved, where only some of the physical processes are included (hopefully the important ones).

A general quasiparticle approach [7-12] can be suggested for the description of the magnetized plasma. It enables one to introduce naturally quasiparticles within the context of the microscopic plasma theory and to achieve reduction of the description for low-frequency plasma processes. Thus, it becomes possible to relate particles and quasiparticles.

The quasiparticle approach can be applied to develop the microscopic theory of magnetized plasmas, and low-frequency phenomena in such plasmas. This was done for the case of potential interaction between non-relativistic particles in uniform and stationary external (background) magnetic fields [9]. Within the context of the latter theory, the reduced Poisson law relates the self-consistent electric field in the plasma to the quasiparticle density in the reduced phase space. While the reduced theory is formulated up to arbitrary orders with respect to a small parameter, the first-order approximation was considered in detail, and some features of the second-order approximation were explained. For example, polarization-drift interpretation of the density perturbation was analysed, as well as nonlinear effects in the reduced Poisson law. It was argued that the description of the effects nonlinear with regard to fluctuations requires going beyond the first-order approximation in this equation. The reduced expression for the microscopic particle flux was derived. The importance of the finite-Larmor-radius effects in particle fluxes was demonstrated. It was shown that the conventional evaluation of the fluctuation-induced particle flux does not take into account effects of the finite Larmor radius correctly. The second-order approximation of the reduced microscopic theory was introduced, as it enables one to explain polarization drift of particles with regard to kinetic (finite-Larmor-radius) effects.

The global energy conservation law for the magnetized plasma can be also investigated within the context of the quasiparticle description. The reduced conservation law was derived for low-frequency plasma motion [10]. Such law can be applied to describe the plasma under the conditions of anomalous transport, when the fluctuations and the fluctuation-induced relaxation of mean plasma characteristics are essential. It makes possible to elucidate the role of polarization particle drift in the overall energy balance. The microscopic expression for the energy was obtained and its mean statistical value was calculated with regard to both contributions from non-uniform mean plasma flows and the fluctuations. The reduced conservation law yields also the sufficient stability conditions for non-equilibrium plasmas and the conclusion about a possible global role of non-uniform mean flows in plasma stability and transport [10].

The purpose of this paper is to elaborate a quasiparticle approach for the case of non-uniformly magnetized plasmas. Exact and reduced continuity equations for the microscopic density in the quasiparticle (reduced) phase space are derived, and the nature of quasiparticles is analyzed. The theory is developed for the general case of relativistic particles in electromagnetic fields (or any other additional force fields), besides non-uniform but stationary magnetic fields. Effects of non-stationary magnetic fields are briefly investigated also.

In Sec. 2, the microscopic density in the particle phase space is introduced. Sec. 3 deals with the microscopic density in the guiding-centre phase space defined by some formal transformation, the equations of guiding-centre motion are derived, and an effective acceleration which takes into account non-uniformity of stationary magnetic fields is found. Sec. 4 presents the microscopic density in the quasiparticle phase space. Here, quasiparticles are introduced, and an exact microscopic continuity equation for the quasiparticle density is analyzed. In Sec. 5, closure is considered, and the reduced theory is formulated to arbitrary orders with respect to a small parameter. Sec. 6 contains a detailed analysis of the first-order adiabatic approximation of the reduced theory. Sec. 7 specifies guiding-centre tarnsformation, the effective acceleration and the guiding-centre velocity. In this section, quasiparticle equations of motion in non-uniformly magnetized plasmas are derived, with the effects of non-uniform magnetic fields being calculated explicitly. The case of potential particle acceleration is illuminated in

Sec. 8, where the renormalized and modified potentials are introduced. The second-order adiabatic invariant is found in Sec. 9. In Sec. 10, the second-order adiabatic invariant is used to find global adiabatic invariants. Sec. 11 relates the plasma energy to the quasiparticle density, and deals with global conservation properties. Sec. 12 introduces some basic equations which describe quasiparticle kinetics and fluctuations in the system of many quasiparticles. Extensions of the theory for the case of non-stationary magnetic fields are discussed in Sec. 13. Sec. 14 contains summary.

Each section has its own numbering of equations, while for example, reference in Sec. 4, Eq. (7.2), sends to Eq. 2 in Sec. 7.

2. Microscopic Density in Particle Phase Space

Let us consider a test-particle with the charge q and the rest mass m in the external stationary but non-uniform (inhomogeneous and curved) magnetic field $\vec{B}(\vec{r}) \equiv B(\vec{r})\vec{b}(\vec{r})$, and in the fields of any other additional forces producing particle acceleration $\vec{a}(\vec{r},\vec{v},t)$. Particle trajectory $\{ \vec{r}_p(t)\ ,\ \vec{v}_p(t)\ \}$ in the phase space $\{\vec{r},\vec{v}\}$ is governed by the following equations of motion:

$$\frac{d\vec{r}_p(t)}{dt} = \vec{v}_p(t)\ , \quad \frac{d\vec{v}_p(t)}{dt} = \vec{v}_p(t) \times \vec{\Omega}(\vec{r}_p(t),\ v_p(t)) + \vec{a}(\vec{r}_p(t),\ \vec{v}_p(t),t)\ , \quad (1)$$

where

$$\vec{\Omega}(\vec{r},v) \equiv \vec{B}(\vec{r})\,\frac{q}{m}\,\sqrt{1 - \frac{v^2}{c^2}}\ ,$$

$\Omega(\vec{r},v)$ is the cyclotron frequency, which in the general case depends on the particle position and the magnitude of its velocity, c is the speed of light in the vacuum. The nature of additional forces can be arbitrary: there can be self-consistent microscopic fields created by particles themselves and externally imposed forces. For relativistic particles in electromagnetic fields:

$$\vec{a}(\vec{r},\vec{v},t) = \frac{q}{m}\sqrt{1 - \frac{v^2}{c^2}}\ \{\ \vec{E} + \vec{v}\times\vec{B}' - \frac{1}{c^2}\,\vec{v}(\vec{v}\cdot\vec{E})\}\ , \quad (2)$$

where $\vec{E}$ is for electric fields, $\vec{B}'$ is for non-stationary magnetic fields.

The microscopic density for a test-particle in the phase space is introduced as usual,

$$F_p(\vec{r},\vec{v},t) = \delta(\vec{r} - \vec{r}_p(t))\ \delta(\vec{v} - \vec{v}_p(t))\ . \quad (3)$$

If a system of many charged particles, such as plasmas, is considered, then a total microscopic distribution for a particular plasma species $F = \sum_p F_p$, and it satisfies the same equation as F_p , but with a different initial condition.

One verifies the microscopic continuity equation:

$$(\ \partial_t + \vec{v}\cdot\vec{\nabla}\ )\,F_p + \partial_{\vec{v}}\cdot\ \{\ [\ \vec{a}(\vec{r},\vec{v},t) + \vec{v}\times\vec{\Omega}(\vec{r},v)\ ]\ F_p\ \} = 0\ , \quad (4)$$

for the particle density in the phase space.

3. Microscopic Density in Guiding-Centre Phase Space

In the case of uniform external magnetic fields [7], in order to allow for finite-Larmor-radius effects associated with the non-uniformity of the acceleration $\vec{a}$ at scales compared to the particle Larmor radius, it was convenient to go to the guiding-centre space according to the well known transformation.

Let us introduce formally a new dynamic variable $\vec{R}_c$ instead of the particle position $\vec{r}_p$:

$$\vec{r}_p = \vec{r}_p(\vec{R}_c, \vec{v}_p) , \tag{1}$$

which from this point will be called just arbitrarily a guiding-centre position. Relation (1) will be specified later, when necessary. And let us exploit the Frenet triad (e.g. [13]) in a new space:

$$\vec{h} = \vec{b}(\vec{R}) , \quad \vec{N} = \rho(\vec{R})\, \vec{h} \cdot \partial_{\vec{R}}\, \vec{h} , \quad \vec{\beta} = \vec{h} \times \vec{N} , \tag{2}$$

where $\rho(\vec{R}) \equiv \mid \vec{h} \cdot \partial_{\vec{R}}\, \vec{h} \mid$ is the local radius of curvature, $\vec{N}$ is the unity vector along the principal normal to a magnetic field line at a position $\vec{R}$, and $\vec{\beta}$ is the unity vector along the binormal to a magnetic field field line at the same position.

One can change to dynamical velocity variables associated with a new space. These are the parallel to $\vec{b}(\vec{R}_c(t))$ component $v_{\parallel c}(t)$ of the particle velocity, and the magnitude $v_{\perp c}(t)$ of the perpendicular component of the particle velocity, or $\epsilon_{\perp c} \equiv v_{\perp c}^2/2$, where the components of the particle velocity are taken in the guiding-centre space, and the gyroangle $\alpha_c(t)$, which is counted from the principal normal in the plane perpendicular to $\vec{b}(\vec{R}_c(t))$,

$$v_{\parallel c}(t) = \vec{b}(\vec{R}_c(t)) \cdot \vec{v}_p(t) , \quad \vec{v}_{\perp c}(t) = \vec{v}_p(t) - v_{\parallel c}(t)\, \vec{b}(\vec{R}_c(t)) =$$

$$= v_{\perp c}(t) \left[\, cos\alpha_c(t)\, \vec{N}(\vec{R}_c(t)) + sin\alpha_c(t)\, \vec{\beta}(\vec{R}_c(t)) \, \right] . \tag{3}$$

