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On the Pais–Treiman method to measure $\pi\pi$ phase shifts in K_{e4} decays

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Abstract

We evaluate theoretical uncertainties to the method of Pais and Treiman to measure the $\pi\pi$ phase shifts in K_{e4} decays. We find that they are very small, below 1%.

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1 Introduction

The measurement of the $\pi\pi$ phase shifts near threshold is a key issue of low energy hadronic physics. First, these phases enter the phenomenological analysis of many different low energy scattering processes, weak decays and, in particular, CP-violating K-decays. A model independent determination of $\pi\pi$ phases would considerably improve our understanding of the corresponding hadronic matrix elements. Furthermore, Chiral Perturbation Theory (CHPT) [1], the systematic low energy expansion of QCD amplitudes, provides a link between the low energy $\pi\pi$ scattering data and the non perturbative chiral structure of the QCD ground state: The standard CHPT has a clean prediction for these quantities [2, 3], and would not be able to explain a large discrepancy with experimental data. On the other hand, the proposed generalization [4] of the usual low energy expansion, called generalized CHPT, expects a somewhat stronger I=0 S-wave $\pi\pi$ interaction. Within the latter scheme, a discrepancy between the standard CHPT and experiment would be interpreted as a manifestation of unusually low values of the quark-antiquark condensate $\langle\bar{q}q\rangle$ and of the ratio of strange to non strange current quark masses.

It is known since a long time that the main source of model independent experimental information on low energy $\pi\pi$ phases comes from K_{e4} decays. In these decays, a complete $\pi\pi$ phase-shift analysis could be performed in principle, assuming nothing more than unitarity or the Watson final state interaction theorem. However, such a complete procedure would require a detailed amplitude analysis of K_{e4} to be performed with respect to all five kinematical variables. To avoid this task, which is rather problematic in practice, Pais and Treiman have proposed [5] to measure $\pi\pi$ phases in a much simpler way which, in addition to the Watson theorem, assumes that higher partial waves beyond the P-wave are small. A variant of the Pais-Treiman method has already been used (together with additional assumptions) in a 1977 high statistics experiment by Rosselet et al. [6], but the data obtained still show very large error bars. Newly planned experiments like KLOE at DAΦNE, will hopefully be able to reduce the errors sizeably enough to decide between the theoretical alternatives mentioned above. In view of this improvement on the experimental side, it is worth to check what kind of uncertainties affect the Pais-Treiman method from the theoretical point of view. This is the purpose of this note.

2 Kinematics and form factors

We discuss the decay

$$K^+(p) \rightarrow \pi^+(p_1)\,\pi^-(p_2)\,e^+(p_l)\,\nu_e(p_\nu) \ , \tag{2.1}$$

(2.2)

and its charge conjugate mode. We do not consider isospin violating contributions and correspondingly set $m_u = m_d = \hat{m}$, $\alpha_{\text{QED}} = 0$.

The full kinematics of this decay requires five variables. We will use the ones introduced by Cabibbo and Maksymowicz [7]. It is convenient to consider three reference frames, namely the K^+ rest system (Σ_K), the $\pi^+\pi^-$ center-of-mass system ($\Sigma_{2\pi}$) and the $e^+\nu_e$ center-of-mass system ($\Sigma_{l\nu}$). Then the variables are s_π, the effective mass squared of the dipion system, s_l, the effective mass squared of the dilepton system, θ_π, the angle of the π^+ in $\Sigma_{2\pi}$ with respect to the dipion line of flight in Σ_K, θ_l, the angle of the l^+ in $\Sigma_{l\nu}$ with respect to the dilepton line of flight in Σ_K, and ϕ, the angle between the plane formed by the pions in Σ_K and the corresponding plane formed by the dileptons. The range of the variables is

$$\begin{aligned} 4M_\pi^2 &\leq s_\pi = (p_1 + p_2)^2 \leq (M_K - m_l)^2 \ , \\ m_l^2 &\leq s_l = (p_l + p_\nu)^2 \leq (M_K - \sqrt{s_\pi})^2 \ , \\ 0 &\leq \theta_\pi, \theta_l \leq \pi, 0 \leq \phi \leq 2\pi. \end{aligned} \quad (2.3)$$

