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Electroweak Interactions on the Lattice

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The Electroweak Interactions

The Standard Model of Particle Physics, with gauge theory as the theoretical framework, has enjoyed unprecedented success in describing and unifying to certain extent the electroweak and strong nuclear forces. However, there remain several longstanding questions at the very foundation of the theory on the one hand; and on the other, phenomena yet to be explained.

The electroweak force distinctly differs from other gauge interactions in its ability to distinguish the handed-ness (chirality) of participating particles and in the massiveness of the vector bosons. In the most direct approach, nevertheless, gauge symmetry is incompatible with massive bosons and chiral gauge symmetry is also inconsistent with massive chiral fermions. In order to overcome these incompatibility and inconsistency, the Higgs mechanism has been introduced as an integral part of the Standard Model. The mechanism, however, is not without its own problems. There is a growing body of field theoretic evidence pointing to the triviality of the mechanism; that is, at best the mechanism can only be treated as an effective, low-energy description of some unknown and yet more fundamental level. Related to the triviality problem is the experimental existence of the Higgs particles demanded by the mechanism. Despite many extensive searches, the Higgs particles have eluded all the efforts to uncover them, pushing the search to higher and higher energy brackets and the validity of the mechanism, imposed by the triviality, to its limit.

Another field theoretic problem of the Standard Model is the lack of a rigorous, non-perturbative definition of chiral gauge theory (CGT). This is, not being overemphasised, one of the most fundamental and important problems of quantum field theory. Because of ultra-violet singularities at short distances, a regulator is necessary to make sense out of a quantum field theory; and the traditional perturbative regulators, which only exist at each order of the perturbative expansion in the coupling, cannot provide an adequate definition. For that purpose, non-perturbative regulators, such as the lattice, are required. But chiral gauge symmetry is generally incompatible with the introduction of the mass scale accompanying the regulators and, in particular, has resisted all the attempts to have it formulated on the lattice. The problem is the lattice fermion doubling phenomenon to be discussed in the next section.

Our point in this paper is that all the seemingly fragmented problems above of the electroweak interactions are in fact related through their connection to the chiral anomaly [1], which is unique to this kind of interactions and separates it from others. It is from this connection that some solutions are proposed for the problems.

The chiral anomaly is the breaking of chiral gauge symmetry at the quantum level due to the quantum fluctuations. One of its manifestations is through the famous triangle Feynman diagram, necessitated by the uv singularities from the quantum fluctuations. Such breaking, however, is undesirable and to be avoided. The gauge symmetry is everything to the gauge theory; it defines the theory, it ensures unitarity and renormalisability of the theory, and without it the mathematical consistency of the theory is lost. The insisted preservation of gauge symmetry imposes stringent constraints on acceptable chiral gauge theory.

It is argued that the constraints are unnecessary because the conventional quantisation of chiral gauge theory has missed out some crucial contributions of the chiral interactions. The subsequently corrected quantisation yields consistent theory in which there is no gauge anomaly and in which various mass terms can be introduced with neither the loss of gauge invariance nor the need for the Higgs mechanism. The new quantisation also provides a solution to the difficulty of how to model the electroweak interactions on the lattice.

Lattice Regularisation and Chiral Fermions

Lattice regularisation is especially important not only because of its fundamental and tradition rôle in defining and regularising quantum field theory but also because it opens the way to rigorous, non-perturbative treatments from the first principles. The lattice is employed as an approximation for the continuum Wick rotated, Euclidean "space-time". The quantum fields then have their values defined at the lattice sites or links, as in the Wilson formulation of lattice gauge fields [2], turning the generating path integral into a mathematically well-defined object of a multi-dimension integral. The continuum limit where continuum physics is recovered is reached at or near to the critical points of the lattice theory.

Fermionic fields, however, are particularly problematic on the lattice. Their lattice transcription suffers the notorious problem of species doubling: the lattice fermions have more species than the continuum counterparts and are always of non-chiral character. For Dirac species of QED and QCD, the problem can be resolved by giving extra lattice species masses of the order of the regulator cut-off, making them decoupled from the physics in the continuum. On the other hand, no solution of this doubling problem has been materialised for chiral fermions of the electroweak interactions. The no-solution situation has been raised to the no-go status by symmetry arguments and formal theorems. Thus, the lattice doubling is not only a technical problem but also a fundamental problem of chiral gauge theory in general; it is as fundamental as the symmetry principle itself.

The Nielsen-Ninomiya no-go theorem [3] stipulates that the lattice fermion doubling is inevitable if the fermion lattice action is bilinear, hermitean in the Euclidean sense, axially symmetric and local and if the lattice has some translational invariance. Several attempts to remove the doublers by removing the various restrictions on the lattice action have not brought much success except when axial symmetry is broken, a situation seemingly incompatible to chiral interactions. To remove the condition of regularity of the lattice, one is led to the consideration of random lattices.

