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ONCE MORE ON THE RADIATIVE CORRECTIONS TO THE NUCLEON STRUCTURE FUNCTIONS IN QCD [1]

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ABSTRACT

A new representation of the next to leading QCD corrections to the nucleon structure functions is given in terms of parton distributions. All $O(\alpha_s)$ corrections to the leading logarithmic approximation (LLA) are included. In contrast to the similar representations in the literature terms of order $O(\alpha_s^2)$ do not attend in our expressions for the nucleon structure functions taken in the next to leading logarithmic approximation. This result is generalized for any order in α_s beyond the LLA. Terms of order $O(\alpha_s^n)$ which belong only to the approximation in consideration are present in such a representation for the structure functions.

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1. Introduction

To make quantitative predictions for hadron processes at present and future high-energy colliders detailed information on the parton distributions inside the nucleon is needed. The study of deep inelastic lepton-nucleon processes is one of the best manners to obtain this information. The increased precision of the deep inelastic scattering data enables us to derive the quark and gluon distributions in the framework of the QCD improved parton model with a good degree of accuracy.

The nucleon structure functions can be written in QCD as suitable convolutions of quark, antiquark and gluon distributions. At the leading logarithmic approximation (LLA) the results of the QCD corrections can be interpreted by saying that the nucleon structure functions $F_i(x,Q^2)$ are given by the naive parton model formulae expressed in terms of Q^2 dependent effective parton distributions obeying to the first order Lipatov-Altarelli-Parisi equations [1].

Beyond the LLA the explicit form of the α_s corrections to the structure functions depends on the definition of the effective parton distributions. Different definitions of the parton distributions will give, of course, the same physical results. In general, there exist two different ways to define these distributions:

i) A-definition [2,3] or so-called DIS scheme: All of the next to LLA α_s corrections to one of the structure functions, for instance F_2, are incorporated in the quark and antiquark distributions. According to such a definition the parton model formula for $F_2(x,Q^2)$ is valid to all orders in perturbation theory. In this case the form of the α_s corrections to the other structure functions is very simple.

ii) B-definition [4,5]: According to this definition of the parton distributions the α_s corrections coming from the Wilson coefficient functions are explicitly factored out. The latter are structure functions dependent.

Notice that in the framework of each of these definitions of the parton distributions different renormalization-prescription schemes can be used for the calculation of the radiative corrections to the structure functions.

The expressions for the nucleon structure functions presented in the literature in terms of parton distributions have a following common feature (independent on the definition). In the next to leading logarithmic approximation (NLLA) of QCD for the structure functions (all $O(\alpha_s)$ corrections to LLA are taken into account) there exist some terms of order $O(\alpha_s^2)$ which do not belong to this approximation. In any case, the presence of these terms which are only a small part of $O(\alpha_s^2)$ corrections is undesirable. First of all, the moments of these structure functions do not coincide with the moments calculated in NLLA in the formal Quantum Field Theory (QFT) approach. Moreover, the quantity of the $O(\alpha_s^2)$ corrections may be changed considerably if all corrections in this order are taken into account. The calculations of the

$O(\alpha_s^2)$ corrections are in progress [6 - 8], but unfortunately, some of them are still unknown.

In this paper we give expressions for the next to leading logarithmic approximation of the nucleon structure functions written in terms of parton distributions in which the unwanted terms of order $O(\alpha_s^2)$ do not appear in this approximation. This representation is generalized also for any order in α_s beyond the LLA. There is one-to-one correspondence between such a representation of the structure functions and the structure functions calculated in the formal QFT approach.

2. Results

In order to simplify the problem we shall restrict our discussion to the nonsinglet part of the structure functions. At the end of the paper we shall present also our results for the complete electromagnetic and neutrino (antineutrino) structure functions.

