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Magneto-Optical Transmission-Reflection Beam Splitter for Multi-level Atoms

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Abstract

An atomic de Broglie wave beam splitter is proposed. The interaction of multi-level atoms ($J_g = 1 - J_e = 0$) with a laser beam in the presence of a static magnetic field leads to the partial transmission and reflection of the atomic beam. The coherent splitting of the atomic beam occurs due to non-adiabatic transitions between different dressed states in the vicinity of avoided crossings. The transition probabilities and populations of split beams are dependent on the value of the magnetic field, laser detuning, and the ratio between different polarization components in the laser beam. For optimal conditions the population of each of the two transmitted and two reflected beams is 25 per cent. For cooled atoms it is possible to obtain splitting angles of 80 mrad. The effect of spontaneous emission during the atom-light interaction was estimated and for a reasonable detuning losses were reduced to less than 10 per cent.

In recent years there has been a great deal of interest in atom optics and, in particular, in atom interferometry [1] . The essential element of an interferometer is a coherent beam splitter. Several atomic beam splitters based on atom-light interaction have been realised [2] [3]. A reflection beam splitter using a standing evanescent laser wave as a grating was proposed [4]. However, theoretical analyses [5] [6] and experiments [7] [8] have demonstrated the difficulty of obtaining significant populations in the diffracted orders. Atomic beam splitters using reflection and transmission of three-level lambda atoms in a laser beam with gaussian profile have been proposed [9] and analysed [10]. Deutschmann et al. [11] have now proposed a reflection beam splitter with up to 50 per cent population in the secondary beam. This scheme is based on the interaction of a multi-level atom (J=2 - J'=2) with a travelling evanescent wave in the presence of a static magnetic field. The evanescent wave is formed on the surface of a dielectric. Polarization dependent non-adiabatic transitions between the dressed states cause the atom to move along two different trajectories. After reflection on each trajectory the emerging atom is split between two magnetic sublevels of the ground state. The transmission of the atomic beam would cause the atoms to be lost on the surface of the dielectric. The splitting angle is determined by the value of the magnetic field.

In this paper we consider an alternative atomic beam splitter. It is based on the interaction of a multi-level atom with a *free* travelling laser beam in the presence of a static magnetic field (Fig. 1). Applying the same principle as Deutschmann [11], this scheme uses *reflection* and *transmission*. There are two advantages: (1) the angle between the split beams is determined by the incident grazing angle as well as the Zeeman splitting and can be considerably greater than in the Deutschmann scheme, (2) two split beams emerge in the ground state $m_g = 0$ hence are are almost unaffected by magnetic field gradients.

To understand the idea of the proposed beam splitter, it is useful to consider the atomic motion along "quasipotentials" (or dressed state energies) [5] [12] (Fig. 2). We consider the interaction of a four level atom (transition $J_g = 1 - J_e = 0$) with a laser beam with a gaussian profile in the presence of a static magnetic field. The laser frequency exceeds the atomic transition frequency. As only coherent interactions are considered, spontaneous emission is neglected. The use of π and σ^+ polarisation components in the travelling laser wave reduces the four level atom to a three level atom. These levels are the $m_g = 0$ and $m_g = -1$ ground states and the $m_e = 0$ excited state (see Fig. 1 inset). The atom enters the laser field in the state $m_g = 0$ and adiabatically follows the quasipotential which asymptotically is labelled $m_g = 0$ (see Fig. 2). In the absence of transitions to other dressed states the atom would travel along this quasipotential and be repelled from the laser field. However, at avoided crossing A [5] (see Fig.2) non-adiabatic transitions to other dressed states occur, hence the atomic wave function is split. The transmitted component travels to point B where it encounters another avoided crossing and is split again. The transmitted atoms emerge from the laser field in the $m_g = 0, -1$ ground state sub-levels. The other component of the atomic wave function is reflected at the classical turning point C (Fig. 2). It is further split at point A hence the reflected atoms will also emerge from the laser field in the $m_g = 0, -1$ ground state sub-levels. The secondary splitting of the atomic wave function at points A and C is undesirable but unavoidable. The angle between reflected (transmitted) $m_g = 0, -1$ components is determined by the Zeeman splitting.

