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THE PARADIGM OF PSEUDODUAL CHIRAL MODELS

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ABSTRACT

This is a synopsis and extension of Phys. Rev. *D49* 5408 (1994). The *Pseudodual Chiral Model* illustrates 2-dimensional field theories which possess an infinite number of conservation laws *but also* allow particle production, at variance with naive expectations—a folk theorem of integrable models. We monitor the symmetries of the pseudodual model, both local and nonlocal, as transmutations of the symmetries of the (very different) usual *Chiral Model.* We refine the conventional algorithm to more efficiently produce the *nonlocal* symmetries of the model. We further find the *canonical transformation* which connects the usual chiral model to its fully *equivalent dual model*, thus contradistinguishing the pseudodual theory.

1. Introduction of the PCM and Outline of its Properties

Many integrable models in two-dimensions evince the limiting feature of *no particle production*, i.e. complete elasticity. There is a variant of the σ-model for which this is not so, however (at least in perturbation theory), the so-called *Pseudodual Chiral Model* of Zakharov and Mikhailov[2], for which all interactions are distilled into a simple, constant torsion term in the lagrangean; it amounts to a delicate Wigner-Inönü contraction of the target manifold in the WZW model in which the "pion decay constant" is taken to infinity in tandem with the topological integer coupling. The essential quantum features of the model were first identified by Nappi[3], who calculated the nonvanishing $2 \to 3$ production amplitude for this model, and who moreover demonstrated that the model was inequivalent to the usual Chiral Model in its behavior under the renormalization group: the Pseudodual Model is not asymptotically free. The physics of the pseudodual model is very different from that of the usual chiral model.

The models were previously compared within the framework of covariant path integral quantization by Fridling and Jevicki, and similarly by Fradkin and Tseytlin[4]. However, the focus of those earlier comparisons was to exhibit (nonabelian-) dualized σ-models with torsion, which were completely equivalent to the usual σ-model. Indeed, it was shown that

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a model fully equivalent but dual to the usual Chiral Model could be constructed, provided both nontrivial torsion and metric interactions were included in the lagrangean.

Here, we focus on the differences between the Pseudodual Model and the usual Chiral Model without enforcing equivalence. We investigate the Pseudodual Model at the classical level and within the framework of canonical quantization, with emphasis on the symmetry structure of the theory. We consider both local and nonlocal symmetries, and compare with corresponding structures in the usual Chiral Model. We present a canonical transformation we have found, which connects the usual Chiral Model with its fully equivalent (nonabelian) dual version, further clarifying the inequivalence of the pseudodual theory† We provide a technically refined algorithm for constructing the conserved nonlocal currents of the pseudodual theory, an algorithm which is particularly well-suited to models with topological currents for which the usual recursive algorithm temporarily stalls at the lowest steps in the recursion before finally producing *genuine nonlocals at the third step and beyond.* In the published paper, we also have considered in detail the current algebra for the full set of local currents in the pseudodual theory, which we omit here. Other, related, more recent investigations can be found in [5,6].

The two-dimensional chiral model (CM) for matrix-valued fields g is defined by

$$\mathcal{L}_1 = \mathrm{Tr}\ \partial_\mu g \partial^\mu g^{-1},$$

with equations of motion which are conservation laws

$$\partial_\mu J^\mu = 0 \qquad \Longleftrightarrow \qquad \partial_\mu L^\mu = 0.$$

$J_\mu \equiv g^{-1}\partial_\mu g$ are the right-, and $L_\mu \equiv g\partial_\mu g^{-1}$ the left-rotation Noether currents of $G_{left} \times G_{right}$, respectively. The pure-gauge form of these currents dictates that the non-abelian field-strength vanishes identically:

$$\partial_\mu J_\nu - \partial_\nu J_\mu + [J_\mu, J_\nu] = 0 \qquad \Longleftrightarrow \qquad \varepsilon^{\mu\nu}\partial_\mu J_\nu + \varepsilon^{\mu\nu} J_\mu J_\nu = 0\ ,$$

and likewise for L_μ. Such curvature–free local currents underlie usual nonlocal-symmetry-generating algorithms[7,8,9,10].

