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Possible reason why leptons are lighter than quarks

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Abstract

The minimal model of spontaneously broken leptonic colour and discrete quark-lepton symmetry predicts that charged leptons have the same masses as their partner charge +2/3 quarks up to small radiative corrections. By invoking a different pattern of symmetry breaking, a similar model can be constructed with the structural feature that charged leptons have to be lighter than their partner quarks because of fermion mixing effects. As well as furnishing a new model-building tool, this is phenomenologically interesting because the scale of the new physics responsible for the quark-lepton mass hierarchy could be as low as several hundred GeV.

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The patterns evident in the mass and mixing angle spectrum of quarks and leptons continue to challenge us to provide an explanation. One may broadly categorise these patterns as hierarchies between generations, between weak-doublet partners and between quarks and leptons. We do not know if these three sub-problems can be solved separately, or if an all-encompassing explanation is necessary. In this paper I will introduce a novel suggestion for why the charged lepton is less massive than the charge +2/3 quark for a given generation. An analysis of the explanatory success of my proposed mechanism will lead us to discuss how the quark-lepton hierarchy problem might be connected with other hierarchy problems.

The obvious place to look for a reason for quarks to be heavier than leptons is in the dynamics of colour. Is there any reason why coloured fermions in a generation should be more massive than colourless fermions? There is a well-known answer to this question in the context of ultra-high-scale unification theories: If quarks and leptons have similar or equal running masses in the 10^{15} GeV to Planck mass regime, then gluonic interactions affect the running to lower energies in such a way as to raise quark masses by roughly the correct amount relative to lepton masses [1]. However, evolution through thirteen orders of magnitude or more in energy is required since the masses run only logarithmically. Although this is an interesting observation, it has the observational disadvantage that the new physics of mass generation would be difficult to test properly. Is there a way of understanding the quark-lepton mass hierarchy through new physics at much lower energy scales, perhaps as low as several hundred GeV? What do we need to do with the dynamics of colour to achieve this end?

One likely avenue is through a spontaneously broken colour group for *leptons* and discrete quark-lepton symmetry (or q-ℓ symmetry for short) [2]. These ideas have been pursued for the last few years [3]. The original motivation for them was to connect the quantum numbers of quarks and leptons by new symmetries that could be spontaneously broken at a relatively low scale such as 1 TeV. However, one of the interesting effects of increasing symmetry beyond the SU(3)⊗SU(2)⊗U(1) of the Standard Model (SM) is that parameters such as coupling constants that were previously unrelated can become connected together. Indeed, it was immediately noticed that in the minimal model discrete q-ℓ symmetry enforced the tree-level mass relations

$$m_{e,\mu,\tau} = m_{u,c,t} \quad \text{and} \quad m^{\text{Dirac}}_{\nu_e,\nu_\mu,\nu_\tau} = m_{d,s,b}. \tag{1}$$

The most constructive way to view the phenomenologically unacceptable charged-lepton–up-quark equality is as a spring-board for further pondering. Although it is unacceptable *per se*, we after all ultimately do want a theory that will relate quark and lepton masses. I will show how this equality can be transformed into an explanation for why charged leptons are less massive than their up quark partners. (The equality between neutrino Dirac masses and down quark masses is perfectly acceptable if one uses the see-saw mechanism [4] to explain why the standard neutrinos have such tiny masses.)

We begin by supposing that the SM gauge group G_{SM} is embedded within the larger gauge group $G_{q\ell}$ where

$$G_{q\ell} \equiv \mathrm{SU}(3)_\ell \otimes \mathrm{SU}(3)_q \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_X, \tag{2}$$

where $\mathrm{SU}(3)_q$ is the usual quark colour group, $\mathrm{SU}(3)_\ell$ is leptonic colour and X is an Abelian charge different from weak hypercharge Y. A fermionic generation is assigned to representations of $G_{q\ell}$ in the following way:

$$\begin{array}{lll} Q_L \sim (1,3,2)(1/3), & u_R \sim (1,3,1)(4/3), & d_R \sim (1,3,1)(-2/3), \\ F_L \sim (3,1,2)(-1/3), & E_R \sim (3,1,1)(-4/3), & N_R \sim (3,1,1)(2/3). \end{array} \tag{3}$$

