

**Testing the Principle of Equivalence by Solar Neutrinos**

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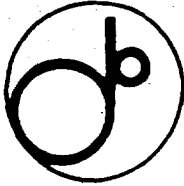
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## Testing the Principle of Equivalence by Solar Neutrinos

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### ABSTRACT

We discuss the possibility of testing the principle of equivalence with solar neutrinos. If there exists a violation of the equivalence principle quarks and leptons with different flavors may not universally couple with gravity. The method we discuss employs a quantum mechanical phenomenon of neutrino oscillation to probe into the non-universality of the gravitational couplings of neutrinos. We develop an appropriate formalism to deal with neutrino propagation under the weak gravitational fields of the sun in the presence of the flavor mixing. We point out that solar neutrino observation by the next generation water Cherenkov detectors can improve the existing bound on violation of the equivalence principle by 3-4 orders of magnitude if the nonadiabatic Mikheyev-Smirnov-Wolfenstein mechanism is the solution to the solar neutrino problem.

Experimental test of the principle of equivalence, one of the fundamental building block of Einstein's general theory of relativity, now has history of more than a century. It was Eötvös [1] who carried out the celebrated torsion balance experiment that elevated the experimental test of the equivalence principle to the realm of precise measurements. Eötvös and his collaborators obtained the bound  $\eta \lesssim 3 \times 10^{-9}$  for the violation of the equivalence of inertial and gravitational masses. Forty years later the Eötvös-type torsion balance experiment was substantially improved by Dicke and his collaborators [2]. They noticed that sun's gravitational field affects this type of experiment by producing torque of 24 hours period in the presence of violation of the equivalence principle. With their extensive efforts in removing systematic uncertainties they achieved the accuracy  $\eta \lesssim 3 \times 10^{-11}$ , an impressive improvement over two orders of magnitude over Eötvös'. Using the similar apparatus with Dicke et al.'s Braginski and Panov [3] reported the bound  $\eta \lesssim 0.9 \times 10^{-12}$ , the most stringent one to date.

In this paper we point out that by using the sun as the source of gravity and at the same time as the source of neutrino beam one can obtain much more stringent constraint for violation of the equivalence principle. We show that the optimal sensitivity one would achieve by the solar neutrino observation by the next-generation water Cherenkov detectors is  $|\Delta f| \simeq 10^{-15} - 10^{-16}$ , an improvement over 3-4 orders of magnitude than the Eötvös-Dicke-type experiment. Here  $\Delta f$  is a measure for violation of the equivalence principle for neutrinos to be defined later.

A mild bound  $|\Delta f| \lesssim 10^{-3}$  for violation of the equivalence principle for neutrinos have been derived [4] by using the small difference between arrival times of photons and neutrinos from SN1987A. The best bound deduced so far for microscopic objects is from the neutron free fall refractometry experiments which led to  $|\eta| < 3 \times 10^{-4}$  [5].

Our discussion in this paper heavily relies on the basic observation by Gasperini [6]. If Einstein's equivalence principle is violated gravity may not universally couple with neutrinos with different flavors. He pointed out that if this occurs neutrino oscillation similar to that of flavor mixing takes place and the effect can be detectable by experiments. After his observation several attempts have been made to sharpen his proposal and to examine the attainable sensitivity for violation of the equivalence principle.

Halprin and Leung [7] observed that one can use solar neutrinos to perform a sensitive test of the equivalence principle and they gave an order-

of-magnitude estimation of the sensitivity,  $|\Delta f| \sim 10^{-14}$ . Iida, Minakata and Yasuda [8] discussed the possibility of doing the similar test by the long-baseline accelerator neutrino experiments and obtained  $|\Delta f| \sim 10^{-14}$  as expected sensitivity. It is accidental that two methods yield the same order of magnitude for the sensitivity since the ingredients involved in these estimations are entirely different with each other. Some related considerations were given in Ref. [9].

If one want to use solar neutrinos as an experimental mean to test the equivalence principle one has to decide his attitude to the solar neutrino problem [10]. One might say that it is due to the fact that our ability of modeling the sun is so poor and the problem may be solved by inventing an appropriate non-standard solar model. Others may argue that it implies a new physics of neutrinos beyond the standard electroweak gauge theory. Someone would be more brave and may propose that it could be explained by the effect of violation of the equivalence principle itself, as has been done in Ref. [9].

In this paper we make an assumption that the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism [11] is the cause of the solar neutrino deficit. It makes our analysis model-dependent but it is inevitable if the solution to the solar neutrino problem involves the physics beyond the standard model of electroweak interactions. In view of the progress in solar neutrino observation done and to be done by the present and the future detectors it would not be too optimistic to speculate that we would finally find the answer to the solar neutrino puzzle. The point we want to stress is that once we have *the* solution one can perform similar analyses as ours to test the equivalence principle and the present work may serve as a prototype for them.

In this paper we will make a substantial improvement in the treatment of possible experimental test of the equivalence principle by solar neutrino observation. A part of this work has been presented in Ref. [12] though in a very preliminary stage.

The points we are going to make in this paper are:

- (1) We shall develop a new formalism which describes neutrino propagation in an arbitrary spherically symmetric metric of a perfect-fluid star. It enables us to calculate neutrino flavor conversion probabilities that are accurate to order  $G$ , the Newton constant.
- (2) Our analysis of the effects of violation of the equivalence principle relies solely on the spectral shape of the  ${}^8\text{B}$  neutrinos. We fully treat the effects of

the MSW transformation due to the neutrino flavor mixing as well as that of the non-universal gravitational couplings. We discuss how one can separate the latter effect from the former's and we estimate the sensitivity by fully taking into account of this mixture of two different flavor transformations.

First let us recall the definition of the equivalence principle. According to the text book by Misner, Thorn and Wheeler [13], the equivalence principle is stated as "In any and every local Lorentz frame, anywhere and anytime in the universe, all the (non-gravitational) laws of physics must take on their familiar special-relativistic forms." It follows from the principle that if the gravitational couplings of different flavor neutrinos are different the equivalence principle is violated. This is because in a local Lorentz frame where electron neutrinos obey the familiar special-relativistic wave equation muon neutrinos do not.

