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Reflection and Diffraction of Atomic de Broglie Waves by Evanescent Laser Waves ------ Bare-State Method

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Abstract

We present two methods for the investigation of the reflection and diffraction of atoms by gratings formed either by standing or travelling evanescent laser waves. Both methods use the bare-state rather than dressed-state picture. One method is based on the Born series, whereas the other is based on the Laplace transformation of the coupled differential equations. The two methods yield the same theoretical expressions for the reflected and diffracted atomic waves in the whole space including the interaction and the asymptotic regions.

1. Introduction

The reflection and diffraction of atomic de Broglie waves by laser radiation have attracted increasing attention in the developing field of atom optics [1-9]. Atomic mirrors and gratings formed by evanescent laser waves are expected to be used as optical elements in highly sensitive atom interferometers [3,4]. Experimental and theoretical studies of the characteristics of such optical elements have been carried out [3-5, 7,8]. A schematic of such a grating is shown in Figure 1.

The original treatment by Cook [1] was semi-classical. Fully quantum mechanical analyses of the reflection and diffraction of atomic de Broglie waves have been undertaken by several authors [3,5,7,8]. The dressed-state approach [5] gives great insight into the mechanisms of reflection and diffraction and with the aid of multi-slice techniques [8] has yielded results consisent with experiment [9]. The successful dressed-state method may be understood as follows. To reflect atoms, a strong laser beam is required to form a high potential barrier for the incoming atomic de Broglie waves. However, the strong laser field also causes a large amount of Rabi oscillations at a frequency proportional to the amplitude of the laser field. As Rabi oscillations represent energy exchanges between the atoms and the laser field, it is also accompanied by high speed atomic population changes between different diffraction orders (defined by the atomic momentum states, i.e. bare-states). The behaviour of the atomic populations is complicated and thus makes direct numerical calculations using the bare-state description difficult. In the dressed-state description, the atomic states are redefined to include the high frequency Rabi oscillations. As a result, the populations of such "dressed atoms" will show less complicated variations in the interaction region and the numerical calculations are much easier than the bare-state case.

On the other hand, in the dressed-state description the populations of each diffraction order are not explicitly expressed in the interaction region. Instead, a mixture of them is included in the dressed-states, and the reflected and diffracted atoms of each diffraction order are obtained by the asymptotic solutions of the dressed-states in the region where the atom-field interaction disappears. If we go to the inside of the interaction region, populations of each diffraction order can only be obtained by a bare-state description.

The dressed-state and the bare-state descriptions are complimentary. In the asymptotic region, each description has states corresponding to the various diffraction orders. However, in the interaction region they are quite different;

this fact allows us to study the diffraction mechanism from different points of view. In the dressed-state description, quasi-potentials are defined for each state enabling us to study the trajectories of the atom along the quasi-potential curves within the interaction region. On the other hand, in the bare-state description each state is defined by a particular momentum (or wave number) in the x direction (Fig. 1), providing us with a way to study the problem in the momentum space.

In this paper we base our studies on the bare-state picture. The traditional Born series [3] is used to solve both the standing and travelling wave gratings. The travelling wave grating is also solved using a Laplace transformation method and is compared directly to the Born series calculation.

2. Atom-Field Interaction Model and Coupled Differential Equations

Fig.1 shows schematically the atomic mirror considered in this paper. A laser beam inside a quartz block is totally reflected at the quartz-vacuum interface with an angle θ larger than the critical angle. Thus on the surface of the quartz block an evanescent travelling wave is produced. Two such counter-propagating laser beams produce an evanescent standing wave. In the case of an evanescent standing wave, the wave vector is given by $Q_x \hat{\imath} + iq\hat{\jmath}$ with

$$Q_x = (\omega/c)N\sin\theta, \qquad \text{and} \qquad q = (\omega/c)(N^2\sin^2\theta - 1)^{1/2},$$

and the electric field can be written as

$$\begin{aligned} E(t,x,y) = \exp(-qy)\,\{\, & E_0\exp[-i(\omega t - Q_x x)] + E_0^*\exp[i(\omega t - Q_x x)] \\ & + E_0\exp[-i(\omega t + Q_x x)] + E_0^*\exp[i(\omega t + Q_x x)]\,\}\,. \end{aligned}$$

Here, the electric field **E** of the laser beam is assumed to be linearly polarised in the z direction with $\mathbf{E} = \hat{k}E$.

