ECOULEMENTS CRITIQUES DANS LES FISSURES

CHOKED FLOW THROUGH CRACKS

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SYNTHÈSE :

Les fuites par les fissures de générateurs de vapeur font l'objet d'une recherche menée en collaboration par EDF et UCL. Un logiciel baptisé ECREVISSE, destiné à la prédiction du débit massique, a été développé et validé avec succès.

L'objet de cette note est de présenter le modèle mathématique utilisé dans ECREVISSE, ainsi que certaines comparaisons établies entre les résultats et les données disponibles actuellement. Le modèle prend en compte la persistance d'un liquide métastable dans la fissure et la forme d'écoulement qui apparaît dans ce genre de géométrie particulière. Bien que le modèle implique l'emploi de plusieurs corrélations (frottement, transfert thermique, ...), aucun ajustement des paramètres par rapport aux données n'a été nécessaire, ni dans la partie concernant l'écoulement monophasique, ni dans celle ayant trait à l'écoulement critique en phase double.

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EXECUTIVE SUMMARY :

The leaks through steam-generator cracks are the subject of a research carried out in cooperation between EDF and UCL. A software called ECREVISSE to predict the mass flow rate has been developed and has been successfully validated.

The purpose of the paper is to present the mathematical model used in ECREVISSE as well as some comparison between the results and the presently available data. The model takes into account the persistence of some metastable liquid in the crack and the special flow pattern which appears in such particular geometry. Although the model involves the use of several correlations (friction, heat transfer, ...), no adjustement of parameters against the data has been needed, neither in the single-phase part of the flow, or in the two-phase part.

1. INTRODUCTION

In Nuclear Power Plants, some local failures can occur in Steam Generators. They are due to stress corrosion and they can involve leaks from the Primary Cooling System to the Secondary Cooling System. The leak rates are often very low and the cracks are characterized by a relatively short transit time of the fluid, a very short width, and a large wall roughness.

The assumption of leak-before-break, i.e. the assumption of the existence of a detectable leakage threshold, is made in relation with the plant operation safety. Continuous operation of the system is allowed until such threshold is reached. Consequently, it is important to accurately predict the leak rate in a tube wall crack. Such prediction involves the calculation of the critical flow.

At present, there are several ways to determine analytically the critical flow rate through cracks. They are based on Fauske's model (1962), Moody's model (1965) and Pana's method (1976). Fauske developed a critical flow model where the critical flow condition corresponds to a maximum flow rate for given momentum flux, pressure and quality at the critical section, whereas in Moody's model the maximum flow rate is calculated for given energy flux, pressure and quality at the critical section. Both models, whose inconsistencies have been demonstrated by Giot and Meunier (1968), assume that the expansion occurs in an equilibrium manner.

Pana proposed a method for calculating the critical mass flow rate with subcooled or saturated water taking into account the fluid friction in the channel. Thermal equilibrium between the phases is also assumed. The mechanical energy balance equation is used for the liquid phase flow part and the homogeneous equilibrium model, or, alternatively Moody's model describes the two-phase flow part.

John et al. (1988) have compared successfully Pana's method with some experimental data obtained with inlet subcooled liquid. However, the use of Pana's method requires the choice between two two-phase models.

Other experimenters like Nabayashi et al. (1991) also use Moody's model under saturated inlet conditions and adjust the crack depth to obtain a good agreement.

No model is available to determine the single-phase flow length when the water is subcooled at the inlet, and to predict the pressure and void fraction distributions along the crack depth. No published model takes into account the variation of the cross-section area along the flow path, nor the simultaneous effects of wall friction and wall heat flux.

Therefore, EDF decided to develop a simplified model which calculates the leak rate in a tube wall crack and the pressure distribution along the crack depth for several thermohydraulic conditions. The variation of the cross-section area and the effects of wall friction and wall heat flux will be taken into account.

As the water is subcooled in the tubes, i.e. at the crack inlet, bubble nucleation and bubble growth take place inside the crack. Therefore some degree of thermal non-equilibrium between the liquid and its saturation state can be expected. Several authors have studied the influence of the thermal non-equilibrium on the critical flow in pipes or nozzles. Let us mention in particular Lackmé (1979), Bilicki and Kestin (1990), Bilicki et al. (1990) and Yan et al. (1990, 1991a and 1991b). The applicability of their models in the case of critical flows through cracks needs to be examined. The model proposed in this paper incorporates some of their ideas, especially those expressed in Lackmé's work.

After presenting in Section 2 a simplified geometrical model of the cracks, the basic assumptions and balance equations of the flow are written in Section 3. The set of equations is complemented by appropriate closure laws. The use of the entropy balance equation appears suitable in the case of the irreversible process of flashing of a metastable liquid : the internal entropy source can be clearly identified, and this helps in understanding the flow evolution. The necessary condition of critical flow is obtained, and the compatibility condition is discussed.

The flow model is the basis of a computer code called ECREVISSE[1]. The calculation procedure is described and some examples of results are given. A comparison between some available data and the results obtained by means of ECREVISSE is presented in Section 4. It is concluded that the proposed model enables to predict the flow through cracks and offers a good potential for interpreting future experimental results.

2. SIMPLIFIED CRACK GEOMETRY

The description of the crack geometry is a basic element necessary to build an analytical model. Cracks cross the tube wall and extend more or less parallel to the tube axis. However, they are very complicated and tortuous. Indeed, the mean surface roughness has the same order of magnitude as the crack width. There are also some local area changes and deviations along the crack depth, but it is difficult to describe them.

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[1]ECREVISSE : this acronym stands in French for Ecoulement CRitique Eau-Vapeur dans les fISSurEs.