In this paper we work with two neutrino flavors and denote them as electron and muon neutrinos. We consider the Lagrangian

$$\begin{aligned} \mathcal{L} = & \sum_{i=1,2} e(G_i) \bar{\nu}_{G_i} i e^{a\mu}(G_i) \gamma_a D_\mu(G_i) \nu_{G_i} \\ & - \sum_{i=1,2} e(G_i) m_i \bar{\nu}_{M_i} \nu_{M_i} \\ & + (\text{interactions with electroweak gauge fields}), \end{aligned} \quad (1)$$

where  $e^{a\mu}(G_i)$  ( $i = 1, 2$ ) are the vierbein fields of some background metric and  $e(G_i) \equiv \det e^{a\mu}(G_i)$ . The Newton constants  $G_i$  ( $i = 1, 2$ ) are allowed to be different for neutrino basis  $\nu_{G_1}$  and  $\nu_{G_2}$ . The last term in (1) is assumed to be written by flavor (or gauge) eigenstate. We take the most general ansatz that the flavor, the gravity and the mass eigenstates are all different with each other. The latter two are related with the flavor eigenstate in the following way:

$$\begin{aligned} \begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix} &= \begin{bmatrix} \cos \theta_G & \sin \theta_G \\ -\sin \theta_G & \cos \theta_G \end{bmatrix} \begin{bmatrix} \nu_{G_1} \\ \nu_{G_2} \end{bmatrix}, \\ &= \begin{bmatrix} \cos \theta_M & \sin \theta_M \\ -\sin \theta_M & \cos \theta_M \end{bmatrix} \begin{bmatrix} \nu_{M_1} \\ \nu_{M_2} \end{bmatrix}. \end{aligned} \quad (2)$$

The mixing angle  $\theta_M$  in (2) is the usual flavor mixing angle which plays an important role in the MSW mechanism.

As a measure for violation of the equivalence principle we define

$$\Delta f = \frac{G_2 - G_1}{\frac{1}{2}(G_2 + G_1)}. \quad (3)$$

We believe that  $\Delta f$  defined in (3) does have a right correspondence with  $\eta$  which is defined in a similar fashion as (3) by replacing  $G_i$  by  $(M/m)i$  as a measure for the in-equivalence of inertial ( $m$ ) to gravitational ( $M$ ) masses. The equivalence of  $\eta$  and  $\Delta f$  comes closer to be true if we are allowed to boldly interpret the Eötvös-Dicke-type experiments as measuring the non-universality of gravitational couplings of  $u$ - and  $d$ -quarks.

We shall derive the neutrino evolution equation under the influence of a weak gravitational field of the sun. For generality we work with the following most general static and spherically symmetric metric,

$$ds^2 = g_{tt}(r)dt^2 + g_{rr}(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (4)$$

taking the center of the sun as the origin of the coordinate. We make the ansatz that the energy-momentum tensor of the solar matter is given by the form of perfect fluid,

$$T_{\mu\nu} = P g_{\mu\nu} + (P + \rho)U_\mu U_\nu, \quad (5)$$

which should give an excellent approximation to the sun. In (5)  $\rho$  and  $P$  denotes the energy density and the pressure, respectively, and  $U^\mu$  is the usual velocity four vector. The metric  $g_{rr}(r)$  and  $g_{tt}(r)$  can be obtained by solving the Einstein equation and are given up to the order of  $G$  by [13]

$$g_{tt}(r) = 1 - 2G \int_r^\infty \left[ \frac{\mathcal{M}(r')}{r'^2} + 4\pi r' P(r') \right] dr', \quad (6)$$

$$-g_{rr}(r) = 1 + 2G \frac{\mathcal{M}(r)}{r}, \quad (7)$$

where

$$\mathcal{M}(r) = \int_0^r dr' 4\pi r'^2 \rho(r'). \quad (8)$$

and it can be interpreted as the total mass contained in the volume with radius  $r$  for Newtonian stars. In case of the sun, we can neglect the pressure

term in equation (6) which is very small compared to the term  $\mathcal{M}(r)/r^2$ . In fact one can show that the pressure term is of order  $G^2$  by noticing the equation of hydrostatic equilibrium,  $r^2[dP(r)/dr] = -G\mathcal{M}(r)\rho$ . This equation holds in most of the stars for which the Newtonian approximation is valid. We cannot neglect the pressure term in such stars as white dwarfs or neutron stars. We will neglect the pressure term in the following discussions.

Here we give some clarifying comments on the theoretical basis and the assumptions we make in doing pilot computations for testing the equivalence principle by using solar neutrinos. First of all we ignore the effects of gravitational field of the earth, where the neutrino detector is located, by assuming that the sun's gravitational field gives a dominant effect. Strictly speaking we assume in our treatment that the detector is located at far apart from the sun where the metric is approximately Minkowskian. We also ignore the possible MSW transformation in the earth which may not give a good approximation with the large-mixing-angle solution of the solar neutrino problem.

One of the most important issue which remains to be explored is the problem of possible frame-dependence in the result. We shall make some comments on it while it will be addressed in more detail in Ref. [14]. Since the general coordinate invariance is broken in the theory defined by (1) there is no guarantee that the bound we obtain is independent of the frame we choose. Namely, we will do our computation by taking the inertial frame of the sun, but the result of neutrino flavor transformation might be different if we do the computation in the inertial frame of the earth even if we keep ignoring the effect of earth's gravitational field.

We argue, however, that the corrections due to the frame-dependence is of the order of  $\Delta f$ , and is negligibly small if we restrict ourselves into, for example,  $|\Delta f| \leq 10^{-10}$ . To this goal let us assume that by taking limit of  $\Delta f \rightarrow 0$  the theory defined by (1) approaches to general relativity coupled with matter-gauge fields. We are to deal with weak gravitational fields of the sun and confine ourselves into the perturbation expansion up to first order in  $G$ . In this weak-field approximation the above assumption should be true. Then in the limit of  $\Delta f \rightarrow 0$  there should be no frame dependence as it is the general relativity. In fact, it is a Newtonian gravity with special relativity. We conclude that within the weak-field approximation the frame dependence comes in as the first-order in  $\Delta f$  and would leave only negligible effects in observables. In particular the coefficient of  $\Delta f$  is frame independent.

Let us consider the case that the electron neutrinos are produced at the



origin and propagate toward the solar surface. In this case we can choose the coordinate with  $\theta = \phi = 0$  to describe the trajectory of neutrino propagation. Under this choice of the coordinate, the Dirac equation is given by

$$[i(e_t^0)^{-1}\gamma_0\partial_t + i(e_r^1)^{-1}\gamma^1\partial_r - m]\psi = 0, \quad (9)$$

where we note that the vierbeins  $e_t^0$  and  $e_r^1$  are given by the metric as  $(e_t^0)^2 = g_{tt}$  and  $(e_r^1)^2 = g_{rr}$ . Here we have ignored the derivative terms in spin connections because it is of the order of  $\sim 1/ER_\odot \sim 10^{-22}$  for  $E=10$  MeV and is negligibly small.