We consider the de Broglie wave of a two-level atom whose level spacing is near the laser frequency. Its incoming momentum is $P_o = (\hbar k_x, \hbar k_y)$. We ignore the motion in the z direction (as the atom has constant momentum in z direction). The wave function in the Schrodinger picture may be written in the rotating wave approximation as

$$\Psi(t,x,y) = \sum_{n=\text{even}} \exp(-i\Omega_o t)\exp[i(k_x + nQ_x)x]\,\phi_n(y)\,\varphi_g$$

$$+ \sum_{n=odd} \exp[-i(\Omega_o + \omega)t] \exp[i(k_x + nQ_x)x]\, \phi_n(y)\, \varphi_e \;,$$

(2-1)

where the initial energy $\hbar\Omega_o = \frac{\hbar^2}{2M}(k_x^2 + k_y^2)$, and ω is the angular frequency of the laser beam. The Hamiltonian governing the motion of the atom is given by

$$\hat{H} = \hat{H}_a + \hat{P}^2/(2M) - \hat{\mu}\, E \;, \tag{2-2}$$

where $\hat{H}_a$ is the Hamiltonian of the atomic internal energy, $\hat{P}$ is the atomic momentum, and $\hat{\mu}$ is the electric dipole moment.

The Schrodinger equation $i\hbar(\partial/\partial t)\Psi = H\Psi$ leads to the following differential equations coupling the wave functions $\phi_n(y)$ of different diffraction orders [3]:

$$\left(\frac{d^2}{dy^2} + k_{yn}^2 \right) \phi_n(y) = - \frac{2M\mu E_o}{\hbar^2} \exp(-qy)\, [\phi_{n+1}(y) + \phi_{n-1}(y)] \;, \tag{2-3}$$

where

$$k_{yn}^2 = (k_y^2 + k_x^2) - (k_x + nQ_x)^2 \;, \qquad \text{(n: even)}$$

$$k_{yn}^2 = (k_y^2 + k_x^2) - (k_x + nQ_x)^2 + \frac{2M}{\hbar}(\omega - \omega_a) \;, \qquad \text{(n: odd)}$$

and we set $\mu = \langle a|\hat{\mu}|b\rangle = \langle b|\hat{\mu}|a\rangle$, and $E_o = E_o^*$. Here, ω_a is the atomic transition frequency, and M is the atomic mass.

3. Solution Based on Born Series
----- The Standing Wave Case

With the Green's function $G_n(y,y_o) = (1/2ik_{yn})\exp(ik_{yn}|y-y_o|)$ which satisfies an outgoing wave boundary condition, the differential equations (2-3) can be transformed to the following integral equations [3]:

$$\phi_n(y) = \delta_{n,o} \exp(-ik_{yo}y)$$

$$- \frac{2M\mu E_o}{\hbar^2} \frac{1}{2ik_{yn}} \int_0^{\infty} \exp(ik_{yn}|y-y_o|) \exp(-qy_o) [\phi_{n+1}(y_o) + \phi_{n-1}(y_o)] dy_o. \quad (3\text{-}1)$$

Here the first term represents the incoming de Broglie wave, and the second term represents the outgoing scattered waves. In the Born series approximation [10], we assume the incoming wave in the ground state $\phi_o(y) = \exp(-ik_{yo}y)$ remains unchanged in the whole interaction region, and use it as a source to generate scattered waves $\phi_{+1}(y)$ and $\phi_{-1}(y)$. These scattered waves become new sources for the generation of waves $\phi_{+2}(y)$ and $\phi_{-2}(y)$ as well as for higher order contributions to the zeroth order wave $\phi_o(y)$. The newly generated $\phi_o(y)$ includes both an outgoing wave component $\exp(ik_{yo}y)$ and an incoming wave component $\exp(-ik_{yo}y)$. Repeating such processes, we obtain $\phi_n(y)$ in the form of Born series:

$$\phi_n(y>0) = \sum_{b=0}^{Nb} \beta_{nb} \exp[(-bq - ik_{yo})y] + \sum_{a=-Na}^{Na} \sum_{b=0}^{Nb} \alpha_{nab} \exp[(-bq + ik_{ya})y].$$