The roles of current conservation and vanishing field strength may be interchanged. A "pseudodual"[4] transformation[2,3] leads to a *different model* for an antisymmetric matrix field ϕ. Define

$$J_\mu = \varepsilon_{\mu\nu}\partial^\nu \phi,$$

conserved identically. But now the curvature-free condition above serves instead as the equation of motion

$$\partial^\mu \partial_\mu \phi - \tfrac{1}{2}\varepsilon_{\mu\nu}[\partial^\mu \phi, \partial^\nu \phi] = 0, \qquad (\P)$$

which follows from the lagrangean of the *Pseudodual Chiral Model* (PCM):

$$\mathcal{L}_2 = -\tfrac{1}{4}\mathrm{Tr}\ \left(\partial^\mu \phi \partial_\mu \phi + \tfrac{1}{3}\phi\varepsilon_{\mu\nu}[\partial^\mu \phi, \partial^\nu \phi]\right).$$

†An abelian penumbrance of this type of canonical transformation has appeared recently in the CERN preprint hepth/9406206 by Álvarez, Álvarez-Gaumé, and Lozano.

Nappi[3] first observed that this model, in contrast to the Chiral Model, is anti-asymptotically free. Actually, this is now possible to establish by inspection, given its subsumption in the general analysis of σ-models with torsion[11]. Introducing a (field-scale) coupling η in the relative normalization of the interaction term, one needs note the complete triviality of the metric (just the kinetic term), $g_{ab} = \delta^{ab}$; the torsion $S_{abc} = \eta f_{abc}\sqrt{g}$ of the interaction term has now collapsed to a constant, merely the structure constant times the coupling, $S_{abc} = \eta f_{abc} = \eta \partial_{[a} e_{bc]}$, for torsion potential $e_{ab} = \eta f_{abc}\phi^c$. This is, in fact, a limiting WZW model—a Wigner-Inönü contraction[12] of the group manifold such that the radius of the target hypersphere (the "pion decay constant") diverges *in tandem* with the integer WZW-term coefficient. To one loop, Braaten, Curtright, and Zachos[11] have shown that e_{ab} evolves by the antisymmetric part of the generalized Ricci tensor, vanishing in this case of constant torsion, so e_{ab} does not renormalize. In contrast, $M\frac{d}{dM}g_{ab} = -S_{acd}S_b{}^{cd}/2\pi = -\eta^2 f_{acd}f_b{}^{cd}/2\pi = -\eta^2\delta_{ab}C/2\pi$, where C is the quadratic adjoint (dual-Coxeter/Casimir) index, e.g. $N-2$ for O(N). Rescaling the kinetic term to canonical normalization amounts to simply *increasing* the interaction coupling as

$$M\frac{d\eta}{dM} = \frac{3\eta}{2}\frac{C\eta^2}{2\pi} = \frac{3}{4\pi}\eta^3 C,$$

in agreement with the original direct calculation[3].

How do the fundamental symmetries generated by these and other currents transmutate? Consider the conserved charge

$$Q = \int dx\ J_0(x).$$

For the CM, the time variation of Q vanishes for field configurations which extremize $\mathcal{L}_1$ by Noether's theorem; while for the PCM, $Q = \phi(\infty) - \phi(-\infty)$, are time-independent for *any* configurations with fixed boundary conditions (ϕ is temporally constant at spatial infinity): Q is a topological "winding" of the field onto the spatial line and hence invariant under the continuous flow of time.

The $\phi \to O^T\phi O$ G_{right}-transformation invariance of $\mathcal{L}_2$ yields the (on-shell conserved) Noether currents

$$R_\mu = [\phi, \tilde{J}_\mu] + \tfrac{1}{3}[\phi,[J_\mu,\phi]] = [\phi, \partial_\mu\phi] + \tfrac{1}{3}\varepsilon_{\mu\nu}[\phi,[\partial^\nu\phi,\phi]],$$

where $\tilde{J}_\mu \equiv \varepsilon_{\mu\nu}J^\nu$. In contrast to the CM, it is these currents, and not J_μ, which generate (adjoint) right-rotations in the PCM.