By taking into account only the positive energy component of the Dirac equation and choosing outgoing wave, we get the evolution equation in the weak gravitational field of the sun for one generation of neutrino,

$$i\frac{d}{dr}\nu = E\left[1 - \frac{m^2}{2E^2} - \phi(r)\right]\nu, \quad (10)$$

where

$$\phi(r) \equiv -G\left[\frac{\mathcal{M}(r)}{r} + \int_r^\infty \frac{\mathcal{M}(r)}{r^2}dr\right]. \quad (11)$$

It is easy to generalize this equation to the two flavor case described by the Lagrangian (1). The evolution equation in the sun for two neutrino flavors is given by

$$i\frac{d}{dr}\begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix} = H\begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix}, \quad (12)$$

where

$$H = \begin{bmatrix} \sqrt{2}G_F N_e + \Delta_M \sin^2 \theta_M + \Delta_G \sin^2 \theta_G & \frac{1}{2}(\Delta_M \sin 2\theta_M + \Delta_G \sin 2\theta_G) \\ \frac{1}{2}(\Delta_M \sin 2\theta_M + \Delta_G \sin 2\theta_G) & \Delta_M \cos^2 \theta_M + \Delta_G \cos^2 \theta_G \end{bmatrix} \quad (13)$$

with

$$\Delta_M \equiv \Delta m^2/2E, \quad (14)$$

$$\Delta_G \equiv \Delta f\phi(r)E. \quad (15)$$

where  $\Delta m^2 = m_2^2 - m_1^2$ . Note the difference in energy dependences of  $\Delta_M$  and  $\Delta_G$ . This makes it possible to distinguish the effect of the violation of

equivalence principle from that of the MSW effect by examining modulation of the neutrino energy spectrum.

With one more ingredient having added to the MSW mechanism the resultant equation of neutrino transformation (12) is far richer than the usual MSW's. We only make a few remarks on some distinctive features of it. The resonance condition is obtained by equating the two diagonal terms in the Hamiltonian matrix (13),

$$\sqrt{2}G_F N_e - \Delta_M \cos 2\theta_M - \Delta_G \cos 2\theta_G = 0 \quad (16)$$

From (16) one observes several notable features: (1) The resonance condition can be satisfied even for  $\Delta m^2 = 0$  if  $\Delta f < 0$ , the gravitational MSW effect.

(2) Resonance can occur at outside the sun where  $N_e = 0$  if  $\Delta m^2 \cdot \Delta f > 0$ . This should not come as a surprise because  $\phi(r)$  plays the role analogous to the position-dependent electron density in the MSW effect.

(3) The mathematical structure of the equation (12) is quite different from that of the flavor MSW mechanism by having  $r$ -dependent term in the off-diagonal element in the Hamiltonian (13). This makes the usual Landau-Zener analysis difficult, or at least highly nontrivial.

Notice that not only the condition (16) but also the adiabaticity condition is needed for sufficient resonant conversion of neutrinos. We can obtain the adiabaticity condition by demanding that the oscillation length at a resonance point is much shorter than the width of the resonance region. For resonance in the interior of the sun the adiabaticity condition reads,

$$\frac{h(\Delta_M \sin 2\theta_M + \Delta_G \sin 2\theta_G)^2}{2\pi(\Delta_M \cos 2\theta_M + \Delta_G \cos 2\theta_G)} \Big|_{r=r_{res}} \gg 1, \quad (17)$$

where

$$\tilde{\Delta}_G \equiv \Delta_G + h(d\Delta_G/dr). \quad (18)$$

To express the adiabaticity condition in the form of (17) we have approximated the electron number density  $N_e$  in the sun as

$$N_e(r) = N_0 \exp(-r/h) \quad (r > 0.1r_\odot). \quad (19)$$

with  $h \sim r_\odot/10$ .

Now we will discuss how sensitively violation of the equivalence principle affect the solar neutrino observation. To compute  $P(\nu_e \rightarrow \nu_x; E_\nu)$ , the probability of observing  $\nu_x$  ( $x = e$  and  $\tau$ ) at the earth, we employ the following

procedure. We numerically integrate the evolution equation (12) from  $r = 0$  to  $r = 10r_\odot$  where the gravity effect is negligible,  $|\Delta_G| \ll \Delta_M$ , for regions of parameters of interest to be specified below. We ignore the term  $\Delta_G$  at  $r > 10r_\odot$  and evaluate  $P(\nu_e \rightarrow \nu_x; E_\nu)$  by

$$P(\nu_e \rightarrow \nu_e; E_\nu) = \left(1 - \frac{1}{2} \sin^2 2\theta_M\right) |A_{\nu_e}|^2 + \frac{1}{2} \sin^2 2\theta_M |A_{\nu_\mu}|^2 - \sin 2\theta_M \cos 2\theta_M \operatorname{Re} [A_{\nu_e} A_{\nu_\mu}^*] \quad (20)$$

$$P(\nu_e \rightarrow \nu_\mu; E_\nu) = 1 - P(\nu_e \rightarrow \nu_e; E_\nu) \quad (21)$$

where  $A_{\nu_e}$  and  $A_{\nu_\mu}$  denote the probability amplitudes of neutrinos being  $\nu_e$  and  $\nu_\mu$  at  $r = 10r_\odot$ , respectively.

We calculate the electron energy spectrum  $f(E_e)$  to be measured at the Super-Kamiokande by the following equations

$$f(E_e) = \sum_{x=e,\mu} \int_0^\infty dE_\nu \varepsilon(E_e) [d\sigma_{\nu_e e}(E_\nu, E_e)/dE_e] \times P(\nu_e \rightarrow \nu_x; E_\nu) [d\phi_{\text{B}}(E_\nu)/dE_\nu], \quad (22)$$

where  $d\sigma_{\nu_e e}(E_\nu, E_e)/dE_e$  ( $x = e, \mu$ ) are the neutrino-electron elastic scattering cross section and  $d\phi_{\text{B}}(E_\nu)/dE_\nu$  is the differential flux of  ${}^8\text{B}$  neutrino calculated by the standard solar model (SSM) [16].

For the Sudbury Neutrino Observatory (SNO) detector we use a similar expression as (22), the one in which one restricts the summation over flavors into electron and replaces the neutrino-electron scattering cross section by the electron energy distribution for a given neutrino energy,  $dD(E_\nu, E_e)/dE_e$ , for the reaction  $\nu_e + d \rightarrow p + p + e^-$  calculated by Nozawa [17]. It includes the effects of (a) the Fermi motions of nucleons in the target deuterons, (b) the final state three-body kinematics, and (c) the final state Coulomb interactions. We have used very simplified form of the detection efficiency  $\varepsilon(E_e)$  as

$$\varepsilon(E_e) = \theta(E_e - E_{th}), \quad (23)$$

with  $E_{th} = 5$  MeV for both of the detectors. Alternative forms of (23) may not alter our conclusion because the effect of gravity exists in high energy part of the neutrino spectrum. We have taken into account the expected energy resolution  $14\%/\sqrt{E/10\text{MeV}}$  for Super-Kamiokande [18] and  $10\%/\sqrt{E/10\text{MeV}}$  for SNO [19].

As a prototype analysis in this paper we pick up the two sets of parameters of flavor mixing as representatives of (i) the nonadiabatic and (ii) the large-mixing-angle MSW solutions: (i)  $\Delta m^2 = 6 \times 10^{-6} \text{eV}^2$ ,  $\sin^2 2\theta_M = 0.01$  and (ii)  $\Delta m^2 = 2 \times 10^{-5} \text{eV}^2$ ,  $\sin^2 2\theta_M = 0.7$ .