(a,b: integer) (3-2)

Here the index a corresponds to the wave number k_{ya} in y direction which contributes to the nth order wave $\phi_n(y)$ through the scattering in the interaction region. These waves do not contribute uniformly throughout the interaction region. The damping term $\exp(-bqy)$ becomes sharper with increasing b. We show in next section for the case of a simple travelling evanescent wave, that the coefficients β_{nb} and α_{nab} converge at large b. Hence we set a upper limit N_b for the index b. We also set a lower limit $-N_a$ and an upper limit $+N_a$ for the index a (i.e. we limit the waves considered in a given calculation).

Instead of doing the iteration as stated above (which is used in a previous paper[3]), we can calculate the coefficients β_{nb} and α_{nab} by solving the following two sets of coupled linear equations which satisfy the integral equations (3-1) or the differential equations (2-3),

$$\beta_{nb} = B_{nb} (\beta_{n+1,b-1} + \beta_{n-1,b-1}) , \quad (b = 1,......,N_b)$$

$$\beta_{no} = \delta_{n,o} ; \quad (b = 0)$$

(3-4)

and

$$\alpha_{nab} = A_{nab} \left(\alpha_{n+1,a,b-1} + \alpha_{n-1,a,b-1} \right) , \qquad (b = 1,...N_b; \; a = -N_a,...,0,...,N_a)$$

$$\alpha_{nao} = \alpha_n \, \delta_{n,a} , \qquad (b = 0; \; a = -N_a,...,0,...,N_a)$$

$$\alpha_n = \sum_{b=1}^{N_b} X_{nb} \beta_{nb} + \sum_{a=-N_a}^{N_a} \sum_{b=1}^{N_b} Y_{nab} \alpha_{nab} , \qquad (b = 0, a = 0)$$

(3-5)

where

$$n = -N_a, ..., 0, ..., N_a .$$

and

$$B_{nb} = \frac{-\eta^2}{[k_{yn}^2 + (-ik_{yo} - bq)^2]} ,$$

$$A_{nab} = \frac{-\eta^2}{[k_{yn}^2 + (ik_{ya} - bq)^2]} ,$$

$$X_{nb} = -\frac{ik_{yn} - ik_{yo} - bq}{2ik_{yn}} , \qquad (3\text{-}6)$$

$$Y_{nab} = -\frac{ik_{yn} + ik_{ya} - bq}{2ik_{yn}} ,$$

$$\eta^2 = \frac{2M\mu E_o}{\hbar^2} .$$

Here, the equations (3-4) for β_{no} and (3-5) for α_{nao} arise from the boundary conditions.

As y approaches infinity, the damping terms disappear, and eq.(3-2) becomes

$$\lim_{y \to \infty} \phi_n(y) = \delta_{n,o} \exp(-ik_{yo}y) + \alpha_n \exp(ik_{yn}y) . \qquad (3\text{-}7)$$

Thus, α_n gives the amplitude of nth order reflected wave.

We notice that the coefficients α_{nab} are not involved in equations (3-4), so we can solve them in two steps. First, on solving the set of $(N_b+1)(2N_a+1)$ equations in (3-4), we get the solution for all the $(N_b+1)(2N_a+1)$ coefficients of β_{nb}. This can be done by calculating an inverse matrix of the size $[(N_b+1)(2N_a+1)]^2$ numerically. With the result for β_{nb}, we can then solve all the α_{nab} in eq.(3-5) in the same way but for a larger matrix of the size $[(N_b+1)(2N_a+1)^2]^2$.

Finally, disregarding the presence of the quartz block for a moment, we may also calculate the outgoing waves in the region $y < 0$, produced by scattering in the interaction region $0 < y \lesssim 1/q$. We suppose that such waves go into the quartz block and are completely absorbed without any reflection into the region $y > 0$. Such an assumption was made in eq.(3-1) [3]. From the original integral equation (3-1) and eq.(3-2), we find

$$\phi_n(y) = \delta_{n,o} \exp(-ik_{yo}y) + \beta_n \exp(-ik_{yn}y), \qquad y<0 \qquad (3\text{-}8)$$

where

$$\beta_n = \sum_{b=1}^{N_b} X^-_{nb}\beta_{nb} + \sum_{a=-N_a}^{N_a}\sum_{b=1}^{N_b} Y^-_{nab}\alpha_{nab} , \qquad (3\text{-}9)$$

$$X^-_{nb} = -\frac{-ik_{yn} - ik_{yo} - bq}{2ik_{yn}} ,$$

$$Y^-_{nab} = -\frac{-ik_{yn} + ik_{ya} - bq}{2ik_{yn}} . \qquad (3\text{-}10)$$

and β_{nb} and α_{nab} are those in eqs.(3-4) and (3-5).