The PCM is also invariant under the nonlinear symmetries[3] $\phi \to \phi + \xi$ with Noether currents

$$Z_\mu = \tilde{J}_\mu + \tfrac{1}{2}[J_\mu,\phi] = \partial_\mu\phi + \tfrac{1}{2}\varepsilon_{\mu\nu}[\partial^\nu\phi,\phi].$$

The conservation law for these currents amounts to the equations of motion Eq.(¶) for the PCM (introduced as a null-curvature condition for the topological J_μ currents of the model). The equations of motion have been transmuted from conservation of J_μ for the CM to conservation of Z_μ for the PCM. These Z_μ currents are not curvature-free, however, but are instead J-covariant-curl-free $\varepsilon^{\mu\nu}\partial_\mu Z_\nu + \varepsilon^{\mu\nu}[J_\mu, Z_\nu] = 0$. The currents Z_μ are contracted vestiges of the axial currents of the WZW model, and we term them "pseudoabelian" since their charges commute among themselves (more precicely, they close into the topological charge, vanishing only for topologically trivial configurations), even though this is not so for the entire current algebra[1]. (Correspondingly, $J_\mu - R_\mu$ are vestiges of the vector currents of the WZW model.)

These "new" local conserved currents, Z_μ and R_μ, are actually transmutations of the usual first and second nonlocal currents of the CM, respectively. All three sets of currents, J_μ, Z_μ, R_μ, transform in the adjoint representation of $O(N)_{right}$ (the charge of R_μ). The left-invariance G_{left} has degenerated: for the field ϕ, left transformations are inert, and thus right, or axial, or vector transformations are all indistinguishable. The $G_{left} \times G_{right}$ symmetry of the chiral model, the axial generators of which are realized nonlinearly, has thus mutated in the PCM. On the one hand it has been reduced by the loss of G_{left}, but on the other hand it has been augmented by the nonlinearly realized pseudoabelian Q_Z charges.

The left-currents L_μ of the CM don't generate left-rotations on the PCM fields ϕ, any more than the J_μ generate right-rotations. In the PCM, L_μ are realized *nonlocally*: $\partial_\mu g = g\ \varepsilon_{\mu\nu}\partial^\nu\phi$, so $\partial_1 g = g\ \partial_0\phi$, integrated at a fixed time,

$$g(x,t) = g_0\ P\exp(\int_x^\infty dy\ \partial_0\phi(y,t))\ ,$$

assuming $g(\infty,t) = g_0$. Consequently,

$$L_\mu = g\partial_\mu g^{-1} = -g\ (g^{-1}\partial_\mu g)\ g^{-1} = -g\ J_\mu\ g^{-1} = -\varepsilon_{\mu\nu}\ g\ \partial^\nu\phi\ g^{-1} =$$

$$= \quad -\varepsilon_{\mu\nu}\partial^\nu(g\ \phi\ g^{-1}) + g\ [\partial_\mu\phi\ ,\phi]\ g^{-1}.$$

These transform in the adjoint of G_{left}, but these transformations only rotate the arbitrary boundary conditions g_0, and do *not* affect ϕ at all. They thus commute with the right-rotations. Discarding g_0 then banishes G_{left} from the theory altogether.

None of the above results hinges on the difference between left- and right-currents. Left↔Right-reflected identical results would have followed upon interchange of left with right.

2. Canonically Equivalent Dual σ–model

The above nonlocal, invertible, fixed-time *map* relating all g and ϕ field configurations is, nevertheless, **not a canonical transformation**. The quantum theories for $\mathcal{L}_1$ and $\mathcal{L}_2$ are thus inequivalent (e.g. perturbation theory assumes canonical variables). As an aside, we find instead a canonical transformation which maps the usual CM onto an *equivalent* **Dual Sigma Model** (DSM), with torsion, different from the PCM, in broad agreement with the result of conventional nonabelian duality transformations[4].