We do not make any attempt to average over the production points of  ${}^8\text{B}$  neutrinos but assume that they are produced at the center of the sun. Given the parameter of interest the resonance points are well outside the production region of  ${}^8\text{B}$  neutrino and therefore our treatment should provide a reasonably good approximation.

To have a feeling on how sensitively violation of the equivalence principle affect the MSW-modulated neutrino spectrum we perform a pilot computation by taking  $\theta_G$  and  $\theta_M$  equal for the case of nonadiabatic MSW parameters (i). In Figs. 1 and 2 we show (a) recoil electron energy spectra, and (b) electron energy spectrum divided by the one predicted by SSM. Fig. 1 is for Super-Kamiokande and Fig. 2 is for SNO. We have done the computation with five parameters,  $\Delta f = 0, \pm 5.0 \times 10^{-16}$ , and  $\pm 1.0 \times 10^{-15}$ . In Figs. 1(b) and 2(b) the ratios are normalized at 9 MeV of electron energy. Super-Kamiokande is expected [18] to have accuracy of 2-3% for the ratio of modulated spectrum to that of SSM after its 3 years operation depending on the energy region and the MSW parameters. Therefore, from Fig. 1 we expect that the effect of  $|\Delta f|$  of the order of  $10^{-15}$  can be distinguished from the pure nonadiabatic MSW solution.

The similar computation can be done with the parameters (ii) of the large-mixing-angle MSW solution. But we do not show the result here for reasons which will be explained after the full analysis with the large-mixing angle MSW solution.

A technical comment is in order. Because of the limitation of the computer time we cut off the lower end of integration over the neutrino energy in (22) at  $E_\nu = 5$  MeV. Since we take into account the energy resolution of about 1 MeV (Super-Kamiokande) and 0.7 MeV (SNO) at electron energy of 5 MeV we would have to include the contribution from neutrino energy  $E_\nu \gtrsim 3$  MeV. It thus provides an unconventional way of smoothing out the effect of the sharp (step function) cut-off (23) of the detection efficiency. We believe that it does not give rise to any serious effects to our analysis because we use the ratio of the energy spectrum to that of SSM.

Now we enter into a full analysis with the nonadiabatic MSW solution of the solar neutrino problem. We aim at identifying the region of parameters

$\Delta f$  and  $\sin 2\theta_G$  in which the spectral shape of  ${}^8B$  neutrinos can be distinguished with 90% confidence level from that predicted by the pure MSW solution with the particular set of parameters (i) and (ii). At the present stage it is of course impossible to pinpoint the MSW parameters even if we assume that it is *the* solution to the solar neutrino problem. The present work with the particular set of parameters, therefore, is only meant to present a proto-typical analysis toward the more complete ones.

We define, as a quantitative measure for the deviation of the electron energy spectrum,

$$\chi^2 = \min \left[ \sum_{i=1}^N \left[ \frac{x_i(\Delta f = 0) - \alpha x_i(\Delta f, \sin 2\theta_G)}{\sigma_i(\text{st})} \right]^2 + \left( \frac{1 - \alpha}{\sigma_i(\text{sys})} \right)^2 \right], \quad (24)$$

following the procedure of Ref. [20]. We divide the energy range into  $N = 19$  bins and calculate the count rate by integrating the electron energy spectrum in each bin.  $x_i$  in (24) implies the ratio of the count rate calculated in this way to the one calculated in the same way but using SSM without any effects of flavor-gravitational transformation.  $\sigma_i(\text{st})$  and  $\sigma_i(\text{sys})$  denote the statistical and the systematic errors, respectively. The symbol *min* in front of the right-hand-side of (24) is to take minimization by varying the parameter  $\alpha$ . The measure  $\chi^2$  is thereby sensitive only to the shape of the energy spectra within the uncertainty of absolute normalization represented by the systematic error.

In our computation we treat the statistical error in the following way. We suppose that the Super-Kamiokande (SNO) detector observed 20000 (6000) and 40000 (12000) solar neutrino events, which roughly corresponds to its operation over 2 and 4 years, respectively. Then, it is rather straightforward to evaluate the statistical error by computing number of events in each bin. We combine the errors of the numerator and of the denominator by assuming that they are independent Gaussian errors. Lacking detailed knowledge of the detector performance we simply assume, as systematic errors of both the detectors,  $\sigma_i(\text{sys}) = \sigma(\text{sys}) = 6\%$ , the systematic error of the Kamiokande-II experiment [20].

We emphasize that our treatment of errors does not have any particular significance and should be taken only as tentative. In particular we did not attempt to make a detailed comparison between the expected sensitivities between the two detectors.

In Fig. 3 we present the results of the analysis for Super-Kamiokande in cases of (a)  $\Delta f > 0$  and (b)  $\Delta f < 0$ . Plotted in Fig. 3 are the regions of parameters on  $\Delta f - \sin 2\theta_G$  plane which are sensitive at 90% confidence level to the effect of violation of equivalence principle. Notice that the relative sign of  $\theta_G$  with respect to  $\theta_M$  is physically meaningful. The open circles and the asterisks are for cases with 40000 and 20000 events, respectively. We observe that the sensitive region goes down to  $|\Delta f| = 10^{-15} - 10^{-16}$  depending upon  $\sin 2\theta_G$ . It is notable that a factor of 2 better statistics achieved by taking the data for 2 more years does not significantly improve the sensitivity. It is largely due to a sharp growth of chi-square near the edge of the sensitivity region.

The same analysis is repeated for the SNO detection and the results are presented in Fig. 4. The notations are exactly the same as in Fig. 3. In spite of the assumed factor of  $\sim \sqrt{3}$  larger statistical errors the sensitivity of SNO to the violation of the equivalence principle is essentially equal to that of Super-Kamiokande. It is due to an advantage of the deuterium detector in which one can probe neutrino spectrum more accurately than the light-water detector by use of the absorption reaction  $\nu_e + d \rightarrow p + p + e^-$ . If we ignore the three effects mentioned before the electron energy is equal to the neutrino energy minus 1.44MeV. On the other hand the measurement of the recoil electron energy implies taking convolution (smearing) of the original neutrino spectrum.

In Fig. 5 and 6 we present the results with the parameter (ii) which corresponds to the large-mixing-angle solution. Fig. 5 is for Super-Kamiokande and Fig. 6 is for SNO. Again the results are almost the same. It is evident that the sensitivity is at most  $\Delta f \sim 10^{-14}$  at some particular values of  $\sin 2\theta_G$  and in general  $|\Delta f| \gtrsim 10^{-13}$ .