In the formal QCD approach the Q^2 dependence of the moments of deep inelastic nonsinglet structure functions is given as

$$\begin{aligned} M_i^{NS}(n,Q^2) &= \int_0^1 dx x^{n-2}\mathcal{F}_i^{NS}(x,Q^2) \\ &= A_n^{NS}(Q_0^2)exp\{-\int_{\alpha_s(Q_0^2)}^{\alpha_s(Q^2)} da \frac{\gamma_n^{NS}(a)}{\beta(a)}\}C_{i,n}^{NS}(1,\alpha_s(Q^2)). \end{aligned} \tag{1}$$

$(i = 1,2,3)$

The functions $\mathcal{F}_i^{NS}(x,Q^2)$ are related to the standard deep inelastic structure functions $F_i^{NS}(x,Q^2)$ by

$$\mathcal{F}_1^{NS}(x,Q^2) = 2xF_1^{NS}(x,Q^2), \quad \mathcal{F}_2^{NS}(x,Q^2) = F_2^{NS}(x,Q^2),$$
$$\mathcal{F}_3^{NS}(x,Q^2) = xF_3^{NS}(x,Q^2). \tag{2}$$

In (1) $A_n^{NS}(Q_0^2)$ stand for the unknown hadronic matrix elements of spin-n nonsinglet operators and $C_{i,n}^{NS}$ are the coefficient functions of these operators in the Wilson expansion [9]. α_s, γ_n^{NS} and β are the the well known effective coupling constant of strong interactions, anomalous dimensions of the nonsinglet operators mentioned above and the standard β function, respectively.

The moments of the nonsinglet structure functions have the following perturbative expansion:

$$M_i(n,Q^2) = \delta_i A_n \alpha_s^{d_n}(Q^2)\{1 + \frac{\alpha_s(Q^2)}{4\pi} f_{i,n}^{(2)} + \frac{\alpha_s^2(Q^2)}{(4\pi)^2} f_{i,n}^{(3)} + ...\}, \tag{3}$$

where

$$A_n = A_n(Q_0^2)\alpha_s^{-d_n}(Q_0^2)\{1 + \frac{\alpha_s(Q_0^2)}{4\pi}Z_n^{(2)} + \frac{\alpha_s^2(Q_0^2)}{(4\pi)^2}[Z_n^{(3)} + \frac{1}{2}(Z_n^{(2)})^2] + ...\}^{-1} \tag{4}$$

and d_n, $Z_n^{(k)}$, $f_{n,i}^{(k)}$ are in principle calculable quantities. The superscript (NS) in (3) and (4) is suppressed.

We remind that the coefficients $Z_n^{(k)NS}$ are connected with the perturbative expansion of

$$\begin{aligned} exp\{-\int_{\alpha_s(Q_0^2)}^{\alpha_s(Q^2)} da \frac{\gamma_n^{NS}(a)}{\beta(a)}\} = (\frac{\alpha}{\alpha_0})^{d_n^{NS}}\{1 &+ \frac{\alpha-\alpha_0}{4\pi}Z_n^{(2)NS} + \frac{\alpha^2-\alpha_0^2}{(4\pi)^2}Z_n^{(3)NS} \\ &+ \frac{(\alpha-\alpha_0)^2}{(4\pi)^2}\frac{1}{2}(Z_n^{(2)NS})^2 + ...\}. \end{aligned} \tag{5}$$

In the RHS of (5) $\alpha = \alpha_s(Q^2)$ and $\alpha_0 = \alpha_s(Q_0^2)$.

In the NLLA (one-loop approximation for the coefficient functions $C_{i,n}^{NS}$ and two-loop approximation for the functions γ_n^{NS}, β) the moments of the structure functions have the form

$$\begin{aligned} M_i^{NS}(n,Q^2) = \delta_i^{NS} A_n^{NS}(Q_0^2)\{1 &+ \frac{\alpha_s(Q^2)}{4\pi}[Z_n^{(2)NS} + c_{i,n}^{(1)NS}] \\ &- \frac{\alpha_s(Q_0^2)}{4\pi}Z_n^{(2)NS}\}\{\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\}^{d_n^{NS}}, \end{aligned} \tag{6}$$