One can easily derive equations of motion for new dynamical variables:

$$\frac{d\vec{R}_c}{dt} = \vec{V} , \tag{4}$$

$$\frac{dv_{\parallel c}}{dt} = \vec{h}\cdot\tilde{\vec{a}} + \vec{h}\cdot\vec{v}_\perp\times\tilde{\vec{\Omega}} + \vec{v}_\perp\cdot(\vec{V}\cdot\partial_{\vec{R}})\,\vec{h} , \tag{5}$$

$$\frac{d\epsilon_{\perp c}}{dt} = \vec{v}_\perp\cdot\tilde{\vec{a}} + v_\parallel\,\vec{v}_\perp\cdot\{\,\vec{h}\times\tilde{\vec{\Omega}} - \vec{V}\cdot\partial_{\vec{R}}\,\vec{h}\,\} , \tag{6}$$

$$\frac{d\alpha_c}{dt} = -\Omega(\vec{R},v) - \nu . \tag{7}$$

Here

$$\nu \equiv -\frac{1}{v_\perp^2}\,(\vec{h}\times\vec{v}_\perp)\cdot\{\,\tilde{\vec{a}} + v_\parallel\,\vec{h}\times\tilde{\vec{\Omega}} - \vec{V}\cdot\partial_{\vec{R}}\,\vec{v}\,\} + \vec{h}\cdot\{\,\tilde{\vec{\Omega}} - \vec{\Omega}(\vec{R},v)\,\} \tag{8}$$

is a frequency shift, an explicit expression for $\vec{V}\equiv\vec{V}(\vec{R},v_\parallel,\epsilon_\perp,\alpha,t)$ is not given as a transformation (1) is not yet specified, for an arbitrary function $Q(\vec{r},\vec{v},t)$ of particle dynamic variables

$$\tilde{Q} \equiv \tilde{Q}(\vec{R},v_\parallel,\epsilon_\perp,\alpha,t) = Q(\vec{r}(\vec{R},\vec{v}),\vec{v},t) , \tag{9}$$

$$\vec{v} \equiv v_\parallel\,\vec{h} + \vec{v}_\perp \equiv v_\parallel\,\vec{h} + v_\perp\,[\,cos\alpha\;\vec{N}(\vec{R}) + sin\alpha\;\vec{\beta}(\vec{R})\,] ,$$

$$\vec{h} \equiv \vec{b}(\vec{R}) , \quad \epsilon_\perp \equiv v_\perp^2/2 , \quad v^2 \equiv v_\parallel^2 + v_\perp^2 ,$$

and all the quantities on the right-hand side of Eqs (5)-(8) are evaluated at the instantaneous guiding-centre position in the phase space, $\vec{R} = \vec{R}_c$, $v_\parallel = v_{\parallel c}$, $\epsilon_\perp = \epsilon_{\perp c}$, $\alpha = \alpha_c$, after taking the derivative $\partial_{\vec{R}}$.

Let us call Eqs (4)-(7) the equations of guiding-centre motion.

In order to have the structure of these equations as close as possible to the equations of guiding-centre motion in uniformly magnetized plasma, one can try to look for some effective acceleration $\vec{A}$ which takes into account non-uniformity of external magnetic fields. Eqs (5)-(8) suggest the following expression for such acceleration:

$$\vec{A} = \tilde{\vec{a}} + \vec{v}\times\{\,\tilde{\vec{\Omega}} - \vec{\Omega}(\vec{R},v)\,\} - \vec{V}\cdot\partial_{\vec{R}}\,\vec{v} . \tag{10}$$

Then, the equations of guiding-centre motion obtain the necessary structure:

$$\frac{d\vec{R}_c}{dt} = \vec{V}\,, \quad \frac{dv_{\parallel c}}{dt} \equiv A_{\parallel} = \vec{h}\cdot\vec{A}\,, \quad \frac{d\epsilon_{\perp c}}{dt} \equiv A_{\epsilon_\perp} = \vec{v}_\perp\cdot\vec{A}\,, \tag{11}$$

$$\frac{d\alpha_c}{dt} = -\Omega(\vec{R},v) - \nu\,, \quad \nu \equiv -\frac{1}{v_\perp^2}\,\vec{h}\times\vec{v}_\perp\cdot\vec{A}\,. \tag{12}$$

Now, the microscopic density F_c' is introduced in the guiding-centre phase space $\{\vec{R}, v_\parallel, \epsilon_\perp, \alpha\}$:

$$F_c' \equiv F_c'(\vec{R}, v_\parallel, \epsilon_\perp, \alpha, t) = \delta(\vec{R} - \vec{R}_c(t))\,\delta(v_\parallel - v_{\parallel c}(t))\,\delta(\epsilon_\perp - \epsilon_{\perp c}(t)) \times$$
$$\times \sum_n \delta(\alpha - \alpha_c(t) + 2\pi n)\,. \tag{13}$$

Its time evolution is governed by the following continuity equation:

$$\partial_t F_c' + \partial_{\vec{R}}\cdot(\vec{V}F_c') + \partial_{v_\parallel}(A_\parallel F_c') + \partial_{\epsilon_\perp}(A_{\epsilon_\perp} F_c') -$$
$$- \partial_\alpha \{ [\,\Omega(\vec{R},v) + \nu\,]\,F_c' \} = 0\,. \tag{14}$$

Non-uniform stationary magnetic fields hide in the guiding-centre velocity, which is not yet specified, and in the effective acceleration, which determines the quantities $A_\parallel$, $A_{\epsilon_\perp}$, and ν. Besides, it is introduced explicitly by the position-dependent cyclotron frequency.

4. Microscopic Density in Quasiparticle Phase Space

According to approach [7], the exact microscopic density G in the quasiparticle phase space is introduced by means of the Fourier series with respect to the gyroangle for the guiding-centre density,

$$F_c' = \frac{1}{2\pi} G + \sum_{n \neq 0} e^{-i\alpha n} F_n \,, \quad F_0 \equiv \frac{1}{2\pi} G \,, \tag{1}$$

as a gyroangle-independent part of the microscopic guiding-centre density. Similarly, one represents the accelerations and the frequency shift:

$$\vec{A} = \sum_n e^{-i\alpha n} \vec{A}_n \,, \quad A_\parallel = \sum_n e^{-i\alpha n} A_{\parallel n} \,,$$

$$A_{\epsilon_\perp} = \sum_n e^{-i\alpha n} A_{\epsilon_\perp n} \,, \quad \nu = \sum_n e^{-i\alpha n} \nu_n \,. \tag{2}$$

Any quantity $\tilde{Q}$ is represented by the Fourier series with the coefficients

$$Q_n = \frac{1}{2\pi} \int_0^{2\pi} d\alpha \; \tilde{Q} e^{in\alpha} \,. \tag{3}$$

It is easy to derive the basic equations for the Fourier coefficients

$$F_n \equiv F_n(\vec{R}, v_\parallel, \epsilon_\perp, t)$$

from Eq. (3.14):

$$(\,\partial_t + in\Omega\,) F_n + \sum_{n_1+n_2=n} \{\, \partial_{\vec{R}} \cdot [\, \vec{V}_{n_1} F_{n_2} \,] + \partial_{v_\parallel} [\, A_{\parallel n_1} F_{n_2} \,] +$$

$$+ \partial_{\epsilon_\perp} [\, A_{\epsilon_\perp n_1} F_{n_2} \,] + in\nu_{n_1} F_{n_2} \,\} = 0 \,, \tag{4}$$

where $\Omega \equiv \Omega(\vec{R}, v)$.

This equation for $n = 0$ yields an equation for the microscopic quasiparticle density in a reduced phase space $\{\vec{R}, v_\parallel, \epsilon_\perp\}$:

$$\partial_t G + \partial_{\vec{R}} \cdot [\, \vec{V}_0 G \,] + \partial_{v_\parallel} [\, A_{\parallel 0} G \,] + \partial_{\epsilon_\perp} [\, A_{\epsilon_\perp 0} G \,] = S \,, \tag{5}$$

where the source term S allows for the effects of the other harmonics of the guiding-centre density:

$$S \equiv -2\pi \sum_{n \neq 0} [\, \partial_{\vec{R}} \cdot (\, \vec{V}_n F_n^* \,) + \partial_{v_\|}(\, A_{\| n} F_n^* \,) + \partial_{\epsilon_\perp}(\, A_{\epsilon_\perp n} F_n^* \,)\,] \,. \quad (6)$$

It is necessary to specify initial conditions. Given the initial guiding-centre position in the phase space, $\{\vec{R}_c(t_0), v_{\|c}(t_0), \epsilon_{\perp c}(t_0), \alpha_c(t_0)\}$, there is a corresponding initial value of the microscopic guiding-centre density. The initial value of G is calculated from the latter:

$$G(\vec{R}.v_\|, \epsilon_\perp, t_0) \;=\; \delta(\vec{R} - \vec{R}_c(t_0))\; \delta(v_\| - v_{\|c}(t_0))\; \delta(\epsilon_\perp - \epsilon_{\perp c}(t_0)) \;, \qquad (7)$$

where the initial guiding-centre position is related to the particle position and the particle velocity by relation (3.1).

This initial condition is in accordance with the suggestion that G describes a quasiparticle in a reduced phase space $\{\vec{R}, v_\|, \epsilon_\perp\}$ with the initial coordinates $\vec{R}_c$, $v_{\|c}$ and $\epsilon_{\perp c}$, related to the initial coordinates of the particle in the original phase space.

If G_q is a microscopic density for one quasiparticle in the reduced phase space, then the total microscopic quasiparticle density $G = \sum_q G_q$, and it satisfies Eq. (5), with the total guiding-centre density $F' = \sum_c F'_c$ entering it. The number of quasiparticles is equal to the number of guiding-centres and the number of particles:

$$\int d\vec{R} dv_\| d\epsilon_\perp \, G \;=\; \int d\vec{R} dv_\| d\epsilon_\perp d\alpha \, F' \;=\; \int d\vec{r} d\vec{v} \, F \,. \qquad (8)$$

When the number of particles is conserved the number of quasiparticles is also conserved. The conservation of quasiparticles has nothing to do with the specific structure of Eq. (5) or its approximations. The exact equation for G (or any of its approximations) complies with the conservation properties of the original continuity equation. The structure of this equation in the case considered implies no local sources or sinks of particles and no factors producing random particle motion (at the very basic level of description no information is lost yet). These particle properties are not to be preserved in general.