It is useful to furthermore introduce the following combinations of four vectors

$$P = p_1 + p_2, \ \ Q = p_1 - p_2, \ \ L = p_l + p_\nu. \quad (2.4)$$

Below we will also use the variables

$$t = (p_1 - p)^2, u = (p_2 - p)^2, \nu = t - u. \quad (2.5)$$

These are related to s_π, s_l and θ_π by

$$\begin{aligned} t + u &= 2M_\pi^2 + M_K^2 + s_l - s_\pi \ , \\ \nu &= -2\sigma_\pi X \cos\theta_\pi \ , \end{aligned} \quad (2.6)$$

where

$$\begin{aligned} \sigma_\pi &= (1 - 4M_\pi^2/s_\pi)^{\frac{1}{2}} \, , \\ X &= \frac{1}{2}\lambda^{1/2}(M_K^2, s_\pi, s_l) \ , \\ \lambda(x,y,z) &= x^2 + y^2 + z^2 - 2(xy + xz + yz) \ . \end{aligned} \quad (2.7)$$

The matrix element is (we always neglect the mass of the electron in what follows)

$$T = \frac{G_F}{\sqrt{2}} V_{us}^\star \bar{u}(p_\nu)\gamma_\mu(1-\gamma_5)\nu(p_l)(V^\mu - A^\mu) \ , \quad (2.8)$$

where

$$\begin{aligned} I_\mu &= <\pi^+(p_1)\pi^-(p_2)\text{out} \mid I_\mu^{4-i5}(0) \mid K^+(p)>; \ I = V, A \ , \\ V_\mu &= -\frac{H}{M_K^3}\epsilon_{\mu\nu\rho\sigma}L^\nu P^\rho Q^\sigma \ , \\ A_\mu &= -i\frac{1}{M_K}\left[P_\mu F + Q_\mu G + L_\mu R\right] \ , \end{aligned} \quad (2.9)$$

and $\epsilon_{0123} = 1$.

The form factors F, G, R and H are analytic functions of the variables s_π, t and u. The partial decay rate for (2.1) can be expressed as

$$\begin{aligned} d\Gamma_5 &= G_F^2 \mid V_{us} \mid^2 N(s_\pi, s_l) J_5(s_\pi, s_l, \theta_\pi, \theta_l, \phi) ds_\pi ds_l d(\cos\theta_\pi) d(\cos\theta_l) d\phi \ , \\ N(s_\pi, s_l) &= \sigma_\pi X/(2^{13}\pi^6 M_K^5) \ , \end{aligned} \tag{2.10}$$

where

$$\begin{aligned} J_5 &= 2\Big[I_1 + I_2 \cos 2\theta_l + I_3 \sin^2\theta_l \cdot \cos 2\phi + I_4 \sin 2\theta_l \cdot \cos\phi \\ &+ I_5 \sin\theta_l \cdot \cos\phi + I_6 \cos\theta_l + I_7 \sin\theta_l \cdot \sin\phi + I_8 \sin 2\theta_l \cdot \sin\phi \\ &+ I_9 \sin^2\theta_l \cdot \sin 2\phi\Big] \ , \end{aligned}$$

