Relativistic Effects on Large Amplitude Nonlinear Langmuir Waves in a Two-Fluid Plasma

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Large amplitude relativistic nonlinear Langmuir waves are analyzed by the pseudo-potential method. The existence conditions for nonlinear Langmuir waves are confirmed by considering relativistic high-speed electrons in a two-fluid plasma. The significant feature of this investigation is that the propagation of nonlinear Langmuir waves depends on the ratio of the electron streaming velocity to the velocity of light, the normalized potential and the ion mass to electron mass ratio. The constant energy is determined by the specific range of the relativistic effect. In the non-relativistic limit, large amplitude relativistic Langmuir waves do not exist. The present investigation predicts new findings of large amplitude nonlinear Langmuir waves in space plasma phenomena in which relativistic electrons are important.

1. Introduction

In the recent space observations, it has been investigated that the high-speed electrons play a major role in the physical mechanism for the nonlinear wave structures. When we assume that the electron energies depend only on the kinetic energy, velocities of plasma particles in the solar atmosphere and the magnetosphere have to attain to relativistic speeds (Scarf *et.al.* 1984a and 1984b). Thus, by considering such relativistic effect as the electron velocities are about $0.01c$—$0.1c$ (c is the velocity of light), we can take the relativistic motion of such particles in the study of nonlinear plasma waves. When the velocity of the particles approaches that of light, the nonlinear waves which occur in the space exhibit a peculiar feature due to the effect of the high-speed electrons.

Although relativistic Langmuir waves have been studied as the subjects of laser-plasma interaction and laboratory experiments (Shukla *et.al.* 1984), the informations on such waves associated with fluid description have not been fully investigated in space plasmas. In the actual situations, there exist high-speed streaming electrons rather than high-speed streaming ions, and they cause the excitation of various kinds of nonlinear waves in the interplanetary space and the Earth's magnetosphere(Lin *et.al.* 1986). Hence, in this paper, we consider large amplitude relativistic nonlinear Langmuir waves composed of the relativistic high-speed electrons and non-relativistic ions. If the problem is solved, we will understand space plasma phenomena associated with high-speed electrons.

The purpose of this paper is to derive the pseudo-potential for relativistic Langmuir waves in a two-fluid plasma and to show that the stationary nonlinear potential structures can be formulated in terms of an integral equation which expresses the same form as the

equation governing the motion of particles in a potential well. It
will be predicted that the existence conditions for large amplitude
relativistic nonlinear Langmuir waves are confirmed by considering the
normalized potential and relativistic high-speed electrons in a two-
fluid plasma.

The layout of this paper is as follows. In § 2, we present the
basic equations for a two-fluid plasma and derive an energy equation
with the pseudo-potential. In § 3, we discuss the existence
condition of large amplitude relativistic nonlinear Langmuir waves
on the basis of an energy equation. The dependency of the pseudo-
potential on the normalized potential and the relativistic effect is
presented. The last section is devoted to the concluding discussion.

2. Basic equations

We consider an unmagnetized, collisionless relativistic two-fluid
plasma consisting of the hot and isothermal electrons and ions. We do
not take into account kinetic effects such as the deviation from the
Maxwell distribution, Landau damping, etc. and assume the electron flow
velocities are relativistic, and thereby there exist high-speed
streaming electrons in an equilibrium state.

The equations of continuity, the equations of motion and Poisson's
equation of a relativistic two-fluid plasma in a one-direction
are described as (Nejoh 1992 and 1994):

$$\frac{\partial}{\partial t}\left[\,\gamma_e\,n_e\,\right] + \frac{\partial}{\partial x}\left[\,\gamma_e\,n_e\,v_e\,\right] = 0 \qquad (1.a)$$

$$\frac{\partial}{\partial t}\,n_i\ + \frac{\partial}{\partial x}\left[\,n_i\,v_i\,\right] = 0 \qquad (1.b)$$

$$m_e \left(\frac{\partial}{\partial t} + v_e \frac{\partial}{\partial x} \right) (\gamma_e v_e) = e \frac{\partial \phi}{\partial x} - \frac{1}{n_e} \frac{\partial P_e}{\partial x} \tag{1.c}$$