Notice that quarks have exactly the same classification under $G_{q\ell}$ as they do under G_{SM} and that $X = Y$ for quarks. This representation pattern yields an anomaly-free theory, and it also enables us to define the discrete symmetry

$$Q_L \leftrightarrow F_L, \quad u_R \leftrightarrow E_R, \quad d_R \leftrightarrow N_R, \quad G_q^\mu \leftrightarrow G_\ell^\mu, \quad W^\mu \leftrightarrow W^\mu, \quad C^\mu \leftrightarrow -C^\mu, \tag{4}$$

between the quarks and the generalized lepton fields F_L, E_R and N_R, and between the various gauge boson multiplets (G_q^μ are the usual gluons, G_ℓ^μ are leptonic colour gluons, W^μ are weak gauge bosons and C^μ is the gauge boson for X).

The standard leptons are located within the generalized multiplets F_L, E_R and N_R. Their precise identification depends on the spontaneous symmetry breakdown pattern of the $\mathrm{SU}(3)_\ell \otimes \mathrm{U}(1)_X$ part of the gauge group. In q-ℓ symmetric models investigated hitherto [2, 3], the breakdown $\mathrm{SU}(3)_\ell \otimes \mathrm{U}(1)_X \to \mathrm{SU}(2)' \otimes \mathrm{U}(1)_Y$ was employed. Weak hypercharge Y was identified with $X + T_8/3$ where $T_8 \equiv \mathrm{diag}(-2, 1, 1)$ is a diagonal generator of leptonic colour. The standard leptons were then seen to be the $T_8 = -2$, $\mathrm{SU}(2)'$ singlet components of F_L, E_R and N_R. The other two leptonic colours were then exotic charge $\pm 1/2$ heavy fermions which lay in doublets of the unbroken, asymptotically-free residual $\mathrm{SU}(2)'$ gauge force. They were confined into integrally-charged, even-body, unstable bound states of $\mathrm{SU}(2)'$. Many aspects of this and related theories have been explored in the literature [2, 3], so I will not dwell on them here.

However, it will be useful to explicitly show how the mass relations of Eq. (1) arose. As in the SM, one introduced a single electroweak Higgs doublet $\phi \sim$

$(1,1,2)(1)$. Because of the q-ℓ discrete symmetry of Eq. (4), its Yukawa interactions were constrained to be

$$\mathcal{L}_{\rm Yuk} = \frac{m_u}{u}(\overline{Q}_L u_R \phi^c + \overline{F}_L E_R \phi) + \frac{m_d}{u}(\overline{Q}_L d_R \phi + \overline{F}_L N_R \phi^c) + \text{H.c.} \tag{5}$$

where $\phi^c \equiv i\tau_2\phi^*$. Electroweak symmetry breakdown via $\langle\phi\rangle = (0,u)^T$ then produced the quark-lepton mass relations because of the discrete q-ℓ symmetry (under which $\phi \leftrightarrow \phi^c$).

Note that the identification of standard leptons as possessors of a unique leptonic colour was important in this derivation. If standard leptons were a superposition of components of *different* leptonic colour, then Eq. (5) would not necessarily produce the mass relations of Eq. (1). The model I construct below is based on this observation.

In order to break the nexus between leptonic colour and standard leptons, one needs to spontaneously break the SU(2)$'$ subgroup of SU(3)$_\ell$ that is left exact in the usual version of the model. I will call the resulting theory the "Completely Broken Leptonic Colour Model." It uses the same gauge group $G_{q\ell}$ and fermion representation content as the usual q-ℓ symmetric model, but a different Higgs sector.