The above analysis reduces to the travelling wave case if we limit the waves to only the 0 and 1 orders by making the change

$$\sum_{a=-N_a}^{N_a} \rightarrow \sum_{a=0}^{1} .$$

This Born series method has the weak field assumption built in from the outset. It is questionable whether it can be extended to the strong field case (as is required for the reflection of the atom) by including sufficiently high order terms in the Born series of eq.(3-2) (i.e., sufficiently large N_b). We will address this question in the next section by solving the original equation (3-2), analytically for the simple travelling evanescent laser wave case. It will be shown that the same wave function of (3-2) is also obtained from the Laplace transformation method (which does not depend on the weak field assumption). Therefore, we might expect that the Born series method is applicable also to the strong field standing wave case if a sufficiently large number of terms are included in eq.(3-2).

4. Solution with Laplace Transformation
---The travelling wave case

For a mirror formed by evanescent travelling waves, we have only two diffraction orders, 0 and +1 (or -1, depending on the relation between the directions of the atomic and the laser beams).Eq.(2-3) becomes

$$\left(\frac{d^2}{dy^2} + k_o^2 \right) \phi_o(y) = -\eta^2 \exp(-qy)\, \phi_1(y) \quad ,$$

$$\left(\frac{d^2}{dy^2} + k_1^2 \right) \phi_1(y) = -\eta^2 \exp(-qy)\, \phi_o(y) \quad ,$$

(4-1)

where $k_o = k_{yo}$ and $k_1 = k_{y1}$, and $\eta^2 = 2 M \mu E_o / \hbar^2$.

The Laplace transformation of eq.(4-1) yield

$$(s^2 + k_o^2)\, \Phi_o(s) + \eta^2 \Phi_1(s+q) = (s - ik_o)\, \phi_o(0) \quad ,$$

$$(s^2 + k_1^2)\, \Phi_1(s) + \eta^2 \Phi_o(s+q) = (s - ik_1)\, \phi_1(0) \quad .$$

(4-2)

We assume that no waves are reflected from the surface of the quartz at $y = 0$ [3], and as $y \to 0^-$ we have $\phi_o(y) \to T_o \exp(-ik_o y)$ and $\phi_1(y) \to T_1 \exp(-ik_1 y)$, which are identified with eq.(3-8). Therefore, we have $\phi_o'(0) = -ik_o \phi_o(0)$ and $\phi_1'(0) = -ik_1 \phi_1(0)$. The function $\Phi_o(s)$ is found to be [11]

$$\Phi_o(s) \equiv \Psi_o(v) = \sum_{m=0}^{\infty} V^{4m}\{ \frac{\phi_o(0)}{(v+i\mu_o)_{m+1}(v-i\mu_o)_m(v+1/2+i\mu_1)_m(v+1/2-i\mu_1)_m}$$

$$- \frac{V^2\phi_1(0)}{(v+i\mu_o)_{m+1}(v-i\mu_o)_{m+1}(v+1/2+i\mu_1)_{m+1}(v+1/2-i\mu_1)_m} \},$$

(4-3)

where $(x)_m = x(x+1)(x+2)......(x+m-1)$, and for the convenience of calculations, the following scalings have been made,

$$v \equiv s/(2q), \qquad z \equiv 2qy,$$

$$\mu_o \equiv k_o/(2q), \qquad \mu_1 \equiv k_1/(2q),$$

$$V \equiv \eta/(2q).$$

Inverting the Laplace transform of $\Psi_o(v)$ (Eq.(4-3)), which consists of a series of poles [11], yields $\psi_o(z)$ as

$$\psi_o(z) = \sum_{m=0}^{\infty} \sum_{\iota=0}^{m} \{ F_{1m\iota} \exp[(-\iota - i\mu_o)z] + F_{2m\iota} \exp[(-\iota + i\mu_o)z]$$

$$+ F_{3m\iota} \exp[(-\iota - 1/2 - i\mu_1)z] + F_{4m\iota} \exp[(-\iota - 1/2 + i\mu_1)z] \}.$$