$$m_i \left(\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x} \right) v_i = -e \frac{\partial \phi}{\partial x} - \frac{1}{n_i} \frac{\partial P_i}{\partial x} \tag{1.d}$$

and

$$\varepsilon_0 \frac{d^2 \phi}{d x^2} = e (n_e - n_i) \tag{1.e}$$

where

$$\gamma_e = \frac{1}{\left[1 - \left(\frac{v_e}{c} \right)^2 \right]^{1/2}}$$

$$\approx 1 + \frac{1}{2} \left(\frac{v_e}{c} \right)^2.$$

The subscript e denotes electrons. The electron and ion pressures are determined by $P_e = \kappa T_e n_e$ and $P_i = \kappa T_i n_i$, respectively, where $T_e (T_i)$ and $n_e (n_i)$ denote the electron (ion) temterature and electron (ion) density. We use the non-relativistic ion pressure as a model.

In the stationary state, the conservation of electron and ion fluxes are obtained as

$$n_e v_e \left[1 + \frac{1}{2} \left(\frac{v_e}{c} \right)^2 \right] = C_1 \tag{2.a}$$

$$n_i v_i = C_2 , \tag{2.b}$$

from eqs. (1.a) and (1.b). The equations for conservation of electron and ion energy are described as:

$$\frac{m_e v_e{}^2}{2} \left[1 + \frac{3}{4} \left[\frac{v_e}{c} \right]^2 \right] - e\phi + \kappa T_e \ln n_e = C_3 \qquad (2.c)$$

$$\frac{m_i v_i{}^2}{2} + e\phi + \kappa T_i \ln n_i = C_4 \qquad (2.d)$$

which are obtained by integrating eqs. (1.c) and (1.d), in the stationary state. $C_1 \sim C_4$ are integration constants. T_e and T_i are assumed to be constant.

In order to obtain the conservation law of total energy, we examine the following calculation.

In the stationary state, eqs. (1.c) and (1.d) become

$$m_e v_e \frac{\partial}{\partial x} \gamma_e v_e - e\frac{d\phi}{dx} + \kappa T_e \frac{1}{n_e} \frac{\partial n_e}{\partial x} = 0 \qquad (3.a)$$

and

$$m_i v_i \frac{\partial}{\partial x} v_i + e \frac{d\phi}{dx} + \kappa T_i \frac{1}{n_i} \frac{\partial n_i}{\partial x} = 0 . \qquad (3.b)$$

It is obvious that the integration of (3.a) is eq. (2.c) and the integration of (3.b) is (2.d).

Multiplying (3.a) by n_e and (3.b) by n_i, we have

$$n_e m_e v_e \frac{\partial}{\partial x} \left[1 + \frac{1}{2} \left[\frac{v_e}{c} \right]^2 \right] v_e - n_e e\frac{d\phi}{dx} + \kappa T_e \frac{\partial n_e}{\partial x} = 0 , \qquad (4.a)$$

and

$$n_i m_i v_i \frac{\partial}{\partial x} v_i + n_i e\frac{d\phi}{dx} + \kappa T_i \frac{\partial n_i}{\partial x} = 0 . \qquad (4.b)$$

Adding eq.(4.a) to eq.(4.b) and using Poisson's equation, we obtain

$$n_e m_e v_e \frac{\partial}{\partial x} \left[1 + \frac{1}{2} \left[\frac{v_e}{c} \right]^2 \right] v_e + n_i m_i v_i \frac{\partial}{\partial x} v_i$$

$$= \varepsilon_0 \frac{d\phi}{dx} \frac{d^2\phi}{dx^2} - \kappa T_e \frac{\partial n_e}{\partial x} - \kappa T_i \frac{\partial n_i}{\partial x} . \qquad (5)$$

We integrate eq.(5) and define its left-hand side and right-hand side as follows;

$$L \equiv L_1 + L_2,$$

$$L_1 \equiv \int \left[n_e m_e v_e \frac{\partial}{\partial x} \left[1 + \frac{1}{2} \left[\frac{v_e}{c} \right]^2 \right] v_e \right] dx ,$$

$$L_2 \equiv \int \left[n_i m_i v_i \frac{\partial}{\partial x} v_i \right] dx ,$$

for the left-hand side and

$$R \equiv \int \left[\varepsilon_0 \frac{d\phi}{dx} \frac{d^2\phi}{dx^2} - \kappa T_e \frac{\partial n_e}{\partial x} - \kappa T_i \frac{\partial n_i}{\partial x} \right] dx$$

for the right-hand side, respectively.

