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DYNAMICS OF AN INHOMOGENEOUS ANISOTROPIC ANTIFERROMAGNETIC SPIN CHAIN

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ABSTRACT

We investigate the nonlinear spin excitations in the two sublattice model of a one dimensional classical continuum Heisenberg inhomogeneous antiferromagnetic spin chain. The dynamics of the inhomogeneous chain reduces to that of its homogeneous counterpart when the inhomogeneity assumes a particular form. Apart from the usual twists and pulses, we obtain some planar configurations representing the nonlinear dynamics of spins.

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1. Introduction:

One dimensional classical continuum Heisenberg spin chains exhibit a rich variety of nonlinear behaviour. In particular, over the past few years, several integrable ferromagnetic models have been identified and the underlying nonlinear spin excitations represented by solitons[1-4]. However, as the ground state of an antiferromagnet is characterized by a doubly degenerate state with adjacent spins antiparallel to each other the system is treated as a two sublattice model and hence the spin dynamics is normally represented by coupled nonlinear evolution equations. Thus probing their dynamics is relatively hard compared to ferromagnetic models. Inspite of this, in the recent years, there was some progress in identifying the underlying nonlinear spin excitations in limiting cases[5-8]. An interesting development in this direction is the identification of twists, pulses and other spin configurations in isotropic and anisotropic antiferromagnetic chains in external fields when the adjacent antiparallel spins are locked together during evolution [11,12]. In the present paper, we extend the ideas of refs.[11,12], and investigate the nonlinear dynamics of the two sublattice model of an inhomogeneous anisotropic antiferromagnetic spin chain in a longitudinal magnetic field.

After constructing the mathematical model for the system (section 2) we derive the equation of motion to represent the spin dynamics and solve them (section 3) to obtain the underlying nonlinear excitations. Towards the end of the paper the results are summarized and concluded (section 4).

2. Two Sublattice Model for an Inhomogeneous Antiferromagnetic chain:

The one dimensional inhomogeneous antiferromagnetic chain having N- nearest neighbour interacting spins, with uniaxial single ion anisotropy in an external magnetic field along the easy axis of magnetization (s^z-direction) can be represented by the Heisenberg Hamiltonian

$$H = -\sum_i [J\ \vec{S}_i \cdot f_i \vec{S}_{i+1} - A\ (S^z_i)^2 + \mu \vec{B}.\vec{S}_i], \quad i=1,2,\ldots,N. \qquad (1)$$

Here J is the exchange integral which takes negative values for antiferromagnets, A is the anisotropic parameter and μ is the gyromagnetic ratio. Further, in eq.(1), $\vec{S}_i$ represents the spin angular momentum operator of the atom or ion at the lattice site i, f_i corresponds to the inhomogeneity that arises at different lattice sites and $\vec{B}$ is the external magnetic field directed along the s^z-direction [$\vec{B}=(0,0,B)$]. When the spin angular momentum

value is large, it is the normal practice to go to the classical limit by replacing the spin angular momentum operator by a classical three component vector $\vec{S}_i=(S^x_i, S^y_i, S^z_i)$. Then treating the spin as a dynamical variable, the classical equation of motion can be written as

$$\frac{d\vec{S}_l}{dt} = \{\vec{S}_l, H\}_{PB}. \qquad (2)$$

The Poisson bracket in eq.(2) for any two arbitary functions A and B of spins can be written in the form [1a]

$$\{A,B\}_{PB} = \sum_{i=1}^{N} \sum_{\alpha,\beta,\gamma=1}^{3} \epsilon_{\alpha\beta\gamma} (\partial A/\partial S^\alpha_i)(\partial B/\partial S^\beta_i) S^\gamma_i . \qquad (3)$$