with

$$\begin{aligned} I_1 &= \frac{1}{4}\left\{|F_1|^2 + \frac{3}{2}\left(|F_2|^2 + |F_3|^2\right)\sin^2\theta_\pi\right\} \ , \\ I_2 &= -\frac{1}{4}\left\{|F_1|^2 - \frac{1}{2}\left(|F_2|^2 + |F_3|^2\right)\sin^2\theta_\pi\right\} \ , \\ I_3 &= -\frac{1}{4}\left\{|F_2|^2 - |F_3|^2\right\}\sin^2\theta_\pi \ , \\ I_4 &= \frac{1}{2}\,\mathrm{Re}(F_1^* F_2)\sin\theta_\pi \ , \\ I_5 &= -\,\mathrm{Re}(F_1^* F_3)\sin\theta_\pi \ , \\ I_6 &= -\,\mathrm{Re}(F_2^* F_3)\sin^2\theta_\pi \ , \\ I_7 &= -\,\mathrm{Im}(F_1^* F_2)\sin\theta_\pi \ , \\ I_8 &= \frac{1}{2}\,\mathrm{Im}(F_1^* F_3)\sin\theta_\pi \ , \\ I_9 &= -\frac{1}{2}\,\mathrm{Im}(F_2^* F_3)\sin^2\theta_\pi \ , \end{aligned} \tag{2.11}$$

and

$$\begin{aligned} F_1 &= X \cdot F + \sigma_\pi (PL)\cos\theta_\pi \cdot G \ , \\ F_2 &= \sigma_\pi \left(s_\pi s_l\right)^{1/2} G \ , \\ F_3 &= \sigma_\pi X \left(s_\pi s_l\right)^{1/2} \frac{H}{M_K^2} \ , \ . \end{aligned} \tag{2.12}$$

The definition of F_1, F_2 and F_3 corresponds to the combinations used by Pais and Treiman [5]. The form factors $I_1, \ldots, I_9$ agree with the expressions given in [5] if one neglects the mass of the electron.

The form factors may be written in a partial wave expansion in the variable θ_π. Suppressing isospin indices, one has for F and G [8]

$$F = \sum_{l=0}^{\infty} P_l(\cos\theta_\pi) f_l e^{i\delta_l} - \frac{\sigma_\pi PL}{X}\cos\theta_\pi G \ ,$$

$$G = \sum_{l=1}^{\infty} P_l'(\cos\theta_\pi) g_l e^{i\delta_l} \ , \tag{2.13}$$

where

$$P_l'(z) = \frac{d}{dz} P_l(z) \ . \tag{2.14}$$

The partial wave amplitudes f_l and g_l depend on s_π and s_l and are real in the physical region of K_{l4} decay (in our overall phase convention). The phases δ_l coincide with the phase shifts in elastic $\pi\pi$ scattering. Accordingly, the form factors F_1 and F_2 have the expansions:

$$\begin{aligned} F_1 &= X \sum_{l=0}^{\infty} P_l(\cos\theta_\pi) f_l e^{i\delta_l} \ , \\ -\sin\theta_\pi F_2 &= \sigma_\pi (s_\pi s_l)^{1/2} \sum_{l=1}^{\infty} P_l^{(1)}(\cos\theta_\pi) g_l e^{i\delta_l} \ , \end{aligned} \tag{2.15}$$

where $P_l^{(m)}$ are the associated Legendre functions.

3 Evaluation of the corrections

The method suggested by Pais and Treiman to measure the $\pi\pi$ phase shifts in K_{e4} decays is very simple and clean. It is based on the observation that the dependence of the differential decay rate on two of the five variables can be worked out analytically under very few general assumptions, as we have seen in the previous section.

In case one neglects all the waves higher than S and P, I_4 and I_7 show a very simple dependence on the phases δ_0 and δ_1. [The same dependence appears in I_5 and I_8 which, however, contain the anomalous form factor H, and are kinematically much more suppressed.] Their integral over $\cos\theta_\pi$ is:

$$\begin{aligned} \bar{I}_4 &\equiv \int_{-1}^{1} d\cos\theta_\pi I_4 = \frac{\pi}{4} X \sigma_\pi (s_\pi s_l)^{1/2} f_0 g_1 \cos(\delta_0 - \delta_1) \ , \\ \bar{I}_7 &\equiv \int_{-1}^{1} d\cos\theta_\pi I_7 = \frac{\pi}{2} X \sigma_\pi (s_\pi s_l)^{1/2} f_0 g_1 \sin(\delta_0 - \delta_1) \ . \end{aligned} \tag{3.1}$$