Eq. (5) is an exact microscopic equation for the quasiparticle density. If its right-hand side is treated as given, then the general solution is the sum of the solution $\tilde{G}$ of the homogeneous equation:

$$\partial_t \tilde{G} + \partial_{\vec{R}} \cdot [\, \vec{V}_0 \tilde{G} \,] + \partial_{v_\parallel} [\, A_{\parallel 0} \tilde{G} \,] + \partial_{\epsilon_\perp} [\, A_{\epsilon_\perp 0} \tilde{G} \,] = 0 , \tag{9}$$

and the particular solution of the inhomogeneous equation. If the homogeneous solution is specified to take into account the initial condition (7), then it can be treated as a microscopic quasiparticle density, and

$$\tilde{G}(\vec{R}, v_\parallel, \epsilon_\perp, t) = \delta(\vec{R} - \vec{R}_q(t))\, \delta(v_\parallel - v_{\parallel q}(t))\, \delta(\epsilon_\perp - \epsilon_{\perp q}(t)) . \tag{10}$$

That is, the quasiparticle is pointwise (it is represented by a point in the reduced phase space), and it is governed by the following equations of motion:

$$\frac{d\vec{R}_q(t)}{dt} = \vec{V}_0 , \quad \frac{dv_{\parallel q}(t)}{dt} = A_{\parallel 0} , \quad \frac{d\epsilon_{\perp q}(t)}{dt} = A_{\epsilon_\perp 0} . \tag{11}$$

Here, the right-hand sides are evaluated at the point $\{\vec{R}_q(t),\ v_{\parallel q}(t),\ \epsilon_{\perp q}(t)\}$ of the reduced phase space. The quasiparticle has initially the same position, parallel velocity and the magnitude of the perpendicular velocity as the original guiding centre. Initial coordinates of the guiding centre and the particle are related by Eq. (3.1). In general, two close particles can be represented by two distant quasiparticles, and vice versa.

5. Closure

Untill now, the theory is not closed, as the microscopic continuity equation for the quasiparticle density contains other harmonics of the guiding-centre density as unknown quantities. The infinite set of harmonics provides complete information on the gyroangle dependence within the context of exact microscopic description. A closure should make it possible to relate guiding-centre (particle) and quasiparticle plasma characteristics. As a result of closure, only incomplete information on the gyroangle dependence can become available in the form of a probability distribution.

Closure can be achieved under the following assumption:

$$F_n \sim \lambda\, G\,, \quad \lambda << 1\,, \tag{1}$$

by means of expanding in powers of the small parameter λ:

$$F_n = \sum_{j=1}^{\infty} F_n^{(j)}\,. \tag{2}$$

Then, the source term appears as an expansion in increasing powers of the small parameter:

$$S = \sum_{j=1}^{\infty} S^{(j)} \equiv \tag{3}$$

$$\equiv -2\pi \sum_{j=1}^{\infty} \sum_{n\neq 0} \{\ \partial_{\vec{R}} \cdot [\ \vec{V}_n^* F_n^{(j)}\ ] \ +\ \partial_{v_\parallel} [\ A_{\parallel n}^* F_n^{(j)}\ ] \ +\ \partial_{\epsilon_\perp} [\ A_{\epsilon_\perp n}^* F_n^{(j)}\ ]\ \}\,.$$

After putting expansion (2) into Eq. (4.4), one finds the first iteration:

$$[\ \hat{D}_0 \ +\ in(\Omega + \nu_0)\ ]\ F_n^{(1)} \ =$$

$$= -\frac{1}{2\pi} [\ in\nu_n G \ +\ \partial_{\vec{R}} \cdot (\ \vec{V}_n G\ ) \ +\ \partial_{v_\parallel}(\ A_{\parallel n} G\ ) \ +\ \partial_{\epsilon_\perp}(\ A_{\epsilon_\perp n} G\ )\ ]\,, \tag{4}$$

and recurrence relations for higher-order iterations:

$$[\ \hat{D}_0 \ +\ in(\Omega + \nu_0)\ ]\ F_n^{(j+1)} \ =\ - \sum_{n_1+n_2=n,\ n_1 n_2 \neq 0} \{\ \partial_{\vec{R}} \cdot [\ \vec{V}_{n_1} F_{n_2}^{(j)}\ ] \ +$$

$$+ \partial_{v_\parallel} [A_{\parallel n_1} F_{n_2}^{(j)}] + \partial_{\epsilon_\perp} [A_{\epsilon_\perp n_1} F_{n_2}^{(j)}] + i n \nu_{n_1} F_{n_2}^{(j)} \} , \qquad (5)$$

where for arbitrary quantity $g \equiv g(\vec{R}, v_\parallel, \epsilon_\perp, t)$,

$$\hat{D}_0 \, g \equiv \partial_t g + \partial_{\vec{R}} \cdot [\vec{V}_0 g] + \partial_{v_\parallel} [A_{\parallel 0} g] + \partial_{\epsilon_\perp} [A_{\epsilon_\perp 0} g] .$$

In the simplest approximation the influence of other harmonics on the time evolution of the quasiparticle density is completely disregarded:

$$\partial_t G + \partial_{\vec{R}} \cdot [\vec{V}_0 G] + \partial_{v_\parallel} [A_{\parallel 0} G] + \partial_{\epsilon_\perp} [A_{\epsilon_\perp 0} G] = 0 ,$$

$$F_c' = \frac{1}{2\pi} G , \qquad (6)$$

with the initial condition (4.7). The discussion of quasiparticle nature and the derivation of the quasiparticle equations of motion presented in Sec. 4 are also appropriate in this case.

The guiding-centre density is not the delta-function of the point in the guiding-centre phase space anymore, and its interpretation as a probability distribution is appropriate. After closure a part of information about the gyroangle, as well as about rapid variations of other coordinates, is lost. The reduced guiding-centre distribution F' contains but incomplete information on gyroangle dependence. An expansion of the phase-space point, which represents the guiding centre (or the particle), to a probability "cloud" occurs. The latter is a circle actually, in the simplest case under consideration.

For simplicity, let us illustrate this point for the case of uniformly magnetized plasma. Within the original exact description, a microscopic number density in the physical space $n(\vec{r}, t) \equiv \int d\vec{v} F = \delta(\vec{r} - \vec{r}_p(t))$. Meanwhile, taking into account the guiding-centre transformation in the usual form $\vec{R} = \vec{r} + \vec{v} \times \vec{b} / \Omega$, one obtains from Eq. (6):

$$n(\vec{r}, t) = \frac{1}{2\pi} \delta(r_\parallel - R_{\parallel q}(t)) \, \delta[(\vec{r}_\perp - \vec{R}_{\perp q}(t))^2 - \rho_L^2] , \qquad (7)$$

where $\rho_L \equiv v_{\perp q}(t)/\Omega$ is the particle Larmor radius. The latter equation enables one to think about the probability circle of the radius ρ_L moving in the physical space with its plane being perpendicular to $\vec{B}$, and its center being at the point $\vec{R}_q(t)$. The probability to find the particle at any point of this circle is the same, according to Eq. (7). In general, this probability circle can be distorted due to the effects of harmonics. Eq. (7) predicts "visually observable" motion of the probability circle with the velocity $\vec{V}_0$, while the velocity of the particle calculated from the reduced distribution in Eq. (6) is $v_{\|q}(t)\ \vec{b}$.

6. First-Order Approximation

In order to allow for perpendicular particle drift, it is necessary to consider the first-order approximation, which involves the source term in the time evolution of the quasiparticle density and the relevant contribution of harmonics into the original distribution of guiding centres with regard to the first iteration:

$$F_n = F_n^{(1)} , \quad S = S^{(1)} , \quad F_c' = \frac{1}{2\pi} G + \sum_{n \neq 0} e^{-i\alpha n} F_n^{(1)} . \tag{1}$$

Thus, the guiding-centre density in the phase space and the quasiparticle density in the reduced phase space have been related.

Let us consider the adiabatic approximation, in which slowly evolving (as compared to the cyclotron frequency) parts of all the quantities are of interest only:

$$[\, \hat{D}_0 + in\nu_0 \,]\, F_n \sim \lambda\, n\Omega\, F_n . \tag{2}$$

In this approximation, Eq. (5.4) yields

$$F_n^{(1)} = \tag{3}$$

$$= \frac{i}{2\pi n\Omega} [\, in\nu_n G + \partial_{\vec{R}} \cdot (\, \vec{V}_n G \,) + \partial_{v_\parallel} (\, A_{\parallel n} G \,) + \partial_{\epsilon_\perp} (\, A_{\epsilon_\perp n} G \,) \,] .$$

After this expression is used to calculate the source term, one discovers that the structure of the microscopic equation for the quasiparticle density remains unchanged:

$$\partial_t G + \partial_{\vec{R}} \cdot [\, \vec{V}_r G \,] + \partial_{v_\parallel} [\, A_{\parallel r} G \,] + \partial_{\epsilon_\perp} [\, A_{\epsilon_\perp r} G \,] = 0 . \tag{4}$$

Here, the renormalized velocity and the renormalized accelerations are obtained with regard to the effects of the source term:

$$\vec{V}_r = \vec{V}_0 - \sum_{n \neq 0} \hat{l}_n\, \vec{V}_n^* ,$$

$$A_{\parallel r} = A_{\parallel 0} - \sum_{n \neq 0} \hat{l}_n \, A^*_{\parallel n} \,, \quad A_{\epsilon_\perp r} = A_{\epsilon_\perp 0} - \sum_{n \neq 0} \hat{l}_n \, A^*_{\epsilon_\perp n} \,, \tag{5}$$

where

$$\hat{l}_n \equiv \frac{1}{\Omega} \left[\frac{i}{n} \left(A_{\parallel n} \partial_{v_\parallel} + \vec{V}_n \cdot \partial_{\vec{R}} + A_{\epsilon_\perp n} \partial_{\epsilon_\perp} \right) + \nu_n \right] +$$
$$+ \frac{i}{n} \left\{ \vec{V}_n \cdot \frac{\partial \ln \Omega}{\partial \vec{R}} + A_{\parallel n} \frac{\partial \ln \Omega}{\partial v_\parallel} + A_{\epsilon_\perp n} \frac{\partial \ln \Omega}{\partial \epsilon_\perp} \right\} . \tag{6}$$

Similarly, the renormalized vector acceleration can be introduced:

$$\vec{A}_r = \vec{A}_0 - \sum_{n \neq 0} \hat{l}_n \, \vec{A}^*_n \,, \tag{7}$$

and for arbitrary $Q(\vec{r}, \vec{v}, t)$ the renormalized (to the first order) quantity Q_r is defined as follows:

$$Q_r \equiv Q_r(\vec{R}, v_\parallel, \epsilon_\perp, t) = Q_0 - \sum_{n \neq 0} \hat{l}_n \, Q^*_n \, . \tag{8}$$

The preservation of the structure of the original continuity equation, but in the reduced phase space, and the initial condition determine the nature of quasiparticles and their equations of motion. However, the structure of the original continuity equation does not preclude in general the effects of random motion, or local sources and sinks for quasiparticles. These effects can be involved within reduced theory approximations.

