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TITLE: THEORIES OF THE ETA-MESON-NUCLEUS INTERACTION

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THEORIES OF THE ETA-MESON-NUCLEUS INTERACTION

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Abstract

It is shown that the pion-nucleon elastic scattering, eta-nucleon scattering length and the cross sections for pion-induced eta production on a nucleon satisfy a set of consistency relations. These relations are used to examine the ηN scattering lengths given by the various models. The nature of the threshold ηN interaction is discussed and recent advancements in η-nucleus reaction theory are reviewed.

1. Introduction

The previous speaker has pointed out the importance of studying η's. In this talk, I will concentrate on the strong-interaction aspect of η physics. In recent years, nuclear production of eta mesons has been studied with high-intensity beams of pions, protons, electrons, and photons. Analyses of these data have yielded new and, in many instances, unexpected information about the eta meson. In turn, this information stimulated many new experimental initiatives. It is, therefore, appropriate for us to review what we have learned and what we still have to understand. I will divide my talk into two parts: the eta-nucleon interaction and eta-nucleus reactions.

2. Eta-Nucleon Interaction

Understanding the basic η-nucleon interaction is of fundamental importance. Based upon the findings of pion-nucleon phase-shift analyses of the past 40 years, one generally believes that low-energy η production proceeds through the formation of spin $\frac{1}{2}$, isospin $\frac{1}{2}$, s-wave N* resonances. The principal properties of relevant s-wave resonances are summarized in Table I, where the percentages denote the decay branching ratios.

Table I: Properties of S_{11} resonances

	N*(1535)	N*(1650)
M	1520–1555 MeV	1640–1680 MeV
Γ	100–250 MeV	145–190 MeV
πN	35–55%	60–80%
ηN	30–50%	1%
$\pi\pi$N	5–20%	5–20%
ΛK	–	7%

In 1986, Bhalerao and Liu[1] employed a coupled-channel one-resonance isobar model to fit the πN S_{11} phase shifts[2] and made predictions on $\pi^- p \to \eta n$ cross sections and ηN scattering. Their analysis predicted an ηN scattering length $a_0 = (0.28 + i0.19)$ fm. In 1992, ASY(Arima, Shimizu, and Yazaki)[3] used a coupled-channel two-resonance isobar model to fit the πN phase shifts. From their fit, they predicted a much larger scattering length: $a_0 = (0.91 + i0.37)$ fm. In 1993, Wilkin[4] employed a first-order K-matrix theory to analyze the newly available $\pi^- p \to \eta n$ reaction at very low energies. Upon neglecting the effective-range effect, he obtained $a_0 = (0.55 \pm 0.20 + i0.30)$ fm. This last scattering length is smaller than the ASY result and has its lower limit compatible with the Bhalerao-Liu scattering length. To understand the very large a_0 obtained by ASY, it is useful to note two important factors. First, the $\pi\pi$N channel was omitted in the ASY analysis. This omission effectively forced the ηN channel to account for all the inelasticity in the πN channel. In so doing, the ASY model inevitably overestimated the ηN interaction. Secondly, results obtained with the use of the two-resonance formalism depend on the model used for the off-shell form factors. I will return to this dependence later.

If I use $H_{\pi NN^*} = \bar{g}_\pi \bar{\psi}_{N^*} \phi_\pi \tau \psi + h.c.$ and $H_{\eta NN^*} = \bar{g}_\eta \bar{\psi}_{N^*} \phi_\eta \psi + h.c.$, then the matrix element t_{ij} for $i \to j$ transition in the one-resonance isobar model is given by

$$t_{ij}(w; k', k) = -\frac{\sqrt{k'k} M_N}{4\pi w} \frac{h_i^\alpha(\Lambda_{i,\alpha}, k') h_j^\alpha(\Lambda_{j,\alpha}, k)}{D_\alpha(w)} , \tag{1}$$

where w is the total c.m. energy and k', k the final and intial c.m. momenta. The indices i and j refer to the channels into which the isobar α decays. I shall consider all three channels: πN,$\pi\pi$N,ηN; and label them, respectively, by $i = 1, 2, 3$. In Eq.(1), The denominator in the equation is defined by