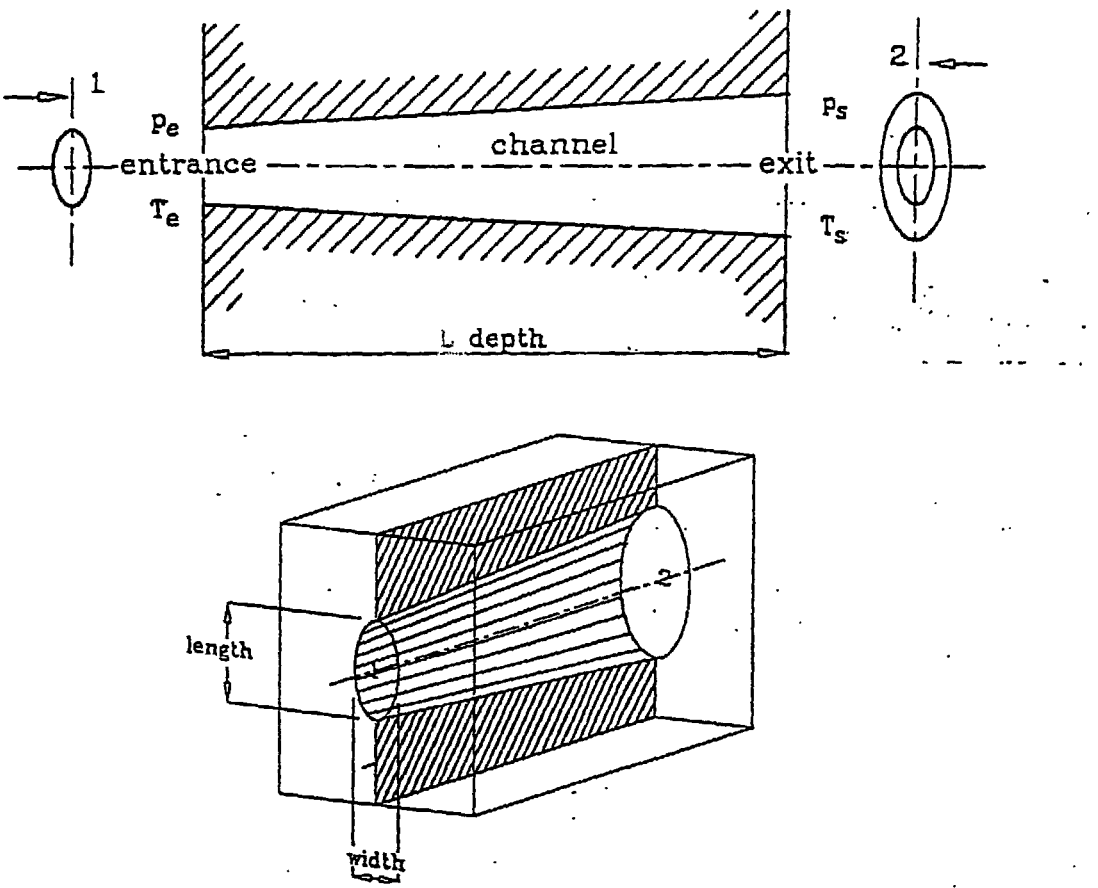

Figure 1. Simplified crack geometry.

Hereunder, the global geometry of the cracks is modeled, whereas the local features are included in the friction factor. As shown in figure 1, the crack is considered as a straight channel, cylindrical, converging or diverging. The crack depth corresponds to the tube wall thickness. The crack width is the smaller dimension (2 to 500 μm). In practice, it appears that the entrance width is generally smaller than the exit one. In most cases, the crack width is not constant along the crack depth. Therefore, we define a convergent or divergent angle. The larger dimension of the crack, that is the crack length, varies between 4 and 15 mm. In this paper, we consider that this length is constant along the crack depth, even if the length at the entrance is in fact slightly larger than at the exit.

The local geometry of the crack which is included in the friction factor takes into account the tortuosity. This global friction factor can be adjusted a priori by a single-phase flow experiment.

3. MODEL

3.1. Assumptions

It is assumed that the flow through the cracks involves two parts : a single-phase liquid flow takes place near the crack inlet and extends to a cross section where nucleation starts. The location of the onset of nucleation corresponds to the achievement of some water superheat. Then the steam bubbles grow and eventually coalesce into flat steam pockets (figure 2). The steam pockets are surrounded by saturated liquid, whereas some superheated

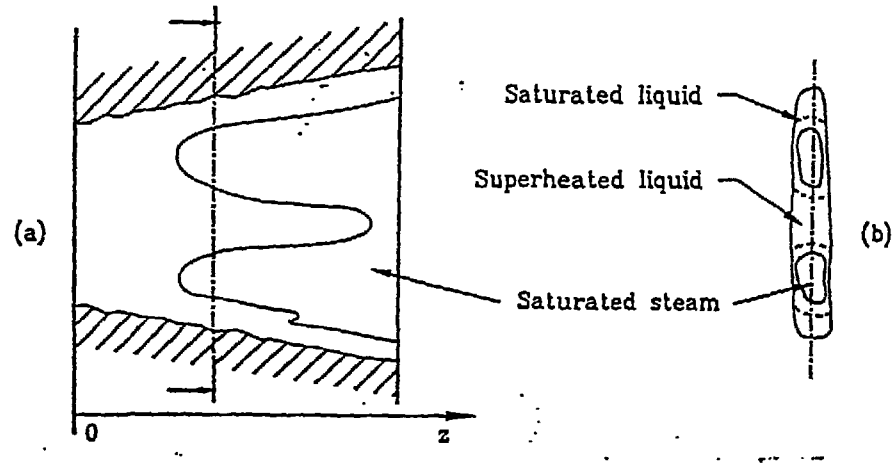

Figure 2. a) Plane view of a crack ; b) Section AA.