(4-4)

where

$$F_{1m\iota} \equiv F_{1m\iota}(\mu_o,\mu_1) = V^{4m}\frac{(-1)^\iota}{\iota!(m-\iota)!}$$

$$\{ \frac{\phi_o(0)}{(-\iota-2i\mu_o)_m(-\iota+1/2-i[\mu_o-\mu_1])_m(-\iota+1/2-i[\mu_o+\mu_1])_m}$$

$$- \frac{V^2\phi_1(0)}{(-\iota-2i\mu_o)_{m+1}(-\iota+1/2-i[\mu_o-\mu_1])_{m+1}(-\iota+1/2-i[\mu_o+\mu_1])_m} \},$$

$$F_{2m\iota} \equiv F_{2m\iota}(\mu_o,\mu_1) = V^{4m}\frac{(-1)^{\iota}}{\iota!(m-\iota)!}$$

$$\left\{ \frac{\phi_o(0)(m-\iota)}{(-\iota+2i\mu_o)_{m+1}(-\iota+1/2+i[\mu_o-\mu_1])_m\,(-\iota+1/2+i[\mu_o+\mu_1])_m} \right.$$

$$\left. - \frac{V^2\phi_1(0)}{[-\iota+2i\mu_o]_{m+1}\,(-\iota+1/2+i[\mu_o-\mu_1])_m\,(-\iota+1/2+i[\mu_o+\mu_1])_{m+1}} \right\},$$

$$F_{3m\iota} \equiv F_{3m\iota}(\mu_o,\mu_1) = V^{4m}\frac{(-1)^{\iota}}{\iota!(m-\iota)!}$$

$$\left\{ \frac{\phi_o(0)(m-\iota)}{(-\iota-2i\mu_1)_m(-\iota-1/2+i[\mu_o-\mu_1])_{m+1}\,(-\iota-1/2-i[\mu_o+\mu_1])_m} \right.$$

$$\left. - \frac{V^2\phi_1(0)}{[-\iota-2i\mu_1]_m\,(-\iota-1/2+i[\mu_o-\mu_1])_{m+1}\,(-\iota-1/2-i[\mu_o+\mu_1])_{m+1}} \right\},$$

$$F_{4m\iota} \equiv F_{4m\iota}(\mu_o,\mu_1) = V^{4m}\frac{(-1)^{\iota}}{\iota!(m-\iota)!}$$

$$(m-\iota)\left\{ \frac{\phi_o(0)}{(-\iota+2i\mu_1)_m(-\iota-1/2-i[\mu_o-\mu_1])_m\,(-\iota-1/2+i[\mu_o+\mu_1])_{m+1}} \right.$$

$$\left. - \frac{V^2\phi_1(0)}{(-\iota+2i\mu_1)_{m+1}\,(-\iota-1/2-i[\mu_o-\mu_1])_{m+1}\,(-\iota-1/2+i[\mu_o+\mu_1])_{m+1}} \right\}.$$

$$(4\text{-}5)$$

In order to compare eq.(4-4) with that of eq.(3-2), we change the order of the summations:

$$\sum_{m=0}^{\infty}\ \sum_{\iota=0}^{m} \equiv \sum_{\iota=0}^{\infty}\ \sum_{m=\iota}^{\infty}.$$

Thus eq.(4-4) can be written in a similar form as eq.(3-2) as

$$\phi_0(y > 0) \equiv \psi_0(z)$$

$$= \sum_{\iota=0}^{\infty} \{ F_{1\iota} \exp[(-2q\iota - i k_0)y] + F_{2\iota} \exp[(-2q\iota + i k_0)y]$$

$$+ F_{3\iota} \exp[(-q(2\iota+1) - i k_1)y] + F_{4\iota} \exp[(-q(2\iota+1) + i k_1)y] \},$$