$$D_\alpha(w) = w - M_\alpha^0 - \sum_j \Sigma_j^\alpha(w) , \tag{2}$$

with Σ_j^α being the self-energy arising from the $\alpha \to j$ decay, and M_α^0 the bare mass of the isobar α. ($\alpha \equiv$ N*(1535) in the one-resonance model.) The dynamical, energy-dependent isobar mass and width are, therefore,

$$M_\alpha(w) = Re[M_\alpha^0 + \sum_j \Sigma_j^\alpha(w)] \tag{3}$$

and

$$\frac{\Gamma}{2} = Im[\sum_j \Sigma_j^\alpha(w)] . \tag{4}$$

The vertex functions h are parametrized by $h_i^\alpha = \beta_i \bar{g}_i^\alpha F_i(\Lambda_{i,\alpha}, k)$, with F_i being the form factor having the range Λ_i^α. For the πN channel $(i, j = 1)$, $\beta_1 = \sqrt{3}$. For the ηN channel $(i, j = 3)$, $\beta_3 = 1$. I will also parametrize the form factor in such a way that $F \to 1$ for the on-shell meson and nucleon. With these conventions, the on-shell πN elastic scattering t-matrix is given by

$$t_{11} \equiv \frac{\bar{\eta} e^{2i\delta} - 1}{2i} = -\frac{p_\pi M_N}{w} \frac{\sqrt{3}\bar{g}_\pi \sqrt{3}\bar{g}_\pi}{4\pi D(w)} , \tag{5}$$

where p_π denotes the on-shell pion momentum. The δ and $\bar{\eta}$ are, respectively, the πN phase shift and elasticity in the S_{11} channel, which are functions of the energy w. The $\pi N \to \eta N$ production cross section is related to the on-shell transition t-matrix through the relations

$$\left(\frac{d\sigma}{d\Omega}\right)_{\eta\pi} = \frac{p_\eta}{p_\pi} \mid f \mid^2 , \tag{6}$$

$$f = \sqrt{\frac{2}{3}} \frac{t_{31}}{\sqrt{p_\eta p_\pi}} , \tag{7}$$

and

$$t_{31} = -\frac{\sqrt{p_\pi p_\eta} M_N}{w} \frac{\sqrt{3}\bar{g}_\pi \bar{g}_\eta}{4\pi D(w)} , \tag{8}$$

where p_η is the on-shell eta momentum. Upon eliminating $D(w)$, I obtain

$$\mid f \mid^2 = \frac{2}{3} \mid t_{11} \mid^2 \frac{\bar{g}_\eta^2}{p_\pi^2 3\bar{g}_\pi^2} . \tag{9}$$

Because[8]

$$\bar{g}_\eta^2 = 4\pi \frac{\gamma^{\eta N}(M_\alpha)}{2} \frac{M_\alpha}{M_N k_\pi^{res}} \tag{10}$$

and

$$3\bar{g}_\pi^2 = 4\pi \frac{\gamma^{\pi N}(E_\alpha)}{2} \frac{M_\alpha}{M_N k_\eta^{res}} , \tag{11}$$

I have

$$\frac{\bar{g}_\eta^2}{3\bar{g}_\pi^2} = \frac{\gamma^{\eta N}(E_\alpha) k_\eta^{res}}{\gamma^{\pi N}(E_\alpha) k_\pi^{res}} . \tag{12}$$

In Eqs.(10)-(12), k_π^{res} and k_η^{res} denote, respectively, the pion and eta c.m. momenta at the resonance energy $w = M_\alpha$.

The significance of Eqs.(9) and (12) lies in the fact that the η production cross sections are completely fixed by the resonance energy (M_α) and the partial widths ($\gamma^{\pi N}$, $\gamma^{\eta N}$); and all these quantities can be measured experimentally.