liquid persists at some distance between the steam pockets. A typical diagram of the pressure and temperature profiles is shown in figure 3. Typically, the wall temperature T_w decreases by 10°C along the wall thickness (variable z). The liquid enters the crack at a temperature T_L slightly higher than $T_w(0)$, and becomes saturated at a location S where $T_L=T_s(p)$. Beyond this point, the liquid is superheated (metastable). Its temperature T_{LM} goes on decreasing due to wall heat transfer. In the pressure versus thickness diagram, p_{ex} denotes the pressure inside the tube, p is the actual local pressure in the crack, and $p_s(T_{LM})$ denotes the saturation pressure corresponding to T_{LM}. The pressure at which the onset of nucleation takes place is denoted p_o, and is somewhat smaller than $p_s(T_{LM})$. Beyond this onset of nucleation point, the remaining liquid is supposed to have a more or less constant temperature.

The following set of assumptions A1 to A6 enables a simple mathematical model to be developed :

$$w_{c1} = -\frac{\partial p}{}$$

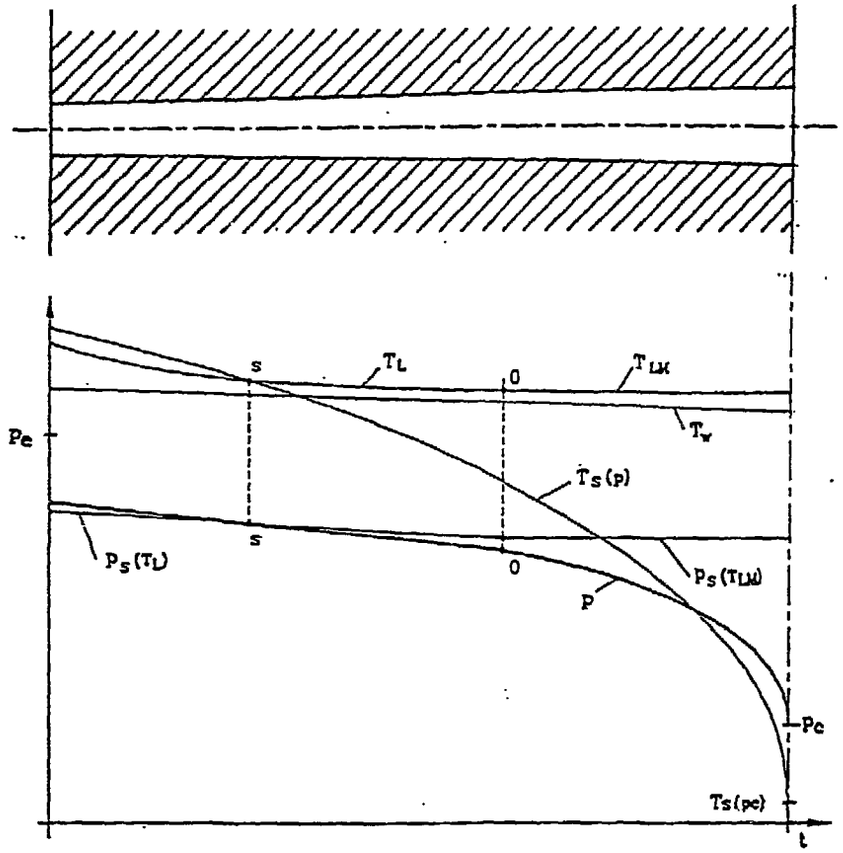

Figure 3. Temperature and pressure profiles along the crack depth.

A1 The flow is in steady state.

A2 The fluid consists of three phases : metastable liquid, saturated liquid and saturated steam, the state variables (temperature, pressure) and the velocities being uniformly distributed in each phase at a given cross section.

A3 There is no slip between the phases. This assumption is supported by the piston-like-flow.

A4 The liquid is subcooled at the crack inlet. This means that the present model cannot deal with the case where a mixture of liquid and vapour would exist along the inner wall of the tubes. Onset of nucleation is assumed to be reached at pressure p_0 given

However, just as in single-phase flow, this condition cannot be fulfilled at any section in the

by the following empirical law (Lackmé, 1979) :

$$p_0 = k_1 p_s(T_{LM}) \tag{1}$$

with $k_1 = 0.95 \ldots 0.97.$

A5 For the sake of simplicity, the variation of temperature T_{LM} versus z is neglected beyond the onset of nucleation.

A6 The flow is horizontal.

Table I presents the consequences of Assumptions A1 to A3 as well as the definitions of the concentrations of the phases. Note that

$$\alpha_{LM} + \alpha_{LS} + \alpha_G = 1$$

	Metastable liquid	Saturated liquid	Saturated vapour
Temperature Pressure Velocity	$T_{LM} = T_{LM}(z)$ $p_{LM} = p(z)$ $w_{LM} = w(z)$	$T_{LS} = T_s(z)$ $p_{LS} = p(z)$ $w_{LS} = w(z)$	$T_G = T_s(z)$ $p_G = p(z)$ $w_G = w(z)$
Mass fraction Area fraction	$1 - y$ α_{LM}	$(1 - x)\, y$ α_{LS}	xy α_G

Table I. Variables and definitions.