Now ,using the Hamiltonian (1) in the equation of motion (2), we obtain

$$\frac{d\vec{S}_l}{dt} = \vec{S}_l \wedge [J(f_l \vec{S}_{l+1} + f_{l-1}\vec{S}_{l-1}) + (\mu B - 2A S_l^z)\vec{k}], \qquad (4)$$

$$\vec{S}_l^2 = S^2 = \text{const.}, \quad \vec{k} = (0,0,1).$$

Redefining the system as a two sublattice model with all the upspins $\vec{S}_p$ put in the sublattice 'p' and all the downspins $\vec{S}_q$ in the sublattice 'q', the equation of motion (4) can be rewritten as

$$\frac{d\vec{S}_{p,l}}{dt} = \vec{S}_{p,l} \wedge [J(f_{p,l}\,\vec{S}_{q,l+1}+f_{q,l-1}\,\vec{S}_{q,l-1}) + (\mu B - 2AS^{z}_{p,l})\vec{k}], \tag{5a}$$

$$\frac{d\vec{S}_{q,l-1}}{dt} = \vec{S}_{q,l-1} \wedge [J(f_{q,l-1}\,\vec{S}_{p,l}+f_{p,l-2}\vec{S}_{p,l-2}) + (\mu B + 2AS^{z}_{q,l-1})\vec{k}]. \tag{5b}$$

Here f_p and f_q represent the inhomogeneities in the 'p' and 'q' sublattices respectively. While writing eqs.(5) the positive and negative S^z-axes have been considered as the easy axes of magnetization for the p and q sublattices respectively.

Now we make the continuum approximation by allowing the lattice parameter 2a of each sublattice to approach zero and assume a slow variation of the spin vectors over the lattice parameter. Thus replacing $\vec{S}_{p,l}(t)$ by $\vec{S}_p(x,t)$ and $\vec{S}_{q,l-1}(t)$ by $\vec{S}_q(x-a,t)$, we introduce the following series expansions.

$$\vec{S}_{q,l+1} = \vec{S}_q + 2a\,\frac{\partial \vec{S}_q}{\partial x} + 2a^2\,\frac{\partial^2 \vec{S}_q}{\partial x^2} + \ldots, \tag{6a}$$

$$\vec{S}_{p,l-2} = \vec{S}_p - 2a\,\frac{\partial \vec{S}_p}{\partial x} + 2a^2\,\frac{\partial^2 \vec{S}_p}{\partial x^2} - \ldots\,. \tag{6b}$$

We also expand $f_{p,l-2}$ and $f_{q,l-1}$ along the lines of (6). Now on using the above expansions for the spin vectors and for the inhomogeneity, in eqs.(5), we find that the dynamics is dominated at O(a) [in ferromagnets it is dominated at $O(a^2)$] and the resultant equation is of the form

$$\frac{\partial \vec{S}_p}{\partial t} = \vec{S}_p \wedge [J((f_p+f_q)\vec{S}_q + 2a\,\frac{\partial \vec{S}_q}{\partial x}) + (\mu B - 2AS^{z}_{p})\vec{k}], \tag{7a}$$

$$\frac{\partial \vec{S}_q}{\partial t} = \vec{S}_q \wedge [J[((f_p+f_q) - 2a\,\frac{\partial f_p}{\partial x})\vec{S}_p - 2af_p\,\frac{\partial \vec{S}_p}{\partial x}] + (\mu B + 2AS^{z}_{q})\vec{k}]. \tag{7b}$$

Having derived the equations of motion the associated spin dynamics can be understood by solving the set of equations (7). However, before actually solving them, in order to reduce the complexity, in accordance with the formulation, we introduce [5,10] a superlattice which contains spin vectors of reduced magnetization