First, we consider L_1, that is,

$$L_1 \equiv \int n_e m_e \frac{\partial}{\partial x} \left[\frac{v_e^2}{2} \right] dx + \frac{3}{2c^2} \int n_e m_e \frac{\partial}{\partial x} \left[\frac{v_e^4}{4} \right] dx$$

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$$= C_1 m_e \int \frac{1}{v_e \left[1 + \frac{1}{2} \left[\frac{v_e}{c}\right]^2\right]} \frac{d}{dx} \left[\frac{v_e^2}{2}\right] dx$$

$$+ \frac{3}{2c^2} C_1 m_e \int \frac{1}{v_e \left[1 + \frac{1}{2} \left[\frac{v_e}{c}\right]^2\right]} \frac{d}{dx} \left[\frac{v_e^4}{4}\right] dx$$

$$= \frac{1}{2} C_1 m_e \left\{ \frac{v_e}{1 + \frac{1}{2} \left[\frac{v_e}{c}\right]^2} + \int \frac{1 + \frac{3}{2} \left[\frac{v_e}{c}\right]^2}{\left[1 + \frac{1}{2} \left[\frac{v_e}{c}\right]^2\right]^2} dv_e \right\}$$

$$+ \frac{3}{8c^2} C_1 m_e \left\{ \frac{v_e^3}{1 + \frac{1}{2} \left[\frac{v_e}{c}\right]^2} + \int \frac{v_e^2 \left[1 + \frac{3}{2} \left[\frac{v_e}{c}\right]^2\right]}{\left[1 + \frac{1}{2} \left[\frac{v_e}{c}\right]^2\right]^2} dv_e \right\} \quad .$$

$$(6)$$

Here, we used eq. (2.a) and the partial integration. The integration in (6) can be solved in the following approximated form

$$\int \frac{1 + \frac{3}{2} \left[\frac{v_e}{c}\right]^2}{\left[1 + \frac{1}{2} \left[\frac{v_e}{c}\right]^2\right]^2} dv_e = \frac{c}{2^{1/2}} \left\{ 3 \int \frac{dX}{X(X-1)^{1/2}} - 2 \int \frac{dX}{X^2(X-1)^{1/2}} \right\}$$

$$= 2^{1/2} c \left\{ 2\arctan \left[\frac{v_e}{2^{1/2}c} \right] - \frac{\frac{v_e}{2^{1/2}c}}{1 + \frac{1}{2} \left[\frac{v_e}{c}\right]^2} \right\}$$

$$= 2^{1/2} c \left\{ 2 \left[\frac{v_e}{2^{1/2}c} - \frac{1}{3} \left[\frac{v_e}{2^{1/2}c}\right]^3 + \frac{1}{5} \left[\frac{v_e}{2^{1/2}c}\right]^5 - \cdots \right] \right.$$

$$\left. -\frac{\dfrac{v_r}{2^{1/2}c}}{1+\dfrac{1}{2}\left[\dfrac{v_c}{c}\right]^2} \right\} \; .$$