The non-electroweak Yukawa Lagrangian is given by $\mathcal{L}'_{\rm Yuk}$ where

$$\begin{aligned}\mathcal{L}'_{\rm Yuk} &= h_1[\overline{(F_L)^c}F_L\chi + \overline{(Q_L)^c}Q_L\chi'] + h_2[\overline{(N_R)^c}E_R\chi + \overline{(d_R)^c}u_R\chi'] \\ &+ h_3[\overline{(N_R)^c}N_R\xi + \overline{(d_R)^c}d_R\xi'] + h_4[\overline{(N_R)^c}E_R\Delta + \overline{(d_R)^c}u_R\Delta'] + \text{H.c.}\end{aligned} \tag{6}$$

and the Higgs bosons are

$$\begin{aligned}&\chi \sim (3,1,1)(2/3) \quad \text{and} \quad \chi' \sim (1,3,1)(-2/3),\\ &\xi \sim (3,1,1)(-4/3) \quad \text{and} \quad \xi' \sim (1,3,1)(4/3),\\ &\Delta \sim (\overline{6},1,1)(2/3) \quad \text{and} \quad \Delta' \sim (1,\overline{6},1)(-2/3).\end{aligned} \tag{7}$$

These Higgs multiplets are grouped into obvious pairs under the q-ℓ discrete symmetry. The electroweak Higgs sector again contains one electroweak doublet ϕ, and the Yukawa Lagrangian is the same as Eq. (5).

Spontaneous symmetry breaking proceeds in at least two stages. First, the fields χ, ξ and Δ gain nonzero vacuum expectation values (VEVs) to break both leptonic colour and the discrete q-ℓ symmetry, leaving electroweak SU(2)$_L\otimes$U(1)$_Y$ unbroken. (The partner Higgs fields χ', ξ' and Δ' must of course have zero VEVs to keep standard colour exact.) The non-standard fermions in the theory gain nonzero masses via $\mathcal{L}'_{\rm Yuk}$ at this stage. The standard leptons (and quarks) are defined to be those fermions that remain massless. This process will split up into distinct symmetry breaking events should there be a hierarchy amongst the VEVs for χ, ξ and Δ. The electroweak gauge symmetry is then broken in the second stage of symmetry breaking via the usual nonzero VEV for ϕ. This also generates nonzero masses for the standard leptons. For phenomenological reasons we will require that $\langle\chi\rangle, \langle\xi\rangle, \langle\Delta\rangle \gg \langle\phi\rangle$.

The VEVs of the leptonically coloured Higgs bosons are induced to take the forms

$$\langle\chi\rangle = \begin{pmatrix} w \\ 0 \\ 0 \end{pmatrix}, \quad \langle\xi\rangle = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix} \quad \text{and} \quad \langle\Delta\rangle = \begin{pmatrix} 0 & a & 0 \\ a & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{8}$$

where Δ is represented by a 3×3 symmetric matrix.[2] A simple calculation shows that this VEV pattern induces the breakdown

$$\mathrm{SU}(3)_\ell \otimes \mathrm{SU}(3)_q \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_X \to \mathrm{SU}(3)_q \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \equiv G_{SM}, \tag{9}$$

where weak hypercharge Y is given by

$$Y = X + \frac{T_8}{3} + T_3, \tag{10}$$

with T_3 being the diagonal generator diag$(0, 1, -1)$ of leptonic colour. The major difference between the Completely Broken Leptonic Colour Model and the usual q-ℓ symmetric model is the presence of T_3 in the formula for Y. In the usual model T_3 is, of course, an unbroken generator and thus has nothing to do with weak hypercharge.

Using Eq. (10) we see that the leptonic colour components of the generalized leptons have weak hypercharges given by

$$Y(F_L) = Y \begin{pmatrix} \ell_L \\ (f_R)^c \\ f_L \end{pmatrix} = \begin{pmatrix} -1 \\ +1 \\ -1 \end{pmatrix}, \quad Y(E_R) = Y \begin{pmatrix} e_{1R} \\ \nu_{2R} \\ e_{3R} \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ -2 \end{pmatrix},$$