By measuring the ratio $\bar{I}_7/2\bar{I}_4$ one has then direct access to $\tan(\delta_0 - \delta_1)$. To experimentally select I_7 and I_4 one could use different methods (fit the distribution in $\cos\theta_l$ and ϕ, measure asymmetries, measure moments ...). Choosing for example to measure the appropriate moments, one would have, all in all, to integrate the distribution over four of the five variables with the weights $\sin\phi$, then $\cos\phi\cos\theta_l$, and then take the ratio. The measurement of $\tan(\delta_0 - \delta_1)$ would be as simple as that.

While on the experimental side there might be detector-dependent problems that could force experimentalists to adapt the Pais-Treiman method to their own peculiar situation (see for example Ref. [6]), it is clear from the theoretical point of view that the beauty of the method is based on neglecting the higher waves. It is the main purpose of this note to try to estimate corrections coming from higher waves to the Pais-Treiman formula:

$$\frac{\bar{I}_7}{2\bar{I}_4} = \tan(\delta_0 - \delta_1) \ . \tag{3.2}$$

For that purpose we shall use the form factors predicted by CHPT at the one loop level.

We start by applying the chiral power counting to the partial waves of the form factors. In general, for any partial wave we may write an expansion in the following way:

$$X_l = \frac{M_K}{\sqrt{2}F_\pi}\left\{X_l^{(0)} + X_l^{(2)} + X_l^{(4)} + \cdots\right\} \ ; \ X = f, g \ , \tag{3.3}$$

where the upper index stands for powers of energies or meson masses. From published calculations of F and G to one-loop [9, 10] we may easily get that only three partial waves start at order $O(E^0)$, while all the others start at order $O(E^2)$:

$$\begin{aligned}
f_0 &= \frac{M_K}{\sqrt{2}F_\pi}\left\{1 + f_0^{(2)} + O(E^4)\right\} \ ; \\
f_1 &= \frac{M_K}{\sqrt{2}F_\pi}\left\{\frac{\sigma_\pi PL}{X} + f_1^{(2)} + O(E^4)\right\} \ ; \\
g_1 &= \frac{M_K}{\sqrt{2}F_\pi}\left\{1 + g_1^{(2)} + O(E^4)\right\} \ ; \\
f_l &= \frac{M_K}{\sqrt{2}F_\pi}\left\{f_l^{(2)} + O(E^4)\right\} \ ; l \geq 2 \\
g_k &= \frac{M_K}{\sqrt{2}F_\pi}\left\{g_k^{(2)} + O(E^4)\right\} \ ; k \geq 2 \ .
\end{aligned} \tag{3.4}$$

Using this chiral power counting is very easy to give a "corrected" Pais-Treiman formula, which is accurate up to and including order $O(E^2)$:

$$\frac{\bar{I}_7}{2\bar{I}_4} \equiv \tan(\delta_0 - \delta_1)\left\{1 + \Delta^{(2)} + O(E^4)\right\} \ , \tag{3.5}$$

$$\begin{aligned}
\Delta^{(2)} &= \left(\frac{\sin\delta_0}{\sin(\delta_0 - \delta_1)} - \frac{\cos\delta_0}{\cos(\delta_0 - \delta_1)}\right)\sum_{k=1}^{\infty} A_{0\,2k+1} g_{2k+1}^{(2)} \\
&+ \left(\frac{\sin\delta_1}{\sin(\delta_0 - \delta_1)} - \frac{\cos\delta_1}{\cos(\delta_0 - \delta_1)}\right)\frac{\sigma_\pi PL}{X}\sum_{k=1}^{\infty} A_{1\,2k} g_{2k}^{(2)} \\
&- \left(\frac{\sin\delta_1}{\sin(\delta_0 - \delta_1)} + \frac{\cos\delta_1}{\cos(\delta_0 - \delta_1)}\right)\sum_{l=1}^{\infty} A_{2l\,1} f_{2l}^{(2)} \ ,
\end{aligned} \tag{3.6}$$