(6.1)

where X is defined by $X = 1 + (1/2)(v_r/c)^2$. Here we expanded $\arctan(v_c/2^{1/2}c)$ under the condition $v_r/2^{1/2}c \ll 1$. Performing the similar calculation, one obtain

$$\int \frac{v_c{}^2\left[1+\dfrac{3}{2}\left[\dfrac{v_c}{c}\right]^2\right]}{\left[1+\dfrac{1}{2}\left[\dfrac{v_c}{c}\right]^2\right]^2}\,dv_r$$

$$= (2^{1/2}c)^3\left\{3\int\frac{dX}{(X-1)^{1/2}} -5\int\frac{dX}{X(X-1)^{1/2}} +2\int\frac{dX}{X^2(X-1)^{1/2}}\right\}$$

$$= (2^{1/2}c)^3\left\{3\frac{v_c}{2^{1/2}c}+\frac{\dfrac{v_r}{2^{1/2}c}}{1+\dfrac{1}{2}\left[\dfrac{v_c}{c}\right]^2}-4\arctan\left[\frac{v_c}{2^{1/2}c}\right]\right\}$$

$$= (2^{1/2}c)^3\left\{3\frac{v_c}{2^{1/2}c}+\frac{\dfrac{v_c}{2^{1/2}c}}{1+\dfrac{1}{2}\left[\dfrac{v_r}{c}\right]^2}\right.$$

$$\left. -4\left[\frac{v_c}{2^{1/2}c}-\frac{1}{3}\left[\frac{v_r}{2^{1/2}c}\right]^3+\frac{1}{5}\left[\frac{v_r}{2^{1/2}c}\right]^5-\cdots\cdots\right]\right\} \; .$$

(6.2)

The use of eqs.(6.1) and (6.2) yields

$$L_1 = C_1 m_e v_e \left[1 - \frac{1}{6} \left[\frac{v_e}{c} \right]^2 \right]$$

$$+ \frac{3}{8c^2} C_1 m_e v_e \left[\frac{v_e{}^2}{1 + \frac{1}{2} \left[\frac{v_e}{c} \right]^2} + 2c^2 \left[\frac{1}{1 + \frac{1}{2} \left[\frac{v_e}{c} \right]^2} \right.\right.$$

$$\left.\left. - \left[1 - \frac{2}{3} \left[\frac{v_e}{c} \right]^2 \right] \right] \right],$$

$$(7.1)$$

where we approximated that $(v_e/c)^4 \approx 0$, $(v_e/c)^6 \approx 0$.

The integration L_2 can be easily obtained,

$$L_2 = C_2 m_i v_i \quad . \tag{7.2}$$

Here we used eq. (2. b) and the approximation $(v_e/c)^4 \approx 0$, $(v_e/c)^6 \approx 0$. Using eqs. (7.1) and (7.2), we can reduce the integration L to

$$L = C_1 m_e v_e \left[1 - \frac{1}{6} \left[\frac{v_e}{c} \right]^2 \right.$$

$$+ \frac{3}{8c^2} \left\{ \frac{v_e{}^2}{1 + \frac{1}{2} \left[\frac{v_e}{c} \right]^2} + 2c^2 \left[\frac{1}{1 + \frac{1}{2} \left[\frac{v_e}{c} \right]^2} \right.\right.$$

$$\left.\left.\left. - 1 + \frac{2}{3} \left[\frac{v_e}{c} \right]^2 \right] \right\} \right]$$

$$+ C_2 m_i v_i \tag{8}$$

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Expansion of $1/[1+(1/2)(v_e/c)^2]$ in (8) under the condition $(v_e/c)^2 \ll 1$ gives rise to

$$L = C_1 m_e v_e \left[1 + \frac{1}{3} \left\{ \frac{v_e}{c} \right\}^2 \right] + C_2 m_i v_i , \qquad (9)$$

where we approximated that $(v_e/c)^4 \approx 0$, $(v_e/c)^6 \approx 0$.

Next, integrating the right-hand side of eq.(5) and using eq.(9), we obtain, from $L = R$,

$$C_1 m_e v_e \left[1 + \frac{1}{3} \left\{ \frac{v_e}{c} \right\}^2 \right] + C_2 m_i v_i$$

$$= \frac{1}{2} \varepsilon_0 \left\{ \frac{d\phi}{dx} \right\}^2 - \kappa T_e n_e - \kappa T_i n_i - C_5 .$$

Using eqs.(2.a) and (2.b) for C_1 and C_2, and putting $(v_e/c)^4 \approx 0$, we finally obtain

$$m_e n_e v_e^2 \left[1 + \frac{5}{6} \left\{ \frac{v_e}{c} \right\}^2 \right] + m_i n_i v_i^2$$

$$+ \kappa T_e n_e + \kappa T_i n_i - \frac{1}{2} \varepsilon_0 \left\{ \frac{d\phi}{dx} \right\}^2 = -C_5. \qquad (10)$$

Equation (10) is the conservation law of total pressure, where C_5 is the integration constant.