Random lattices have an advantage over regular lattices in that they may approach the continuum quicker than the later could afford. Random lattices also feature prominently in the school of thoughts that lattice is not an approximation but could well be the structure of space-time at the most fundamental level. For our purposes here, random lattices are interesting as they might not be contaminated with fermion doublers. Some studies of *free* fermions on random lattices have claimed that this is in fact the case [6]. However, our results [7] for fermions *with gauge interactions* on a kind of random lattices are disappointing.

The fermion doubling can manifest itself mathematically in the path-integral fermion determinant, resulted when fermionic fields are integrated out. The lattice determinant, properly renormalised, in the continuum limit is the continuum determinant, also properly renormalised, raised to a power which is exactly the number of lattice species including the doublers. Our results, as summarised in the figure, indicate that there is doubling when gauge invariance is maintained on the lattice and no-doubling when gauge symmetry is broken. The no doubling result is not a proof that doubling can be avoided with broken gauge symmetry because it is only in two dimensions and for abelian gauge fields. It is also unclear whether gauge symmetry can be recovered in the continuum limit without further fine tunings. On the other hand, it is certain that there is doubling if there is gauge invariance on our random lattices.

Even though our lattices are of a new kind of randomness not employed before, they do satisfy the crucial condition of no translational invariance and consequently are outside the jurisdiction of the no-go theorem. We thus expect that our negative results are also applicable to the more conventional random lattices. To confirm this expectation, we are currently working on these latter lattices.

The doubling on random lattices is disappointing news to lattice chiral fermions and to the belief that space-time is of discrete (and random) nature if gauge invariance is to be maintained at all scales. Chiral fermions and discrete space-time and gauge invariance do not seem to be compatible. The next section sketches a possible way out of this dilemma.

Quantisation and Lattice Simulations

Quantisation is synonymous with the derivation of Feynman rules in perturbation context or the path integral in general. The conventional quantisation of chiral gauge theory, as of non-chiral gauge theory, leads to a path-integral action identical to the classical action, except Faddeev-Popov gauge fixing terms. From this path-integral action follows

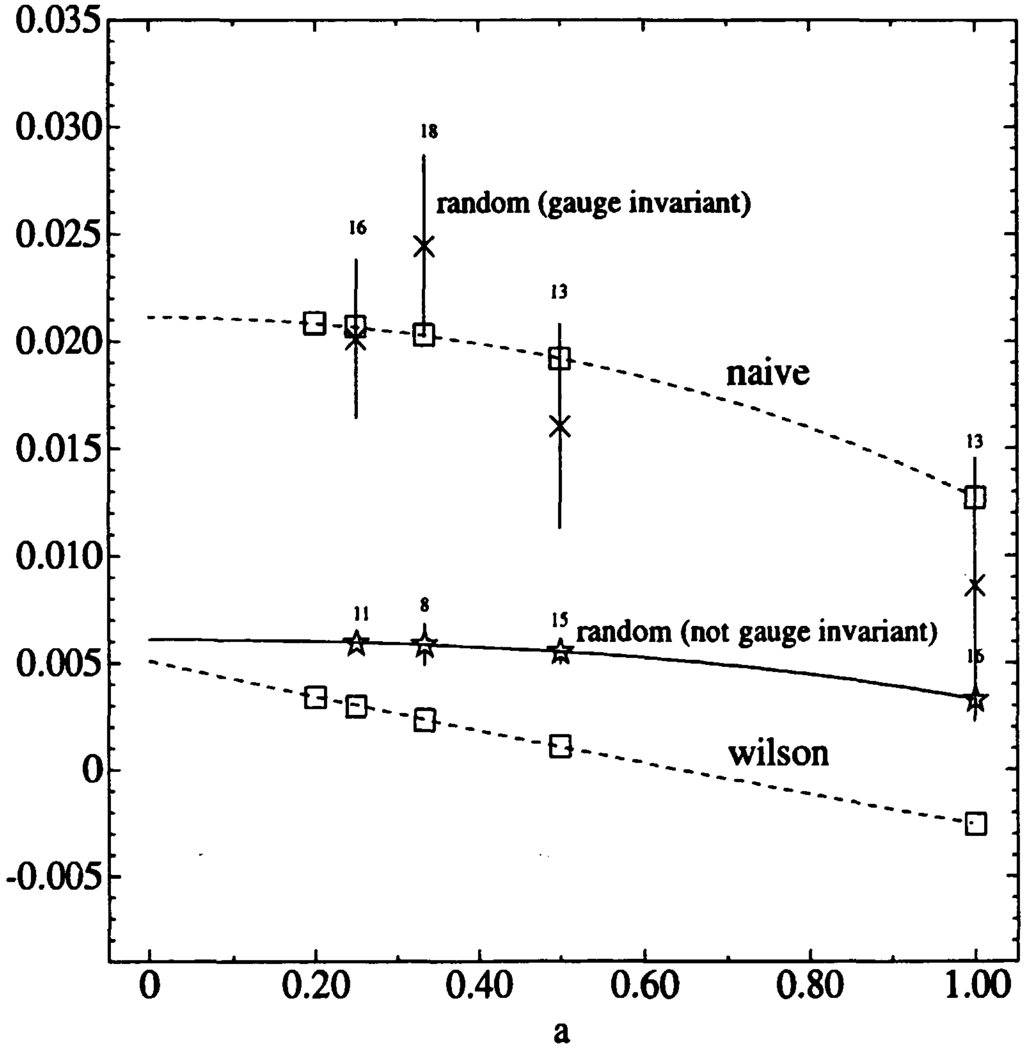

Figure 1: Various lattice fermion determinants in the continuum limit with fixed background gauge fields. The lower branches approach the right continuum behaviour but the lattice actions are either not gauge invariant or not axially symmetric.

the anomalous breaking of the cherished chiral gauge symmetry if the gauge group or its representation is not anomaly-free.