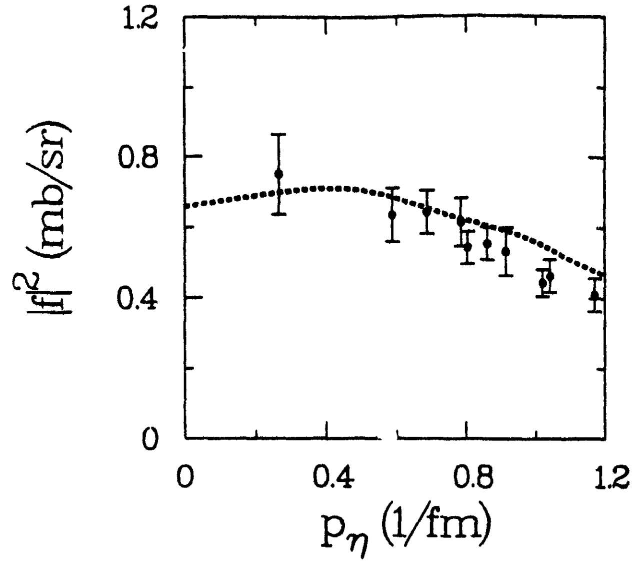

Figure 1: Eta production strength as a function of c.m. η momentum.

In a similar manner, I can derive the relation

$$\frac{t_{33}}{p_\eta} = \frac{t_{11}}{p_\pi}\left(\frac{\bar{g}_\eta^2}{3\bar{g}_\pi^2}\right), \tag{13}$$

In the limit of $p_\eta \to 0$ (*i.e.* $w \to w_0 \equiv M_\eta + M_N$), the left hand side of Eq.(13) is, by definition, the ηN scattering length a_0. Substituting Eq.(12) for the ratio of the coupling constants, I obtain from Eq.(13)

$$a_0 = \left(\frac{t_{11}}{p_\pi}\right)_{w=w_0} \frac{\gamma^{\eta N}(E_\alpha)k_\eta^{res}}{\gamma^{piN}(E_\alpha)k_\pi^{res}} . \tag{14}$$

Hence, the ηN scattering length is also completely fixed by the πN elastic t-matrix in the S_{11} channel at the thereshold energy w_0. An equivalent relation between a_0 and the πN resonance parameters can be found in ref.[7].

Using the 1993 Arndt phase shifts, M_α = 1535 MeV, and $\gamma^{\pi N} : \gamma^{\pi\pi N}\gamma^{\eta N} = 0.47 : 0.15 : 0.38$, I have calculated the energy dependence of $|\ f\ |^2$ with the aid of Eq.(9). The results are compared with the data[4] and shown as the dotted curve in fig.1. With the aid of Eq.(13), this set of parameters gives rise to an ηN scattering length of $a_0 = 0.35 + i0.33$ fm. Here, the positive sign of the real and imaginary parts of a_0 indicates that the low-energy ηN interaction is attractive. We may note that this scattering length is slightly larger than the 1986 Bhalerao-Liu result. This difference is mainly caused by the modification of the Arndt 1993 phase-shift solution[6] from its 1984 values[2] at energies $w > w_0$.

If I use the value $M_\alpha = 1500$ MeV as advocated by Nefkens[5], then the ratios $\gamma^{\pi N} : \gamma^{\pi\pi N} : \gamma^{\eta N} = 0.60 : 0.15 : 0.25$ have to be used in order to obtain the same curve as in fig.1. Thus, a careful measurement of the branching ratios and $(d\sigma/d\Omega)_{\eta\pi}$ would help pin down the resonance energy, M_α, of the N*.

When the two-resonace isobar model is used, Eq.(1) is replaced by

$$t_{ij}(w;k',k) = -\frac{\sqrt{k'k}M_N}{4\pi w}\sum_{\alpha,\alpha'} h_i^{(\alpha)}(\Lambda_{i,\alpha},k')\left(\Delta^{-1}\right)_{\alpha\alpha'} h_j^{(\alpha')}(\Lambda_{j,\alpha},k) \ , \tag{15}$$