We consider the two-dimensional motion of the atom in the x-y plane. For quantitative analysis the parameters of the $2^3S_1 - 2^3P_0$ transition in metastable Helium were used (see table). In order to obtain the correct ratio of the π and σ^+ polarization components, the magnetic field should be aligned at an angle θ to the direction of laser propagation (Fig. 1). If the laser beam has a mixture of left hand circular polarization with amplitude $A = A_+ \mathcal{E}_0 \exp(-y^2/y_0^2)/\cos\theta$ and linear polarization along the z axis with amplitude $A(1 - \cos\theta)/\sqrt{2}$ the resulting electric field is

$$\mathbf{E}(t,x,y) = (A_\pi \boldsymbol{\epsilon}_\pi + A_+ \boldsymbol{\epsilon}_+)\mathcal{E}_0 \exp(-y^2/y_0^2)\cos(Kx - \omega t) \tag{1}$$

where $\boldsymbol{\epsilon}_\pi$ and $\boldsymbol{\epsilon}_+$ are the π and σ^+ polarization vectors respectively and the ratio $A_\pi/A_+ = \tan\theta/\sqrt{2}$. A decelerated monochromatic atomic beam with velocity 70 m/s is incident on the laser beam at an angle of 30 mrad. To reduce the laser beam divergence a laser beam waist of 12 μm was chosen.

The Hamiltonian for an atom moving through a travelling laser wave is

$$H = H_0 + \frac{\hat{p}^2}{2M_A} - \hat{\boldsymbol{\mu}}.\mathbf{E}(t, x, y) \tag{2}$$

where H_0 governs the internal atomic motion, $\hat{p}$ is the centre of mass momentum operator, M_A is the atomic mass, and $\hat{\mu}$ is the electric dipole operator. The atomic wave function can be expanded using Bloch's theorem in the form

$$\Phi(t, x, y, r) = \exp[-iWt/\hbar]\exp[ik_x x](\Psi_{0,g}(y)\phi_{0,g}(r) + \Psi_{-1,g}(y)\phi_{-1,g}(r))$$
$$+ \exp[-i(W/\hbar + \omega)t]\exp[i(k_x + K)x]\Psi_{0,e}(y)\phi_{0,e}(r) \tag{3}$$

The wave functions $\Psi_{m,n}(y)$ and $\phi_{m,n}(r)$ ($m = 0, -1$, $n = g, e$) control the internal atomic motion and the y-axis centre-of-mass motion respectively. The atoms initial energy is

$$W = \frac{\hbar^2}{2M_A}(k_x^2 + k_y^2) \tag{4}$$

where $\hbar k_x$ and $\hbar k_y$ are the x and y components of the incoming atomic momentum. Substituting the expansion Φ into the Schrödinger equation (in the rotating-wave approximation) yields a set of coupled equations for $\Psi_{m,n}$ which, written in matrix form, are

$$\frac{d^2}{dy^2}\begin{pmatrix} \Psi_{0,g} \\ \Psi_{-1,g} \\ \Psi_{0,e} \end{pmatrix} = -M(y)\begin{pmatrix} \Psi_{0,g} \\ \Psi_{-1,g} \\ \Psi_{0,e} \end{pmatrix}. \tag{5}$$

where

$$M(y) = \begin{bmatrix} k_{0,g}^2 & 0 & \frac{M_A}{\hbar}\beta(y) \\ 0 & k_{-1,g}^2 & \frac{M_A}{\hbar}\alpha(y) \\ \frac{M_A}{\hbar}\beta(y) & \frac{M_A}{\hbar}\alpha(y) & k_{0,e}^2 \end{bmatrix} \tag{6}$$

the Rabi frequencies are

$$\alpha(y) = \frac{A_+ d\mathcal{E}_0 \exp(-y^2/y_0^2)}{\sqrt{3}\hbar}$$
$$\beta(y) = \frac{A_\pi d\mathcal{E}_0 \exp(-y^2/y_0^2)}{\sqrt{3}\hbar} \tag{7}$$

and d is the matrix element of the dipole operator between the ground and excited states.

The diagonal elements, as a result of the conservation of energy, are given by

$$\begin{aligned} k^2_{0,g} &= k^2_y \\ k^2_{-1,g} &= k^2_y + \frac{2M_A}{\hbar}\omega_L \\ k^2_{0,e} &= k^2_y - K^2 - 2Kk_x + \frac{2M_A}{\hbar}(\omega - \omega_0) \end{aligned} \tag{8}$$

where $\hbar\omega_0$ is the energy difference between $m = 0$ ground and excited states, and ω_L is the Larmor frequency.