3.2. Single-phase flow

Let us consider the single-phase part of the flow, located between the inlet of the crack (z=0) and the onset of flashing ($z=z_0$). The *total pressure drop* is given by the sum of the longitudinal and singular pressure losses :

$$-\Delta p = p(0) - p(z_0) = \Delta p_{long} + \Delta p_{sing} . \tag{2}$$

The longitudinal pressure losses are calculated by the classical expression

the following input data :

.048 mm at the inlet to 0.0375 mm at the outlet

$$\Delta p_{long} = \int_0^{z_0} C_f \frac{P_w}{A} \frac{\rho_{LM} w^2}{2} dz ,$$

where C_f is the friction factor given by Churchill's correlation (Churchill, 1977), P_w the wetted perimeter, A the cross-section area and ρ_{LM} the density of the metastable liquid. The inlet singular pressure losses, including the inlet acceleration term, are given by :

$$\Delta p_{sing} = (1 + \xi) \frac{\rho_{LM} w^2(0)}{2} ,$$

where $\xi = 0,5$ is the head loss coefficient.

The evolution of the temperature of the metastable liquid --and in particular its value at the depth z_0 - is given by a simple heat balance :

$$c_p [T_{LM}(z_0) - T_{LM}(0)] = \int_0^{z_0} \frac{h_c P_h [T_W(z) - T_{LM}(z)]}{A \rho_{LM} w} dz . \qquad (3)$$

In this equation, h_c denotes the heat transfer coefficient between the flowing liquid and the crack wall, P_h is the heated perimeter (P_h=P_w), and c_p is the specific heat of the liquid. The coefficient h_c can be deduced from standard heat transfer correlations like those of Colburn, Sieder and Tate. Then equation (3) requires a step by step integration along the flow path.

The set of equations (2) and (3) is complemented by equation (1) which enables to determine z_0 if the mass flow rate ($A\rho_{LM}w$) is known or vice-versa (see section 4.1).

3.3. Balance equations and closure laws for the region where $p \leq p_b$

In this region, the flow consists of three phases.

3.3.1. MASS BALANCE EQUATIONS

The mass conservation equations for the metastable liquid, saturated liquid and saturated vapour are, respectively,

$$\frac{d}{dz}(\alpha_{LM} A \rho_{LM} w) = -\dot{M} \frac{dy}{dz} , \qquad (4_{LM})$$

8×10^{-3} .

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$$\frac{d}{dz}(\alpha_{LS} A \rho_{LS} w) = \dot{M} \frac{d}{dz}[(1-x)y] , \qquad (4_{LS})$$

$$\frac{d}{dz}(\alpha_G A \rho_G w) = \dot{M} \frac{d}{dz}(xy) , \qquad (4_G)$$

where $\dot{M}$ denotes the mass flow rate.

By summing up the phasic balance equations, we obtain the three-phase mass balance equation :

$$\sum_k \frac{d}{dz}(A \alpha_k \rho_k w) = 0 \qquad k = LM, LS, G . \qquad (4)$$

Introducing the mixture density ρ_m defined by

$$\rho_m \triangleq \sum_k \alpha_k \rho_k ,$$

equation(4) becomes

$$\frac{d}{dz}(A \rho_m w) = 0 , \qquad (5)$$

or

$$\frac{1}{w}\frac{dw}{dz} - \frac{1}{v_m}\frac{dv_m}{dz} = -\frac{1}{A}\frac{dA}{dz} , \qquad (5')$$

where the volume v_m per unit mass, which is a function of x, y, p and T_{LM}, is calculated as

$$v_m \triangleq \frac{1}{\rho_m} = (1-y)v_{LM} + xyv_G + (1-x)y v_{LS} . \qquad (6)$$

3.3.2. MOMENTUM BALANCE EQUATIONS

The momentum balance equations for the metastable liquid, saturated liquid and saturated vapour, are :

$$\alpha_{LM} A \frac{dp}{dz} + \frac{d}{dz}(\alpha_{LM} A \rho_{LM} w^2) = -P_{wILM} \tau_{wILM} - \dot{M} w \frac{dy}{dz} , \qquad (7_{LM})$$

$$\alpha_{LS} A \frac{dp}{dz} + \frac{d}{dz}(\alpha_{LS} A \rho_{LS} w^2) = -P_{w/LS} \tau_{w/LS} + \dot{M} w \frac{d}{dz}[(1-x)y] , \qquad (7_{LS})$$

$$\alpha_G A \frac{dp}{dz} + \frac{d}{dz}(\alpha_G A \rho_G w^2) = -P_{w/G} \tau_{w/G} + \dot{M} w \frac{d}{dz}(xy) , \qquad (7_G)$$

where $P_{w/k}$ denotes the fraction of the cross section perimeter occupied by phase k and $\tau_{w/k}$ the wall shear-stress of phase k. By summing up the phasic balance equations, we obtain the three-phase momentum balance equation :

$$A \frac{dp}{dz} + \sum_k (A \alpha_k \rho_k w^2) = -\sum_k P_{w/k} \tau_{w/k} , \qquad (7)$$

or

$$\frac{dp}{dz} + \frac{1}{A} \frac{d}{dz}(A \rho_m w^2) = -\frac{P_w}{A} \tau_w , \qquad (7')$$

where P_w is the wetted perimeter, and τ_w is defined by :

$$\tau_w = \frac{\sum_k P_{w/k} \tau_{w/k}}{P_w} .$$

By combining equations (5) and (7'), one obtains :

$$\frac{dp}{dz} + \rho_m w \frac{dw}{dz} = -\frac{P_w}{A} \tau_w . \qquad (8)$$

3.3.3. ENERGY BALANCE EQUATIONS

The energy balance equations for the metastable liquid, saturated liquid and saturated vapour, are :

$$\frac{d}{dz}[\alpha_{LM} A \rho_{LM} w (h_{LM} + \frac{w^2}{2})] = P_{I/LM} \dot{q}_{LM} - \dot{M}(h_{LM} + \frac{w^2}{2}) \frac{dy}{dz} , \qquad (9_{LM})$$