What is the reason for such (relative) great insensitivity in this case? In what follows we give a very simple-minded argument which apparently explains this behavior. For the parameter set (ii) with  $\Delta f \sim 10^{-14}$ , the adiabatic condition is almost completely satisfied except for the case  $\theta_M = \theta_G$ . An angle which diagonalizes the Hamiltonian matrix (13) is given by

$$\sin 2\theta = \frac{\Delta_M S_M + \Delta_G S_G}{\sqrt{(\Delta_M S_M + \Delta_G S_G)^2 + (\Delta_M C_M + \Delta_G C_G - \sqrt{2} G_F N_e)^2}}, \quad (25)$$

where  $S_{M,G} \equiv \sin 2\theta_{M,G}$  and  $C_{M,G} \equiv \cos 2\theta_{M,G}$ . By using this  $\theta$ , the two

energy eigenstate  $\nu_1$  and  $\nu_2$  of  $H$  is given by

$$\begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix}, \quad (26)$$

At the center of the sun,  $\theta \sim \pi/2$  because the matter effect  $N_e$  dominates over the mass term and the gravitational term (see eq. (25)). This means that  $\nu_e$  produced in the center of the sun is essentially equal to the state  $\nu_2$  (see eq. (26)). On the other hand, at the earth  $\theta \sim \theta_M$  because there is no matter effect ( $N_e = 0$ ) and  $\Delta G \ll \Delta_M$  for the parameter set (ii) with  $\Delta f \sim 10^{-14}$ . Hence if the resonance occurs adiabatically the probability of observing  $\nu_e$  at the earth is given by

$$P(\nu_e) \sim |\langle \nu_e | \nu_2 \rangle|_{\text{earth}}^2 = \sin^2 \theta_{\text{earth}} \sim \sin^2 \theta_M. \quad (27)$$

Of course this is the same expression as we obtain in the large mixing angle MSW mechanism by which the flux of solar neutrinos is uniformly reduced independent of energy. Thus we expect the non-universality of the neutrino gravitational couplings is unlikely to produce detectable effect in modulation of the neutrino energy spectrum with  $|\Delta f|$  smaller than  $10^{-14}$ , if adiabaticity condition holds. Far better sensitivity with the parameter (i) is then understood to be due to the non-adiabaticity of the neutrino flavor-gravitational transformation.

If we increase the value of  $|\Delta f|$  beyond  $\sim 10^{-13}$  the sensitivity to the non-universal gravitational coupling of neutrinos depends on the sign of  $\Delta f$ . As we see in Fig. 5 and 6 it produces an appreciable difference with the large-mixing-angle parameter (ii). Let us examine negative  $\Delta f$  case first to understand the reasons for the difference. In view of the resonance condition (16) the resonance ceases to occur if  $\Delta f < 0$  and  $|\Delta f| \gtrsim 10^{-13}$  because the gravity effect (third term of (16)) tends to dominate over the matter effect (first term). If the resonance does not occur the spectrum of course deviates from the one expected by the MSW mechanism. It explains the region of sensitivity which starts to develop at  $-\Delta f \simeq 5 \times 10^{-14}$  in Figs. 5b and 6b.

In case of positive  $\Delta f$  the situation is quite different. If  $\Delta f \gtrsim 10^{-14}$  the resonance can not occur inside the sun because both the first and the third terms of (16) give positive contributions. So the resonance occurs between the sun and the earth. For this resonance the adiabaticity condition is fulfilled and the argument leading to (27) holds. The insensitivity exhibited

in Figs. 5a and 6a follows because of the difficulty in perturbing the adiabatic large-mixing-angle mechanism by the effect of  $\Delta f$ . We should note that our computation is not very accurate in regions  $|\Delta f| \gtrsim 10^{-14}$  for these particular figures because we only integrate the equation (12) up to  $r = 10r_\odot$ .

In the analysis in this paper we have assumed that the MSW mechanism is the cause of the solar neutrino deficit. How reliable is this assumption in the light of existing data of ongoing four experiments? An extensive analysis by Hata and Langacker [21] seems to indicate that the MSW mechanism is the most plausible solution to the solar neutrino problem. Of course, we have to wait for the next generation experiments, Super-Kamiokande, SNO, and BOREXINO, to see if this really is the case. At the present stage of solar neutrino experiments it is worthwhile to examine the expected sensitivity with other solutions to the solar neutrino problem. We hope that we will be able to return to this problem in the near future.

In summary we have discussed the possibility of testing the equivalence principle by using solar neutrinos. We have presented an appropriate formalism for treating neutrino propagation under the influence of the weak gravitational fields of the sun. Assuming the MSW mechanism as the origin of the observed solar neutrino deficit we analyzed a complex process of neutrino flavor transformation with coexisting effects of the flavor mixing and the assumed non-universal gravitational couplings of neutrinos. We have obtained, as expected sensitivities,  $|\Delta f| \sim 10^{-15} - 10^{-16}$  for a parameter corresponding to the nonadiabatic solution and  $|\Delta f| \gtrsim 10^{-13}$  for the large-mixing angle solution. We emphasize that it opens a remarkable possibility that solar neutrino observation by next generation (light- or heavy-) water Cherenkov detectors can improve the experimental bound on violation of the principle of equivalence by 3-4 orders of magnitude.

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## References

- [1] R. v. Eötvös, *Math. nat. Ber. Ungarn*, **8**, 65 (1890); R. v. Eötvös, D. Pekár, and E. Fekete, *Ann. Phys.* **68**,11 (1922).
- [2] P. G. Roll, R. Krotkov, and R. H. Dicke, *Ann. Phys. (N. Y.)*, **26**, 442 (1964).
- [3] V. B. Braginski and V. I. Panov, *Sov. Phys. JETP* **34**, 463 (1972).
- [4] M. J. Longo, *Phys. Rev. Lett.* **60**, 173 (1988); L. M. Krauss and S. Tremaine, *ibid.* **60**, 176 (1988).
- [5] L. Koester, *Phys. Rev.* **D14**, 907 (1976); V. F. Sears, *Phys. Rev.* **D25**, 2023 (1982); J. Schmiedmayer, *Nucl. Instr. and Meth.* **A284**, 59 (1989).
- [6] M. Gasperini, *Phys. Rev.* **D38**, 2635 (1988); **D39**, 3606 (1989).
- [7] A. Halprin and C. N. Leung, *Phys. Rev. Lett.* **47**, 1833 (1991).
- [8] K. Iida, H. Minakata and O. Yasuda, *Mod. Phys. Lett.* **A8**, 1037 (1993).
- [9] J. Pantaleone, A. Halprin and C. N. Leung, *Phys. Rev.* **D47**, R4199 (1993). See, however, M. N. Butler, S. Nozawa, R. A. Malaney, and A. I. Boothroyd, *Phys. Rev.* **D47**, 2615 (1993).
- [10] J. N. Bahcall, *Neutrino Astrophysics* Cambridge University Press, New York, 1989).
- [11] S. P. Mikheyev and A. Smirnov, *Sov. J. Nucl. Phys.* **42**, 913 (1985); L. Wolfenstein, *Phys. Rev.* **D17**, 2369 (1978).
- [12] H. Minakata, in *Festschrift in Honor of Tetsuro Kobayashi's 63rd Birthday*, edited by H. Minakata (World Scientific, Singapore, 1994).
- [13] C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation* (W. H. Freeman and Co., San Francisco, 1973).
- [14] K. Iida, H. Minakata and O. Yasuda, in preparation.
- [15] J. N. Bahcall and R. K. Ulrich, *Rev. Mod. Phys.* **60**, 297 (1988).
- [16] S. Nozawa, private communications.
- [17] Y. Totsuka, in *Proceedings of International Symposium on Underground Physics Experiments*, edited by K. Nakamura (Institute for Cosmic Ray Research, Tokyo, 1990).
- [18] E. Beier and N. Hata, private communications.
- [19] K. S. Hirata et al. , *Phys. Rev. Lett.* **65**, 1301 (1990).
- [20] N. Hata and P. Langacker, preprint UPR-0592T, hep-ph/9311214.