The above equations are solved using the multi-slice method [13]. The essence of this method is to consider the function $\alpha(y)$, $\beta(y)$ in the matrix M as a piecewise constant function, and to numerically solve equation (5) for each piece (or slice). That is, if the wavefunction and its derivative are known for any value of y corresponding to the edge of a particular slice then the numerical solution to equation (5) yields the wavefunction and its derivative at the opposite edge of the slice. Repeating this procedure, using the fact that the wavefunction and its derivative must be continuous at the edges of successive slices, one can calculate the output wavefunction. This yields the population in the outgoing reflected and transmitted beams. No approximations of the underlying physics are made and the accuracy of the method can be systematically improved by decreasing the slice thickness.

In the multi-slice approach the M matrix is diagonalized for each slice. From the eigenvalues of M quasipotentials $V(y)$ [5] can be easily computed using

$$V_i(y) = T_0 - \frac{\hbar^2 M_i(y)}{2M_A} \tag{9}$$

where T_0 is the initial kinetic energy of the atom in a frame moving with velocity $v_x = \hbar k_x/M_A$ and is given by

$$T_0 = \frac{\hbar^2}{2M_A}k^2_{0,g} \tag{10}$$

$M_i(y)$ are the y-dependent eigenvalues of the matrix M (see Fig. 2). Each quasipotential corresponds to a particular dressed state

$$\Psi^D_{m,n}(y) = a_{m,n}(y)\Psi_{0,g} + b_{m,n}(y)\Psi_{-1,g} + c_{m,n}(y)\Psi_{0,e} \tag{11}$$

where $m = 0, -1$ for the ground state $n = g$, and $m = 0$ for the excited state $n = e$. As the coordinate y approaches infinity each dressed state $\Psi^D_{m,n}(y)$ asymptotes to the corresponding "bare" states $\Psi_{0,g}, \Psi_{-1,g}, \Psi_{0,e}$. For convenience, the bare ground states are labelled $m_g = 0$ and $m_g = -1$. Assuming the atom moves adiabatically along these quasipotentials and any transitions between dressed states occur only at avoided crossings, the result is a simple physical picture.

The population of the split atomic beams is crucially dependent on the ratio between the amplitudes of π and σ^+ polarization (Fig. 3). If the laser polarization is purely circular, $A_\pi/A_+ = 0$, the atomic beam in the $m_g = 0$ is not affected by the laser and is completely transmitted. However, even when the π polarization is small the result is a non zero probablity of transition from one dressed state to another at avoided crossing A (Fig. 2). Consequently, the atoms can now be reflected at point C and emerge in the $m_g = 0$ and $m_g = -1$ states. For the chosen parameters of the laser and magnetic field the optimal ratio of A_π/A_+ is 0.0294. For this value the the population of each of the reflected and transmitted beams is 25 per cent. As the ratio A_π/A_+ is increased the dressed states are no longer nearly degenerate at the avoided crossing A and consequently the probablity of transition decreases to zero. The incoming atom in state $m_g = 0$ then moves completely adiabatically along the repulsive quasipotential (Fig. 2) to point C and is reflected. It emerges entirely in the $m_g = 0$ state.

The dependence of the population of outgoing atomic beams on the Larmor frequency is shown in Fig. 4. In the absence of a magnetic field the ground state sub-levels become degenerate. This results in an identical case to that of coherent population trapping in a lambda atom [10]. The dressed states $\Psi^D_{0,g}(y)$ and $\Psi^D_{-1,g}(y)$ are degenerate at an infinite distance from the laser field and are still in a linear superposition of the the bare states. Since there is no avoided crossing, the population of the reflected components of $\Psi^D_{0,g}(y)$ and $\Psi^D_{-1,g}(y)$ are given by $A_+^2/(A_+^2 + A_\pi^2)$ and $A_\pi^2/(A_+^2 + A_\pi^2)$ respectively. As the Larmor frequency is increased an avoided crossing is formed. In this case, as above, the population of the outgoing beams is determined by transitions between dressed states in the vicinity of the avoided crossing. For the optimal Larmor frequency of $\omega_L/2\pi = 14.3$ MHz the populations of the reflected and transmitted beams in the states $m_g = -1$ and $m_g = 0$ are 25 per cent each. As the Larmor frequency further increases the distance between quasipotentials in the avoided crossings increases. This causes the probability of a transition between dressed states to drop quickly to zero, hence an atom initially on the $m_g = 0$ quasipotential remains so and moves adiabatically along this repulsive potential (Fig. 2), is reflected at point C and emerges entirely in the $m_g = 0$ state.