(4-6)

with

$$F_{k\iota} \equiv F_{k\iota}(\mu_0,\mu_1) = \sum_{m=\iota}^{\infty} F_{km\iota}(\mu_0,\mu_1), \qquad k=1,2,3 \text{ and } 4.$$

(4-7)

The summations in eq.(4-7) should converge for fixed μ_0, μ_1, V and ι, at large values of m, i.e. $(m - \iota) > V$. This is ensured by the coefficient $V^m/(m - \iota)!$ appearing in $F_{1m\iota} \sim F_{4m\iota}$ in eq.(4-5) and by the denomenators which grow factorially. Furthermore, $\iota!$ appearing in the denominators in eq.(4-5) ensures the convergence of the coefficients $F_{1\iota} \sim F_{4\iota}$ in eq.(4-6) at large values of ι.

Exchanging μ_0 and μ_1 (or k_0 and k_1) in $\phi_0(y)$, and positions of $\phi_0(0)$ and $\phi_1(0)$, of eq.(4-5) we get the solution for $\phi_1(y)$ as

$$\phi_1(y>0) = \sum_{\iota=0}^{\infty} \{ G_{1\iota} \exp[(-2q\iota - i k_1)y] + G_{2\iota} \exp[(-2q\iota + i k_1)y$$

$$+ G_{3\iota} \exp[(-q(2\iota+1) - i k_0)y] + G_{4\iota} \exp[(-q(2\iota+1) + i k_0)y]\},$$

(4-8)

with

$$G_{k\iota} = F_{k\iota}(0 \leftrightarrow 1), \quad k=1,2,3 \text{ and } 4. \qquad (4\text{-}9)$$

In eqs.(4-6) and (4-8), the numbers 2ι and $2\iota + 1$ correspond to the number b in eq.(3-2). We notic that in eqs.(4-6) and (4-8) there are additional terms for the wave $\exp(-ik_1y)$ in comparison with eq.(3-2). The existence of such higher order waves propagating toward the quartz surface seems to be reasonable. Indeed, if we add terms $\beta_{n1b}\exp[(-bq - ik_1)y]$ $(n = 0,1)$ to eq.(3-2), it still satisfies the original integral equation (3-1). However, it is interesting that we find the initial condition $\beta_{n10} = 0$ leads to $\beta_{n1b} = 0$ (for $b \geq 1$) which means that terms like $\beta_{n1b}\exp[(-bq - ik_1)y]$ exist neither in $\phi_0(y)$ nor in $\phi_1(y)$. Thus, we can say that in eq.(4-6)

$$F_{3\iota} \equiv 0, \quad \text{and} \quad G_{1\iota} \equiv 0. \qquad (\text{for any } \iota \geq 0) \qquad (4\text{-}10)$$

The above eq.(4-10) may also be proved by substituting the wave functions of eqs.(4-6) and (4-8) into the original differential equations of (4-1). Also, corresponding to the index b given by eq.(3-2), we have either an even number 2ι or an odd number $2\iota + 1$ in eqs. (4-6) and (4-8). It may be easily proved that the number b in eq.(3-2) (in which $a = 0,1$ only, for a mirror formed by travelling waves, as opposed to the standing wave case where $a \in [-N_a, +N_a]$) will also be either even or odd for a fixed value of the index a. Therefore, the wavefunctions of eqs(4-6) and (4-8) derived from Laplace transformation are identified with those of eq.(3-2) based on Born series.

In the asymptotic region $y \to +\infty$, only the non-evanescent terms survive. Thus, eqs. (4-6) and (4-8) become

$$\phi_0(y) = F_{1o}\exp(-ik_oy) + F_{2o}\exp(+ik_oy) \quad y \to +\infty,$$

$$\phi_1(y) = G_{1o}\exp(-ik_1y) + G_{2o}\exp(+ik_1y) \quad y \to +\infty. \qquad (4\text{-}11)$$