$$\frac{d}{dz}[\alpha_{LS} A \rho_{LS} w (h_{LS} + \frac{w^2}{2})] = P_{I/LS} \dot{q}_{LS} + \dot{M}(h_{LS} + \frac{w^2}{2}) \frac{d}{dz}[(1-x)y] , \qquad (9_{LS})$$

$$\frac{d}{dz}[\alpha_G A \rho_G w(h_G + \frac{w^2}{2})] = P_{hG}\dot{q}_G + \dot{M}(h_G + \frac{w^2}{2})\frac{d}{dz}(xy) , \qquad (9_G)$$

where P_{hk} denotes the fraction of the heated perimeter occupied by phase k, and $\dot{q}_k$ the wall heat-flux to phase k. By summing up the phasic balance equations, we obtain the three-phase energy balance equation :

$$\sum_k \frac{d}{dz}[A\,\alpha_k \rho_k w(h_k + \frac{w^2}{2})] = P_h \dot{q}_W , \qquad (9)$$

where P_h is the heated perimeter, and $\dot{q}_W$ is defined by

$$\dot{q}_W = \frac{\sum_k P_{hk}\dot{q}_k}{P_h} .$$

Let us define the mixture enthalpy per unit mass as

$$h_m \triangleq \frac{1}{\rho_m}\sum_k \alpha_k \rho_k h_k .$$

Then, using equation (5), equation (9) can be rewritten in the form

$$\frac{d}{dz}(h_m + \frac{w^2}{2}) = \frac{\dot{q}}{\dot{M}} . \qquad (10)$$

We note that the mixture massic enthalpy, which is a function of x, y, p and T_{LM}, can be calculated by the following expression :

$$h_m = (1-y)h_{LM} + xyh_G + (1-x)y\,h_{LS} . \qquad (11)$$

3.3.4. ENTROPY BALANCE EQUATIONS

The entropy balance equation for the metastable liquid is

$$\frac{d}{dz}(\alpha_{LM} A \rho_{LM} w s_{LM}) = -\dot{M} s_{LM}\frac{dy}{dz} + \dot{M}(1-y)\frac{ds_{LM}}{dz} , \qquad (12_{LM})$$

with

$$T_{LM}\frac{ds_{LM}}{dz} = \frac{dh_{LM}}{dz} - \frac{1}{\rho_{LM}}\frac{dp}{dz} . \qquad (13_{LM})$$

From equation (9_{LM}) combined with equation (4_{LM}), one may derive

$$\frac{dh_{LM}}{dz} = \frac{P_{WLM}\,\dot{q}_{LM}}{(1-y)\dot{M}} - w\frac{dw}{dz}\,, \qquad (14_{LM})$$

and equation (7_{LM}) combined with equation (4_{LM}) leads to

$$\frac{dp}{dz} = -\frac{P_{W/LM}\,\tau_{W/LM}}{\alpha_{LM}A} - \rho_{LM}w\frac{dw}{dz}\,. \qquad (15_{LM})$$

Hence, equation (13_{LM}) can be written as

$$T_{LM}\frac{ds_{LM}}{dz} = \frac{P_{WLM}\,\dot{q}_{LM}}{(1-y)\dot{M}} + \frac{P_{W/LM}\,\tau_{W/LM}}{\alpha_{LM}A\,\rho_{LM}}\,, \qquad (16_{LM})$$

and finally, equation (12_{LM}) becomes

$$\frac{d}{dz}(\alpha_{LM}A\,\rho_{LM}ws_{LM}) = -\dot{M}s_{LM}\frac{dy}{dz} + \frac{P_{WLM}\,\dot{q}_{LM}}{T_{LM}} + \frac{w}{T_{LM}}P_{W/LM}\,\tau_{W/LM}\,. \qquad (17_{LM})$$

Following a similar procedure for the saturated liquid, one obtains

$$\frac{d}{dz}(\alpha_{LS}A\,\rho_{LS}ws_{LS}) = \dot{M}s_{LS}\frac{d}{dz}[(1-x)y] + \frac{P_{W/LS}\,\dot{q}_{LS}}{T_S} + \frac{w}{T_S}P_{W/LS}\,\tau_{W/LS}\,, \qquad (17_{LS})$$

and for the saturated vapour, one can write :

$$\frac{d}{dz}(\alpha_G A\,\rho_G ws_G) = \dot{M}s_G\frac{d}{dz}(xy) + \frac{P_{WG}\,\dot{q}_G}{T_S} + \frac{w}{T_S}P_{W/G}\,\tau_{W/G}\,. \qquad (17_G)$$

By summing up the phasic entropy balance equations, one obtains the three-phase entropy balance equation,

$$\frac{d}{dz}\sum_k(A\,\alpha_k\,\rho_k\,ws_k) = \dot{M}(s_{LS}-s_{LM})\frac{dy}{dz} + \dot{M}(s_G-s_{LS})\frac{d}{dz}(xy) + \sum_k\frac{P_{Wk}\dot{q}_k}{T_k} + w\sum_k\frac{P_{W/k}\,\tau_{Wk}}{T_k}\,. \qquad (17)$$

As the heat corresponding to the transfer of a mass flux $\dot{M}dy$ from the metastable phase to the saturated liquid is equal to the vaporization heat, we can write

$$\dot{M} h_{lv} \frac{d}{dz}(xy) = \dot{M} (h_{LM} - h_{LS}) \frac{dy}{dz} , \tag{18}$$

and hence, equation (17) becomes

$$\frac{d}{dz} \sum_k (A \alpha_k \rho_k w s_k) = \dot{M} (s_{LS} - s_{LM}) \frac{dy}{dz} + \dot{M} \frac{h_{LM} - h_{LS}}{T_S} \frac{dy}{dz} + \sum_k \frac{P_{Ik} q_k}{T_k} + w \sum_k \frac{P_{Ik} \tau_{Ik}}{T_k} . \tag{19}$$