where

$$A_{lk} = \frac{2}{\pi}\int_{-1}^{1} d\cos\theta_\pi P_l(\cos\theta_\pi) P_k^{(1)}(\cos\theta_\pi) \ , \qquad (3.7)$$

and where all the phases of waves higher than the P have been put to zero (this is again consistent at the order at which we are working). The ratios of sines and cosines start at order $O(E^0)$, and will have also contributions of higher order, that we neglect at this level of accuracy. We may thus use the leading order CHPT expressions:

$$\delta_0 = \frac{1}{32\pi F_\pi^2}\sigma_\pi\left(2s_\pi - M_\pi^2 + 5\epsilon M_\pi^2\right) + O(E^4) \qquad (3.8)$$

$$\delta_1 = \frac{1}{96\pi F_\pi^2}\sigma_\pi^3 s_\pi + O(E^4) \ . \qquad (3.9)$$

The standard CHPT predicts $\epsilon = 0$, whereas in generalized CHPT ϵ is an arbitrary parameter, $0 \leq \epsilon \leq 1$, related to the quark mass ratio $r = m_s/\hat{m}$ [4]. Eq. (3.6) then becomes

$$\begin{aligned}\Delta^{(2)} &= \left(\frac{s_\pi - 4M_\pi^2}{5s_\pi + M_\pi^2 + 15\epsilon M_\pi^2}\right)\sum_{k=1}^{\infty} A_{0\,2k+1} g_{2k+1}^{(2)} \\ &- \left(\frac{4s_\pi + 5M_\pi^2 + 15\epsilon M_\pi^2}{5s_\pi + M_\pi^2 + 15\epsilon M_\pi^2}\right)\frac{\sigma_\pi PL}{X}\sum_{k=1}^{\infty} A_{1\,2k} g_{2k}^{(2)} \\ &- \left(\frac{6s_\pi - 3M_\pi^2 + 15\epsilon M_\pi^2}{5s_\pi + M_\pi^2 + 15\epsilon M_\pi^2}\right)\sum_{l=1}^{\infty} A_{2l\,1} f_{2l}^{(2)} \ . \end{aligned} \qquad (3.10)$$

Before evaluating numerically the correction we would like to stress that the low energy expansion has been performed only inside the braces of eq. (3.5).

The explicit calculation shows that both in standard and in generalized CHPT $\Delta^{(2)}$ is numerically very small over the whole phase space. It barely reaches 0.5 %. The dependence on s_l is rather weak, and there is no difficulty in repeating the same analysis for the ratio of $\bar{I}_7$ and $2\bar{I}_4$ not treated as functions of s_l but averaged over it (as suggested by Pais and Treiman). At this order in the low energy expansion the correction turns out to be just $\langle\Delta^{(2)}\rangle$, where with the symbol $\langle\rangle$ we mean the following average:

$$\langle A\rangle \equiv \frac{1}{\int_{s_l^{min}}^{s_l^{max}} ds_l X^2\sqrt{s_l}}\int_{s_l^{min}}^{s_l^{max}} ds_l X^2\sqrt{s_l}A(s_l) \qquad (3.11)$$

The plot of $\langle\Delta^{(2)}\rangle$ as a function of s_π is shown in fig. 1.

Since the corrections are so small, we would like to understand whether this is just a typical size of higher waves, or an effect depending on some miraculous cancellation.