where α and α' take the value 1 and 2, corresponding to N*(1535) and N*(1650), respectively. Δ^{-1} is the inverse of the matrix Δ which has its matrix elements equal to $\Delta_{\alpha\alpha'} = (w - M_\alpha^0)\delta_{\alpha\alpha'} - \sum_j \Sigma_j^{(\alpha\alpha')}(w)$. Here, $\Sigma_j^{(\alpha\alpha)}$ are the self-energies due to the $\alpha \to j$ decay. The $\Sigma_j^{(\alpha\alpha')}$ are the off-diagonal self-energies. I should mention that $\Sigma^{(\alpha\alpha')}$ and, hence, the $\Delta^{(\alpha\alpha')}$ are functionals of h. Eq.(15) can be rewritten as

$$t_{ij}(w;,k',k) = -\frac{\sqrt{k'k}M_N}{4\pi w}\ N_{ij}(w;k',k)/D(w) \ , \tag{16}$$

with

$$N_{ij}(w;k',k) = h_i^{(1)}\Delta_{11}h_j^{(1)} - h_i^{(1)}\Delta_{12}h_j^{(2)} - h^{(2)}\Delta_{21}h_j^{(1)} + h_i^{(2)}\Delta_{22}h_j^{(2)} \tag{17}$$

and $D(w) = \Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21}$. Again, the η production cross section and the ηN scattering length are related to πN elastic scattering. The relations are

$$t_{31}(w,p_\eta,p_\pi) = t_{11}(w;p_\pi,p_\pi)\sqrt{\frac{p_\eta}{p_\pi}\frac{N_{31}(w;p_\eta,p_\pi)}{N_{11}(w;p_\pi,p_\pi)}} \ ; \tag{18}$$

$$\mid f \mid^2 = \frac{2}{3} \mid \frac{t_{31}}{\sqrt{p_\eta p_\pi}} \mid^2 \ ; \tag{19}$$

and

$$t_{33}(w;p_\eta,p_\eta) = t_{11}(w;p_\pi,p_\pi)\frac{p_\eta}{p_\pi}\frac{N_{33}(w;p_\eta,p_\eta)}{N_{11}(w;p_\pi,p_\pi)} \ ; \tag{20}$$

$$a_0 = lim_{p_\eta \to 0}\frac{t_{33}}{p_\eta} \ . \tag{21}$$

Upon fitting the 1993 Arndt phase shifts with a two-resonance isobar formalism, I obtained a preliminary result of $a_0 = (0.37 \pm 0.10) + i(0.33 \pm 0.04)$ fm.

I emphasize that, unlike Eqs.(9) and (14) which contain only the observables, Eqs.(18) and (20) are the functionals of the off-shell form factors h. Hence, in the two-resonance theory, predictions of $(d\sigma/d\Omega)_{\eta\pi}$ and a_0 depend on the model used for the form factors. It is, therefore, important to carefully examine the dependence of

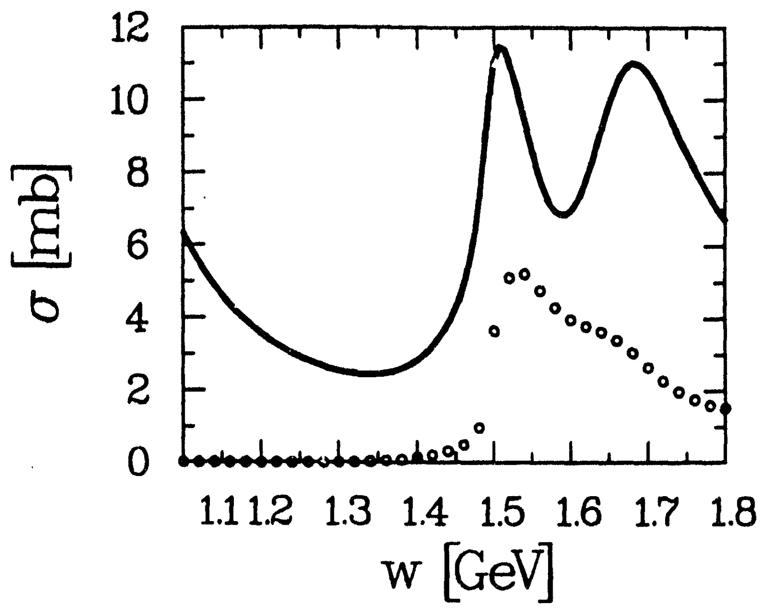

Figure 2: Energe dependences of total (solid curve) and reaction (open circles) cross sections in the πN S_{11} channel.

the prediction on the form factors. Such dependence is currently under investigation and will be reported elsewhere.