The effective detuning is given by

$$\Delta = \omega - \omega_0 - \frac{\hbar}{M_A} K k_x - \frac{\hbar}{2M_A} K^2 \tag{12}$$

where the third and fourth terms represent the Doppler shift and the frequency shift due to the photon recoil respectively. The effect of the detuning on the populations of the split beams is shown in Fig. 5. The other parameters are as indicated above. To reduce the loss due to spontaneous emission the detuning must necessarily be far greater than the natural linewidth of the excited state. At a detuning of 1 GHz the resulting populations in the outgoing reflected beams are close to 20 per cent. The populations of the transmitted beams are appproximately 20 percent for the $m_g = -1$ beam and 40 per cent for the $m_g = 0$ beam. The loss however, at this detuning is considerable. As the detuning is increased the population in the transmitted $m_g = 0$ beam falls slowly until we have 25 per cent population in each beam at a detuning of 7.96 GHz. A large detuning also reduces the potential barrier formed by the $m_g = 0$ quasipotential, hence reducing the maximum y-component of atomic velocity allowed in order to obtain reflection.

In order to estimate the losses due to spontaneous emission it is necessary to know the amount of time an atom is in the excited state. Using the adiabatic assumption, the probability of being in the excited state $\Psi_{0,e}$ for a particular dressed state is given by $c^2_{m,n}$ (see eqn. (11)). A plot of $c_{m,n}$ (see eqn. (11)) for the $\Psi^D_{0,g}(y)$ and the $\Psi^D_{-1,g}(y)$ dressed states is shown in Fig. 6. Therefore, a good estimate of the losses can be made. In Fig. 6 the avoided crossing at point A in Fig. 2 is shown as well as the classical turning point at point C. The incoming atom follows the $m_g = 0$ quasipotential (Fig. 2) and is almost exclusively in the ground $\Psi_{0,g}$ state until the avoided crossing at A. This is expected as the amplitude of π polarization is very small (i.e. $c_{0,e}^2$ is practically zero). In the region of the avoided crossing A the adiabatic assumption no longer holds. However, this region is very narrow and an estimate for the loss can be made. After avoided crossing A a transmitted atom will now be in the $\Psi_{-1,g}^D(y)$ dressed state. Again this dressed state is mainly in the ground state $\Psi_{0,g}$ hence the losses will be negligible. In contrast an atom in dressed state $\Psi_{0,g}^D(y)$ after avoided crossing A will have the much larger probability of $c_{0,g}^2$ of being in the excited state. In this case the atom is reflected at point C. The traversal time from avoided crossing A to the classical turning point at C can be estimated, hence the loss while the atom is in this region can be calculated. For a detuning of 7.96 GHz the losses are about 5 percent. A reflected or transmitted atom emerging in the $m_g = 0$ state will have suffered practically no further loss after avoided crossing A because (as with the incident atom) $c_{0,e}^2$ is nearly zero. A reflected or transmitted atom emerging in the $m_g = -1$ state (see Fig. 6) has a far greater probability of loss ($c_{-1,g}^2 > 0$ on the asymptotic side of avoided crossings A and B) but it is still only a few The total losses are about seven per cent for atoms emerging in the $m_g = 0$ state and less than ten per cent for atoms emerging in the $m_g = -1$ state.

In conclusion, we proposed an atomic beam splitter based on the partial transmission and reflection of atoms travelling through a laser beam in the presence of a static magnetic field. The coherent splitting of the atomic beam occurs due to non-adiabatic transitions between different dressed states in the vicinity of avoided crossings. The beam is split into four: two transmitted components (in ground states $m_g = 0$ and $m_g = -1$) and two reflected components ($m_g = 0$ and $m_g = -1$). The angle between transmitted (reflected) components in the states $m_g = 0$ and $m_g = -1$ is determined by the value of the magnetic field. For a magnetic field of 5 gauss, (the required field for the optimal Larmor frequency) the angle is $\phi = 8.5$mrad. The angle between transmitted amd reflected components in the state

$m_g = 0$ is twice the incident angle. (60 mrad in our case). The angle between transmitted and reflected components in the state $m_g = -1$ is twice the incident angle plus twice the angle ϕ. This gives a total angle of approximately 80 mrad. This is considerably greater than previously proposed beam splitters. The other advantage of the the proposed beam splitter is that two components are in the state $m_g = 0$. These components are not affected by unavoidable spatial fluctuations (gradients) of the static magnetic field hence they are not subject to loss of coherence during propagation from beam splitter to recombiner [14].