The solution of Eq. (4) with the initial condition (4.7) can be represented in the form (4.10):

$$G(\vec{R}, v_\parallel, \epsilon_\perp, t) = \delta(\vec{R} - \vec{R}_q(t)) \, \delta(v_\parallel - v_{\parallel q}(t)) \, \delta(\epsilon_\perp - \epsilon_{\perp q}(t)) \,, \tag{9}$$

which suggests point quasiparticles governed by the equations of motion:

$$\frac{d\vec{R}_q(t)}{dt} = \vec{V}_r \,, \quad \frac{dv_{\parallel q}(t)}{dt} = A_{\parallel r} \,, \quad \frac{d\epsilon_{\perp q}(t)}{dt} = A_{\epsilon_\perp r} \, . \tag{10}$$

In order to elucidate the relation of renormalized (quasiparticle) quantities to the particle, let us take an arbitrary function $Q(\vec{r}, \vec{v}, t)$ of particle dynamical variables and derive a reduced expression for it. It is convenient to introduce the following notations:

$$\langle ... \rangle_F \equiv \int d\vec{r} d\vec{v} (...) F_p \ , \quad \langle ... \rangle_{F'} \equiv \int d\vec{R} dv_{\parallel} d\epsilon_{\perp} d\alpha (...) F'_c \ ,$$

$$\langle ... \rangle_G \equiv \int d\vec{R} dv_{\parallel} d\epsilon_{\perp} (...) G \ . \tag{11}$$

So,

$$Q(t) \equiv Q(\vec{r}_p(t), \vec{v}_p(t), t) \equiv \langle Q \rangle_F = \langle \tilde{Q} \rangle_{F'} =$$

$$= \langle Q_0 \rangle_G + 2\pi \sum_{n \neq 0} \int d\vec{R} dv_{\parallel} d\epsilon_{\perp} Q_n^* F_n \ .$$

Then, the reduced expression follows immediately:

$$\langle Q \rangle_F = \langle Q_0 \rangle_G + 2\pi \sum_{j=1}^{\infty} \sum_{n \neq 0} \int d\vec{R} dv_{\parallel} d\epsilon_{\perp} Q_n^* F_n^{(j)} \ . \tag{12}$$

In the adiabatic first-order approximation:

$$\langle Q \rangle_F = \langle Q_r \rangle_G \ . \tag{13}$$

Clearly, $Q(t) = Q_r(\vec{R}_q(t), v_{\parallel q}(t), \epsilon_{\perp q}(t), t)$ if the quasiparticle is pointwise.

One can notice the following exact relations:

$$\vec{R}_c(t) \equiv \langle \vec{R} \rangle_{F'} = \langle \vec{R} \rangle_G \ , \tag{14}$$

$$v_{\parallel c}(t) \equiv \langle v_{\parallel} \rangle_{F'} = \langle v_{\parallel} \rangle_G \ , \quad v_{\perp c}(t) \equiv \langle v_{\perp} \rangle_{F'} = \langle v_{\perp} \rangle_G$$

(and similar relations for any other functions of guiding-centre variables except the gyroangle). That is, the guiding-centre position is exactly the average quasiparticle position, and in the reduced theory it is so to arbitrary orders in the small parameter. Within the adiabatic approximation, when the quasiparticle is pointwise, $\vec{R}_c(t) = \vec{R}_q(t)$: the quasiparticle and guiding-centre positions coincide. Similar relationship holds for the parallel velocities and the magnitudes of the perpendicular velocities of the guiding centre and the quasiparticle.

After taking into account Eq. (3) in Eq. (1), one finds that

$$F_c' = \frac{1}{2\pi} \; G \, (\, 1 \; - \; \frac{\nu - \nu_0}{\Omega} \,) \; + \; \frac{1}{2\pi\Omega} \int^{\alpha} d\alpha \; \{ \; \partial_{\vec{R}} \cdot \; [\; G \; (\; \vec{V} - \vec{V}_0 \;) \;] \; +$$

$$+ \; \partial_{v_\parallel} \; [\; G \; (\; A_\parallel - A_{\parallel 0} \;) \;] \; + \; \partial_{\epsilon_\perp} \; [\; G \; (\; A_{\epsilon_\perp} - A_{\epsilon_\perp 0} \;) \;] \; \} \; , \qquad (15)$$

which is the relation between the guiding-centre and the quasiparticle within the adiabatic first-order approximation of the reduced theory.

7. Specifying Transformation to Guiding-Centre Space

So far, relation (3.1) remains unspecified, and so do the guiding-centre velocity $\vec{V}$, which appears in the microscopic continuity equations, and the effective acceleration $\vec{A}$. When external magnetic fields are uniform, the guiding-centre transformation is well known: $\vec{r} = \vec{R} - \vec{v} \times \vec{b}/\Omega$. In the case of non-uniformly magnetized plasma, one has some freedom. Without going into appropriate speculations, let us take one from the simplest choices:

$$\vec{r} = \vec{R} - \frac{1}{\Omega(\vec{R}, v)} \vec{v} \times \vec{b}(\vec{R}) . \tag{1}$$

Then,

$$\vec{V} = v_{\|} \vec{h} + \frac{1}{\Omega} \{ \tilde{\vec{a}} \times \vec{h} + [\vec{v} \times (\tilde{\vec{\Omega}} - \vec{\Omega})] \times \vec{h} \} + \vec{v} \times (\vec{V} \cdot \partial_{\vec{R}}) \frac{\vec{h}}{\Omega} , \tag{2}$$

where $\vec{\Omega} \equiv \vec{\Omega}(\vec{R}, v)$. This is an equation for the guiding-centre velocity. It is convenient to transform it, with the effective acceleration being introduced:

$$\vec{V} = v_{\|} \vec{h} + \frac{1}{\Omega} \vec{A} \times \vec{h} - \vec{V} \cdot \partial_{\vec{R}} \frac{\vec{h} \times \vec{v}}{\Omega} . \tag{3}$$

When the gradient of the magnetic field is small, because of either weak non-uniformity or a small strongly non-uniform part of the magnetic field, then a small parameter associated with the last term on the right-hand side of Eq. (2) can be introduced, and this equation can be solved by the method of iterations. Eqs (2) and (3.10) can be used to calculate the guiding-centre velocity and the effective acceleration to arbitrary orders of the small parameter.

If the first term on the right-hand side of Eq. (2) is assumed to be dominant, while the other terms have magnitudes of the order λ (other orderings are possible in general), then

$$\vec{V} = v_{\|} \vec{h} + \frac{1}{\Omega} \vec{A} \times \vec{h} - v_{\|} \vec{h} \cdot \partial_{\vec{R}} \frac{\vec{h} \times \vec{v}}{\Omega} + \dots , \tag{4}$$

$$\vec{A} = \tilde{\vec{a}} + \vec{v} \times \{ \tilde{\vec{\Omega}} - \vec{\Omega} \} - v_{\parallel} \vec{h} \cdot \partial_{\vec{R}} \vec{v} + \dots . \tag{5}$$

Here, terms with the magnitude of the order λ^2 are not retained. These equations yield explicitly the quasiparticle velocity,

$$\vec{V}_0 = v_{\parallel} \vec{h} + \frac{1}{\Omega} \vec{A}_0 \times \vec{h} + \dots , \tag{6}$$

$$\vec{A}_0 = \vec{a}_0 + v_{\parallel} \vec{h} \times \vec{\Omega}_0 + \{ \vec{v}_{\perp} \times \tilde{\vec{\Omega}} \}_0 - v_{\parallel}^2 \vec{h} \cdot \partial_{\vec{R}} \vec{h} + \dots , \tag{7}$$

and the quasiparticle accelerations,

$$A_{\parallel 0} = \vec{h} \cdot \vec{a}_0 + \vec{h} \cdot \{ \vec{v}_{\perp} \times \tilde{\vec{\Omega}} \}_0 + \dots , \tag{8}$$

$$A_{\epsilon_{\perp} 0} = \{ \vec{v}_{\perp} \cdot \tilde{\vec{a}} \}_0 - v_{\parallel} \vec{h} \cdot \{ \vec{v}_{\perp} \times \tilde{\vec{\Omega}} \}_0 + \dots . \tag{9}$$

Thus, it is necessary to calculate $\vec{\Omega}_0$ and $\{ \vec{v}_{\perp} \times \tilde{\vec{\Omega}} \}_0$.