E.g. consider the standard O(4) $\simeq$ O(3)×O(3) $\simeq$ SU(2)×SU(2) CM, with $g = \varphi^0 + i\tau^j\varphi^j$, φ^0, φ^j $(j = 1,2,3)$, and $(\varphi^0)^2 + \varphi^2 = 1$, where $\varphi^2 \equiv \sum_j (\varphi^j)^2$. Resolve $\varphi^0 = \pm\sqrt{1-\varphi^2}$, to get the CM,

$$\mathcal{L}_1 = \frac{1}{2}\left(\delta^{ij} + \frac{\varphi^i\varphi^j}{1-\varphi^2}\right)\partial_\mu\varphi^i\partial^\mu\varphi^j.$$

This is **canonically equivalent** to the DSM:

$$\mathcal{L}_3 = \frac{1}{1+4\psi^2}\left(\tfrac{1}{2}\left(\delta^{ij} + 4\psi^i\psi^j\right)\partial_\mu\psi^i\partial^\mu\psi^j - \varepsilon^{\mu\nu}\varepsilon^{ijk}\psi^i\partial_\mu\psi^j\partial_\nu\psi^k\right),$$

which differs from the PCM, $\mathcal{L}_2$, but reduces to it in the weak ψ field limit, i.e. it contracts to it similarly to the Wigner-Inönü contraction of the WZW model. However, *no* such canonical transformation may lead to the PCM instead.

The generator for a canonical transformation relating φ and ψ at any fixed time is $F[\psi,\varphi]=\int_{-\infty}^{\infty}dx\;\psi^i J_i^1[\varphi]$, (where we choose[‡] the right, $V+A$, J_μ),

$$F[\psi,\varphi]=\int_{-\infty}^{+\infty}dx\;\psi^i\left(\sqrt{1-\varphi^2}\,\overleftrightarrow{\frac{\partial}{\partial x}}\varphi^i+\varepsilon^{ijk}\varphi^j\frac{\partial}{\partial x}\varphi^k\right).$$

The conjugate momentum of ψ^i:

$$\begin{aligned}\pi_i=\frac{\delta F[\psi,\varphi]}{\delta\psi^i} &= \sqrt{1-\varphi^2}\,\frac{\partial}{\partial x}\varphi^i-\varphi^i\frac{\partial}{\partial x}\left(\sqrt{1-\varphi^2}\right)+\varepsilon^{ijk}\varphi^j\frac{\partial}{\partial x}\varphi^k=\\ &= \left(\sqrt{1-\varphi^2}\,\delta^{ij}+\frac{\varphi^i\varphi^j}{\sqrt{1-\varphi^2}}-\varepsilon^{ijk}\varphi^k\right)\frac{\partial}{\partial x}\varphi^j=J_i^1.\end{aligned}$$

The conjugate of φ^i:

$$\begin{aligned}\varpi_i=-\frac{\delta F[\psi,\varphi]}{\delta\varphi^i} &= \left(\sqrt{1-\varphi^2}\,\delta^{ij}+\frac{\varphi^i\varphi^j}{\sqrt{1-\varphi^2}}+\varepsilon^{ijk}\varphi^k\right)\frac{\partial}{\partial x}\psi^j\\ &+\left(\frac{2}{\sqrt{1-\varphi^2}}\left(\varphi^i\psi^j-\psi^i\varphi^j\right)-2\varepsilon^{ijk}\psi^k\right)\frac{\partial}{\partial x}\varphi^j,\end{aligned}$$

Substitute for π_i and ϖ_i, in terms of $\frac{\partial}{\partial t}\varphi^j$ and $\frac{\partial}{\partial t}\psi^j$, as follows from $\mathcal{L}_1$ and $\mathcal{L}_3$:

$$\pi_i=\frac{1}{1+4\psi^2}\left(\left(\delta^{ij}+4\psi^i\psi^j\right)\frac{\partial}{\partial t}\psi^j+2\varepsilon^{ijk}\psi^j\frac{\partial}{\partial x}\psi^k\right),\qquad \varpi_i=\left(\delta^{ij}+\frac{\varphi^i\varphi^j}{1-\varphi^2}\right)\frac{\partial}{\partial t}\varphi^j.$$

The resulting covariant pair of first-order, nonlinear, partial differential equations for φ and ψ constitute a Bäcklund transformation connecting the two theories. Consistency of this Bäcklund transformation is equivalent to the classical equations of motion for φ and ψ.