where $Z_n^{(2)NS}$, d_n^{NS} and $c_{i,n}^{(1)NS}$ are well known numbers (e.g. [10]) from the perturbative calculations of the functions γ_n^{NS}, β and $C_{i,n}^{NS}(1,\alpha_s)$. In (6) δ_i^{NS} are weak or electromagnetic charge factors and $\alpha_s(Q^2)$ is given in the NLLA

$$\alpha_s(Q^2) = \frac{1}{\beta_0 ln(Q^2/\Lambda^2)} - \frac{\beta_1}{\beta_0^3}\frac{lnln(Q^2/\Lambda^2)}{ln^2(Q^2/\Lambda^2)} \tag{7}$$

with

$$b_0 = 11 - \frac{2}{3}N_f, \qquad b_1 = 102 - \frac{38}{3}N_f,$$

where N_f is the number of flavors.

Let us now rewrite the moments of the nonsinglet structure functions via the valence quark distributions $V(x,Q^2)$. Their moments are defined as

$$V(n,Q^2) = \int_o^1 dx x^{n-1} V(x,Q^2)\ . \tag{8}$$

According to the A-definition the Q^2 evolution of the moments of the valence quark distributions is given as

$$V^{(a)}(n,Q^2) = V^{(a)}(n,Q_0^2)L_n^{(a)}(Q^2,Q_0^2)\ , \tag{9}$$

where

$$V^{(a)}(n,Q_0^2) = A_n^{NS}(Q_0^2)[1+\frac{\alpha_s(Q_0^2)}{4\pi}c_{2,n}^{(1)NS}] \tag{10}$$

and

$$L_n^{(a)}(Q^2,Q_0^2) = \{1+\frac{\alpha(Q^2)-\alpha_s(Q_0^2)}{4\pi}(Z_n^{(2)NS}+c_{2,n}^{(1)NS})\}\{\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\}^{d_n^{NS}}\ , \tag{11}$$

i.e. in the case of this definition of the parton distributions the radiative corrections from the Wilson coefficient functions $C_{2,n}^{NS}(1,\alpha_s)$ are included in the evolution of their moments. In (9) $V^{(a)}(n,Q_0^2)$ are the moments of the valence quark distributions at some fixed value of $Q^2 = Q_0^2$, which must be determined from the experimental data. The superscript (a) means that A-definition is used.

Taking into account this definition of the parton distributions the moments of the structure functions are usually given in the following form:

$$M_i^{NS}(n,Q^2) = \delta_i^{NS}V^{(a)}(n,Q^2)\{1+\frac{\alpha_s(Q^2)}{4\pi}(c_{i,n}^{(1)NS}-c_{2,n}^{(1)NS})\}. \tag{12}$$

It is seen from (12) that in the case $i=2$

$$M_2^{NS}(n,Q^2) = \delta_2^{NS}V^{(a)}(n,Q^2), \tag{13}$$

i.e. the naive parton formula for $F_2(x,Q^2)$ is valid in the NLLA too.

According to the B-definition the Q^2 evolution of the moments of the valence quark distributions is given as

$$V^{(b)}(n,Q^2) = V^{(b)}(n,Q_0^2)L_n^{(b)}(Q^2,Q_0^2), \tag{14}$$

where

$$V^{(b)}(n,Q_0^2) = A_n^{NS}(Q_0^2) \tag{15}$$

and

$$L_n^{(b)}(Q^2,Q_0^2) = \{1+\frac{\alpha_s(Q^2)-\alpha_s(Q_0^2)}{4\pi}Z_n^{(2)NS}\}\{\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\}^{d_n^{NS}}\ . \tag{16}$$

Taking into account the B-definition of the parton distributions the moments of the structure functions are usually given by

$$M_i^{NS}(n,Q^2) = \delta_i^{NS}V^{(b)}(n,Q^2)[1+\frac{\alpha_s(Q^2)}{4\pi}c_{i,n}^{(1)NS}]. \tag{17}$$

Using (9-11) and (14-16) for A and B-definition of the valence quark distributions, respectively, one can see that the Q^2 evolution equations for the moments of the structure functions (12) and (17) contain $O(\alpha_s^2)$ terms, which do not attend in

(6) and do not belong to NLLA for the moments.