$$\vec{n} = (\vec{S}_p + \vec{S}_q)/2S\epsilon, \tag{8a}$$

and another sublattice with magnetization

$$\vec{m} = (\vec{S}_p - \vec{S}_q)/[2S(1-\epsilon^2)^{1/2}]. \tag{8b}$$

Here $\vec{m}$ and $\vec{n}$ are three component unit vectors

$$\vec{m} = (m^x, m^y, m^z), \quad \vec{n} = (n^x, n^y, n^z) \tag{9a}$$

and

$$\vec{m}.\vec{n}=0, \tag{9b}$$

and also

$$\epsilon^2 = \frac{1}{2}\,[1+(\vec{S}_p.\vec{S}_q)/S^2]. \tag{9c}$$

In view of the above, we rewrite eqs.(7a) and (7b) by adding and subtracting them and then use eqs.(8) and (9), to obtain the evolution equations for the unit vectors $\vec{m}$ and $\vec{n}$. As the low energy configurations correspond to $|\vec{S}_p-\vec{S}_q| \approx 2S$ and $|\vec{S}_p+\vec{S}_q| \approx 0$, corresponding to $\epsilon \ll 1$, we find that the dynamics is dominated at O(a) by the following set of equations.

$$(1-f+2af_x)\,(\vec{m}_x\wedge\vec{m})=0, \tag{10a}$$

$$\vec{m}_t = -\vec{m}\wedge[(1+f-2af_x)\vec{m}_x + (Am^z-\mu B)\vec{k}]. \tag{10b}$$

While writing eq.(10), we have rescaled the time and redefined the parameters A and μ and the inhomogeneous function $f_p(f_p\text{-->}f)$. Further, in eqs.(10), the suffices 'x' and 't' represent the partial derivatives with respect to x and t respectively. Also from the constraint on the length of the spin vectors namely $\vec{S}^2$=const, $\vec{m}^2$=1 one can write [10]

$$(1-f+2af_x)\ \vec{m}\wedge\vec{m}_t = 0, \tag{11a}$$

$$\vec{m}_x = \vec{m}\wedge[(1+f-2af_x)\vec{m}_t+(Am^z-\mu B)\ \vec{k}]. \tag{11b}$$

The set of eqs.(10) and (11) represent the spin dynamics of the inhomogeneous system when the adjacent antiparallel spins are locked together during evolution.

4. Dynamics of the Inhomogeneous Spin Chain:

In order to understand the nature of spin excitations we now try to solve the equations of motion (10) and (11). From eqs.(10a) and (11a), we demand that

$$1-f+2af_x = 0\ . \tag{12}$$

On integrating eq.(12), we obtain

$$f = 1+c_1e^{(x/2a)}. \tag{13}$$

where c_1 is the arbitary constant of integration. On using eq.(12) or (13) in eqs.(10) and (11) and after another rescaling of time and redefinition of parameters, we obtain

$$\vec{m}_t = -\vec{m}\wedge[\ \vec{m}_x+(Am^z-\mu B)\ \vec{k}\ ], \tag{14a}$$

$$\vec{m}_x = \vec{m}\wedge[\ \vec{m}_t+(Am^z-\mu B)\ \vec{k}\ ]. \tag{14b}$$

In fact, eqs.(14) is same as the equation for a homogeneous anisotropic antiferromagnetic chain in a longitudinal magnetic field [For details see ref.[12]]. Thus, the dynamics of the inhomogeneous anisotropic antiferromagnetic spin chain in a longitudinal magnetic field reduces to that of its homogeneous counterpart when the inhomogeneity 'f_p' of the p-sublattice (= f) assumes the form $f = 1+c_1 e^{(x/2a)}$.

In order to solve eqs.(14), we write them in the component form

$$\vec{m} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta), \tag{15}$$

and define

$$m^{\pm} = m^x \pm i m^y, \tag{16}$$

$$\xi = t+ix, \qquad \xi^* = t-ix . \tag{17}$$

Thus, upon using eqs.(15-17) in eqs.(14), after lengthy calculations, we obtain

$$\theta_\xi = i\sin\theta\, \phi_\xi + \frac{(1-i)\sin\theta}{(1-\cos\theta)}[A\cos\theta-\mu B], \tag{18a}$$

$$\theta_{\xi^*} = -i\sin\theta\, \phi_{\xi^*} + \frac{(1+i)\sin\theta}{(1-\cos\theta)}[A\cos\theta-\mu B]. \tag{18b}$$

It may be noted that eq.(18b) is the complex conjugate of eq.(18a).