Since F and G are dimensionless functions of s_π, s_l and ν, and depend on $\cos\theta_\pi$ only through ν, their higher waves projections are suppressed by powers of the kinematical factor:

$$\frac{\sigma_\pi X}{M_K^2} = 2\frac{p_\pi p_l}{M_K^2} \leq 0.15 \tag{3.12}$$

where p_π and p_l stand for the momenta of individual pions and of the dilepton, respectively, in the dipion center of mass frame. The bound in (3.12) holds over the whole phase space.

The dominant contribution to the sum $\Delta^{(2)}$, eq. (3.10), comes from the D-waves $g_2^{(2)}$ and $f_2^{(2)}$. The latter contains contributions both from the F and G form factors:

$$f_2^{(2)} = f_d^{(2)} + 2\frac{\sigma_\pi PL}{X} g_2^{(2)} \ , \tag{3.13}$$

where $f_d^{(2)}$ stands for the D-wave of F alone. Making explicit the factors (3.12), we may define:

$$\begin{aligned} f_d^{(2)} &\equiv \left(\frac{\sigma_\pi X}{M_K^2}\right)^2 \widetilde{f_d}^{(2)} \ , \\ g_2^{(2)} &\equiv \frac{\sigma_\pi X}{M_K^2} \widetilde{g_2}^{(2)} \ , \end{aligned} \tag{3.14}$$

where $\widetilde{f_d}^{(2)}$ and $\widetilde{g_2}^{(2)}$ are expected to be smooth functions of s_π and of s_l. Since D-waves are a loop effect, we may guess that $\widetilde{f_d}^{(2)}$ and $\widetilde{g_2}^{(2)}$ will be of the order $M_K^2/(16\pi^2 F_\pi^2)$. An explicit calculation shows that this is indeed the case and, for a possible later use, we may even write the following simplified expressions for them: Standard CHPT gives

$$\begin{aligned} \widetilde{f_d}^{(2)} &= \frac{M_K^2}{16\pi^2 F_\pi^2}(-0.28)\left[1 + 0.12\left(\frac{s_l}{4M_\pi^2} - q^2\right)\right] \ , \\ \widetilde{g_2}^{(2)} &= \frac{M_K^2}{16\pi^2 F_\pi^2}(-0.62)\left[1 + 0.08\left(\frac{s_l}{4M_\pi^2} - q^2\right)\right] \ , \end{aligned} \tag{3.15}$$

where

$$q^2 = (s_\pi - 4M_\pi^2)/4M_\pi^2 \ . \tag{3.16}$$

The above evaluation of $\widetilde{f_d}^{(2)}$ is parameter free. On the other hand, a substancial part of the contribution to $\widetilde{g_2}^{(2)}$ comes from the low energy constant $L_3 + 4L_2$, whose determination brings in some uncertainty [11]: The corresponding error in (3.15) is however smaller than 50%. Finally, $\Delta^{(2)}$ can be expressed in terms of the smooth functions $\widetilde{f_d}^{(2)}$ and $\widetilde{g_2}^{(2)}$:

$$\begin{aligned} \Delta^{(2)} &= \frac{3}{8}\left(\frac{2s_\pi - M_\pi^2 + 5\epsilon M_\pi^2}{5s_\pi + M_\pi^2 + 15\epsilon M_\pi^2}\right)\left(\frac{\sigma_\pi X}{M_K^2}\right)^2 \widetilde{f_d}^{(2)} \\ &- \frac{3}{2}\left(\frac{s_\pi + 3M_\pi^2 + 5\epsilon M_\pi^2}{5s_\pi + M_\pi^2 + 15\epsilon M_\pi^2}\right)\frac{\sigma_\pi^2 (PL)}{M_K^2}\widetilde{g_2}^{(2)} + \cdots \end{aligned} \tag{3.17}$$

where the ellipsis stands for $l > 2$ waves. This expression gives clearly account of the size of the corrections. The second term is dominant with respect to the first one, and its typical scale is $\sigma_\pi^2(PL)/(16\pi^2 F_\pi^2)$, which, after averaging over s_l, is at most 1.6% over the whole phase space. The remaining coefficients reduce this number by a factor 4. Moreover, we have calculated the dependence of $\widetilde{f_d}^{(2)}$ and $\widetilde{g_2}^{(2)}$ on $m_s/\hat{m}$, in generalized CHPT[3]. We have found that $\widetilde{f_d}^{(2)}$ is affected by no more than 10%, whereas the form factor $\widetilde{g_2}^{(2)}$ may be modified by at most a factor one half, further reducing $\langle\Delta^{(2)}\rangle$.