We notice that there are two unknown coefficients $\phi_0(0)$ and $\phi_1(0)$ still involved in F_{1o}, F_{2o}, G_{1o}, and G_{2o} in eq.(4-11) as can be seen from eqs (4-5) and (4-7). This can be solved by the boundary conditions that we have only an incoming wave in the ground state:

$$F_{1o} = 1, \quad \text{and} \quad G_{1o} = 0. \qquad (4\text{-}12)$$

Here the second one is also included in eq.(4-10). In eq.(4-11) the terms $\exp(+ik_oy)$ and $\exp(+ik_1y)$ represent the reflected de Broglie waves, and the amplitudes R_o of the 0 order and R_1 of the +1 order reflected waves are given by

$$R_0 = F_{2o}, \quad R_1 = G_{2o}. \qquad (4\text{-}13)$$

After simple algebraic calculations for eqs.(4-12) and (4-13), we find

$$R_0 = \frac{S_{+o}W_{-1} - S_{+1}W_{-o}}{S_{-o}W_{-1} - S_{-1}W_{-o}} \quad ,$$

$$R_1 = \frac{W_{+o}W_{-1} - W_{+1}W_{-o}}{S_{-o}W_{-1} - S_{-1}W_{-o}} \quad .$$

where we have set

$$F_{1o} = S_{-o}\,\phi_o(0) + S_{-1}\,\phi_1(0) \ ,$$

$$F_{2o} = S_{+o}\,\phi_o(0) + S_{+1}\,\phi_1(0) \ ,$$

$$G_{1o} = W_{-o}\,\phi_o(0) + W_{-1}\,\phi_1(0) \ ,$$

$$G_{2o} = W_{+o}\,\phi_o(0) + W_{+1}\,\phi_1(0) \ ,$$

and definitions (4-5) and (4-7) give

$$S_{-o} \equiv S_{-o}(\mu_o,\mu_1) = \sum_{m=0}^{\infty} \frac{V^{4m}}{m!} \frac{1}{(-2i\mu_o)_m(1/2 - i[\mu_o - \mu_1])_m(1/2 - i[\mu_o + \mu_1])_m} \ ,$$

$$S_{-1} \equiv S_{-1}(\mu_o,\mu_1) = \sum_{m=0}^{\infty} \frac{V^{4m}}{m!} \frac{-V^2}{(-2i\mu_o)_{m+1}(1/2 - i[\mu_o - \mu_1])_{m+1}(1/2 - i[\mu_o + \mu_1])_m} \ ,$$

$$S_{+o} \equiv S_{+o}(\mu_o,\mu_1) = \sum_{m=0}^{\infty} \frac{V^{4m}}{m!} \frac{m}{(2i\mu_o)_{m+1}(1/2 + i[\mu_o - \mu_1])_m(1/2 + i[\mu_o + \mu_1])_m} \ ,$$

$$S_{+1} \equiv S_{+1}(\mu_o,\mu_1) = \sum_{m=0}^{\infty} \frac{V^{4m}}{m!} \frac{-V^2}{(2i\mu_o)_{m+1}(1/2 + i[\mu_o - \mu_1])_m(1/2 + i[\mu_o + \mu_1])_{m+1}} \ ,$$

and

$$W_{-o} = S_{-1}(\mu_1,\mu_o)\ ,$$

$$W_{-1} = S_{-0}(\mu_1,\mu_o)\ ,$$

$$W_{+o} = S_{+1}(\mu_1,\mu_o)\ ,$$

$$W_{+1} = S_{+0}(\mu_1,\mu_o)\ .$$

5. Conclusion

Two different bare-state methods for the investigation of the reflection and diffraction of atomic de Broglie waves by evanescent laser waves were presented. The Born series for the diffraction and reflection of atomic beams from a standing evanescent wave was derived and expressions for solutions in the bare-state picture were obtained for the whole space including the interaction region and the asymptotic region. Of particular interest is the question of convergence of the series in the strong field case. The simpler case of a travelling wave grating was considered in both the Born series approach and a direct solution by Laplace transformation, which is valid for all field strengths. The two methods give the same form of the wavefunction indicating that the Born series can be extended to strong fields, for the travelling wave case. We forsee no reasons why this observation will not be valid for the standing wave case.

To test the convergence of the Born series for the standing wave case a great deal of numerical work must be undertaken. Such calculations will supplement numerical calculations based on the multi-slice method in which the interaction region is divided into slices and the coupled differential equation (2-3) is solved with the assumption that the laser light intensity is constant inside each of the slice[7,8]. Thus the quantum mechanical analysis of the standing evanscent wave grating is nearing completion.