In order to do this, let us represent the magnetic field as a sum of the weakly non-uniform part $\vec{B}_s$, which varies slowly in space, and the small part $\vec{B}_f$ with fast space variations:

$$\vec{B} \equiv \vec{B}_s + \vec{B}_f , \quad B_f \sim \lambda B_s , \tag{10}$$

$$\vec{\Omega} \equiv \vec{\Omega}_s + \vec{\Omega}_f ,$$

$$B \equiv B_s + \delta B_f , \quad \vec{h} \equiv \vec{h}_s + \delta \vec{h}_f ,$$

where in approximation (10),

$$\delta B_f = \vec{h}_s \cdot \vec{B}_f + \dots , \quad \delta \vec{h}_f = \vec{B}_{f\perp} / B_s + \dots .$$

After expanding the part with slow time variation, one gets:

$$\tilde{\vec{B}} = \vec{B}_s + \tilde{\vec{B}}_f - \frac{1}{B} \vec{v}_{\perp} \times \vec{h} \cdot \partial_{\vec{R}} \vec{B}_s + \dots ,$$

where in the last term on the right-hand side, B and $\vec{h}$ can be interplaced with B_s and $\vec{h}_s$ within the accuracy considered. Hence,

$$\vec{B}_0 = \vec{B}_s + \vec{B}_{f0} , \tag{11}$$

$$\{ \vec{v}_\perp \times \tilde{\vec{\Omega}} \}_0 = - \frac{1}{B_s} \{ \vec{v}_\perp \times [\vec{v}_\perp \times \vec{h}_s \cdot \partial_{\vec{R}}] \vec{B}_s \}_0 + \{ \vec{v}_\perp \times \tilde{\vec{\Omega}}_f \}_0 + \dots =$$

$$= - \epsilon_\perp \partial_{\vec{R}} \ ln B + \{ \vec{v}_\perp \times \tilde{\vec{\Omega}}_f \}_0 + \dots .$$

Therefore, Eqs (7)-(9) yield:

$$\vec{A}_0 = \vec{a}_0 - \epsilon_\perp \partial_{\vec{R}} \ ln B_s - v_\parallel^2 \vec{h}_s \cdot \partial_{\vec{R}} \vec{h}_s +$$

$$+ v_\parallel (\vec{h}_s \times \vec{\Omega}_{f0} + \vec{\Omega}_f \times \vec{h}_s) + \{ \vec{v}_\perp \times \tilde{\vec{\Omega}}_f \}_0 + \dots , \tag{12}$$

$$A_{\parallel 0} = \vec{h} \cdot \vec{a}_0 - \epsilon_\perp \vec{h}_s \cdot \partial_{\vec{R}} \ ln B_s + \vec{h}_s \cdot \{ \vec{v}_\perp \times \tilde{\vec{\Omega}}_f \}_0 + \dots , \tag{13}$$

$$A_{\epsilon_\perp 0} = \{ \vec{v}_\perp \cdot \tilde{\vec{a}} \}_0 + v_\parallel \epsilon_\perp \vec{h}_s \cdot \partial_{\vec{R}} \ ln B_s - v_\parallel \vec{h}_s \cdot \{ \vec{v}_\perp \times \tilde{\vec{\Omega}}_f \}_0 + \dots . \tag{14}$$

After substituting Eq. (12) into Eq. (6), one derives a final approximate expression for the quasiparticle velocity:

$$\vec{V}_0 = v_\parallel \vec{h}_s + \frac{v_\parallel}{B_s} \{ \vec{B}_{f0} - \vec{h}_s \vec{h}_s \cdot \vec{B}_{f0} \} + \frac{1}{\Omega} \vec{a}_0 \times \vec{h} +$$

$$+ \frac{1}{\Omega_s} \vec{h}_s \times \{ \epsilon_\perp \partial_{\vec{R}} \ ln B_s + v_\parallel^2 \vec{h}_s \cdot \partial_{\vec{R}} \vec{h}_s \} - \frac{1}{B_s} \{ \vec{v}_\perp \vec{h}_s \cdot \tilde{\vec{B}}_f \}_0 + \dots . \tag{15}$$

The first term on the right-hand side describes quasiparticle streaming along $\vec{B}_s$. The second term introduces a magnetic drift velocity for the quasiparticle with regard to finite-Larmor-radius effects, and the third term is the velocity of quasiparticle drift motion in crossed fields with regard to finite-Larmor-radius effects as well. The fourth term represents gradient-drift motion of the quasiparticle, while the fifth one allows for centrifugal-drift motion of the quasiparticle (the curvature effect). Finally, the last term is due to finite-Larmor-radius effects associated with space variations of the magnetic field $\vec{B}_f$.

The second term in Eq. (14) can be approximated as

$$v_{\|}\epsilon_{\perp}\,\vec{h}_s\cdot\partial_{\vec{R}}\;ln\,B_s\;=\;\epsilon_{\perp}\,\frac{d}{dt}\;ln\,B_s\;+\;\ldots\,,$$

therefore, the equation of motion for $\epsilon_{\perp q}$ can be transformed:

$$\frac{d}{dt}\,\frac{\epsilon_{\perp q}}{B_s}\;=\;\frac{1}{B_s}\,\{\,\vec{v}_{\perp}\cdot\tilde{\vec{a}}\,\}_0\;-\;\frac{v_{\|}}{B_s}\,\vec{h}_s\cdot\{\,\vec{v}_{\perp}\times\tilde{\vec{\Omega}}_f\,\}_0\;+\;\ldots\,. \qquad (16)$$

This equation can be interpreted as the conservation law for the magnetic momentum.

8. Potential Particle Acceleration

The particle equations of motion can include various physical effects, for example friction or diffusion, or compressibility, etc. When the perpendicular component of the additional acceleration can be described by a potential $\theta(\vec{r}, v, t)$,

$$\vec{a}_\perp = \vec{\nabla}_\perp \theta , \tag{1}$$

then this acceleration does not affect time evolution of $\epsilon_\perp$ for the quasiparticle:

$$\frac{d\epsilon_{\perp q}(t)}{dt} = v_\parallel \vec{h} \cdot \{ \vec{V} \cdot \partial_{\vec{R}} \vec{v}_\perp - \vec{v}_\perp \times \tilde{\vec{\Omega}} \}_0 , \tag{2}$$

because $\{\vec{v}_\perp \cdot \tilde{\vec{a}}\}_0 = 0$. Here, for an arbitrary quantity $q(\vec{R}, v_\parallel, \epsilon_\perp, \alpha, t)$, $\{ q \}_n$ means the n-order harmonic in its Fourier series,

$$\{ q \}_n \equiv \frac{1}{2\pi} \int_0^{2\pi} d\alpha \; q(\vec{R}, v_\parallel, \epsilon_\perp, \alpha, t) \; e^{in\alpha} .$$

Then, the magnitude of the perpendicular velocity of the quasiparticle evolves in time due to non-uniformity of magnetic fields.

In the case of uniformly magnetized relativistic plasma, the magnitude of the perpendicular velocity of the quasiparticle is an integral of motion (an adiabatic invariant). This is also true for more general potentials, $\theta = \theta(\vec{r}, v, v_\parallel, t)$.

Let us consider the case of uniformly magnetized plasma in more detail following [7, 10]. One immediately notices that in the lowest-order approximation, the perpendicular component of the reduced acceleration is also described by a potential,

$$\vec{a}_{\perp 0} = \partial_{\vec{R}\perp} \theta_0 + \dots , \quad \theta = \theta(\vec{r}, v, v_\parallel, t) . \tag{3}$$

That is, the potential of the reduced acceleration and the reduced potential of the acceleration coincide in this approximation.

Now, let us consider the first-order adiabatic approximation, but limit ourselves to the non-relativistic limit, and $\theta = \theta(\vec{r}, t)$. In this case,

$$F_n^{(1)} = -\frac{1}{2\pi} [\, \theta_n \, \partial_{\epsilon_\perp} G - \frac{i}{n\Omega} (\, \vec{V}_n \cdot \partial_{\vec{R}} G + a_{\|n} \, \partial_{v_\|} G \,) \,] \,, \qquad (4)$$

where one takes into account that

$$\nu_n = -\frac{1}{v_\perp^2} \{ \, \vec{b} \times \vec{v} \cdot \partial_{\vec{R}} \, \tilde{\theta} \, \}_n = - \{ \, \Omega \, \partial_{\epsilon_\perp} \tilde{\theta} \, \}_n = - \, \Omega \, \partial_{\epsilon_\perp} \theta_n \,,$$

$$a_{\epsilon_\perp n} = \{ \, \vec{v}_\perp \cdot \partial_{\vec{R}} \, \tilde{\theta} \, \}_n = - \{ \, \Omega \, \partial_\alpha \tilde{\theta} \, \}_n = in\Omega \, \theta_n \,, \qquad (5)$$

and the flow $\vec{V}_n$ is incompressible, $\partial_{\vec{R}} \cdot \vec{V}_n = 0$, as a consequence of the property (8) (see below). The relation between the guiding centre and the quasiparticle takes this form:

$$F_c' = \frac{1}{2\pi} \{ \, G - (\, \tilde{\theta} - \theta_0 \,) \, \partial_{\epsilon_\perp} G +$$

$$+ \frac{1}{\Omega} \int^\alpha d\alpha \, [\, (\vec{V} - \vec{V}_0) \cdot \partial_{\vec{R}} \, G + (\tilde{a}_\| - a_{\|0}) \, \partial_{v_\|} G \,] \, \} \,. \qquad (6)$$

Eq. (6.7), in which

$$\hat{l}_n = \frac{i}{n\Omega} (\, a_{\|n} \partial_{v_\|} + \vec{V}_n \cdot \partial_{\vec{R}} \,) - \theta_n \, \partial_{\epsilon_\perp} - \frac{\partial \theta_n}{\partial \epsilon_\perp} \,, \qquad (7)$$

yields the following expression for the reduced accelerations:

$$\vec{a}_{r\perp} = \partial_{\vec{R}\perp} \, \theta_m \,, \quad a_{\epsilon_\perp r} = 0 \,. \qquad (8)$$

Here, the modified potential appears:

$$\theta_m = \theta_0 + \frac{1}{2} \sum_{n \neq 0} \{ \, \partial_{\epsilon_\perp} \mid \theta_n \mid^2 + \frac{i}{n\Omega^2} \, (\partial_{\vec{R}} \, \theta_n) \times (\partial_{\vec{R}} \, \theta_n^*) \cdot \vec{b} \, \} \,, \qquad (9)$$

or

$$\theta_m = \theta_0 + \frac{1}{2} \, \partial_{\epsilon_\perp} (\, \tilde{\theta} - \theta_0 \,)^2 -$$

$$- \frac{1}{2\Omega^2} [\, \partial_{\vec{R}} (\, \tilde{\theta} - \theta_0 \,) \,] \cdot \int^\alpha d\alpha \, [\, \partial_{\vec{R}} (\, \tilde{\theta} - \theta_0 \,) \,] \times \vec{b} \,.$$

In the first-order adiabatic approximation, the renormalized value of the perpendicular component of the acceleration appears to be potential as in the lower order. Therefore, the magnitude of the perpendicular velocity of the quasiparticle remains to be an integral of motion in the adiabatic first-order approximation, and the continuity equation for the quasiparticle density simplifies:

$$(\partial_t + \vec{V}_r \cdot \vec{\nabla} + a_{\parallel r} \, \partial_{v_\parallel}) \, G = 0 \, . \tag{10}$$

Here

$$\vec{V}_r = v_\parallel \vec{b} + \frac{1}{\Omega} \, \vec{b} \times \vec{a}_r \, . \tag{11}$$

That is, when relativistic effects are disregarded, and the perpendicular component of the particle acceleration is governed by a potential $\theta = \theta(\vec{r}, t)$, $\vec{a} = \vec{\nabla}\theta$, then the reduced value of the perpendicular acceleration component is also governed by the potential, which is θ_m in Eq. (9). Another consequence of potential acceleration is that the magnitude of the perpendicular velocity of the quasiparticle is an adiabatic invariant, and the quasiparticle density satisfies the Liouville-type equation.