We shall show now that there exists a representation for the structure functions written in terms of parton distributions, in which this weak point is overcome.

Let us use the A-definition (9-11) for the moments of the valence quark distributions. Then from (6) more precise expressions for the moments of nonsinglet structure functions in NLLA of QCD can be obtained:

$$M_i^{NS}(n,Q^2) = \delta_i^{NS}\{V^{(a)}(n,Q^2)+\frac{\alpha_s(Q^2)}{4\pi}(c_{i,n}^{(1)NS}-c_{2,n}^{(1)NS})V(n,Q^2)_{\text{"}LO\text{"}}\}, \tag{18}$$

where

$$V(n,Q^2)_{\text{"}LO\text{"}} = V^{(a)}(n,Q_0^2)\{\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\}^{d_n^{NS}}, \tag{19}$$

i.e. $V(n,Q^2)_{\text{"}LO\text{"}}$ satisfy the leading order (LLA) evolution equations for the moments of the valence quark distributions, in which for $\alpha_s(Q^2)$ the NLLA (7) is taken.

We would like to stress that $O(\alpha_s^2)$ terms are absent in (18) and there is one-to-one correspondence between the moments of the structure functions calculated in NLLA in formal QFT approach (Eq. (6)) and the moments of the same functions in the same approximation when the latter are expressed in terms of parton distributions (Eq. (18)). The difference between our representation (18) and that one given in the literature (Eq. (11)) is in the second term. Instead of $V^{(a)}(n,Q^2)_{NLLA}$ we have $V(n,Q^2)_{\text{"}LO\text{"}}$.

We present now our formulae for the Q^2 evolution of the moments of the nonsinglet structure functions when the next to NLLA $O(\alpha_s^2)$ corrections are taken into account:

$$\begin{aligned} M_i^{(3)}(n,Q^2) = \delta_i V^{(a)}(n,Q^2)_{NNLLA} &+ \delta_i V(n,Q^2)_{\text{"}LO\text{"}}\{\frac{\alpha_s}{4\pi}(c_{i,n}^{(1)}-c_{2,n}^{(1)}) \\ &- \frac{\alpha_s\alpha_{so}}{(4\pi)^2}(Z_n^{(2)}+c_{2,n}^{(1)})(c_{i,n}^{(1)}-c_{2,n}^{(1)}) \\ &+ \frac{\alpha_s^2}{(4\pi)^2}[(c_{i,n}^{(2)}-c_{2,n}^{(2)})+(c_{i,n}^{(1)}-c_{2,n}^{(1)})Z_n^{(2)}]\}, \end{aligned} \tag{20}$$

where the moments of the valence quark distributions $V^{(a)}(n,Q^2)_{NNLLA}$ evolve according to

$$\begin{aligned} V^{(a)}(n,Q^2)_{NNLLA} = &\ V^{(a)}(n,Q_0^2)\{1+\frac{\alpha_s-\alpha_{so}}{4\pi}(Z_n^{(2)}+c_{2,n}^{(1)}) \\ &+\frac{\alpha_s^2-\alpha_{so}^2}{(4\pi)^2}[Z_n^{(3)}+\frac{1}{2}(Z_n^{(2)})^2+c_{2,n}^{(2)}+c_{2,n}^{(1)}Z_n^{(2)}] \\ &+\frac{\alpha_{so}(\alpha_{so}-\alpha_s)}{(4\pi)^2}(Z_n^{(2)}+c_{2,n}^{(1)})^2\}\{\frac{\alpha_s}{\alpha_{so}}\}^{d_n} \end{aligned} \tag{21a}$$

and

$$V^{(a)}(n,Q_0^2)=\{1+\frac{\alpha_{so}}{4\pi}c_{2,n}^{(1)}+\frac{\alpha_{so}^2}{(4\pi)^2}c_{2,n}^{(2)}\}A_n(Q_0^2). \qquad (21b)$$

The superscript NS in (20) and (21a,b) is suppressed. $\alpha_s(Q^2)$ in the above equations is given by the three-loop approximation of the β function. Ixepting a part of $Z_n^{(3)NS}$ connected with the three-loop approximation of the anomalous dimensions γ_n^{NS} all coefficients in (20) and (21) are already known. There is a big progress now in the calculations of $\gamma_n^{(3)NS}$ [8]. Note also that all combinations of the coefficients in front of α_s, $\alpha_s\alpha_{so}$ and α_s^2 do not depend on the renormalization scheme used to calculate them.