[2]It is important to check that this VEV pattern can be the minimum of the Higgs potential for a range of parameters. The Higgs potential V for this model is quite complicated, and I will not write it down in this paper since I want to focus on the issue of fermion mass. Ideally, one would like to write V in the sum-of-squares form $\Sigma\lambda_i|\text{quadratic form}|^2$, where the λ_i's are chosen to be positive and "quadratic form" is a quadratic function of the Higgs fields. The global minimum of V is then obtained by simply making each quadratic form zero. My analysis shows that most of the terms in V can be written in this manner in such a way that the required alignment of nonzero VEVs ensues. There are a few terms that I have not succeeded in writing thus, so the required region of parameter space presumably forces the coefficients of these recalcitrant terms to be somewhat smaller than the λ_i's. If these coefficients are zero, then there is an unwanted global U(1) symmetry in V and an unwanted pseudo-Goldstone boson is produced. A rigorous analysis would need to show that this potentially light boson is made sufficiently heavy when the recalcitrant terms are switched on. This issue is beyond the scope of this paper.

$$Y(N_R) = Y \begin{pmatrix} \nu_{1R} \\ (e_{2L})^c \\ \nu_{3R} \end{pmatrix} = \begin{pmatrix} 0 \\ +2 \\ 0 \end{pmatrix}, \tag{11}$$

where we have used a suggestive notation for the colour components of F_L, E_R and N_R. As a further piece of notation, let the weak-isospin components of the colours of F_L be given by

$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad (f_R)^c = \begin{pmatrix} (\epsilon_R)^c \\ (n_R)^c \end{pmatrix} \quad \text{and} \quad f_L = \begin{pmatrix} n_L \\ \epsilon_L \end{pmatrix}. \tag{12}$$

Equation (11) reveals that the generalized leptons contain, per generation, (i) the standard leptons, (ii) mirror or vector-like pairs of ℓ_L-like and e_R-like states and (iii) two additional ν_R-like particles. After the first stage of symmetry breakdown the vector-like pairs form massive Dirac fermions, while the ν_R-like states all become massive. The standard ℓ_L and e_R leptons are defined to be the remaining massless states.

Inputting the VEV pattern of Eq. (8) into the Yukawa Lagrangian of Eq. (6), and combining this with $\langle\phi\rangle$ inputted into the electroweak Yukawa Lagrangian, we find that the charged lepton mass matrix is given by

$$\mathcal{L}^{\text{ch lept}}_{\text{Yuk}} = \begin{pmatrix} \overline{e}_L & \overline{e}_{2L} & \overline{\epsilon}_L \end{pmatrix} \begin{pmatrix} m_u & 0 & 0 \\ M_4 & M_2 & m_d^T \\ 0 & m_u & M_1 \end{pmatrix} \begin{pmatrix} e_{1R} \\ e_{3R} \\ \epsilon_R \end{pmatrix} + \text{H.c.} \tag{13}$$

where

$$M_1 \equiv h_4 a, \quad M_2 \equiv h_2 w \quad \text{and} \quad M_1 \equiv (h_1 + h_1^T) w. \tag{14}$$

The terms m_u, m_d, $M_{1,2,4}$ are all 3×3 matrices in generation space. Let us call the full mass matrix in Eq. (13) M_-.

To get a feel for what this mass matrix does, let us turn off the generation structure for the moment. In the absence of the electroweak contributions m_u and m_d we see that:

(1) The $Q = -1$ field e_L is massless and thus identified as the standard left-handed electron. The other charged members $\epsilon_{L,R}$ of F_L form the left- and right-handed components of a Dirac fermion of mass M_1. (Actually the whole weak-doublet $f = f_L + f_R$ of Dirac fermions has mass $M_1 w$.)

(2) The fields $\epsilon'_L \equiv e_{2L}$ and $\epsilon'_R \equiv \sin\phi\, e_{3R} + \cos\phi\, e_{1R}$ form a $Q = -1$ Dirac fermion ϵ' of mass $\sqrt{M_2^2 + M_4^2}$, where $\tan\phi = M_4/M_2$. The right-handed field orthogonal

to ϵ'_R is massless and thus identified as the standard right-handed electron: $e_R \equiv \cos\phi\, e_{3R} - \sin\phi\, e_{1R}$.