However, Eq. (6.8), where $\hat{l}_n$ has the form (7), yields the following result for the reduced potential,

$$\theta_r = \theta_0 + \sum_{n \neq 0} \{ \partial_{\epsilon_\perp} \mid \theta_n \mid^2 + \frac{i}{n\Omega^2} (\partial_{\vec{R}} \, \theta_n) \times (\partial_{\vec{R}} \, \theta_n^*) \cdot \vec{b} \, \} \, . \tag{12}$$

That is, the potential of the reduced acceleration is not equal to the reduced potential of the original acceleration.

9. Second-Order Quasiparticle Adiabatic Invariant

The significance of the difference between the renormalized and modified potentials was investigated in [10], within the context of global energy conservation, and it was shown that it is related to the second-order adiabatic invariant, associated with quasiparticle motion.

The second-order approximation of the reduced description takes into account the second-order iterations $F_n^{(2)}$ of harmonics of the guiding-centre density. Then, it becomes possible to treat the polarization drift of particles in non-stationary force fields, as well as its modification in strongly non-uniform force fields due to the finite-Larmor-radius effects [9, 10]. Within this approximation, the conservation of the magnitude of the particle (quasiparticle) perpendicular velocity is violated.

Let us give a simpler but at the same time more general derivation of the second-order adiabatic invariant, without relying on the law of global energy conservation in the plasma.

First of all, one can use exact relations of the type (6.14) for the part of the particle kinetic energy associated with perpendicular motion:

$$\frac{v_{\perp p}^2}{2} \equiv \langle \frac{v_\perp^2}{2} \rangle_F = \langle \epsilon_\perp \rangle_{F'} = \langle \epsilon_\perp \rangle_G \, .$$

Therefore, time evolution of this quantity within any approximation of the reduced theory could be found by taking the $\epsilon_\perp$-moment of the microscopic equation (4.5) for the quasiparticle density, in which the reduced expression (5.3) for the source term enters. This requires to consider the second-order iterations $F_n^{(2)}$.

An easier way to achieve the same goal is to transform a bit the time derivative of the interesting quantity, before taking into account explicitly equations of the reduced theory:

$$\frac{d}{dt}\frac{v_{\perp p}^2}{2} = \vec{v}_{\perp p}(t)\cdot\vec{a}(\vec{r}_{\perp p}(t)) \equiv \langle\vec{v}_\perp\cdot\vec{a}(\vec{r},t)\rangle_F =$$

$$= \langle\vec{v}_\perp\cdot\vec{\nabla}\theta\rangle_F = -\int d\vec{r}d\vec{v}\,\theta\,\vec{v}_\perp\cdot\vec{\nabla}F_p\,, \tag{1}$$

where integration by parts over $\vec{r}$ has been carried out. Then, the microscopic continuity equation (2.4) for F_p can be taken into account:

$$\frac{d}{dt}\langle\epsilon_\perp\rangle_G = -\int d\vec{r}d\vec{v}\,\theta\,\vec{v}_\perp\cdot\vec{\nabla}F_p = \int d\vec{r}d\vec{v}\,\theta\,(\,\partial_t + v_\parallel\,\vec{b}\cdot\vec{\nabla}\,)\,F_p =$$

$$= \int d\vec{r}d\vec{v}\,\{\,\partial_t\,(\theta F_p) - F_p\,(\,\partial_t + v_\parallel\,\vec{b}\cdot\vec{\nabla}\,)\,\theta\,\} \equiv$$

$$\equiv \frac{d}{dt}\langle\theta\rangle_F - \langle(\,\partial_t + v_\parallel\,\vec{b}\cdot\vec{\nabla}\,)\,\theta\rangle_F\,, \tag{2}$$

where integration by parts over $\vec{r}$ has been carried out again.

Now, it becomes especially convenient to go to the guiding-centre space,

$$\frac{d}{dt}\langle\epsilon_\perp\rangle_G = \frac{d}{dt}\langle\tilde{\theta}\rangle_{F'} - \langle(\,\partial_t + v_\parallel\,\vec{b}\cdot\vec{\nabla}\,)\,\tilde{\theta}\rangle_{F'} \tag{3}$$

(this is an exact equation), and then to the quasiparticle space by means of expansion (5.2). The lower-order contribution from this expansion to the right-hand side of Eq. (3) is non-vanishing already. Therefore, it is sufficient to approximate F_c' as in Eq. (6.1), and the property (6.13) can be exploited immediately to transform Eq. (3):

$$\frac{d}{dt}\langle\epsilon_\perp\rangle_G = \frac{d}{dt}\langle\theta_r\rangle_G - \langle(\,\partial_t + v_\parallel\,\vec{b}\cdot\partial_{\vec{R}}\,)\,\theta_m\rangle_G\,, \tag{4}$$

as $(\partial_t\theta)_r = \partial_t\theta_m$, and $(\vec{\nabla}\theta)_r = \partial_{\vec{R}}\theta_m$. The right-hand side of the latter equation is transformed by means of integrating by parts over $\vec{R}$:

$$\frac{d}{dt}\langle\epsilon_\perp\rangle_G = \frac{d}{dt}\langle\theta_r - \theta_m\rangle_G + \int d\vec{R}dv_\parallel d\epsilon_\perp\,\theta_m\,(\,\partial_t + v_\parallel\,\vec{b}\cdot\partial_{\vec{R}}\,)\,G\,.$$

Eq. (8.10) is used to find that the last term on the right-hand side vanishes:

$$\int d\vec{R} dv_{\|} d\epsilon_{\perp}\, \theta_m \,(\, \partial_t \; + v_{\|}\, \vec{b} \cdot \partial_{\vec{R}}\,)\, G \;=\; - \int d\vec{R} dv_{\|} d\epsilon_{\perp}\, \theta_m\, \vec{V}_r \cdot \partial_{\vec{R}\perp} G \;=$$

$$=\; \langle \vec{V}_r \cdot \vec{a}_{r\perp} \rangle_G \;=\; 0 \; .$$

Thus, Eq. (4) takes the final form

$$\frac{d}{dt} \langle \epsilon_{\perp} \rangle_G \;=\; \frac{d}{dt} \langle \theta_r \;-\; \theta_m \rangle_G \;, \tag{5}$$

which introduces the quantity

$$s_q \;\equiv\; \langle \epsilon_{\perp} \;+\; \theta_m \;-\; \theta_r \rangle_G \;, \tag{6}$$

associated with one quasiparticle, as a second-order adiabatic invariant. For a point quasiparticle, $\epsilon_{\perp} \;+\; \theta_m \;-\; \theta_r$ is invariant along the quasiparticle trajectory in the reduced phase space.

10. Global Adiabatic Invariants

For the system of many charged particles with the self-consistent microscopic electric field $\vec{E} = -\vec{\nabla}\Phi$, the quasiparticle invariant (9.6) is rewritten as

$$\langle m\epsilon_\perp \; + \; q\,(\Phi_r \; - \; \Phi_m)\rangle_G \; . \tag{1}$$

One can sum up such quantities for all quasiparticles in order to consider a global adiabatic invariant, associated with particle motion across the magnetic field:

$$S \;\equiv\; \sum \langle m\epsilon_\perp \; + \; q\,(\Phi_r \; - \; \Phi_m)\rangle_G \; , \tag{2}$$

where the summation is over all quasiparticles. This expression can be transformed after the explicit forms of the potentials Φ_r and Φ_m are taken into account and integration by parts is carried out:

$$S \;=\; K_\perp \;+\; \sum \frac{q^2}{4\pi m} \int d\vec{R} dv_\| d\epsilon_\perp d\alpha \; [\; \tilde{\phi}^2 \; \partial_{\epsilon_\perp} G \; - \tag{3}$$

$$- \frac{1}{\Omega^2} \; \tilde{\phi} \int^\alpha d\alpha (\partial_{\vec{R}} \tilde{\phi}) \cdot \vec{b} \times \partial_{\vec{R}} G \;] \; ,$$

where

$$K_\perp \;\equiv\; \sum \langle m\epsilon_\perp\rangle_G \tag{4}$$

is the part of the kinetic energy of particles, associated with their perpendicular motion,

$$\tilde{\phi} \;\equiv\; \Phi(\vec{R} - \frac{1}{\Omega}\,\vec{v} \times \vec{b}, t) \; - \; \Phi_0 \; . \tag{5}$$

Let us introduce the total quasiparticle density G_a for a particular plasma species a,

$$G_a \;\equiv\; \sum{}' G \; , \tag{6}$$

where summation is over quasiparticles from the plasma species a, with the same charge q_a and mass m_a. Then, invariant (3) takes this form:

$$S = K_\perp + \sum_a \frac{q_a^2}{4\pi m_a} \int d\vec{R} dv_\| d\epsilon_\perp d\alpha \, [\, \tilde{\phi}_a^2 \, \partial_{\epsilon_\perp} G_a - \tag{7}$$

$$- \frac{1}{\Omega_a^2} \, \tilde{\phi}_a \int^\alpha a\alpha (\partial_{\vec{R}} \tilde{\phi}_a) \cdot \vec{b} \times \partial_{\vec{R}} G_a \,] \, ,$$

where the quantity $\tilde{\phi}_a$ depends on plasma species as the cyclotron frequency enters its definition (5).