Applying the convolution theorem to (18) we obtain for the nonsinglet structure functions $\mathcal{F}_i^{NS}(x,Q^2)$ (NLLA approximation)

$$\mathcal{F}_i^{NS}(x,Q^2)=\delta_i^{NS}x\{V^{(a)}(x,Q^2)+\frac{\alpha_s(Q^2)}{2\pi}\int_x^1\frac{dy}{y}[c_i^{NS}(\frac{x}{y})-c_2^{NS}(\frac{x}{y})]V(y,Q^2)_{"LO"}\}, \qquad (22)$$

where

$$c_1^{NS}(\frac{x}{y})-c_2^{NS}(\frac{x}{y})=-\frac{8}{3}\frac{x}{y},\quad c_3^{NS}(\frac{x}{y})-c_2^{NS}(\frac{x}{y})=-\frac{4}{3}(1+\frac{x}{y}). \qquad (23)$$

In (22) $V(x,Q^2)_{"LO"}$ and $V^{(a)}(x,Q^2)$ obey the first and second order Lipatov-altarelli-Parisi equations, respectively:

$$Q^2\frac{dV(x,Q^2)_{"LO"}}{dQ^2}=\frac{\alpha_s(Q^2)}{2\pi}\int_x^1\frac{dy}{y}P^{(0)}(\frac{x}{y})V(y,Q^2)_{"LO"}, \qquad (24a)$$

$$Q^2\frac{dV^{(a)}(x,Q^2)}{dQ^2}=\frac{\alpha_s(Q^2)}{2\pi}\int_x^1\frac{dy}{y}[P^{(0)}(\frac{x}{y})+\frac{\alpha_s(Q^2)}{2\pi}P^{(1)}(\frac{x}{y})]V^{(a)}(y,Q^2), \qquad (24b)$$

$$V(x,Q_0^2)_{"LO"}=V^{(a)}(x,Q_0^2). \qquad (24c)$$

Notice that $P^{(1)}(x)$ in (24b) is the modified [3] second order AP kernel, which corresponds to A-definition of the parton distributions. We would like to mention also that in [10] formula similar to (22) has been obtained for the longitudinal structure function $F_L=F_2-2xF_1$. However, in [10] instead of $V(y,Q^2)_{"LO"}$ a pure LLA of $V(y,Q^2)$ has been used. We consider that in the case of NLLA to use $V(y,Q^2)_{"LO"}$ is more correct.

Finally we present our expressions for the moments of the singlet part of the structure functions:

$$\begin{aligned}M_i^S(n,Q^2)=\delta_i^{NS}\{\Sigma^{(a)}(n,Q^2) &+ \frac{\alpha_s(Q^2)}{4\pi}(c_{i,n}^{(1)S}-c_{2,n}^{(1)S})\Sigma(n,Q^2)_{"LO"}\\ &+ \frac{\alpha_s(Q^2)}{4\pi}2N_f(c_{i,n}^{(1)G}-c_{2,n}^{(1)G})G(n,Q^2)_{"LO"}\}.\end{aligned} \qquad (25)$$

Here $G(n,Q^2)$ are the moments of the gluon distributions and $\Sigma(n,Q^2)$ are the moments of the singlet quark distribution

$$\Sigma(x,Q^2)=\sum_f[(q_f(x,Q^2)+\bar{q}_f(x,Q^2)]. \qquad (26)$$

Applying the convolution theorem to (25) it is a simple task to obtain the expressions for the singlet part of the structure functions themselves.