When the electroweak terms m_u and m_d are switched on, e_L and e_R are connected by a diagonal mass and they also mix with the heavy exotic electron-like states ϵ and ϵ'. The mass of the physical standard electron is obtained by calculating the magnitude of the smallest eigenvalue of M_- (or we could work with $M_- M_-^\dagger$ if we wanted to). Continuing to ignore generation structure, we see that $\det M_- = m_u(M_1 M_2 - m_u m_d)$. From the phenomenologically motivated hierarchy between the non-electroweak VEVs and the electroweak VEV, we expect that $M_{1,2,4} \gg m_{u,d}$ so that $\det M_- \simeq m_u M_1 M_2$. To zeroth order in $m_{u,d}$ the large eigenvalues are still M_1 and $\sqrt{M_2^2 + M_4^2}$, so see we that

$$\text{smallest eigenvalue} \equiv m_e \simeq m_u \cos\phi \leq m_u, \tag{15}$$

where

$$\cos\phi \equiv \frac{M_2}{\sqrt{M_2^2 + M_4^2}}. \tag{16}$$

This equation illustrates the central result of this paper: *Mixing between electron-like states of different leptonic colour lowers the electron mass from that of its q-ℓ symmetric partner the up quark.*[3]

In the multi-generation real world, it is clear that each of the ratios m_e/m_u, m_μ/m_c and m_τ/m_t can be separately adjusted to fit the measurements. This can be trivially seen by supposing we have three generations but no inter-generational mixing. Each generation can have different values for their corresponding M_2 and M_4 matrices, so the corresponding values for $\cos\phi$ can be different.

The neutrino sector before electroweak symmetry breakdown splits into massless left-handed neutrinos ν_L, massive fermions $n \sim n_L + n_R$ which are degenerate with the ϵ's, plus a right-sector mass matrix given by

$$\mathcal{L}^{\text{neut}}_{\text{mass}} = \begin{pmatrix} \overline{(\nu_{1R})^c} & \overline{(\nu_{2R})^c} & \overline{(\nu_{3R})^c} \end{pmatrix} \frac{1}{2} \begin{pmatrix} 0 & M_4 & M_3 \\ M_4^T & 0 & -M_2^T \\ -M_3 & -M_2 & 0 \end{pmatrix} \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix} + \text{H.c.} \tag{17}$$

where $M_3 \equiv (h_3^T - h_3)v$ and $M_{2,3,4}$ are 3×3 matrices in generation space. Note that M_3 is antisymmetric in generation space. Diagonalization of the whole right-sector neutrino mass matrix yields all eigenvalues as nonzero and of order M_i. The

[3]This type of result was first explicitly calculated by M. de Jonge [5] in the context of a quark-lepton symmetric model with a Higgs sector different from the one I use here. The primary characteristic of the Higgs sector used in Ref. [5] was the non-minimal combination of two χ-type leptonic triplets.

submatrix M_3 is crucial for this. If M_3 is zero then there are zero eigenvalues. This is an interesting connection between neutrino masses and the multi-generation structure. With all nine right-handed neutrinos shown to be massive Majorana particles, the usual see-saw mechanism [4] for neutrinos will ensue when electroweak symmetry breakdown occurs.

The eight gauge bosons of leptonic colour acquire masses of the order of $g_s(\Lambda)\Lambda$ where g_s is the strong coupling constant and $\Lambda \sim \langle\chi\rangle$, $\langle\xi\rangle$ and $\langle\Delta\rangle$. Neutral current and other phenomenology will typically require that these gauge bosons are heavier than about 1 TeV, which means that the leptonic colour breaking VEVs should be of the order of a few TeV or more. Note that the $\cos\phi$ suppression factor can be large even if the leptonic colour breaking scale is much higher than a few TeV. This is because the mixing between electron-like states of different leptonic colour is controlled by M_4 and thus it increases with Λ.