## Figures Captions

Fig. 1. Modulation of (a) recoil electron energy spectrum and (b) the ratio of electron energy spectrum to that of SSM to be seen at the Super-Kamiokande detector are indicated. The ratios in (b) are normalized at  $E_e = 9$  MeV. The flavor mixing parameters are taken as (i)  $\Delta m^2 = 6 \times 10^{-8}$  eV<sup>2</sup> and  $\sin^2 2\theta_M = 0.01$  as a representative of the nonadiabatic MSW solution to the solar neutrino problem. The presented five lines are for  $\Delta f = 0, \pm 5.0 \times 10^{-16}$ , and  $\pm 10^{-15}$  with corresponding line symbols shown in the figure and  $\theta_G$  is taken to be equal to  $\theta_M$ . The upper solid line in Fig. 1 (a) which is denoted as SSM represents the energy spectrum expected by SSM.

Fig. 2. The same as in Fig. 1 but for SNO.

Fig. 3. The regions of sensitivity at 90 % confidence level to the violation of the equivalence principle are drawn on the parameter plane of  $\Delta f - \sin 2\theta_G$ . The result is for Super-Kamiokande and, 40000 (open circles) and 20000 (asterisks) solar neutrino events are assumed, roughly corresponding to its operation over 4 and 2 years, respectively. The flavor mixing parameter for nonadiabatic MSW solution are taken as (i)  $\Delta m^2 = 6 \times 10^{-8}$  eV<sup>2</sup> and  $\sin^2 2\theta_M = 0.01$ .

Fig. 4. The same as in Fig. 3 but for SNO. The numbers of events are taken as 12000 (open circles) and 6000 (asterisks), roughly corresponding to its operation over 4 and 2 years, respectively.

Fig. 5. The 90 % confidence level regions of sensitivity are drawn for flavor mixing parameter (ii)  $\Delta m^2 = 2 \times 10^{-5}$  eV<sup>2</sup> and  $\sin^2 2\theta_M = 0.7$  of large-mixing angle MSW solution. The result is for Super-Kamiokande and assumes 40000 (open circles) and 20000 (asterisks) solar neutrino events.

Fig. 6. The same as in Fig. 5 but for SNO. The numbers of events are taken as 12000 (open circles) and 6000 (asterisks).