From eq.(10), one obtain

$$\frac{1}{2} \left\{ \frac{d\phi}{dx} \right\}^2 + V = 0 \qquad (11)$$

where the pseudo-potential is given by

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$$V = U - W \tag{12}$$

and

$$U = -\frac{1}{\varepsilon_0} \left\{ \kappa T_e n_e + \kappa T_i n_i \right.$$

$$\left. + m_e n_e v_e^2 \left[1 + \frac{5}{6} \left(\frac{v_e}{c} \right)^2 \right] + m_i n_i v_i^2 \right\} \tag{13}$$

where

$$W = \frac{C_5}{\varepsilon_0} \tag{14}$$

Equation (11) represents an energy equation for a classical particle moving with the velocity $d\phi/dx$ in a potential $V(\phi)$.

We consider the nonlinear potential structures for Langmuir wave with eqs. (11) and (12) in the following section.

3. L a r g e   a m p l i t u d e   r e l a t i v i s t i c   L a n g m u i r   w a v e s

We consider large amplitude relativistic Langmuir waves in the case that the electron inertia is important.

We assume that the electron flux and the ion flux are equal, that is,

$$n_e v_e = n_i v_i = \varphi_0 . \tag{15}$$

In the case of $v_e = v_i = v_0$ at $\phi = 0$ and $T_e = T_i = 0$, from eqs.

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(2.c) and (2.d), the integration constants C_3 and C_4 are given as

$$C_3 = \frac{m_e v_0^2}{2} \left[1 + \frac{3}{4} \left[\frac{v_0}{c} \right]^2 \right] \quad,$$

$$C_4 = \frac{m_i v_0^2}{2} \quad,$$

where v_0 is the constant velocity. In this case, eqs. (2.c) and (2.d) reduce to

$$\frac{m_e v_e^2}{2} \left[1 + \frac{3}{4} \left[\frac{v_e}{c} \right]^2 \right] - e\phi = \frac{m_e v_0^2}{2} \left[1 + \frac{3}{4} \left[\frac{v_0}{c} \right]^2 \right], \quad (16.\,a)$$

and

$$\frac{m_i v_i^2}{2} + e\phi = \frac{m_i v_0^2}{2} \quad. \tag{16.\,b}$$

Using eqs. (16.a) and (16.b), we get, from eqs. (2.c) and (2.d),

$$\frac{v_e}{v_0} \approx \left[1 + \frac{e\phi}{\frac{m_e v_0^2}{2} \left[1 + \frac{3}{4} \left[\frac{v_0}{c} \right]^2 \right]} \right]^{1/2} \quad, \tag{17.\,a}$$

$$\frac{v_i}{v_0} \approx \left[1 - \frac{e\phi}{\frac{m_i v_0^2}{2}} \right]^{1/2} \quad, \tag{17.\,b}$$

where we approximated that $[1 + (3/4)(v_0/c)^2] \approx [1 + (3/4)(v_e/c)^2]$. Then the potential function $U(\phi)$ reduces to

$$U(\phi) = -\omega_e^2 v_0^2 \left[\frac{m_e}{e} \right]^2 \left\{ \left[1 + \frac{e\phi}{\frac{m_e v_0^2}{2} \left[1 + \frac{3}{4} \left[\frac{v_0}{c} \right]^2 \right]} \right]^{1/2} \right.$$

$$\times \left[1 + \frac{5}{6} \left[\frac{v_0}{c} \right]^2 \left[1 + \frac{e\phi}{\frac{m_e v_0^2}{2} \left[1 + \frac{3}{4} \left[\frac{v_0}{c} \right]^2 \right]} \right] \right] \right\}$$

$$+\frac{m_i}{m_e}\left[1-\frac{e\phi}{\frac{m_i}{m_e}\frac{m_e v_0^2}{2}}\right]^{1/2}\Bigg\}$$

(18)

by using eqs. (17.a) and (17.b), where $\omega_e=(n_e e^2/\varepsilon_0 m_e)^{1/2}$ is the electron plasma frequency.