In ref.[12] it was shown that in the limit $A=\mu B$ and for appropriate choice of the integration constant eqn.(18) admits the twist solution

$$\vec{m} = \operatorname{sech} g\,[\cos\phi\, \hat{e}_x + \sin\phi\, \hat{e}_y] + \tanh g\, \hat{e}_z, \tag{19}$$

[g:a real function related to ϕ] of the isotropic chain.

Defining $A(1+i)=c$, $A(1-i)=c^*$, eqs.(18) in the limit $A=\mu B$, after integration becomes

$$\tan\frac{\theta}{2}\, e^{-(i\phi - c\xi)} = c_2\,(\xi^*), \tag{20a}$$

$$\tan\frac{\theta}{2}\, e^{(i\phi + c^*\xi^*)} = c_3\,(\xi). \tag{20b}$$

where $c_2(\xi^*)$ and $c_3(\xi)$ are arbitary functions of ξ^* and ξ respectively. Eqs.(20) represent a general class of solutions to eqs.(18) in the limit $A=\mu B$. Now, rewriting eq.(20b) as

$$\tan\frac{\theta}{2}\, e^{i\phi} = f(\xi,\xi^*), \tag{21}$$

where

$$f(\xi,\xi^*) = c_3\,(\xi)\, e^{-c^*\xi^*}, \tag{22}$$

and using the identity $\tan^2(\frac{\theta}{2}) = \frac{1-\cos\theta}{1+\cos\theta}$ therein, we obtain

$$\cos\theta = \frac{1-f^2 e^{-2i\phi}}{1+f^2 e^{-2i\phi}} . \tag{23}$$

Again using the identity $\cos 2\phi = \frac{1-\tan^2\phi}{1+\tan^2\phi}$ and $\sin 2\phi = \frac{2\tan\phi}{1+\tan^2\phi}$ and choosing $f = (1-i\tan\phi)^{-1}$, $\phi = ig$ we obtain the pulse-like solution

$$\cos\theta = \frac{\operatorname{sech}^2 g-1}{\operatorname{sech}^2 g+1}, \tag{24}$$

reported in the case of the isotropic chain [11].From eq.(24) the spin components can be constructed explicitly.

We now try to find the spin excitations in the more general case by removing the constraint $A=\mu B$. However, as the general discussion in this case is found to be difficult we look for planar spin configurations by restricting the spin excitations to specific planes.

(i) Spin Excitations in the (m^x-m^y) Plane:

The spin excitations can be confined to the (m^x-m^y)-plane by choosing $\theta=\pi/2$. Now eq.(18b) can be written as

$$i\phi_{\xi^*}+(1+i)\mu B=0. \qquad (25)$$

On integrating eq.(25), we obtain

$$\phi=(i-1)\mu B\xi^*+c_4(\xi). \qquad (26)$$

where $c_4(\xi)$ is an arbitary function depending on ξ ($c_4=d_1+id_2$; d_1,d_2:real). Hence m^+ can be written as

$$m^+=e^{i\phi}=c_4e^{-(1+i)\mu B\xi^*}. \qquad (27)$$

Using $\xi^*=t-ix$ (eq.(17)) in eq.(27), the components of $\vec{m}$ can be written as

$$m^x=[d_1\cos\mu B(x-t)-d_2\sin\mu B(x-t)]e^{-\mu B(x+t)}, \qquad (28a)$$

$$m^y=[d_1\sin\mu B(x-t)+d_2\cos\mu B(x-t)]e^{-\mu B(x+t)}, \qquad (28b)$$

$$m^z=0. \qquad (28c)$$

From eqs.(28), we observe that the spin oscillations in the (m^x-m^y)- plane are getting damped if the magnetic field applied normal to the plane of oscillation is large. Hence, in order to sustain the oscillations the strength of the magnetic field should be maintained low.