Moreover, the relations

$$\pi\cdot\pi=\varphi'\cdot\varphi'+\frac{(\varphi\cdot\varphi')^2}{1-\varphi^2},$$

$$\psi'\cdot\psi'=\varpi^2-(\varphi\cdot\varpi)^2+4\pi^2\psi^2-4(\pi\cdot\psi)^2-4\sqrt{1-\varphi^2}\;\varepsilon^{ijk}\varpi_i\psi_j\pi_k-4\varphi\cdot\psi\;\varpi\cdot\pi+4\varphi\cdot\pi\;\varpi\cdot\psi,$$

$$\varepsilon^{ijk}\psi_i\pi_j\psi'_k=-2\psi^2\pi^2+2(\psi\cdot\pi)^2+\sqrt{1-\varphi^2}\varepsilon^{ijk}\varpi_i\psi_j\pi_k+\varphi\cdot\psi\pi\cdot\varpi-\varphi\cdot\pi\;\psi\cdot\varpi,$$

may be combined to demonstrate the equivalence of the hamiltonian densities in the respective theories:

$$\mathcal{H}_3=4\varepsilon^{ijk}\psi_i\pi_j\psi'_k+\pi^2+\psi'\cdot\psi'+4\psi^2\pi^2-4(\psi\cdot\pi)^2=\varpi^2-(\varphi\cdot\varpi)^2+\varphi'\cdot\varphi'+\frac{(\varphi\cdot\varphi')^2}{1-\varphi^2}=\mathcal{H}_1.$$

[‡]N.B. Left-rotations on φ alone do nothing to this F; ψ^i is a left-transformation singlet, just like its conjugate quantity, $J_i^1[\varphi]$, and $F[\psi,\varphi]$ is left-invariant.

Now, in the DSM, **what is the conserved, curvature–free current?** In contrast to the PCM, where it was essentially *forced* to be a topological current, here a topological current by itself will not suffice; neither will a conserved Noether current. (Under isospin transformations, $\delta\psi^i = \varepsilon^{ijk}\psi^j\omega^k$, the Noether current of $\mathcal{L}_3$ is $I_i^\mu = \delta\mathcal{L}_3/\delta(\partial_\mu\omega^i)$ so $I_i^0 = \varepsilon^{ijk}\psi^j\pi_k$, but it is not curvature-free.)

Instead, the conserved, curvature-free current $\mathcal{J}_i^\mu[\psi,\pi] = J_i^\mu[\varphi,\varpi]$ (identified with J_i^μ of the CM) is a *mixture* of the Noether isocurrent and a topological current: $\mathcal{J}_i^\mu = 2I_i^\mu - \varepsilon^{\mu\nu}\partial_\nu\psi^i$, so that $\mathcal{J}_i^1 = \pi_i$. Both conservation and curvature-freedom now hold on-shell.

$$\mathcal{J}_i^\mu = \frac{-1}{1+4\psi^2}\left(\left(\delta^{ij}+4\psi^i\psi^j\right)\varepsilon^{\mu\nu}\partial_\nu\psi^j + 2\varepsilon^{ijk}\psi^j\partial^\mu\psi^k\right).$$

$$\mathcal{J}_i^1 \equiv \pi_i = \left(\sqrt{1-\varphi^2}\,\delta^{ij} + \frac{\varphi^i\varphi^j}{\sqrt{1-\varphi^2}} - \varepsilon^{ijk}\varphi^k\right)\frac{\partial}{\partial x}\varphi^j \equiv J_i^1,$$

$$\mathcal{J}_i^0 \equiv -\frac{\partial}{\partial x}\psi^i - 2\varepsilon^{ijk}\psi^j\pi_k = -\sqrt{1-\varphi^2}\,\varpi_i - \varepsilon^{ijk}\varphi^j\varpi_k \equiv J_i^0.$$