The oscillatory solution of the nonlinear Langmuir wave exist when the following two conditions are satisfied.

(i)The potential $U(\phi)$ or $V(\phi)$ has a minimum value W_{min} at $\phi=0$. The minimum energy W_{min} is as follows:

$$W_{min}\fallingdotseq U(\phi=0)$$

$$\fallingdotseq-\omega_e^2 v_0^2\left\{\frac{m_e}{e}\right\}^2\left\{1+\frac{m_i}{m_e}+\frac{5}{6}\left\{\frac{v_0}{c}\right\}^2\right\}.$$

(19)

Nonlinear Langmuir waves exist provided that the constant energy W in eq. (14) exceeds a minimum value W_{min}.

(ii)The maximum potential W_{max} holds when $W_{max}=U(\phi_c)$, where $\phi_c=-(m_e v_0^2/2e)\left[1+(3/4)(v_0/c)^2\right]$. W_{max} is given as

$$W_{max}\fallingdotseq U(\phi_c)$$

$$\fallingdotseq-\omega_e^2 v_0^2\left\{\frac{m_e}{e}\right\}^2\frac{m_i}{m_e}\left[1+\frac{m_e}{m_i}\left\{1+\frac{3}{4}\left\{\frac{v_0}{c}\right\}^2\right\}\right]^{1/2}.$$

(20)

Oscillatory nonlinear Langmuir solutions exist if W is less than the maximum value W_{max}. From eqs. (19) and (20), we obtain

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the relativistic nonlinear Langmuir wave in the case of W lies in
the range

$$-\omega_e^2 v_0^2 \left(\frac{m_e}{e}\right)^2 \left[1+\frac{m_i}{m_e}+\frac{5}{6}\left(\frac{v_0}{c}\right)^2\right] \;\; < \;\; W$$

$$< \;\; -\omega_e^2 v_0^2 \left(\frac{m_e}{e}\right)^2 \frac{m_i}{m_e}\left\{1+\frac{m_e}{m_i}\left[1+\frac{3}{4}\left(\frac{v_0}{c}\right)^2\right]\right\}^{1/2}. \;\; (21)$$

Large amplitude nonlinear Langmuir waves can propagate when W
satisfies eq.(21). It is noted that the energy
$W/(\omega_e^2 v_0^2(m_e/e)^2)$ of the Langmuir wave depends on m_i/m_e
and the relativistic effect v_0/c.

It should be noted that $U(\phi)$ is real, if

$$-\left[1+\frac{3}{4}\left(\frac{v_0}{c}\right)^2\right] < \frac{e\phi}{\dfrac{m_e v_0^2}{2}} < \frac{m_i}{m_e} \;.$$

Figure 1 illustrates the dependency on the normalized potential
$e\phi/(m_e v_0^2/2)$ and the effect of the streaming velocity v_0/c of
the pseudo-potential $V(\phi)/(\omega_e^2 v_0^2(m_e/e)^2)$, where $W=1837.0$
and $V(\phi)= U-W$. From figure 1, when W nearly equal to
$-(1+m_i/m_e)\sim-1837$ in the case that the ion is proton(H^+), we
can understand the following:
(1) If $0< v_0/c <0.144$, $V(\phi)/(\omega_e^2 v_0^2(m_e/e)^2)$ becomes positive
 almost everywhere.
(2) When v_0/c lies in the range $0.144< v_0/c <0.167$, the curve of
 $V(\phi)$ forms the potential well and $V(\phi)$ approaches from the
 positive value to the negative value, and finally to positive
 value at $e\phi/(m_e v_0^2/2)=1836$. In this case, stationary
 relativistic Langmuir waves exist.

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(3) When v_0/c becomes large more than 0.168, $V(\phi)$ becomes negative almost everywhere.