Higher orders will certainly modify the above numbers, but not the orders of magnitude, which are essentially given by the kinematics. In a sense, the limited effect of the uncertainties in the values of the low energy constants L_i may be considered as an indication in this direction. Furthermore, since the generalized CHPT differs from the standard CHPT by a different ordering in the perturbative expansion (that is, at every finite order it includes terms which in the standard scheme are relegated to higher orders), the weak dependence of $\widetilde{f_d}^{(2)}$ and of $\widetilde{g_2}^{(2)}$ on $m_s/\hat{m}$ is a further indication that it is very unlikely that higher orders would overwhelm the strong kinematical suppression of higher waves.

Our conclusion is then that the Pais-Treiman formula (3.2) is free of corrections up to the percent level, over the whole accessible range of energy.

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References

[1] For recent reviews on CHPT see e.g.
H. Leutwyler, in: Proc. XXVI Int. Conf. on High Energy Physics, Dallas, 1992, edited by J.R. Sanford, AIP Conf. Proc. No. 272 (AIP, New York, 1993) p. 185;
U.G. Meißner, Rep. Prog. Phys. 56 (1993) 903;
A. Pich, Lectures given at the V Mexican School of Particles and Fields, Guanajuato, México, December 1992, preprint CERN-Th.6978/93 (hep-ph/9308351);
G. Ecker, Lectures given at the 6th Indian-Summer School on Intermediate Energy Physics Interaction in Hadronic Systems Prague, August 25 - 31, 1993, to appear in the Proceedings (Czech. J. Phys.), preprint UWThPh -1993-31 (hep-ph/9309268).

[3] The generalized CHPT one loop K_{l4} form factors will be given elsewhere [12]

[2] J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) 158 (1984) 142.

[3] J. Gasser and H. Leutwyler, Nucl. Phys. B250 (1985) 465.

[4] J. Stern, H. Sazdjian, N.H. Fuchs, Phys. Rev. D47 (1993) 3814

[5] A. Pais and S.B. Treiman, Phys. Rev. 168 (1968) 1858.

[6] L. Rosselet et al., Phys. Rev. D15 (1977) 574.

[7] N. Cabibbo and A. Maksymowicz, Phys. Rev. B137 (1965) B438; Phys. Rev. 168 (1968) 1926 E.

[8] F.A. Berends, A. Donnachie and G.C. Oades, Phys. Lett. 26B (1967) 109; Phys. Rev. 171 (1968) 1457.

[9] J. Bijnens, Nucl. Phys. B337 (1990) 635.

[10] C. Riggenbach, J. Gasser, J.F. Donoghue and B.R. Holstein, Phys. Rev. D43 (1991) 127.

[11] J. Bijnens, G. Colangelo and J. Gasser, K_{l4}-decays beyond one loop, preprint BUTP-94/4 and ROM2F 94/05 (hep-ph/9403390).

[12] M. Knecht and J. Stern, to appear.

Figure Caption

Fig. 1: $\langle \Delta^{(2)} \rangle$, as a function of s_π. The plot refers to the standard CHPT case: it is calculated using eq. (3.10) with $\epsilon = 0$ and the F and G form factors given by [9, 10], with the central values $L_2 = 1.35 \times 10^{-3}$, $L_3 = -3.5 \times 10^{-3}$ of [11]. The same curve can be obtained using eq. (3.17) with $\epsilon = 0$, and (3.15).

Fig. 1