This last equation may also be integrated directly to yield ψ in terms of φ, given the pure-gauge (zero curvature) feature of $J_\mu[\varphi] = g^{-1}\partial_\mu g$, on which the canonical transformation was predicated:

$$\frac{\partial}{\partial x}\psi = \psi J_1 - J_1\psi - J_0 \qquad \Longrightarrow \qquad \frac{\partial}{\partial x}(g\psi g^{-1}) = -gJ_0g^{-1} = g\partial_0 g^{-1}\ .$$

The argument of the r.h.s. has reduced to a *left* current component. This equation readily integrates to

$$\psi(x) = g^{-1}(x)g(0)\psi(0)g^{-1}(0)g(x) + g^{-1}(x)\Big(\int_0^x dy g(y)\partial_0 g^{-1}(y)\Big)g(x)\ .$$

N.B. Field-parity properties: under $\varphi \to -\varphi$, the right current for the CM converts to the left current, so that $F[-\psi,-\varphi]$ generates a canonical transformation which projects onto right-invariants, instead.

The connections among the four models discussed are summarized in the diagram:

$$\begin{array}{ccc} WZW & \xrightarrow{\ contraction\ } & PCM\ \mathcal{L}_2 \\ {\scriptstyle null\ integer\ coupling}\big\downarrow & & \big\uparrow{\scriptstyle contraction} \\ CM\ \mathcal{L}_1 & \xleftrightarrow{\ canonical\ equivalence\ } & DSM\ \mathcal{L}_3\ . \end{array}$$

3. Nonlocal Currents and Charges for the Pseudodual Model

The full set of nonlocal conservation laws [7,8,9,10,13] follows from any conserved, curvature-free currents such as J_μ, irrespective of the specific model considered. Introduce a dual boost[14] spectral parameter κ to define

$$C_\mu(x,\kappa) = -\frac{\kappa^2}{1-\kappa^2}J_\mu - \frac{\kappa}{1-\kappa^2}\tilde{J}_\mu\ ,$$

where $\tilde{J}_\mu \equiv \varepsilon_{\mu\nu} J^\nu$. Given these properties of J_μ, it follows that

$$(\partial^\mu + C^\mu)\,\tilde{C}_\mu = 0 \ .$$

This serves as the consistency condition for the two equations

$$\partial_\mu \chi^{ab}(x) = -C_\mu^{ac}\,\chi^{cb}(x) \ ,$$

or, equivalently,

$$\varepsilon_{\mu\nu}\partial^\nu \chi = \kappa\ (\partial_\mu + J_\mu)\ \chi \ ,$$

which are solvable recursively[8] in κ. Equivalently, the solution χ can be expressed as a path-ordered exponential (Polyakov's path-independent disorder[10] variable)

$$\chi(x,\kappa) = P\exp\Big(-\int_{-\infty}^{x} dy\ C_1(y,t)\Big) \equiv \mathbb{1} + \sum_{n=0}^{\infty} \kappa^{n+1}\chi^{(n)} \ .$$

These ensure conservation of an **antisymmetrized** nonlocal "master current":

$$\mathfrak{J}^\mu(x,\kappa) \equiv \frac{1}{2\kappa}\ \varepsilon^{\mu\nu}\partial_\nu\ \Big(\chi(x,\kappa) - \chi^T(x,\kappa)\Big) \equiv \sum_{n=0}^{\infty} \kappa^n\ J^\mu_{(n)}(x) \ .$$