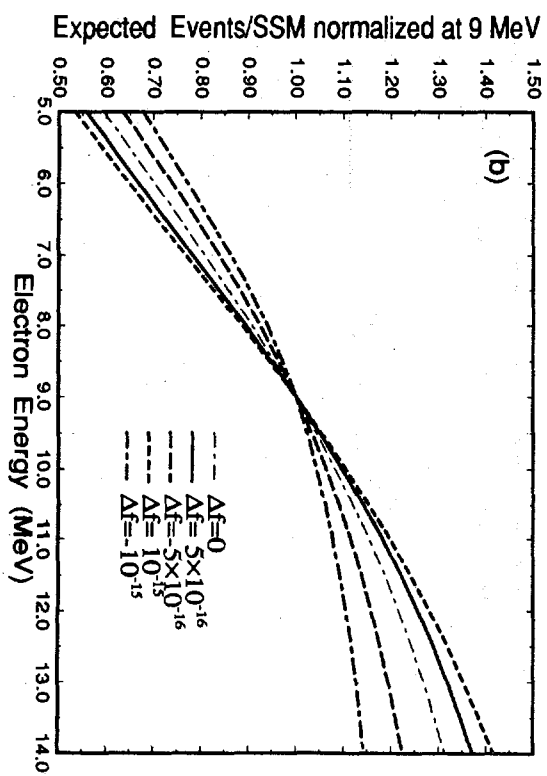
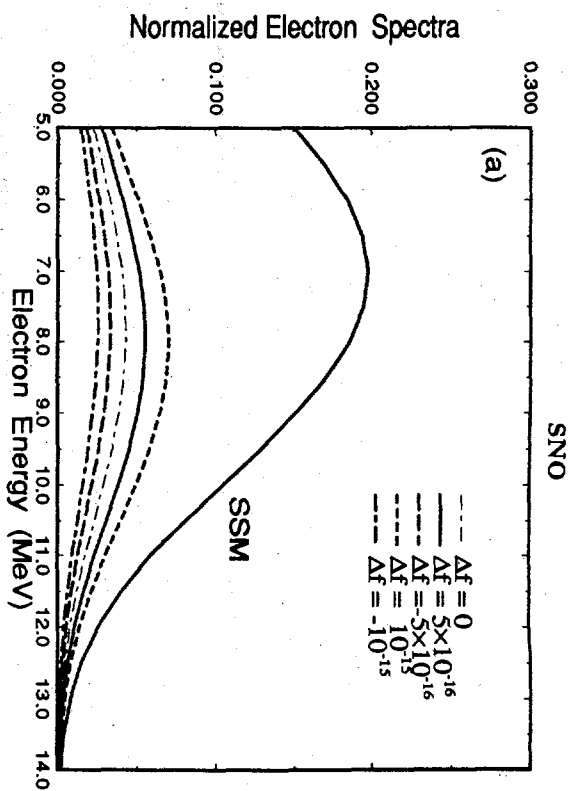
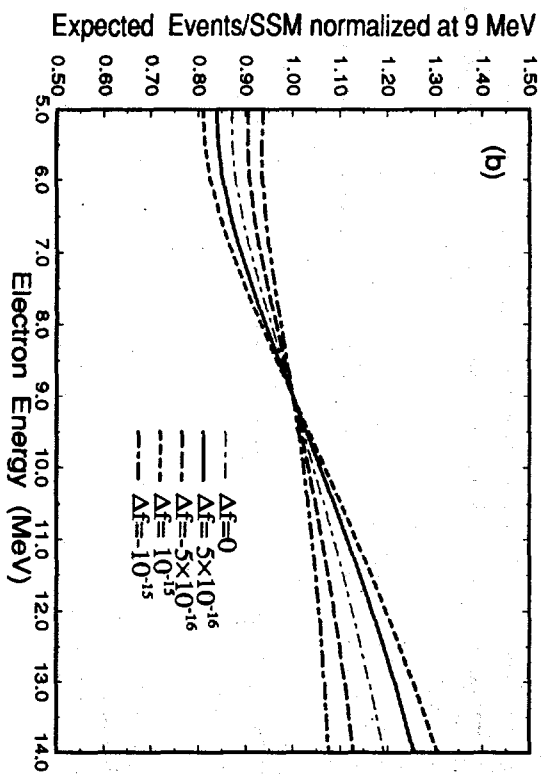
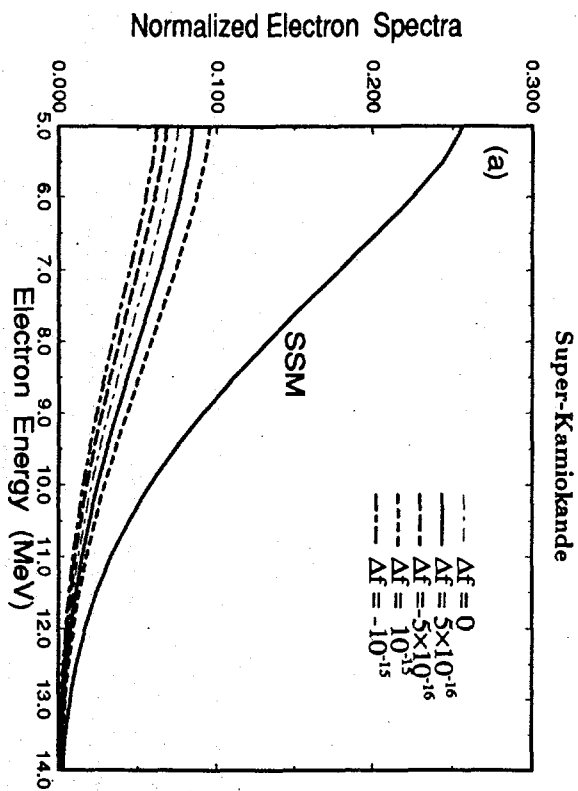
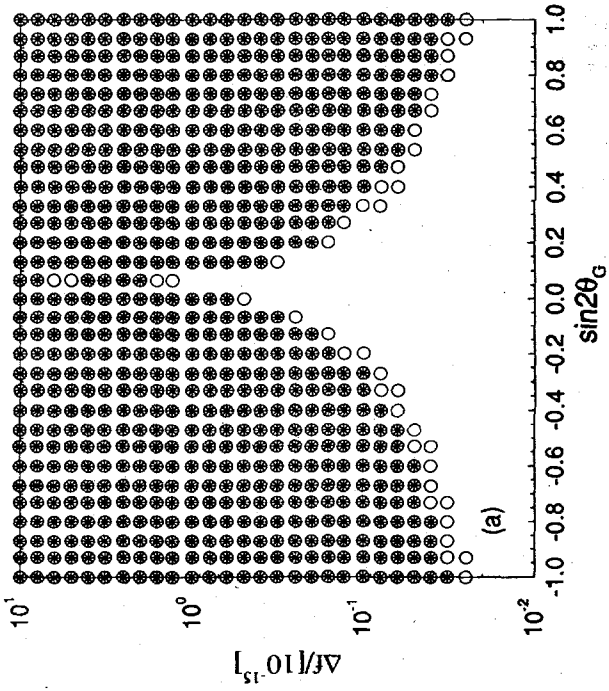


Fig. 1

Fig. 2

Super-Kamiokande



SNO

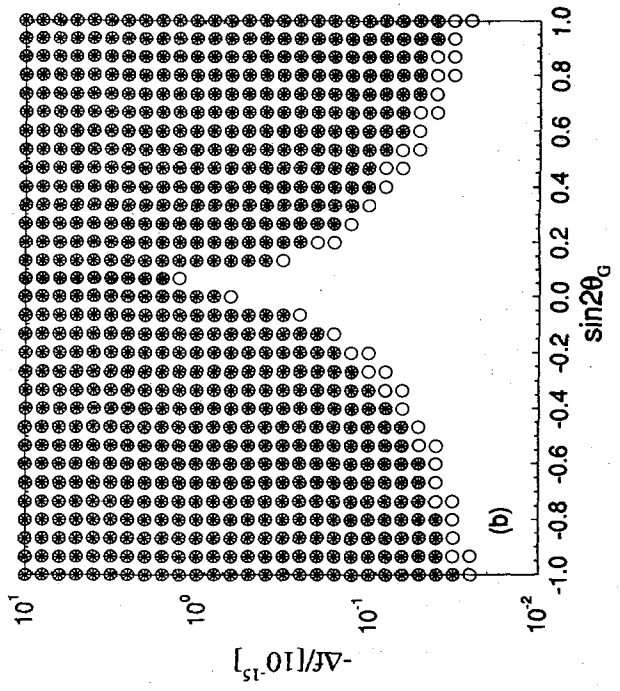
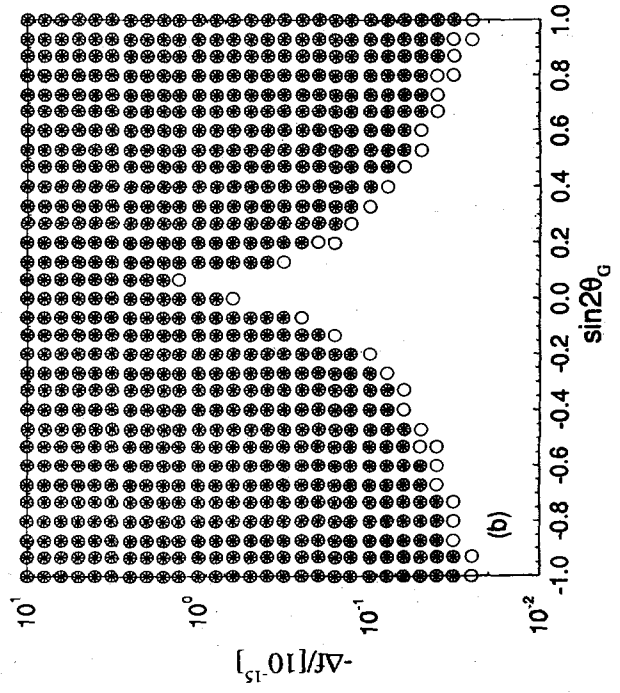
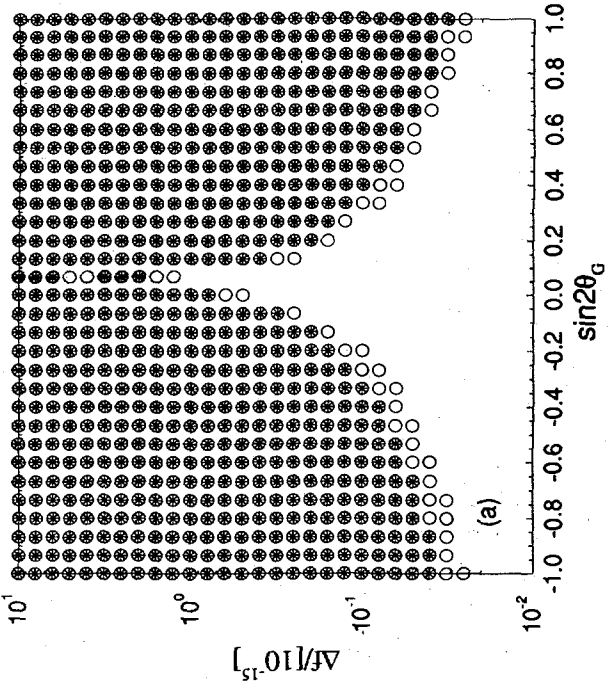