The two first terms at the r.h.s. of equation (19) consist in an "internal" entropy source $\Delta_{IS}(dy/dz)$ due to the irreversible process involving the mass transfer from the metastable to the saturated liquid phase on the one hand :

$$\dot{M} (s_{LS} - s_{LM}) \frac{dy}{dz} = \dot{M} c_{pL} Ln \frac{T_S}{T_{LM}} \frac{dy}{dz} ,$$

and the heat transfer associated with this mass transfer through a temperature discontinuity on the other hand :

$$\dot{M} \frac{h_{LM} - h_{LS}}{T_S} \frac{dy}{dz} = \dot{M} c_{pL} T_{LM} \left(\frac{1}{T_S} - \frac{1}{T_{LM}}\right) \frac{dy}{dz} .$$

The sum Δ_{IS} (dy/dz) is always positive or zero.
The last two terms at the r.h.s. of equation (19) consist in an "external" entropy flux due to wall heat flux and an "external" entropy source due to wall friction. The sum is denoted Δ_{ES}.
Let us define the mixture entropy per unit mass by :

$$s_m \triangleq \frac{1}{\rho_m} \sum_k \alpha_k \rho_k s_k ,$$

Then equation (19) can be written as

$$\frac{d}{dz} (A \rho_m w s_m) = \Delta_{IS} \frac{dy}{dz} + \Delta_{ES} , \tag{20}$$

or, using equation (5), one obtains

$$\frac{ds_m}{dz} = \frac{1}{\dot{M}} \left(\Delta_{IS} \frac{dy}{dz} + \Delta_{ES}\right) . \tag{20'}$$

The mixture massic entropy, which is a function of x, y, p and T_{LM}, can be calculated as

$$s_m = (1-y)s_{LM} - xys_G + (1-x)ys_{LS} \,. \tag{21}$$

3.3.5. PRACTICAL SET OF EQUATIONS

If we assume that τ_{wz} and $\dot{q}_z$ depend only on the variables, and not on their gradients, the set of equations (5'), (7') and (20') complemented by an equation of state

$$v_m = v_m(p, s_m, y) \,, \tag{22}$$

and by a closure law for y

$$\frac{dy}{dz} = f(p, y, T_{LM}) \,, \tag{23}$$

can be written in a matrix form :

$$\begin{bmatrix} 0 & 1 & -\frac{w}{v_m} & 0 & 0 \\ 1 & \frac{w}{v_m} & 0 & 0 & 0 \\ \frac{\partial v_m}{\partial p} & 0 & -1 & \frac{\partial v_m}{\partial s_m} & \frac{\partial v_m}{\partial y} \\ 0 & 0 & 0 & 1 & -\frac{\Delta_{IS}}{\dot{M}} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{dp}{dz} \\ \frac{dw}{dz} \\ \frac{dv_m}{dz} \\ \frac{ds_m}{dz} \\ \frac{dy}{dz} \end{bmatrix} = \begin{bmatrix} -\frac{w}{A}\frac{dA}{dz} \\ -\frac{P_W}{A}\tau_W \\ 0 \\ \frac{\Delta_{ES}}{\dot{M}} \\ f(p,y,T_{LM}) \end{bmatrix} \,. \tag{24}$$

3.3.6. CLOSURE LAWS

Saturated steam-water mixture fraction

In addition to τ_w and $\dot{q}_w$ which have to be determined by some two-phase flow friction and heat transfer correlations (see below), a closure law is used to define the evolution of the saturated steam-water mixture fraction y along the crack (Hardy and Mali, 1983) :

$$\frac{dy}{dz} = k_2 (1-y) [p_s(T_{LM}) - p]^{1/4} , \qquad (25)$$

where k_2 is a constant for a given pipe geometry. According to this expression, the fraction dy of liquid which is transferred from the metastable phase to the saturated liquid phase per unit length, is proportional to the remaining quantity of metastable liquid (1-y) and to some function of the metastability expressed by means of a pressure difference.

As the nucleation sites consist mainly in wall microcavities, it is assumed that different pipe geometries and wall roughnesses would lead to different values of the constant k_2. For example, assuming the same material, the number of active sites per unit volume V of pipe is proportional to the wall surface S_w. Therefore, we suppose that k_2 is proportional to the ratio :

$$\frac{S_W}{V} = \frac{\int P_W dz}{\int A dz} .$$

Using the data resulting from the Moby-Dick experiments, Hardy and Mali (1983) have obtained $k_2 = 1/15$ for a cylindrical pipe having a ratio $P_w/A = 200$ m^{-1}. For other geometries, we propose to use :

$$k_2 = \frac{1}{3000} \frac{P_W}{A} .$$

Due to the lack of experimental evidence, the dependence of k_2 with respect to the roughness is not taken into account explicitely in this expression.