One could be interested in the mean statistical value of the global invariant (7). In order to find it, the quasiparticle density, the potential, and other quantities are represented as sums of the mean statistical value and the fluctuation,

$$G_a \equiv \langle G_a \rangle + \delta G_a \, , \quad \Phi \equiv \langle \Phi \rangle + \delta \Phi \, , \quad \ldots \, , \tag{8}$$

and the following approximations are used:

$$\delta G_a \sim \lambda G_a \, , \quad \lambda << 1 \, , \tag{9}$$

$$\partial_{\vec{R}} \langle G_a \rangle \sim \partial_{\vec{R}} \delta G_a \, , \quad \langle \Phi \rangle \sim \delta \Phi \, . \tag{10}$$

Then, after statistical averaging of expression (7), one obtains:

$$\langle S \rangle = \langle K_\perp \rangle + \tag{11}$$

$$+ \sum_a \frac{q_a^2}{4\pi m_a} \int d\vec{R} dv_\| d\epsilon_\perp d\alpha \, [\, \langle \tilde{\phi}_a \rangle^2 + \langle (\delta \tilde{\phi}_a)^2 \rangle \,] \, \partial_{\epsilon_\perp} \langle G_a \rangle \, .$$

When the condition, which is usually associated with no instability sources in the velocity space,

$$\partial_{\epsilon_\perp} \langle G_a \rangle < 0 \, , \tag{12}$$

holds, invariant (11) is not definite-positive. This feature has important implications for energy exchange between particles and fields, as both terms in Eq. (11), one of which is associated with the particles, while another with the fields, can grow or diminish simultaneously only. When parallel particle motion is disregarded, the excitation of electric fields cannot occur due to decrease of the particle kinetic energy.

Thus, inequality (12) can be taken as a sufficient condition of non-equilibrium plasma stability, with regard to kinetic and nonlinear effects. It is clear, that instability might be associated with small-scale motion only, for which the typical size is comparable to particle Larmor radii. This follows from the observation that the contribution from large-scale motion in (11) is negative, as it equals to

$$- \sum \frac{\bar{n}_a m_a}{2B^2} \int d\vec{r} \; \{ \; \langle \vec{E} \rangle^2 \; + \; \langle (\delta \vec{E})^2 \rangle \; \} \; . \qquad (13)$$

Here $\bar{n}_a$ is a non-perturbed number density for a particular plasma species.

Invariant (11) suggests also that the only possible mechanism for decrease of the kinetic energy might be cascade of electric field oscillations from large scales to small scales. And vice versa, inverse cascade of electric field oscillations results in increase of the particle kinetic energy, and, if the total energy in the system is conserved, in decrease of the electric field energy.

Let us analyze effects of parallel particle motion, and consider the relevant part of the kinetic energy:

$$K_{\parallel} \; = \; \sum \langle \frac{m v_{\parallel}^2}{2} \rangle_G \; . \qquad (14)$$

Then, condition (12) is not sufficient for plasma stability as excitation of electric field oscillations can be possible at the expense of the kinetic energy due to parallel particle motion. The time variation of the statistical mean value of the latter quantity:

$$\frac{d}{dt} \langle K_{\parallel} \rangle \; = \; \sum_a q_a \int d\vec{R} dv_{\parallel} d\epsilon_{\perp} \; v_{\parallel} \; [\; \langle E_{\parallel r} \rangle \langle G_a \rangle \; + \; \langle \delta E_{\parallel r} \delta G_a \rangle \;] \; . \qquad (15)$$

In the approximation of weak plasma inhomogeneity along the external magnetic field, when

$$\nabla_{\|}\langle\Phi\rangle \sim \lambda\nabla_{\|}\delta\Phi \,, \tag{16}$$

the contribution from the first term in the integrand of Eq. (15) is negligible, while one can use the continuity equation for quasiparticles,

$$\{\, \partial_t + \vec{V}_r\cdot\partial_{\vec{R}} \,\}\, \delta G_a \; = \; -\frac{q_a}{m_a}\, \delta E_{\|r}\, \partial_{v_\|}\, \langle G_a\rangle - \delta\vec{V}_r\cdot\partial_{\vec{R}}\langle G\rangle + \langle\delta\vec{V}_r\cdot\partial_{\vec{R}}\delta G_a\rangle \,, \tag{16}$$

to calculate the correlation function $\langle\delta E_{\|r}\delta G_a\rangle$. The last term on the right-hand side of the latter equation does not contribute, and the contribution from the second term can be disregarded under the assumption that either the mean quasiparticle distribution function is homogeneous or the mean plasma flows dominate during the evolution of the quasiparticle distribution function ($\delta\vec{V} \sim \lambda\langle\vec{V}\rangle$). Then, Eq. (14) can be integrated in time, which introduces another global adiabatic invariant:

$$P \;\equiv\; \langle K_{\|}\rangle \;+\; \frac{1}{2}\sum_a \int d\vec{R}dv_{\|}d\epsilon_{\perp}\, \frac{mv_{\|}\langle(\delta G_a)^2\rangle}{\partial_{v_\|}\langle G_a\rangle}, \tag{18}$$

associated with particle motion along the magnetic field.

11. Global Energy Conservation

The invariant (9.6) can be used to derive a global energy conservation law [10]. For the system of many charged particles with the self-consistent microscopic electric field $\vec{E} = -\vec{\nabla}\Phi$, the total energy W is represented as the sum of the electric field energy,

$$W_E \equiv \frac{1}{8\pi}\int d\vec{r}\, E^2 , \tag{1}$$

and the kinetic energy of particles:

$$W = W_E + \sum \frac{1}{2} \langle mv^2 \rangle_F = W_E + \sum \langle \frac{1}{2} m(v_{\|}^2 + 2\epsilon_{\perp}) \rangle_G , \tag{2}$$

where summation is over all quasiparticles. From Eq. (9.6):

$$W = S + W_E + \sum \langle \frac{1}{2} mv_{\|}^2 + q(\Phi_m - \Phi_r) \rangle_G . \tag{3}$$

Now, the mean statistical value of the plasma energy:

$$\langle W \rangle = \langle S \rangle + \langle W_E \rangle + \sum \langle \langle \frac{1}{2} mv_{\|}^2 + q(\Phi_m - \Phi_r) \rangle_G \rangle , \tag{4}$$

where

$$\langle W_E \rangle = \frac{1}{8\pi}\int d\vec{r}\, [\, \langle \vec{E} \rangle^2 + \langle (\delta \vec{E})^2 \rangle \,] \tag{5}$$

is the statistical mean value of the electric field energy.

For conditions (9) and (10),

$$\langle W \rangle = \langle S \rangle + \langle W_E \rangle + \sum_a \langle \langle \frac{1}{2} m_a v_{\|}^2 \rangle_{G_a} \rangle -$$

$$- \sum_a \frac{q_a^2}{4\pi m_a} \int d\vec{R} dv_{\|} d\epsilon_{\perp} d\alpha\, [\, \langle \tilde{\phi}_a \rangle^2 + \langle (\delta \tilde{\phi}_a)^2 \rangle \,]\, \partial_{\epsilon_{\perp}} \langle G_a \rangle . \tag{6}$$

This equation determines the mean energy of the plasma with both mean flows and fluctuations. It is in accordance with the result [14, 15], derived for the collisionless plasma (in the Vlasov approximation). Eq. (6) can be applied under conditions of anomalous plasma transport, when fluctuations are essential, as well as fluctuation-induced changes in the mean plasma characteristics (such as the density, the velocity, the temperature).

If the mean energy is conserved and parallel particle motion can be disregarded in the energy balance, then the following quadratic functional of electric fields is an invariant of motion:

$$J\{\Phi\} \equiv 8\pi \langle W_E \rangle -$$

$$- \sum_a \frac{2q_a^2}{m_a} \int d\vec{R} dv_\| d\epsilon_\perp d\alpha \, [\, \langle \tilde{\phi}_a \rangle^2 + \langle (\delta \tilde{\phi}_a)^2 \rangle \,] \, \partial_{\epsilon_\perp} \langle G_a \rangle \, . \qquad (7)$$

When the condition (10.12) holds, this functional is definite-positive. This supports the interpretation of the latter condition within the context of plasma stability.

According to Eqs (7) and (10.12), the level of mean oscillations and fluctuations of the electric field is limited, and the kinetic energy of particles cannot decrease due to excitation of electric field oscillations. Also, simultaneous cascade of electric field oscillations to large and small scales is possible only, as unidirectional cascade would violate energy conservation.

Another consequence is that for a fixed value of the mean energy, the level of fluctuations (and hence, as it can be expected, the level of fluctuation-induced transport) will be lower in the plasma with developed non-uniform mean flows. It is to be noted, that similar phenomena seem to be observed in the toroidally confined plasmas under the conditions of improved confinement (H-mode) or transition to such confinement, when near the plasma edge there emerge zonal, poloidal, particle flows. The latter is some kind of a barrier for the fluctuation-induced radial particle fluxes. According to energy conservation, such flows can affect the plasma not only locally, but their stabilizing influence on the whole plasma volume is possible.

For conditions, when the adiabatic invariant (10.18) exists,

$$\langle W \rangle \; = \; \langle S \rangle \; + \; P \; + \; \langle W_E \rangle \; - \tag{8}$$

$$- \frac{1}{4\pi} \sum_a \int d\vec{R} dv_{\|} d\epsilon_{\perp} d\alpha \; \{ \; \frac{q_a^2}{m_a} [\; \langle \tilde{\phi}_a \rangle^2 \; + \; \langle (\delta \tilde{\phi}_a)^2 \rangle \;] \; \partial_{\epsilon_\perp} \langle G_a \rangle \; + \; \frac{m v_{\|} \langle (\delta G)^2 \rangle}{\partial_{v_{\|}} \langle G_a \rangle} \; \} \; .$$

Therefore, conditions (10.12) and

$$v_{\|} \partial_{v_{\|}} \langle G_a \rangle \; < \; 0 \tag{9}$$

are sufficient conditions for non-equilibrium plasma stability, with regard to kinetic and nonlinear effects.

12. Quasiparticle Kinetics and Fluctuations

Let us consider a system of many charged particles, such as the plasma, and a corresponding system of quasiparticles with the microscopic distribution function $G = \sum_p G_p$, when additional force fields contain a self-consistent microscopic part associated with particles themselves. The relation between the particle (guiding-centre) density in the phase space and the quasiparticle density in the reduced phase space makes it possible to relate self-consistent fields to the quasiparticle density.