Acknowledgements

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REFERENCES

[1] Special Issue on "Optics and Interferometry with Atoms" . *Appl. Phys.* **B54** (1992) 5

[2] T. Sleator, T. Pfau, V. Balykin, O.Carnal and J. Mlynek. Phys. Rev. Lett. **68** (1992) 1996

[3] T. Pfau, C.H. Kurtsiefer, C.S. Adams, M. Siegel and J. Mlynek, *Phys. Rev. Lett.* **71** (1993) 3427

[4] J.V. Hajnal and G.I. Opat, *Opt. Commun.* **71** (1989) 119

[5] R. Deutschmann, W. Ertmer and H. Wallis, *Phys. Rev. A.* **47** (1993) 4665

[6] J. Murphy, L.C.L. Hollenberg and A. Smith, *Phys Rev. A.* **49** (1994) 3100

[7] M. Christ, A. Scholz, M. Schiffer, R. Deutschmann and W. Ertmer , *Opt. Commun.* **107** (1993) 211

[8] B. Stenlake, I.C.M. Littler, H.-A. Bachor, K.G.H. Baldwin, *Phys. Rev. A* **49** (1994) R16-R19

[9] W. P. Zhang and D.F. Walls, *Phys. Rev. A* **47** (1993) 626

[10] S.M. Tan and D.F. Walls, *to be published* (1994)

[11] R. Deutschmann, W. Ertmer and H. Wallis, *Phys. Rev. A.* **48** (1993) R4023

[12] C. Cohen-Tannoudji, J. Dupont-Roc and G. Grynberg, *Atom-Photon Interactions* (J. Wiley and Sons, New York, 1992)

[13] J. Murphy, P. Goodman and A. Smith *J. Phys.: Condens. Matter* **5** (1993) 4665-4676

[14] B.G. Englert, J. Schwinger and M.O. Scully, *Found. of Phys.* **18** (1988) 1045

TABLES

TABLE I. List of parameters used in the calculation.

$\mathcal{E}_0$	500000 V/m
M_A	6.64×10^{-27} kg
d	4.2×10^{-30} C m
$k = \omega/c$	5.81×10^{6} m^{-1}
$\Gamma/2\pi$	1.6 MHz

Figure Captions

Figure 1. Schematic diagram of transmission-reflection beam splitter (inset: level scheme for J=1 - J'=0).

Figure 2. Spatial dependence of quasipotentials aymptotically connected with ground states $m_g = 0, -1$ for the parameters $A_\pi/A_+ = 0.0294$, effective detuning $\Delta/2\pi = 7.96$ GHz, Larmor frequency $\omega_L/2\pi = 14.3$ MHz.

Figure 3. Populations for the transmitted and reflected beams as the ratio A_π/A_+ is varied. The transmitted and reflected beams are denoted by m_T, m_R respectively. Effective detuning $\Delta/2\pi = 7.96$ GHz, Larmor frequency $\omega_L/2\pi = 14.3$ MHz.

Figure 4. Populations for the transmitted and reflected beams as the Larmor frequency $\omega_L/2\pi$ is varied. The transmitted and reflected beams are denoted by m_T, m_R respectively. Effective detuning $\Delta/2\pi = 7.96$ GHz. the ratio A_π/A_+=0.0294.

Figure 5. Dependence of the populations for the transmitted and reflected beams on the effective detuning. The transmitted and reflected beams are denoted m_T, m_R respectively. Larmor frequency $\omega_L/2\pi = 14.3$ MHz, the ratio A_π/A_+=0.0294.

Figure 6. Spatial dependence of the coefficients $c^2_{0,g}(y)$ and $c^2_{-1,g}(y)$ of the excited state $\Psi_{0,e}$ in the dressed states $\Psi^D_{0,g}(y)$ and $\Psi^D_{-1,g}(y)$. Larmor frequency $\omega_L/2\pi = 14.3$ MHz, the ratio A_π/A_+=0.0294, effective detuning $\Delta/2\pi = 7.96$GHz.