The conserved master current acts as the generating functional of all currents $J^\mu_{(n)}$ (separately) conserved order-by-order in κ. E.g. the lowest 4 orders yield:

$$\begin{aligned}
\mathfrak{J}_\mu(x,\kappa) \ &= \ J_\mu(x) \ + \ \kappa\Big(\tilde{J}_\mu(x) + \tfrac{1}{2}\Big[J_\mu(x)\ ,\ \int_{-\infty}^{x} dy\ J_0(y)\Big]\Big) \ + \\
&\quad + \kappa^2\Big(\tilde{J}^{(1)}_\mu(x) + \tfrac{1}{2}(J_\mu(x)\ \chi^{(1)} + \chi^{(1)T}\ J_\mu(x))\Big) \ + \\
&\quad + \kappa^3\Big(\tilde{J}^{(2)}_\mu(x) + \tfrac{1}{2}(J_\mu(x)\ \chi^{(2)} + \chi^{(2)T}\ J_\mu(x))\Big) \\
&\quad + \mathcal{O}(\kappa^4) \ .
\end{aligned}$$

This yields a conserved "master charge"

$$\mathfrak{Q}(\kappa) = \int_{-\infty}^{+\infty} dx\ \mathfrak{J}_0(x,\kappa) \equiv \sum_{n=0}^{\infty} \kappa^n\ Q_{(n)} \ .$$

$Q_{(0)}$ is the conventional symmetry charge, while $Q_{(1)}, Q_{(2)}, Q_{(3)}, \ldots$ are the well-known nonlocal charges, best studied for $\sigma-$models[7,8,10], the Gross-Neveu model[15], and supersymmetric combinations of the two[9].

However, for the PCM,

$$J^{(0)}_\mu = J_\mu = \varepsilon_{\mu\nu}\partial^\nu\phi, \qquad \Longrightarrow \qquad \chi^{(0)}(x) = \phi(x) - \phi(-\infty), \qquad \leadsto$$

$$J^{(1)}_\mu = \partial_\mu\phi + \tfrac{1}{2}\varepsilon_{\mu\nu}[\partial^\nu\phi, \phi] - \tfrac{1}{2}[J_\mu, \phi(-\infty)] = Z_\mu - \tfrac{1}{2}[J_\mu, \phi(-\infty)].$$

Recall $\phi(-\infty)$ is taken to be time-independent, and thus each piece of this current is separately conserved. So, the CM$\leftrightarrow$PCM transmutation has yielded **a local** current for the first nonlocal hopeful! Moreover,

$$\chi^{(1)}(x) = \int_{-\infty}^{x} dy \,\left(\partial_0\phi(y) + \partial_1\phi(y)\;\phi(y)\right) - \phi(x)\phi(-\infty) + \phi(-\infty)^2 \,.$$

Likewise, $J_\mu^{(2)} =$

$$= \varepsilon_{\mu\nu}\partial^\nu\phi + [\partial_\mu\phi, \phi] - \phi\;\varepsilon_{\mu\nu}\partial^\nu\phi\;\phi + \tfrac{1}{2}\varepsilon_{\mu\nu}\partial^\nu\left(\phi\;\chi^{(1)} + \chi^{(1)T}\phi\right)$$
$$-\tfrac{1}{2}[Z_\mu, \phi(-\infty)] + \;\tfrac{1}{4}\varepsilon_{\mu\nu}\partial^\nu(\phi^2\phi(-\infty) + \phi(-\infty)\phi^2) \;=$$
$$= J_\mu - R_\mu - \tfrac{1}{3}\varepsilon_{\mu\nu}\partial^\nu(\phi^3) \;+ \tfrac{1}{2}\varepsilon_{\mu\nu}\partial^\nu\left(\phi\chi^{(1)} + \chi^{(1)T}\phi\right) - \tfrac{1}{2}[Z_\mu, \phi(-\infty)]$$
$$+\;\tfrac{1}{4}\varepsilon_{\mu\nu}\partial^\nu(\phi^2\phi(-\infty) + \phi(-\infty)\phi^2).$$

On-shell properties of the currents have been used. However, this second "nonlocal" current is also effectively *local*: the skew-gradient term, which might appear to contribute a nonlocal piece to the charge via $\chi^{(1)}$, only contributes $[\phi(\infty), Q_Z]/2$, i.e. a trivial piece based on a local current.