The formulae for the complete electromagnetic and neutrino (antineutrino) structure functions are listed in the Appendix.

Let us now generalize our formulae for the structure functions for any order in α_s beyond the LLA. For simplicity we present only the expressions for the nonsinglet structure functions (A-definition of the parton distributions is used):

$$\begin{aligned}\mathcal{F}_i^{(n)NS}(x,Q^2) &= \delta_i^{NS}x\{V_{(a)}^{(n)}(x,Q^2)\\ &+\sum_{k=1}^{n-1}\sum_{m=0}^{k-1}\frac{\alpha_s^{k-m}(Q^2)\alpha_s^m(Q_0^2)}{(2\pi)^k}\int_x^1\frac{dy}{y}a_i^{(k,m)NS}(\frac{x}{y})V(y,Q^2)_{"LO"}\}.\end{aligned} \qquad (27)$$

In the case of $O(\alpha_s^{n-1})$ corrections to LLA the valence quark distributions $V_{(a)}^{(n)}(x,Q^2)$ satisfy the (n)-order LAP equation. $V(y,Q^2)_{"LO"}$ in the above equation obey the first order LAP equation, in which $\alpha_s(Q^2)$ is given by n-loop approximation of the β function. In (27) $\alpha_s(Q^2)$ is taken in the same approximation.

If B-definition of the parton distributions is used then in (27) instead of $V_{(a)}^{(n)}(x,Q^2)$ and $a_i^{(k,m)NS}$, $V_{(b)}^{(n)}(x,Q^2)$ and $b_i^{(k,m)NS}$ have to be taken, respectively.

We remind that the valence parton distributions $V_{(a)}^{(n)}(x,Q^2)$ and $V_{(b)}^{(n)}(x,Q^2)$ satisfy different LAP equations. The coefficient functions $b_i^{(k,m)NS}$ are also different from those one in the case of A-definition.

Notice that the above representations of the structure functions are perturbation theory selfconsistent. For any order in α_s beyond the LLA the terms which belong only to this approximation are present in (27).

3. Conclusion

We give a new representation of the next to leading corrections to the nucleon structure functions in terms of parton distributions. The terms which belong only to this approximation attend in our expressions for the structure functions. This representation is generalized for the higher-order QCD corrections beyond NLLA, which we expect to become noticeable at the current (HERA) and future (LEP*LHC) experiments. Finally, we consider such a representation of the structure functions to be useful for more precise determination of the parton distribution from the deep inelastic data, and especially, in the case of cross-section data analysis [11].

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Appendix

1. A-definition of the parton distributions

a) Electromagnetic structure functions in NLLA

$$\begin{aligned}\mathcal{F}_i^{\mu(e)N}(x,Q^2) &= \sum_f e_q^2[q_f^{(a)}(x,Q^2)+\bar{q}_f^{(a)}(x,Q^2)]_{NLLA}\\ &+\frac{\alpha_s(Q^2)}{2\pi}x\int_x^1\frac{dy}{y}\{\sum_f e_f^2[c_i^{(q)}(\frac{x}{y})-c_2^{(q)}(\frac{x}{y})]\\ &\cdot[q_f^{(a)}(y,Q^2)+\bar{q}_f^{(a)}(y,Q^2)]_{\text{"}LO\text{"}}\\ &+<e^2>2N_f[c_i^{(G)}(\frac{x}{y})-c_2^{(G)}(\frac{x}{y})]G(y,Q^2)_{\text{"}LO\text{"}}\},\end{aligned} \qquad (A1)$$

$(i=1,2;\quad N=p,n)$.

b) Neutrino and antineutrino structure functions in NLLA
We quote the formulae for isoscalar targets.

$$\begin{aligned}\mathcal{F}_i^{\nu(\bar\nu)} = x\Sigma^{(a)}(x,Q^2)_{NLLA} &+ \frac{\alpha_s(Q^2)}{2\pi}x\int_x^1\frac{dy}{y}\{[c_i^{(q)}(\frac{x}{y})-c_2^{(q)}(\frac{x}{y})]\Sigma(y,Q^2)_{\text{"}LO\text{"}}\\ &+ [c_i^{(G)}(\frac{x}{y})-c_2^{(G)}(\frac{x}{y})]2N_fG(y,Q^2)_{\text{"}LO\text{"}}\},\end{aligned} \qquad (A2)$$