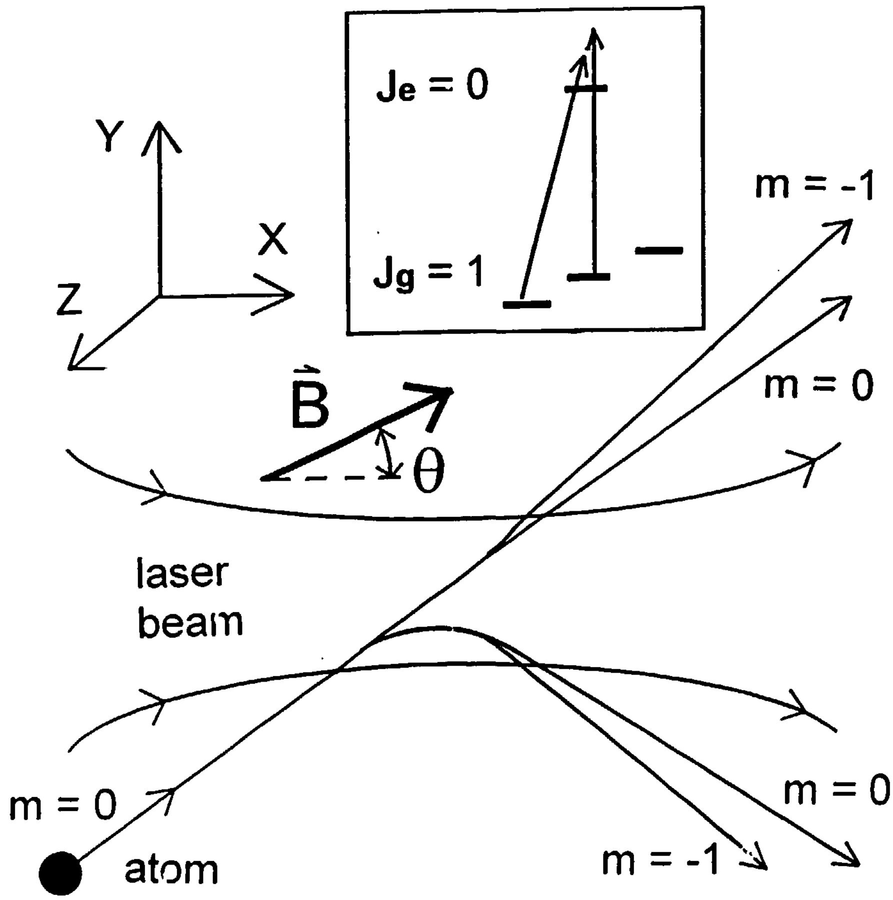
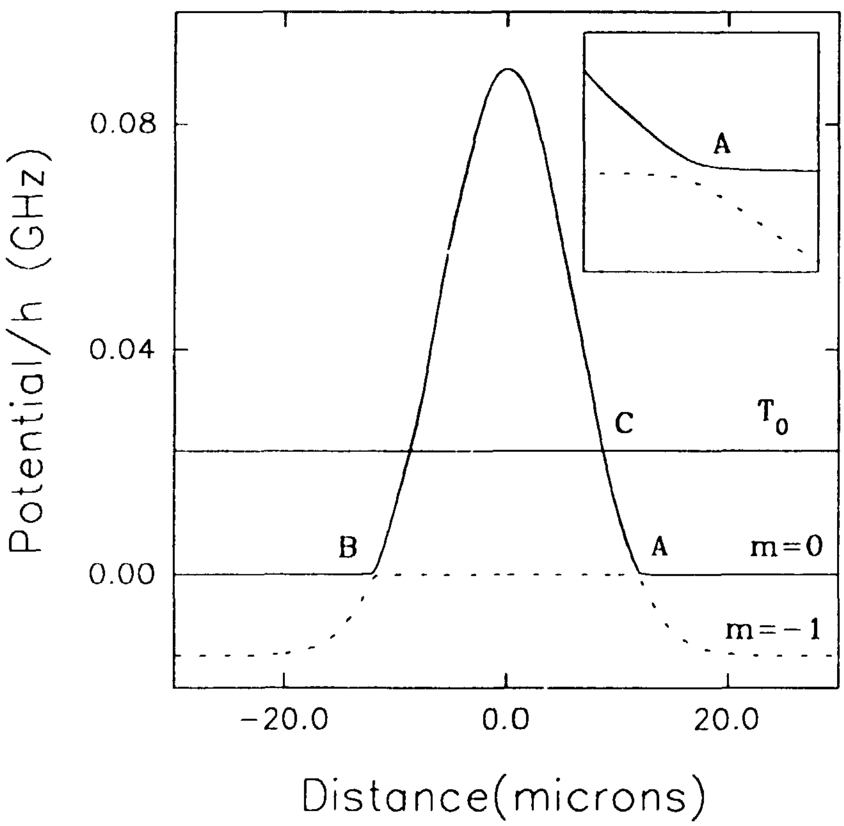


FIGURE 1



0.08
0.04
0.00
Potential/h (GHz)
A
C
T_0
B
A
m=0
m=−1
−20.0
0.0
20.0
Distance(microns)