Upon inspecting fig.1, I conclude that good measurements of pion-induced eta production on a single nucleon in the threshold region (with a c.m. momentum $k_\eta < 0.25$ fm^{-1}) are extremely valuable. From the measured near-threshold k_η-dependence, one can conclude whether the one-resonance formalism is adequate. In order to do so, very small error bars (say, about one-third of those shown in fig.1) are indispensable. In other words, one prefers one good near-threshold measurement to several measurements with larger error bars.

An open question: As we can see from Table I, while N*(1535) has an ηN branching ratio of about 50%, the nearby $N^*(1650)$ has an ηN branching ratio of 1% only. Why are the branching ratios of these two resonances, which have the same quantum numbers, so disproportionate? The conventional answer to this question is that these two resonances are the effective resonances and are the result of the destructive interference between two normal-mode resonances. Indeed, this was the viewpoint adopted in the non-relativistic quark model.[9] Could there be other alternatives? To this end, it is worth examining the total and reaction cross sections of the πN interaction in the S_{11} channel[6]. Their energy dependences are shown in fig.2. The two peaks present in the total cross sections are traditionally ascribed to N*(1535) and N*(1650) resonances. A closer examination reveals, however, that the lower-energy peak is mostly caused by the peak in the reaction cross sections, in marked contrast to the situation of the higher-energy peak. Because the currently accepted N* resonance energy(1535±15 MeV) is so close to the ηN threshold(1488

MeV), I tend to believe that the lower-energy peak in fig.2 is not entirely due to the N*(1535) resonance. More likely, it is caused jointly by the resonance and by the cusp effect due to the opening of the ηN channel. The dynamical origin of the cross-section peak in the πN S_{11} channel in the 1500 MeV region is of fundamental importance and deserves a careful study. (Note added after presentation: Prof. B.M.K. Nefkens informed me that Prof. Höhler had also raised the question about the interpretation of the N*(1535) resonance.)

Summary: In the one-resonance isobar theory, the ηN scattering length and near-threshold η production cross sections can be directly related to the πN S_{11} phase shifts and observables. In the two-resonance isobar theory, these relations are no longer direct and become dependent on the models of the form factors. In this latter case, one has to be cautious in inferring the ηN scattering length and η production cross sections from fits to πN phase shifts. My analyses have indicated that both the one- and two-resonance theories lead to an *attractive* low-energy ηN interaction. On the other hand, we still need to understand the effects of off-shell form factors and non-resonant dynamics on the ηN interaction.

3. Etas in Nuclei

The eta-nucleus interaction can add a new dimension to η physics. First, it can allow us to study the behavior of η and N* in a many-nucleon environment, further extending our knowledge of the η-nucleon interaction. Secondly, it can provide premises for extracting fundamental parameters that cannot be obtained with meson-nucleon interactions in free space.

As an example of the first type of theoretical studies, I would like to mention the work by Krippa and Londergan[10] who used an isobar-hole model to calculate the modification of N* propagation in nuclei. Upon comparing their calculations with the data of the inclusive ^{12}C$(\pi,\eta)X$ reaction, they concluded that the width of the N*(1535) inside ^{12}C is considerably broadened and that the bulk of the broadening is due to the N* $\rightarrow \pi\pi$N decay which produces strongly absorptive low-energy pions. This finding confirms the result of an earlier study by Chiang, Oset, and Liu.[8] As to the second type of theoretical studies, there is a recent work by Lopez and Oset who have advocated that coherent eta production from nuclei can be used to determine the ηNN coupling constant.[11] It will be useful to measure cross sections for coherent nuclear eta production and to see if the ηNN coupling constant extracted from these cross sections agrees with that given by the analysis of eta photonuclear production.[12]

Before concluding this section, I want to emphasize that for η-nucleus reactions, theories based on impulse approximation (or the one-nucleon mechanism) are deficient. This is because the (π,η) reaction involves large momentum transfers. Consequently, multi-nucleon processes become important as they favor the sharing of momentum transfer. The importance of the multi-nucleon effect has been noted by