Fig. 3

Fig. 4



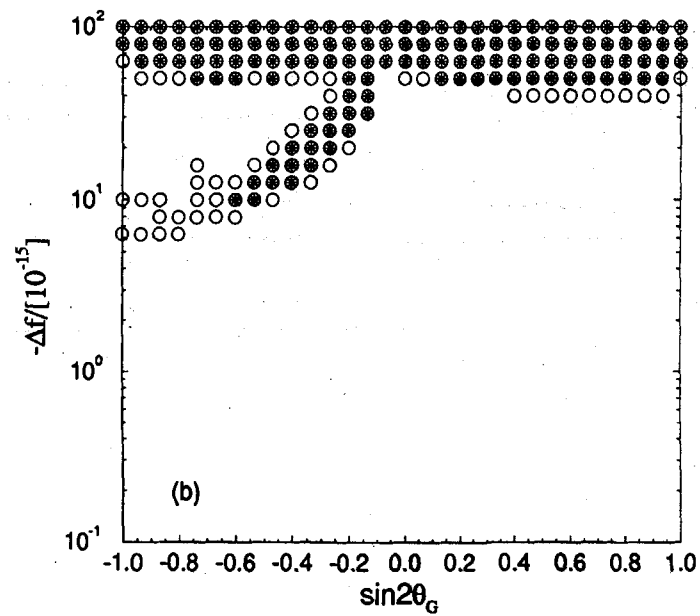
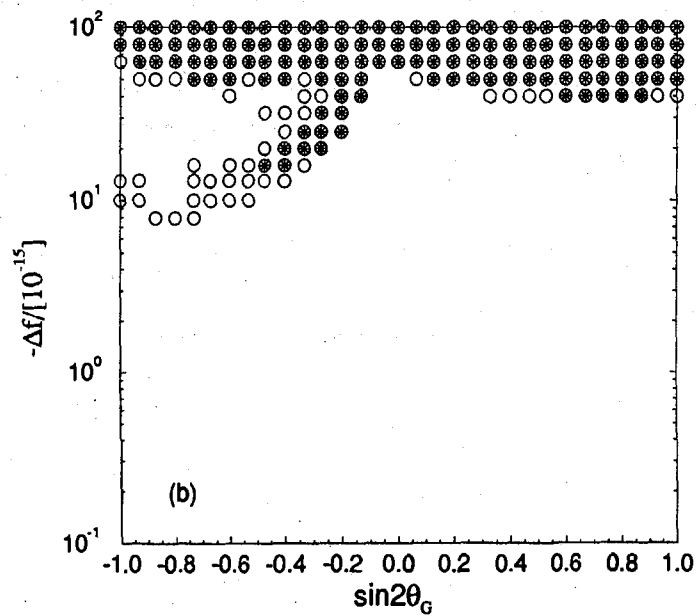
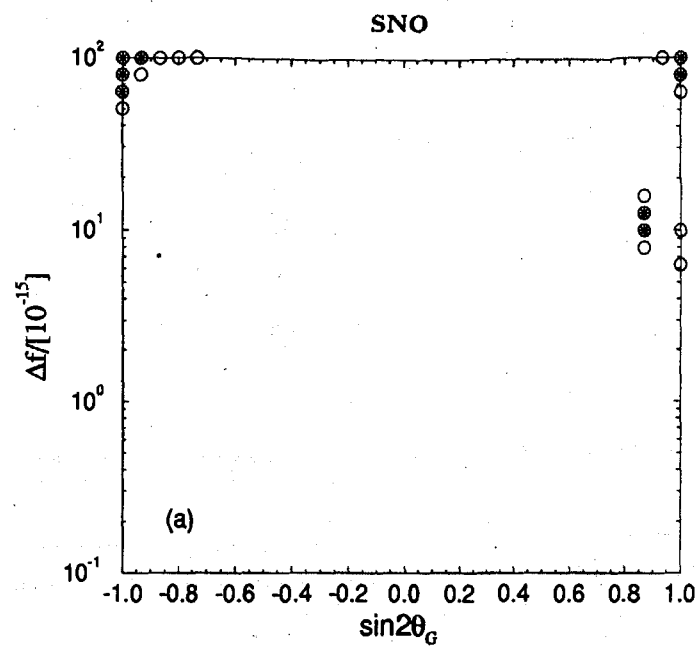
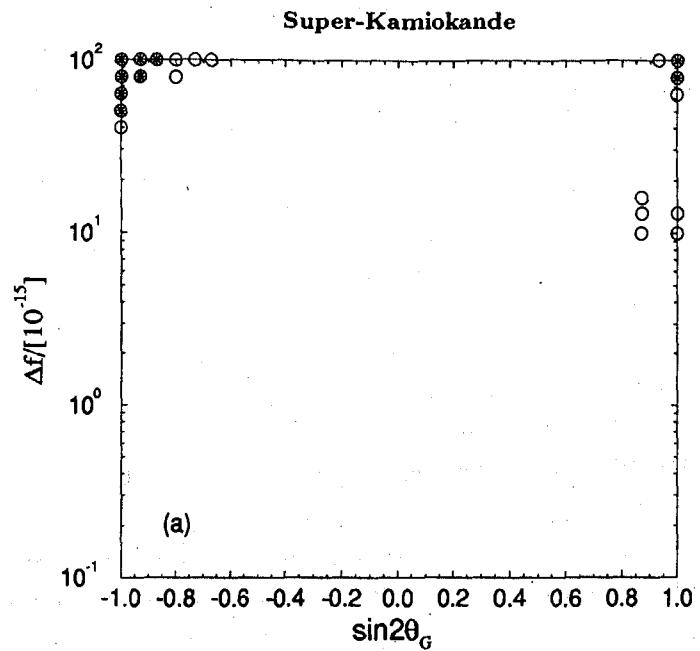


Fig. 5

Fig. 6