FIGURE 2

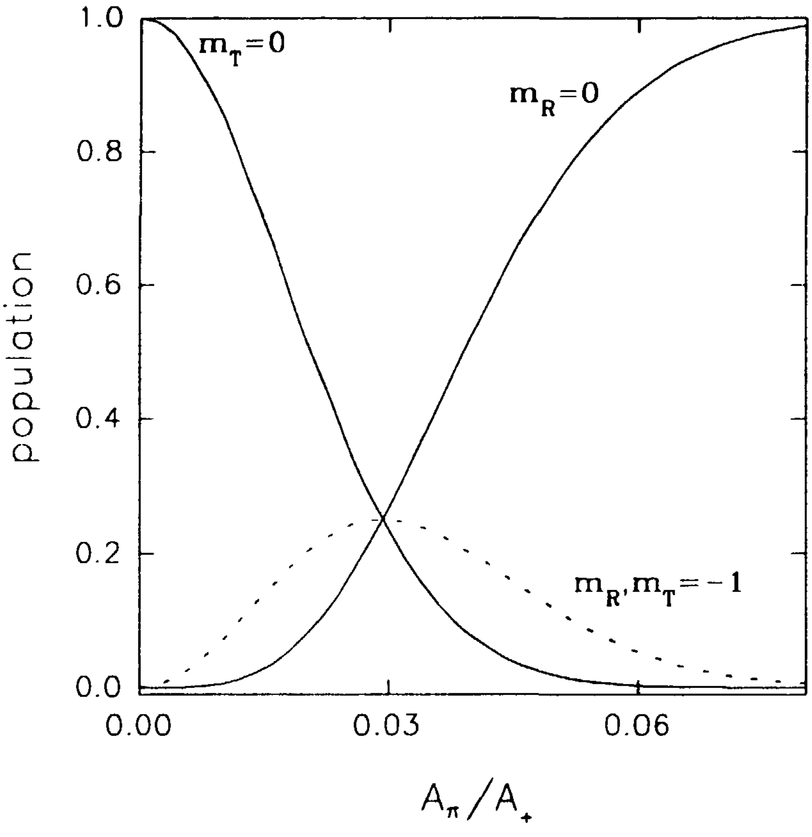


Figure 3

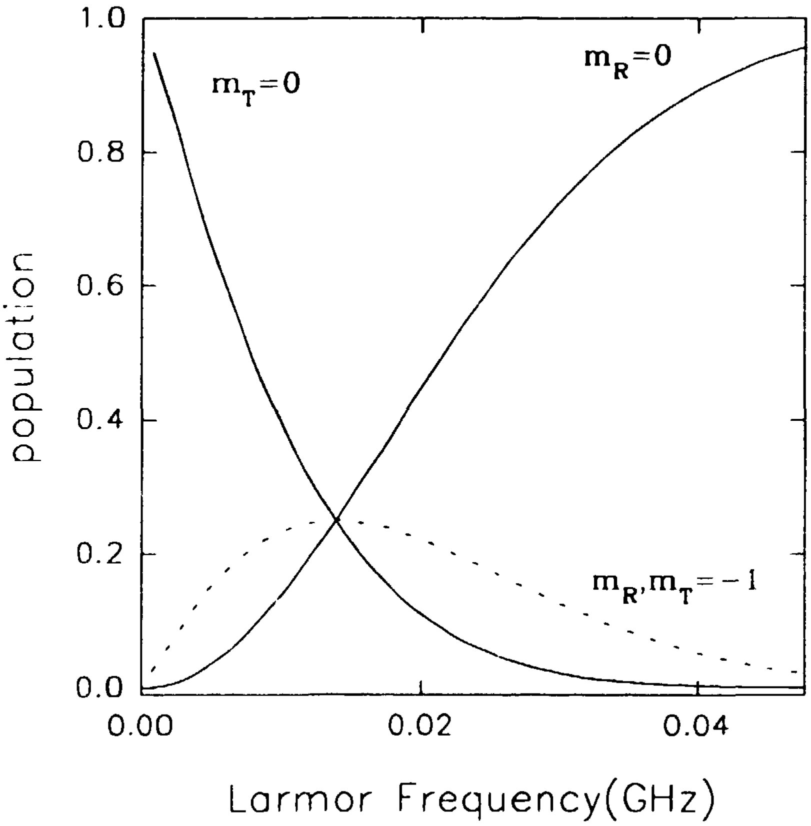

1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.02
0.04
population
Larmor Frequency(GHz)
m_T=0
m_R=0
m_R, m_T=−1