$(i=1,2)$

$$\begin{aligned}F_3^{\nu+\bar\nu}(x,Q^2) &= \sum_f[q_f^{(a)}(x,Q^2)-\bar{q}_f^{(a)}(x,Q^2)]_{NLLA} \qquad (A3)\\ &+\frac{\alpha_s(Q^2)}{2\pi}\int_x^1\frac{dy}{y}[c_3^{(q)}(\frac{x}{y})-c_2^{(q)}(\frac{x}{y})]\sum_f[q_f(y,Q^2)-\bar{q}_f(y,Q^2)]_{\text{"}LO\text{"}}.\end{aligned}$$

Note that we use notations

$$c_i^{(q)}=c_i^{(1)S}=c_i^{(1)NS},\qquad c_i^{(G)}=c_i^{(1)G}.$$

2. B-definition of the parton distributions

a) Electromagnetic structure functions in NLLA

$$\begin{aligned}\mathcal{F}_i^{\mu(e)N}(x,Q^2) &= \sum_f e_f^2 x[q^{(b)}(x,Q^2)+\bar{q}_f^{(b)}(x,Q^2)]_{NLLA}\\ &+\frac{\alpha_s(Q^2)}{2\pi}x\int_x^1\frac{dy}{y}\{c_i^q(\frac{x}{y})\sum_f e_f^2[q_f(y,Q^2)+\bar{q}_f(y,Q^2)]_{\text{"}LO\text{"}}\\ &+<e^2>2N_fc_i^{(G)}(\frac{x}{y})G(y,Q^2)_{\text{"}LO\text{"}}\}.\end{aligned} \qquad (A4)$$

$(i=1,2;\ N=p,n)$

b) Neutrino and antineutrino structure functions in NLLA

$$\begin{aligned}\mathcal{F}^{\nu(\bar\nu)}(x,Q^2)=x\Sigma^{(b)}(x,Q^2)_{NLLA} &+ \frac{\alpha_s(Q^2)}{2\pi}x\int_x^1\frac{dy}{y}\{c_i^{(q)}(\frac{x}{y})\Sigma(y,Q^2)_{\text{"}LO\text{"}}\\ &+ 2N_fc_i^{(G)}(\frac{x}{y})G(y,Q^2)_{\text{"}LO\text{"}}\},\end{aligned} \qquad (A5)$$

$(i=1,2)$

$$\begin{aligned}F_3^{\nu+\bar\nu}(x,Q^2) &= \sum_f[q_f^{(b)}(x,Q^2)-\bar{q}_f^{(b)}(x,Q^2)]_{NLLA}\\ &+\frac{\alpha_s(Q^2)}{2\pi}\int_x^1\frac{dy}{y}c_3^{(q)}(\frac{x}{y})\{\sum_f[q_f(y,Q^2)-\bar{q}_f(y,Q^2)]_{\text{"}LO\text{"}}\}.\end{aligned} \qquad (A6)$$

In (A1-A6) the quark $q_f^{a(b)}(x,Q^2)$, antiquark $\bar{q}_f^{a(b)}(x,Q^2)$ and $\Sigma^{a(b)}(x,Q^2)$ distributions satisfy the second order LAP equations. In the case of A-definition of the parton distributions the usual second order LAP kernels have to be modified (see [3]). In (A1-A6) the parton distributions $q_f(x,Q^2)_{\text{"}LO\text{"}}$, $\bar{q}_f(x,Q^2)_{\text{"}LO\text{"}}$, $\Sigma(x,Q^2)_{\text{"}LO\text{"}}$ and $G(x,Q^2)_{\text{"}LO\text{"}}$ satisfy the first order LAP equations, in which for $\alpha_s(Q^2)$ the NLLA (7) is taken.

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