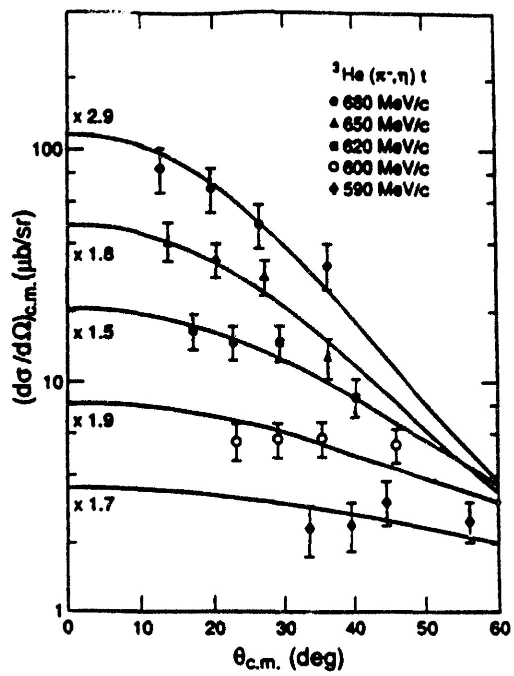

Figure 3: The ^{3}He(π^-, η)t cross sections. Theoretical curves are the results of the one-nucleon mechanism. Data are from Peng *et al.*

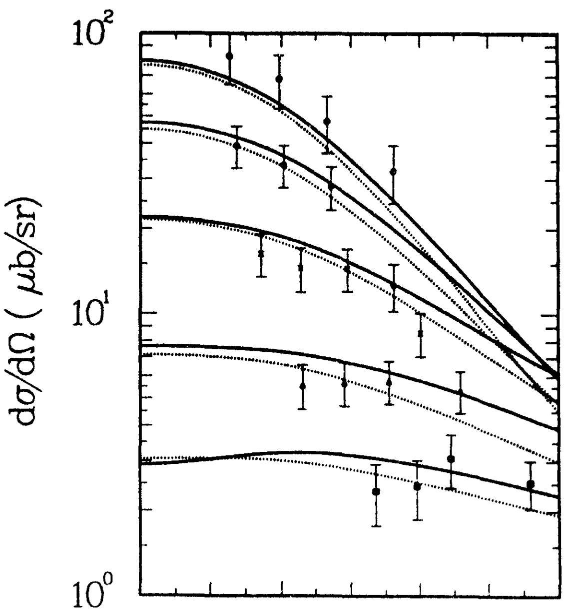

Figure 4: Theoretical ^{3}He(π^-, η)t cross sections obtained with the inclusion of two-nucleon mechanisms (dotted curves) and with the inclusion of two- and three-nucleon mechanisms (solid curves).

Laget and Lecolley in their qualitative analysis of the $p(d,\eta)^3$He reaction.[13] A more quantitative study of multi-nucleon effects was given by Liu.[14] He showed that the inclusion of two-nucleon processes can explain the experimental cross sections for the $He^3(\pi^-,\eta)t$ reaction at a wide range of "sub-threshold" energies without the need for introducing any adjustable parameters. In fig.3, I show the results given by the one-nucleon mechanism alone. The numbers in the parentheses are the multiplicative factors needed to obtain the theoretical curves shown. The results obtained with the inclusion of multi-nucleon mechanisms are given in fig.4. As we can see, good agreement with the data was obtained without the need of multiplicative factor. It was further shown that three-nucleon processes tend to cancel each other and only produce a small effect. I refer you to ref.[15] for detailed figures. Since the experiments analyzed by Laget and Lecolley and by Liu were performed at energies corresponding to a πN c.m. energy below the ηN threshold, where solutions of πN phase-shift analyses are quite stable, the above-mentioned multi-nucleon effects are genuine. It is my belief that only after the inclusion of these realistic, though calculationally more involved, reaction mechanisms, one can reliably discuss the need for exotic dynamics in η-nucleus reactions.

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