FIGURE 4

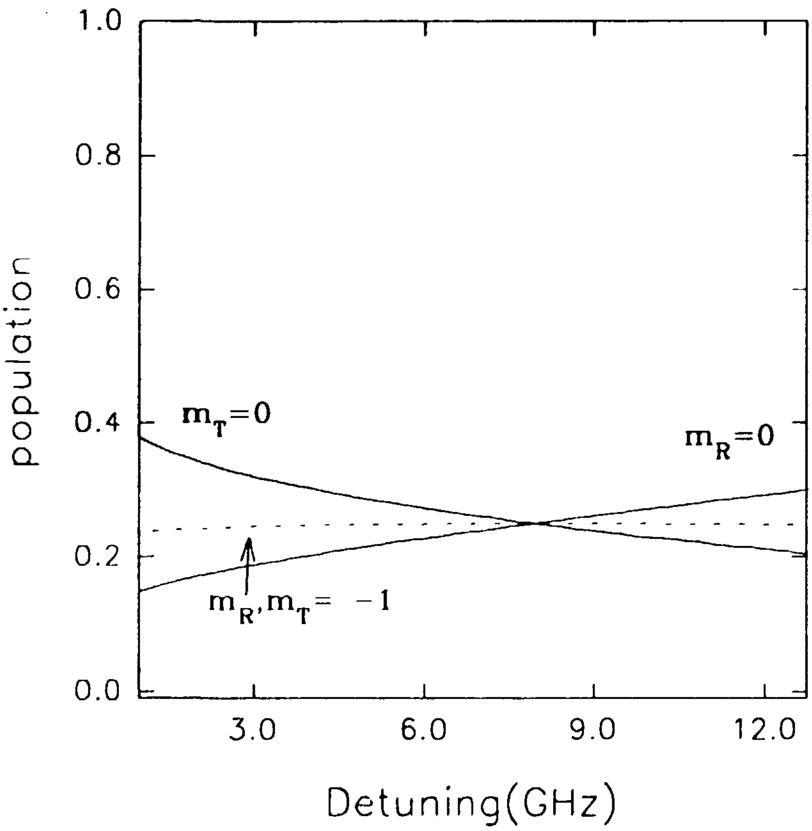

1.0
0.8
0.6
0.4
0.2
0.0
population
m_T=0
m_R=0
m_R,m_T= −1
3.0
6.0
9.0
12.0
Detuning(GHz)

FIGURE 5

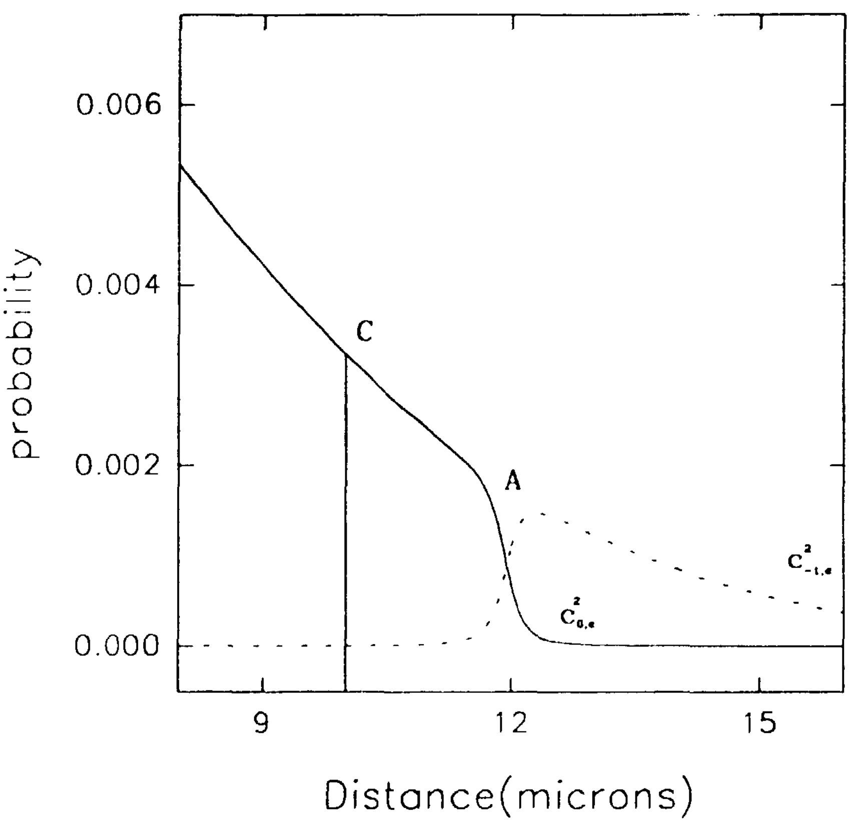

0.006
0.004
0.002
0.000
probability
C
A
$C^2_{-1,e}$
$C^2_{0,e}$
9
12
15
Distance(microns)

FIGURE 6