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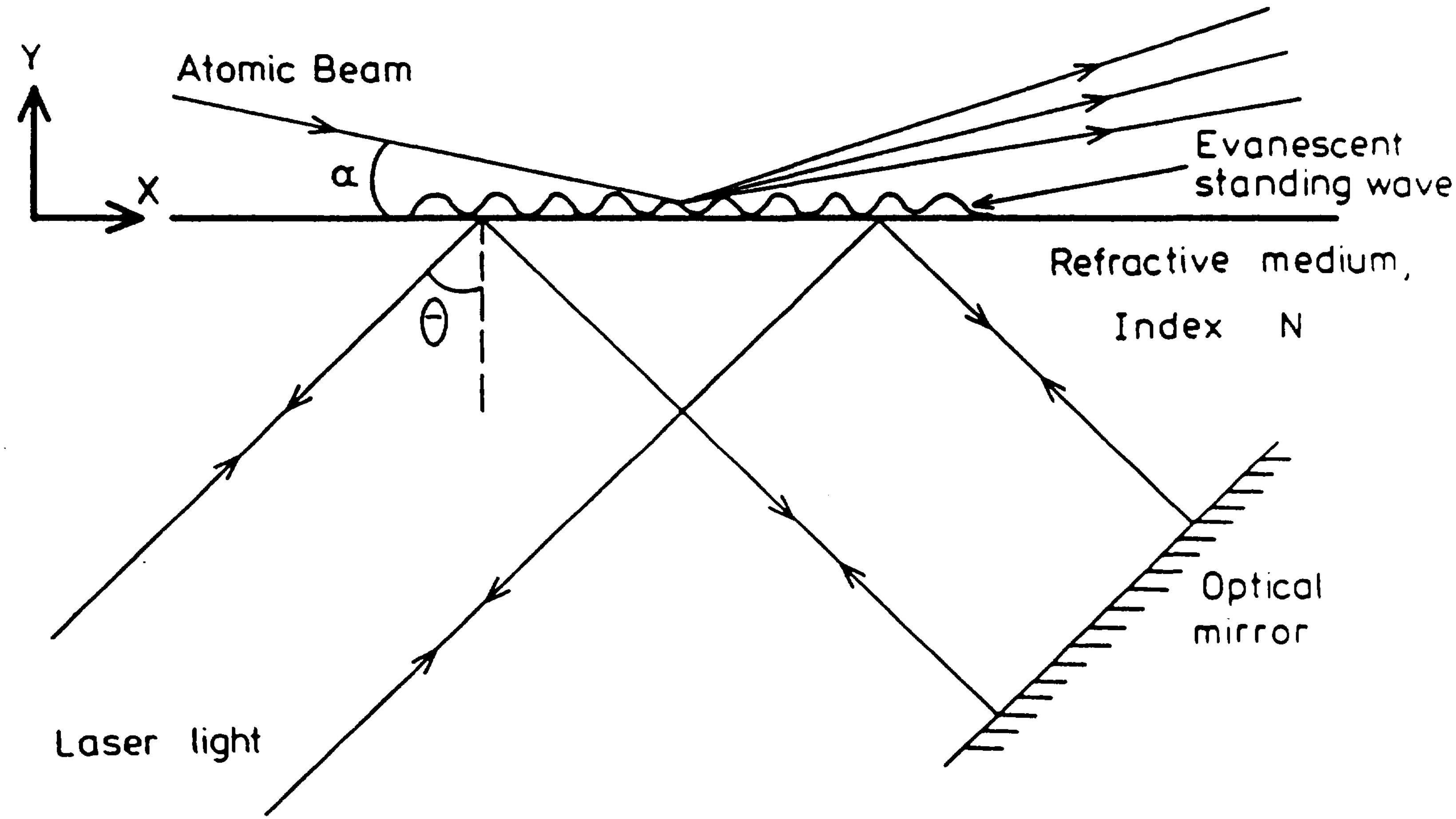
Figure Caption

Fig.1 Schematic diagram of the atomic reflection grating. The grating consisting of an evanescent wave produced by total internal reflection of two counter-propagating laser beams.



Y
X
Atomic Beam
α
Θ
Evanescent standing wave
Refractive medium, Index N
Optical mirror
Laser light