In order to describe fluctuations in the system of quasiparticles, one can apply a statistical approach and formulate an infinite hierarchy of equations for quasiparticle correlation functions, similar to the Boholiubov-Born-Green-Kirkwood-Yvon hierarchy for the particle correlation functions. The first equation of the hierarchy is for the statistical mean value $\langle G \rangle \equiv g$ of the microscopic quasiparticle distribution $G \equiv g + \delta g$, where $\langle \delta g \rangle = 0$ by definition. For simplicity, the kinetic equation for quasiparticles is presented in the adiabatic first-order approximation:

$$\partial_t\, g \;+\; \partial_{\vec{R}} \cdot \;[\; \langle \vec{V}_r \rangle\; g \;] \;+\; \partial_{v_{\parallel}} \;[\; \langle A_{\parallel r} \rangle g \;] \;+\; \partial_{\epsilon_{\perp}} \;[\; \langle A_{\epsilon_{\perp} r} \rangle\; g \;] \;\equiv$$

$$\equiv\; \langle \hat{D}_R \rangle\; g \;=\; -\; \langle \delta \hat{D}_R\; \delta g \rangle \;, \qquad (1)$$

where

$$\langle \delta \hat{D}_R\; \delta g \rangle \;=\; \partial_{\vec{R}} \cdot \langle \delta \vec{V}_r \delta g \rangle \;+\; \partial_{v_{\parallel}} \langle \delta A_{\parallel r} \delta g \rangle \;+\; \partial_{\epsilon_{\perp}} \langle \delta A_{\epsilon_{\perp} r} \delta g \rangle \;, \qquad (2)$$

and there are contributions to $\langle \vec{V}_r \rangle$, $\langle A_{\parallel r} \rangle$, $\langle A_{\epsilon_{\perp} r} \rangle$ due to fluctuations,

$$\langle Q_r \rangle \;=\; \langle Q_0 \rangle \;-\sum_{n \neq 0} [\; \langle \hat{l}_n \rangle\; \langle Q_n^* \rangle \;+\; \langle \delta \hat{l}_n\; \delta Q_n^* \rangle \;] \,. \qquad (3)$$

The effects of quasiparticle collisions anf fluctuation-induced relaxation can be described in terms of low-frequency incoherent fluctuations in the plasma due to stochastic motion of individual quasiparticles, and collective fluctuations. The theory can be extended to allow for high-frequency incoherent and collective fluctuations.

In the case of potential interaction between particles and uniform magnetic fields, when the additional acceleration is due to self-consistent electric fields, $\vec{E} = -\vec{\nabla}\Phi$:

$$a_{\epsilon_\perp r} = 0 , \quad a_{\|r} = -\frac{q}{m}\,\vec{b}\cdot\partial_{\vec{R}}\Phi_m , \quad \vec{V}_r = v_\|\vec{b} + \frac{1}{B}\,\vec{b}\times\partial_{\vec{R}}\Phi_m . \tag{4}$$

Equations for quasiparticle correlation functions can be derived from the nonlinear equation for fluctuations:

$$\hat{D}_R\delta g - \langle\delta\hat{D}_R\,\delta g\rangle = -\,\delta\hat{D}_R\,g \equiv \tag{5}$$

$$\equiv -\,\partial_{\vec{R}}\cdot[\,\delta\vec{V}_r\,g\,] - \partial_{v_\|}[\,\delta A_{\|r}\,g\,] - \partial_{\epsilon_\perp}[\,\delta A_{\epsilon_\perp r}\,g\,] .$$

In the linear with regard to fluctuations approximation:

$$\hat{D}_R \simeq \langle\hat{D}_R\rangle , \quad \delta g \simeq -\,\langle\hat{D}_R\rangle^{-1}\,\delta\hat{D}_R\,g . \tag{6}$$

If collisions are disregarded, as well as fluctuation-induced relaxation of the mean quasiparticle distribution function,

$$\partial_t\,g + \partial_{\vec{R}}\cdot[\,\langle\vec{V}_r\rangle\,g\,] + \partial_{v_\|}[\,\langle A_{\|r}\rangle g\,] + \partial_{\epsilon_\perp}[\,\langle A_{\epsilon_\perp r}\rangle\,g\,] = 0 , \tag{7}$$

where

$$\langle Q_r\rangle \simeq \langle Q_0\rangle - \sum_{n\neq 0}\langle\hat{I}_n\rangle\,\langle Q_n^*\rangle . \tag{8}$$

This is the kinetic equation for quasiparticles in the self-consistent field approximation (Vlasov-type kinetic equation), various limiting cases of such equation were derived, and analyzed analytically and via computational models [16-27].

13. Non-Stationary Magnetic Field

It is possible to generalize above results for the case of non-stationary magnetic fields, $\vec{B} \equiv \vec{B}(\vec{r},t)$, when the local reference frame (Frenet triad) becomes time dependent, as do the local radii of curvature and torsion. The transformation to the guiding-centre space can be taken also with explicit time dependence:

$$\vec{r}_p = \vec{r}_p(\vec{R}_c, \vec{v}_p, t) . \tag{1}$$

In fact, one could include all the magnetic fields into $\vec{\Omega}$ in particle equations of motion, putting $\vec{B}' = 0$ in Eq. (2.2).

These features can be taken into account by substituting formally in Eqs (3.5)-(3.8),

$$\vec{V} \cdot \partial_{\vec{R}} \rightarrow \partial_t + \vec{V} \cdot \partial_{\vec{R}} , \tag{2}$$

and setting

$$\vec{v} \equiv v_{\|} \vec{h} + \vec{v}_{\perp} \equiv v_{\|} \vec{h} + v_{\perp} [\, cos\alpha \, \vec{N} + sin\alpha \, \vec{\beta} \,] ,$$

$$\vec{h} \equiv \vec{h}(\vec{R},t) , \quad \vec{N} \equiv \vec{N}(\vec{R},t) , \quad \vec{\beta} \equiv \vec{\beta}(\vec{R},t) . \tag{3}$$

Then, all the quantities on the right-hand side of Eqs (3.5)-(3.8) are evaluated at the instantaneous guiding-centre position in the phase space, $\vec{R} = \vec{R}_c$, $v_{\|} = v_{\|c}$, $\epsilon_{\perp} = \epsilon_{\perp c}$, $\alpha = \alpha_c$, after taking the derivatives $\partial_{\vec{R}}$ and ∂_t. The effective acceleration $\vec{A}$ with regard to non-stationary magnetic fields follows in the form (3.10), with substitution (2) being carried out:

$$\vec{A} = \tilde{\vec{a}} + \vec{v} \times \{ \tilde{\vec{\Omega}} - \vec{\Omega}(\vec{R},v,t) \} - (\partial_t + \vec{V} \cdot \partial_{\vec{R}}) \, \vec{v} . \tag{4}$$

It is to be noted, that the equations (4.11) and (8.2) for the time evolution of $\epsilon_{\perp q}$ (and $v_{\|q}$) remain formally unchanged as $\{ \partial_t \, \vec{v}_{\perp} \}_0 \equiv 0$. In these equations, non-stationary magnetic fields can enter through the guiding-centre velocity, only.

If the transformation to the guiding-centre space is specified similar to (7.1),

$$\vec{r} = \vec{R} - \frac{1}{\Omega(\vec{R}, v, t)} \vec{v} \times \vec{b}(\vec{R}, t) , \tag{5}$$

then, all the equations in Sec. 9 could be extended to include non-stationarity of magnetic fields by means of the formal substitution (2), or

$$v_{\|}\vec{h} \cdot \partial_{\vec{R}} \vec{h} \rightarrow (\partial_t + v_{\|}\vec{h} \cdot \partial_{\vec{R}}) \vec{h} \tag{6}$$

(see Eqs (7.4), (7.5), (7.7), (7.12), and (7.15)).

For example, the equation for the guiding-centre velocity modifies like this:

$$\vec{V} = v_{\|} \vec{h} + \frac{1}{\Omega} \vec{A} \times \vec{h} - (\partial_t + \vec{V} \cdot \partial_{\vec{R}}) \frac{\vec{h} \times \vec{v}}{\Omega} , \tag{7}$$

where $\Omega \equiv \Omega(\vec{R}, v, t)$. Eq. (7.6) is not changed formally. The conservation law (7.16) remains unaffected, and so does expression (7.13). At the same time, the additional term, $-v_{\|}\partial_t\vec{h}$, appears on the right-hand side of expression (7.12) for the reduced acceleration, and the additional term, $(v_{\|}/\Omega)\vec{h} \times \partial_t\vec{h}$, is to be accepted on the right-hand side of expression (7.15) for the quasiparticle velocity.

14. Summary

The basic quantities of the theory is the microscopic density in the particle phase space, and the microscopic density in the guiding-centre phase space, which can be introduced by some formal transformation.

The equations of guiding-centre motion are derived, and an effective acceleration which takes into account non-uniformity of stationary magnetic fields is found without specifying explicitly the guiding-centre transformation.

The microscopic density in the quasiparticle phase space is introduced, and an exact microscopic continuity equation for the quasiparticle density is formulated. The point quasiparticles are introduced within its context.

Closure is suggested, and the reduced theory is formulated to arbitrary orders with respect to a small parameter. The detailed analysis of the adiabatic first-order approximation of the reduced theory is carried out, basic equations and relations of this approximation are derived, with a guiding-centre transformation being yet unspecified.

A simple form of guiding-centre tarnsformation is suggested, and the equation for the guiding-centre velocity is derived. The quasiparticle equations of motion in non-uniformly magnetized plasmas are derived, with the effects of non-uniform magnetic fields being calculated explicitly.

The case of potential particle acceleration is illuminated, and the renormalized and modified potentials are introduced. The significance of the difference between them is explained, as it is shown to determine the second-order adiabatic invariant.

The second-order adiabatic invariant is used to find global adiabatic invariants associated with particle kinetic energy, and to relate the plasma energy to the quasiparticle density. Global conservation properties are analyzed and sufficient conditions of non-equilibrium plasma stability with regard to kinetic and nonlinear effects are obtained.

Some basic equations which describe quasiparticle kinetics and fluctuations in the system of many quasiparticles are formulated.

Extensions of the theory for the case of non-stationary magnetic fields is suggested.

Acknowledgements

The author appreciates discussions with Prof. Jan Weiland, Dr. Jaroslav Kolesnychenko, and M. Dominique Grésillon.

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