　　We show the existence region of large amplitude relativistic Langmuir waves in figure 2. It is noted that Langmuir waves exist in the region A but do not exist in the region B. Figure 3 shows the existence region of the energy W depending on the relativistic effect v_0/c.

　　We now understand that large amplitude relativistic Langmuir waves can propagate under the proper conditions, mentioned above.

4. C o n c l u d i n g　D i s c u s s i o n

　　The nonlinear wave structures of large amplitude relativistic nonlinear Langmuir waves is investigated in a two-fluid plasma composed with relativistic electrons and cold non-relativistic ions. We predicted the existence conditions for large amplitude stationary relativistic Langmuir waves, by analyzing the structure of pseudo-potential, which is illustrated in figures 1-3. The author finds the following facts:

(a) The existence conditions for relativistic Langmuir waves depend on the ratio of the ion mass to electron mass m_i/m_e, the relativistic effect v_0/c and the normalized potential $e\phi/(m_e v_0^2/2)$. It is noted that m_i/m_e is constant in the case that the ion is proton (H^+) but v_0/c is a variable.

(b) In the case that $v_0/c < 0.144$, since the pseudo-potential takes almost positive values, nonlinear Langmuir waves do not exist.

(c) Large amplitude relativistic Langmuir waves propagate if the condition $0.144 < v_0/c < 0.167$ is satisfied for the pseudo-

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potential.   This implies that the existence condition of relativistic Langmuir waves depends on the relativistic effect. The existence region of the energy W depending on the relativistic effect is shown.

(d) If the value of v_0/c is more than 0.167, the pseudo-potential is always negative.   In this case, nonlinear Langmuir waves cannot propagate.

(e) It is shown that the existence region of the normalized potential for relativistic nonlinear Langmuir waves is large as long as a certain value of the relativistic effect, but it becomes narrower as the relativistic effect decreases.

(f) If the velocity of electrons is non-relativistic, large amplitude Langmuir waves do not exist.

We emphasize that the pseudo-potential for the relativistic nonlinear Langmuir wave depends on the fixed ratio of the ion mass to electron mass and on the relativistic effect.   The nonlinear wave structure varies according to the relativistic effect, normalized potential and the ion mass to electron mass ratio.   Although we have no direct observational data of high-speed energetic events, it appears reasonable to assume that we can apply this theory to energetic large amplitude relativistic nonlinear Langmuir waves which occur in space.   Therefore, since the results obtained here have importance when we discuss nonlinear Langmuir waves in plasmas with relativistic high-speed electrons, the present investigation may be applicable to another plasma physical systems.

Acknowledgement

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Captions of figures

Figure 1:

The curves of the pseudo-potential $V/(\omega_e^2 v_0^2 (m_e/e)^2)$ for large amplitude relativistic Langmuir waves represented by eq. (11) with eqs. (12)-(14) under the conditions of $0.140 < v_0/c < 0.170$ and $0 < m_e v_0^2/2 < 1836.$ $W = 1837.0$. Several values of $V/(\omega_e^2 v_0^2 (m_e/e)^2)$: (a) $v_0/c = 0.140$, (b) $v_0/c = 0.146$, (c) $v_0/c = 0.150$, (d) $v_0/c = 0.155$, (e) $v_0/c = 0.160$, (f) $v_0/c = 0.165$, (g) $v_0/c = 0.170$.

Figure 2:

The existence region of large amplitude relativistic Langmuir shock and/or solitary waves depending on the normalized potential $e\phi/(m_e v_0^2/2)$ and the relativistic effect v_0/c.

Figure 3:

The existence region of the energy W depending on the relativistic effect v_0/c.
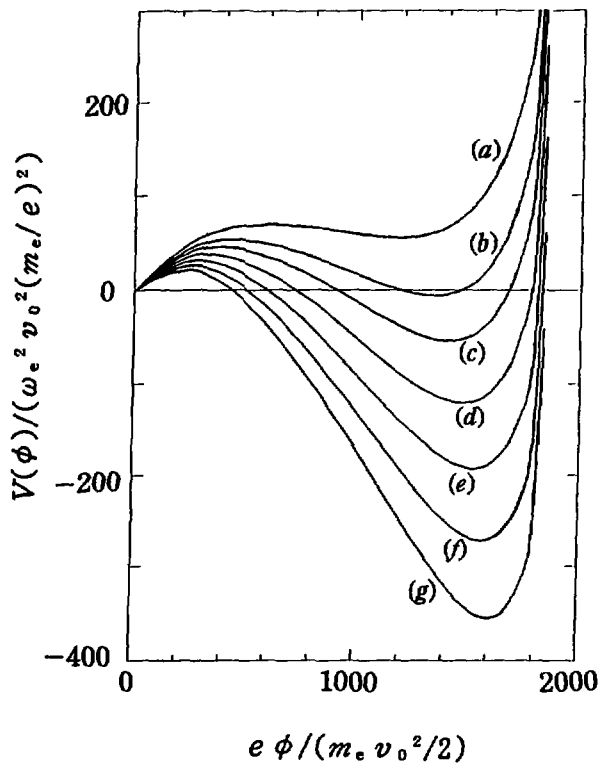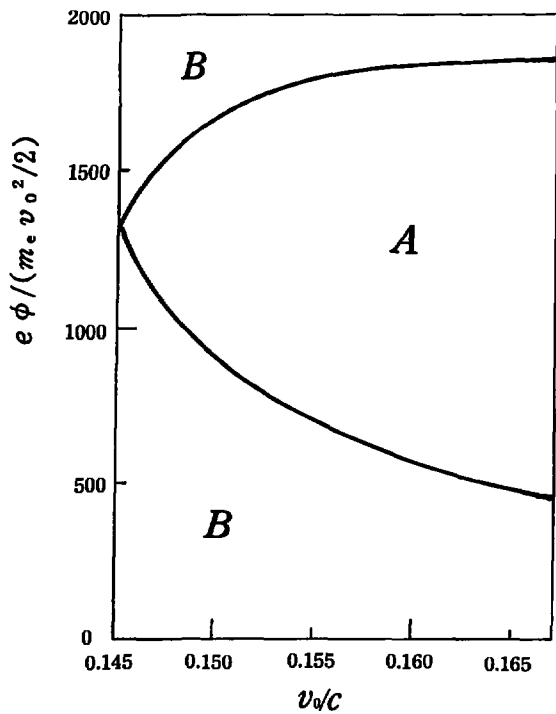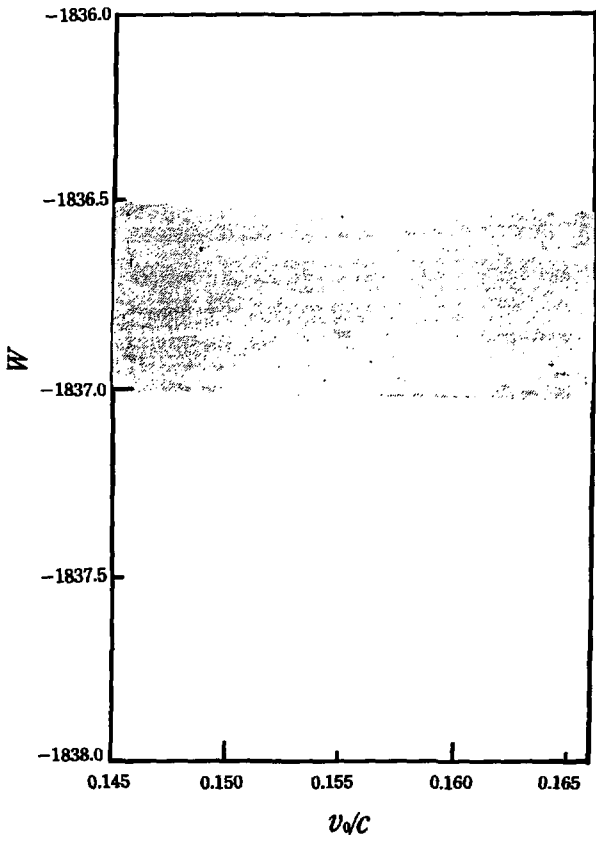
Figure 1

Figure 2

Figure 3

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