(ii) Spin Excitations in the (m^x-m^z) Plane:

Next, we assume that the spin excitations are confined to the (m^x-m^z)- plane. This can be achieved by choosing $\phi=0$. Now eq.(18b) can be written as

$$\theta_{\xi^*}=(1+i)\frac{A\sin\theta}{(1-\cos\theta)}[\cos\theta-\frac{\mu B}{A}] \qquad (29)$$

In the limit that the strength of the applied magnetic field is very small compared to the strength of the anisotropy (i.e) in the limit $\mu B \ll A$ the last term in the right hand side of eq.(29) drops out. Hence eq.(29) can be written as

$$\theta_{\xi^*}=\frac{(1+i)\sin\theta\cos\theta}{(1-\cos\theta)} \qquad (30)$$

which on integration gives

$$\cos\theta\equiv m^z=\frac{1}{2}[\tanh[\frac{(1+i)}{2}A\xi^*+c_5(\xi)]-1], \qquad (31)$$

where $c_5(\xi)$ is an arbitary function of ξ. Using eq.(31), the component $m^x=\pm(1-(m^z)^2)^{1/2}$ can be written as

$$m^x=\pm\frac{1}{2}[3+2\tanh[\frac{(1+i)}{2}A\xi^*+c_5(\xi)]-\tanh^2[\frac{(1+i)}{2}A\xi^*+c_5(\xi)]]^{1/2}. \qquad ...(32)$$

Further

$$m^y = 0. \quad (33)$$

The nature of solutions (31) and (32) suggest that the spin excitations in the $(m^x\text{-}m^z)$-plane are in the form of domain wall-like configurations.

In a similar way, one can also find the spin excitations in the $(m^y\text{-}m^z)$-plane by choosing $\phi = \pi/2$. We find that in the limit $\mu B \ll A$ the spin excitations are once again governed by the same type of domain wall-like configurations as in the $(m^x\text{-}m^z)$ plane. Finally, it is worth pointing out that when the magnetic field is applied in a direction normal to the easy axis of magnetization, the dynamics becomes complicated and it is not possible to find similar type of spin excitations there.

4.Summary and Conclusions:

In this paper, we investigated the nonlinear spin dynamics of a one dimensional inhomogeneous anisotropic Heisenberg antiferromagnetic chain in the presence of a longitudinal magnetic field in the classical continuum limit.Treating the inhomogeneous antiferromagnetic chain as a two sublattice model, we derived the classical equation of motion at the lowest order of continuum approximation (O(a)) for the sublattice magnetization with the adjacent antiparallel spins locked together during evolution.

Analysis showed that the dynamics of the inhomogeneous anisotropic chain reduces to that of its homogeneous counterpart when the upspin sublattice inhomogeneity (f_p) assumes the form $f_p=1+c_1e^{(x/2a)}$. Then we found that the twist and pulse-like solutions of the isotropic chain follows when the strength of the anisotropy matches with the external field (i.e) when $A=\mu B$. On the otherhand when $A\neq\mu B^z$, we obtained interesting spin configurations when the spins are restricted to specific planes. For instance, when the spins are confined to the $(m^x\text{-}m^y)$-plane the excitations are governed by spin waves and we observe that these spin oscillations sustain only when the external magnetic field applied normal to the plane of oscillation $(m^x\text{-}m^y)$ is sufficiently small. Otherwise the oscillations are getting damped. On the otherhand, when the spins are confined to the $(m^y\text{-}m^z)$ and $(m^x\text{-}m^z)$ planes, the spin excitations are governed by domain wall-like configurations in the limit $\mu B \ll A$. As the above spin excitations correspond to a model in which the adjacent antiparallel spins are locked together in pair it is natural to know about the nature of the nonlinear spin excitations and its dynamics when the adjacent spins are unlocked allowing complete freedom for the spins. The work in this direction is under progress and the results will be reported elsewhere.

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