But the third step in the recursive algorithm is different:

$$J_\mu^{(3)} = \tfrac{1}{2}\left(Z_\mu\chi^{(1)} + \chi^{(1)T}Z_\mu\right) + \dots\,.$$

(...) terms contribute only local pieces to the charge, whereas the term written contributes ineluctable nonlocal pieces. Thus $J_\mu^{(3)}$ is *genuinely nonlocal*, like all higher currents. The action of $Q^{(3)}$ (slightly improved to Q_N, as detailed below) on the field changes the boundary condition at $x = \infty$ to a different one than at $-\infty$, and thereby switches its topological sector, which is quantified by $Q^{(0)}$:

$$[\![\, Q_N, \phi^{ab}(y)\,]\!] \;= -[[M^{ab}, \phi(y)], \phi(y)] + 2\int_{-\infty}^{+\infty} dx\;\varepsilon(y-x)[Z_0(x), M^{ab}],$$

where $\;(M^{ab})_{cd} \equiv \delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc},\;$ and $[\![\,,]\!]$ represents Poisson brackets, in contrast to matrix commutators $[,]$.

In summary, for the pseudodual model, the charge $Q^{(0)}$ is topological, while $Q^{(1)}$ generates pseudoabelian shifts, $Q^{(2)}$ generates right rotations, and $Q^{(n\geq 3)}$ appear genuinely nonlocal.

4. Refinements and Remarks

The above master current construction starts off with a non-Noether (topological) current, then "stalls" twice at the first two steps before finally producing genuine non-locals at the third step and beyond. Here is an improved algorithm which begins with the lowest nontopological (Noether) current Z_μ to produce an alternate conserved master current which only stalls once. Define[13]

$$W_\mu(x,\kappa) \equiv Z_\mu + \kappa\tilde{Z}_\mu,$$

which is C-covariantly conserved:

$$\partial^\mu W_\mu + [C^\mu, W_\mu] = 0 \ .$$

This condition then empowers W_μ to serve as the seed for a new and improved conserved master-current

$$\mathfrak{W}_\mu(x,\kappa) = \chi^{-1} W_\mu \, \chi = Z_\mu + \kappa\,(T_\mu - [Z_\mu, \phi(-\infty)]) +$$
$$+\kappa^2 \left(N_\mu - [T_\mu, \phi(-\infty)] + \tfrac{1}{2}[[Z_\mu, \phi(-\infty)], \phi(-\infty)]\right) + \mathcal{O}(\kappa^3),$$

where we have introduced

$$T_\mu \equiv J_\mu - \tfrac{3}{2} R_\mu = \tilde{Z}_\mu + [Z_\mu, \phi],$$

and where now the terms of second order and higher are genuinely nonlocal; e.g.

$$N_\mu = [T_\mu, \phi] - \tfrac{1}{2}[[Z_\mu, \phi], \phi] + [Z_\mu, \int_{-\infty}^{x} dy \ Z_0(y)] \ .$$

This is a refined equivalent of $J^{(3)}_\mu$ above. The terms in $\mathfrak{W}_\mu$ involving the constant matrices $\phi(-\infty)$ are separately conserved.

In general, the seeds for such improved master currents only need be conserved currents, such as Z_μ above, which also have a vanishing J-covariant-curl. E.g. the previous nonlocal currents themselves may easily be fashioned to satisfy J-covariant-curl-free conditions and thereby seed respective conserved master currents.

In summary, at tree level (and thus for massless excitations), it has been made evident that particle production is not prevented by nonlocal conservation laws, as holds for the CM[7], but is often thought to automatically occur in general[5]. In our paper, we further work out the current algebra of the currents discussed, and we moreover list the known *local* sequence of conserved currents predicated on conserved, curvature-free currents such as J_μ. But, in this case, elasticity theorems[16] on the prevention of particle production as a consequence of Lorentz tensor charges such as those are evaded, since they require massive states, which are absent at the semiclassical level considered here.

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