Equation (25) can be compared to the relaxation equation proposed by Bilicki and Kestin (1990). Let us define the quality :

$$X \triangleq xy .$$

Then, equation (18) can be integrated at a given pressure between an equilibrium state characterized by a value $\overline{X}$ of the quality, and y=1 on the one hand and the actual non-equilibrium state on the other hand :

$$h_{Lv}(X - \overline{X}) = -(h_{LM} - h_{LS})(1-y) .$$

This equation can be written as

$$X - \overline{X} = -\Delta h^* (1-y) ,$$

where Δh^* denotes the non-dimensional quantity similar to the Jakob number :

$$\Delta h^* = \frac{h_{LM} - h_{LS}}{h_{lv}} .$$

Equation (25) becomes

$$\frac{dy}{dz} = \frac{d}{dz}\left[\Delta h^{*-1}(X - \overline{X})\right] = -k_2(X - \overline{X})\,\Delta h^{*-1}(p_s - p)^{1/4} . \qquad (26)$$

If we accept the two following approximations :

$$\Delta h^* = constant \quad and \quad \overline{X} = constant ,$$

which correspond to the flashing of a stagnant fluid, then equation (26) yields :

$$\frac{dX}{dz} = -k_2(p_s - p)^{1/4}(X - \overline{X}) .$$

This expression can be identified to the relaxation equation whose use has been proposed by Bilicki and Kestin :

$$\frac{dX}{dz} = \frac{X - \overline{X}}{w\theta} ,$$

by writing :

$$-k_2(p_s - p)^{1/4} = \frac{1}{w\theta} .$$

The above development suggests that the validity of Bilicki and Kestin relaxation equation could be limited to the flashing of a stagnant liquid. The structure of equation (26) is better suited for a flowing fluid subject to flashing.

Wall shear-stress

Three correlations for predicting the gas-liquid friction pressure gradient have been used and compared in the present study

(i) the *Lockhart-Martinelli correlation*, modified by Richardson, which yields :

$$\Phi_L^2 = (1 - \alpha_G)^{-1.75} , \qquad (27)$$

where Φ_L^2 denotes the ratio between the gas-liquid friction pressure gradient and the single-phase friction pressure gradient where the liquid flows at the same flow-rate as in the multiphase flow.

(ii) *Chisholm's correlation :*

$$\Phi_{LO}^2 = 1 + (\tau^2 - 1)\{B[X(1-X)]^{(2-n)/2} + X^{2-n}\}, \qquad (28)$$

where Φ_{LO}^2 denotes the ratio between the gas-liquid friction pressure gradient and the single-phase friction pressure gradient when the liquid flows at velocity w. The other parameters are defined as follows :

$$B \triangleq \frac{21\tau - 2^{2-n} + 2}{\tau^2 - 1},$$

$$\tau^2 \triangleq \left(\frac{\rho_L}{\rho_G}\right)\left(\frac{\mu_G}{\mu_L}\right)^n,$$

$$n = 0.25 .$$

(iii) A correlation proposed for *capillary tubes* by Lin et al. (1989) and based on Churchill's correlation for single-phase flow, the Reynolds number being calculated with a mixture viscosity μ_m :

$$\mu_m \triangleq \left[\frac{1}{\mu_L} + X^{1.4}\left(\frac{1}{\mu_G} - \frac{1}{\mu_L}\right)\right]^{-1}. \qquad (29)$$

Wall heat transfer

Three correlations have been tested for the prediction of the wall heat flux :

(a) *Chen's correlation* (1966). The two-phase gas-liquid heat transfer coefficient h_{TP} is given by :

$$h_{TP} = h_c F + h_b S, \qquad (30)$$

where h_c and h_b refer respectively to convection and nucleate boiling, and F and S are wheighing factors. The convection coefficient is calculated by the Dittus-Boelter correlation, whereas the nucleate boiling coefficient is determined from Forster-Zuber's correlation. The factor F is expressed as a function of the Martinelli parameter X_{tt} :

$$F = 1 \qquad \text{if} \quad X_{tt}^{-1} < 0.1$$

$$F = 2.35\,(X_{tt}^{-1} + 0.213)^{0.736} \qquad \text{if} \quad X_{tt}^{-1} \geq 0.1 .$$

Finally,

$$S = (1 + 2.53\,10^{-6}\,Re_{TP}^{1.17})^{-1} ,$$

with

$$Re_{TP} \triangleq F^{1.25}\,Re_L .$$

(b) *Klimenko's correlation* (1988). This correlation is as follows :

$$Nu_c = h_c \frac{D_h}{k_L} = 0.087\,Re_m^{0.6}\,Pr_L^{1/6}\,(\frac{\rho_G}{\rho_L})^{0.2}\,(\frac{k_W}{k_L})^{0.09} , \qquad (31)$$

with

$$Re_m \triangleq \frac{\rho_L w D_h}{\mu_L} .$$

(c) *Johnson and Abou-Sabe correlation* (1952). When the phase change is only due to a longitudinal pressure gradient, these authors suggest the use of the following correlation :

$$h_{TP} = Z_G\,h_L \frac{\psi^{0.1}}{(1-\alpha)^{0.9}} , \qquad (32)$$

where

$$\psi = \Phi_L^{3.33}\,(1-\alpha)^4 ,$$

$$Z_G = [1 + 0.006\,(\frac{\rho_m w D_h}{\mu_G})^{0.5}]^{-1} ,$$

and h_L is calculated by the Dittus-Boelter correlation.

3.4. Critical flow condition and compatibility condition

3.4.1. CRITICAL FLOW CONDITION

The critical flow condition (Bouré and al., 1976) is the vanishing condition of the determinant of the set of equations (24). It can be easily seen that this determinant is equal to its minor involving the three first lines and the three first columns. One obtains then

$$(\frac{w_c}{v_m})^2 = -(\frac{\partial p}{\partial v_m})_{s_m,y} . \quad (33)$$

The critical velocity w_c is thus given by the classical expression of the speed of sound :

$$w_c = \sqrt{(\frac{\partial p}{\partial \rho_m})_{s_m,y}} . \quad (34)$$

Here, the isentropic condition implies not only the absence of wall friction and wall heat flux, but also the absence of mass transfer inside the mixture, i.e. a constant value for y.