I have reconsidered such an 'anomalous' chiral gauge theory [4] and found that the conventional canonical quantisation in the interaction picture is not correct. The chiral gauge interactions introduce, in two dimensions, non-vanishing commutators and, in higher dimensions, also spoil the double and triple commutator identities, such as the Jacobi and Malcev identities, involving fermion gauge currents. The immediate consequence is now the path-integral action is substantially different from the classical action by a non-local term; and it is this new term that renders the theory consistent and anomaly-free without the conventional constraint of anomaly-free gauge group or representation. The non-locality does not affect the micro-causality in the sense of vanishing commutators at space-like separation. Together with the preserved chiral gauge symmetry, a gauge can be fixed in which the action is not more non-local than the usual gauge fixing procedure, and renormalisability and unitarity should not be violated as explicitly checked in abelian theory. Mass terms, furthermore, for the gauge bosons and chiral fermions can be introduced into the theory without referring to the Higgs mechanism and its associated particles.

The new quantisation above can also be seen directly in the path-integral formalism through the contribution of the Berry's phase [5].

Armed with this new quantisation of chiral gauge theory and particularly the admissibility of a chiral fermion mass, we propose to remove the lattice doublers in the same way as in non-chiral lattice gauge theory [8]. With respect to the no-go theorem, our lattice action, even though possessing chiral gauge invariance, is not globally axially symmetric and thus is not subjected to its consequences. Some appropriate gauge is fixed for a local lattice action. The lattice model is then not manifestly gauge invariant but physical quantities should be since the Ward identities are observed. We expect, and will have to confirm, that doublers are removed and chiral fermions survive the continuum limit. Such decoupling of chiral fermion doublers would normally leave behind a non-local, anomaly-canceling (Wess-Zumino) term, but that term vanishes in our chosen gauge.

We are currently working on a numerical simulation of a two dimensional chiral gauge model to try out the lattice formulation. In general, the lattice chiral fermion determinant is of complex value and this presents a technical difficulty because the determinant is part of the sampling weights (probability distribution) for the simulation. But with appropriate gauge fixing, the imaginary parts should be of the order of lattice spacing or smaller, which is vanishing in the continuum limit. We thus propose to employ the complex Langevin approach [9] and hope that it might work, by virtue of analytic continuation, for sampling weights of small imaginary parts. The outcome of the simulation will be reported elsewhere.

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References

[1] H Fukuda and Y Miyamoto, **Prog Theo Phys 4** (1949) 347;

J Steinberg, **Phys Rev 76** (1949) 1180;
S L Adler, **Phys Rev 177** (1969) 2426;
J Bell and R Jackiw, **Nuovo Cim 60A** ((1969) 47.

[2] K G Wilson, in *New Developments in Quantum Field Theory and Statistical Mechanics*, eds M Lévy and P Mitter (Plenum Press, New York, 1980).

[3] H B Nielsen and M Ninomiya, **Nucl Phys B185** (1981) 20; *ibid* **B193** (1981) 173; *ibid* **B195** (1982) 541; **Phys Lett 105B** (1981) 219.

[4] T D Kieu, **Phys Rev D44** (1991) 2548;
Int J Mod Phys A7 (1992) 177;
Report UM-P-94/58 and references therein;
M Danos and L C Biedenharn, **Phys Rev D36** (1987) 3069.

[5] T D Kieu, **Phys Lett 218B** (1989) 221.

[6] D Espiru, M Gross, P E L Rakow and J F Wheater, **Nucl Phys B275[FS17]** (1986) 39;
Y Pang and H-C Ren, **Phys Lett 195B** (1987) 223;
T W Chiu, **Phys Lett B245** (1990) 570.

[7] C J Griffin and TD Kieu, **Phys Rev Lett 70** (1993) 3844;
Nucl Phys B30 (Proc Supp) (1993) 629;
Report UM-P-93/05, to appear in *Annals of Physics.*

[8] T D Kieu, **Nucl Phys B20** (Proc Supp) (1991) 251 and references therein.

[9] G Parisi, **Phys Lett 131B** (1983) 393;
J R Klauder and W P Petersen, **J Stat Phys 39** (1985) 53;
H Gausterer and J R Klauder, **Phys Lett 164B** (1985) 127;
Phys Rev D33 (1986) 3678.