Let us now evaluate the successes and failures of the above scenario:

(i) We have clearly succeeded in constructing a quark-lepton symmetric model that has both a minimal electroweak Higgs sector and acceptable quark-lepton mass relations. In the usual q-ℓ symmetric model, one can evade the relations in Eq. (1) by postulating two electroweak Higgs doublets rather than one [6]. Although we have evaded the mass relations by expanding the Higgs sector (in order to completely break leptonic colour), each of the Higgs multiplets ϕ, χ, ξ and Δ has different quantum numbers so in that sense the Higgs sector is "minimal."

(ii) But the most important achievement is the fact that charged leptons are forced to be less massive than up-quarks by a *structural ingredient of the model.* This provides us with an interesting new technique in model-building, and is the main point I want to make in this paper. In this regard, my mechanism is closely related in spirit to the see-saw mechanism [4] and the universal see-saw mechanism [7]. The former is a way of using fermion mixing to understand why neutrinos are much lighter than any other fermion, while the latter is a way of employing fermion mixing to understand why fermions are generally much lighter than the electroweak scale. My mechanism, on the other hand, is a way to employ fermion mixing to understand why charged leptons are lighter than up quarks. Furthermore, it is a low-energy (or potentially low-energy) alternative to the running mass idea alluded to in the introductory paragraphs. I would also like to stress that the mechanism itself is almost certainly of more interest than the specific model I have chosen by way of illustration here. (This is after all also true of the see-saw and universal see-saw mechanisms.) For instance, there are non-minimal q-ℓ symmetric models [8] that have $m_e = m_d$ rather than $m_e = m_u$ which can also probably be modified to incorporate my mechanism.

This could be of great interest since the charged-lepton–down-quark hierarchy is less severe than the charged-lepton–up-quark hierarchy for the second and third generations, and thus it might be easier to explain.

(iii) If one is going to explain why leptons are lighter than quarks, then one should to begin with have a reason for connecting their masses. This entails a symmetry principle, and if one is interested in a low-energy explanation then discrete q-ℓ symmetry via leptonic colour is the primary candidate. Thus, my mechanism presupposes these particular additional gauge and discrete symmetries and thus much new phenomenology also. Note that the see-saw mechanism can be implemented without any extra symmetries (although one could argue that it finds its most natural implementation within the left-right symmetric model), while the universal see-saw mechanism requires left-right symmetry.

(iv) The specific model examined here easily incorporates both the see-saw mechanism for neutrinos and my new mechanism in a reasonably coherent theoretical structure.

(v) The model, however, fails to account for the quark-lepton mass hierarchy in quantitative detail. In the one-generation case of Eqs. (15) and (16) we require M_4 to be significantly larger than M_2 in order to reproduce any of m_e/m_u, m_μ/m_c or m_τ/m_t. It is interesting that M_4 is proportional to the sextet Higgs VEV while M_2 is proportional to a triplet Higgs VEV. This indicates that the quark-lepton mass hierarchy might be related to a VEV hierarchy and we would have to search for a fundamental reason for the sextet Higgs to have a larger VEV than the triplet Higgs. However, such a VEV hierarchy is not enough since the additional hierarchy $m_e/m_u > m_\mu/m_c > m_\tau/m_t$ can only be incorporated by adjusting Yukawa coupling constants. However, it is clearly unfair to demand that my mechanism explain the quark-lepton hierarchy in this much detail. It would be very surprising in my view if a theory *perfectly* explained one type of hierarchy in the quark-lepton sector but left the others accomodated but unexplained. I have deliberately made no attempt to address the generation, mixing angle and down-up quark hierarchies in the present model. It is quite likely that only those models that seek to explain the whole lot can adequately explain any one specific hierarchy. For instance, one can speculate that the M_2 and M_4 parameters in $\cos\phi$ perhaps should come from different generations so that $M_4 > M_2$ for the same general reason that, say, $m_c > m_u$. My point in this introductory article is simply to present my new mixing mechanism in an unencumbered context. The wider, deeper and much more challenging issues raised above are clearly matters for the future. (Of course, these types of observations can also be made about the see-saw and universal see-saw mechanisms. Neither has anything to say about generations or mixing angles.)

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