3.4.2. COMPATIBILITY CONDITION

The compatibility condition is derived from the vanishing condition of a secondary determinant of the matrix of equation (24) at the critical section. This condition is necessary to guarantee a physical solution at the critical section. From equation (24), the compatibility equation is given by

$$\frac{v_m}{A}(\frac{w_c}{v_m})^2\frac{dA}{dz} - \frac{P_W}{A}\tau_W - \frac{\Delta_{ES}}{\dot{M}_c}(\frac{w_c}{v_m})^2\frac{\partial v_m}{\partial s_m} - f(p,y,T_{LM})(\frac{w_c}{v_m})^2(\frac{\partial v_m}{\partial y} + \frac{\Delta_{IS}}{\dot{M}_c}\frac{\partial v_m}{\partial s_m}) = 0 . \quad (35)$$

Taking equations (20') and (23) into account, the compatibility equation can be rewritten :

$$\frac{v_m}{A}(\frac{w_c}{v_m})^2\frac{dA}{dz} - \frac{P_W}{A}\tau_W - (\frac{w_c}{v_m})^2(\frac{\partial v_m}{\partial s_m}\frac{ds_m}{dz} + \frac{\partial v_m}{\partial y}\frac{dy}{dz}) = 0 . \quad (36)$$

Further, as the expression of the gradient of the specific volume is

$$\frac{dv_m}{dz} = \frac{\partial v_m}{\partial p}\frac{dp}{dz} + \frac{\partial v_m}{\partial y}\frac{dy}{dz} + \frac{\partial v_m}{\partial s_m}\frac{ds_m}{dz} , \quad (37)$$

taking into account equation (33), equation (36) becomes

$$\frac{v_m}{A}(\frac{w_c}{v_m})^2\frac{dA}{dz} - \frac{P_W}{A}\tau_W - (\frac{w_c}{v_m})^2[\frac{dv_m}{dz} + (\frac{v_m}{w_c})^2\frac{dp}{dz}] = 0 . \quad (38)$$

This equation is formally identical to the compatibility condition for a single-phase compressible flow. In particular, if this flow is frictionless and the cross section uniform, then equation (38) is reduced to

$$\frac{dp}{dv_m} = (\frac{\partial p}{\partial v_m})_{s_m,y} . \quad (39)$$

However, just as in single-phase flow, this condition cannot be fulfilled at any section in the pipe under these circumstances : thus, the critical section is located at the pipe outlet. This conclusion holds in the case where $dA/dz < 0$.

4. CALCULATION PROCEDURE AND RESULTS

4.1. Calculation procedure

A computer code called ECREVISSE has been developed to calculate the flow through the crack (critical or not) according to the model presented in section 3.

An iterative procedure is used in this code to converge towards the boundary conditions, i.e. the inlet pressure and temperature of the subcooled water at one end and the pressure of the secondary side at the other end.

The first step consists in determining the flow rate, assuming that the flow remains liquid throughout the depth of the crack (Lc), considering the outlet pressure equal to $p_b = 0.95\ p_{sat}(T_{LM})$. For this first estimated flow rate, the set of balance equations for three-phase flow is then integrated step by step using a Newton-Raphson adaptative method until either the critical flow condition or the outlet pressure is reached for a calculated length L_{TP} of the three-phase flow zone. This integration can be performed for different pressure drop correlations (Martinelli-Richardson, Chisholm, capillary tubes), and for different wall heat transfer correlations (Chen, Klimenko, Johnson-Abou-Sabe). The choice between these correlations is an option in the computer code.

New flow conditions are then calculated with a single-phase, zone length equal to $L_c - L_{TP}$ according to the procedure described above. This procedure is repeated until all the following parameters converge within a given accuracy :

- flow rate,
- lengths of single phase and three-phase flow zone,
- quality X at the outlet cross-section,
- mass fraction y at the outlet cross-section.

This iterative method is valid only if the water at the inlet of the crack is subcooled.

As an example of results, figures 4 and 5 give the profiles of some variables in the three-phase flow zone corresponding to all possible options considered one by one, and for

the following input data :

- crack width : 0.048 mm at the inlet to 0.0375 mm at the outlet
- crack length : 8.075 mm
- crack depth : 2.54 mm
- inlet temperature : 282°C
- inlet pressure : 100 bar
- outlet pressure : 1 bar.

The calculated mass flow rate corresponding to this case (59.6 10^{-3} kg s^{-1}) is practically independent of the correlations used for the pressure drop and for the heat transfer in the three-phase flow.

4.2. Comparison between experimental tests and ECREVISSE results

Two types of tests have been selected : (i) slits with John et al. tests and (ii) cracks with Schrock et al.

4.2.1. JOHN ET AL. TESTS

These tests have been chosen because their results are very coherent and seem to have been made in good conditions even if the size of the slits are larger than the cracks we want to study.

The slits are rectangular and their dimensions are well known. The depth of the slits is 46mm and the length 80mm. The width ranges from 0.25 to 0.44 mm. Several roughnesses have been tested and range from 5.3 to 287 μm.

For each slit, some single fluid flow experiments have been performed in cold water to determine the global friction factor. The results have been used without corrections in ECREVISSE.

For each slit, two-phase flow tests have been done for different upstream pressures (40, 60, 80 and 100 bar), and, for each pressure, several upstream subcooled temperatures (2 to 60°C) have been tested. The exit pressure ranged from 4 to 9 bar.

More than 70 tests have been compared with ECREVISSE results (fig.6). The present model shows a reasonable agreement with the data (better than $\pm$ 12%). We note

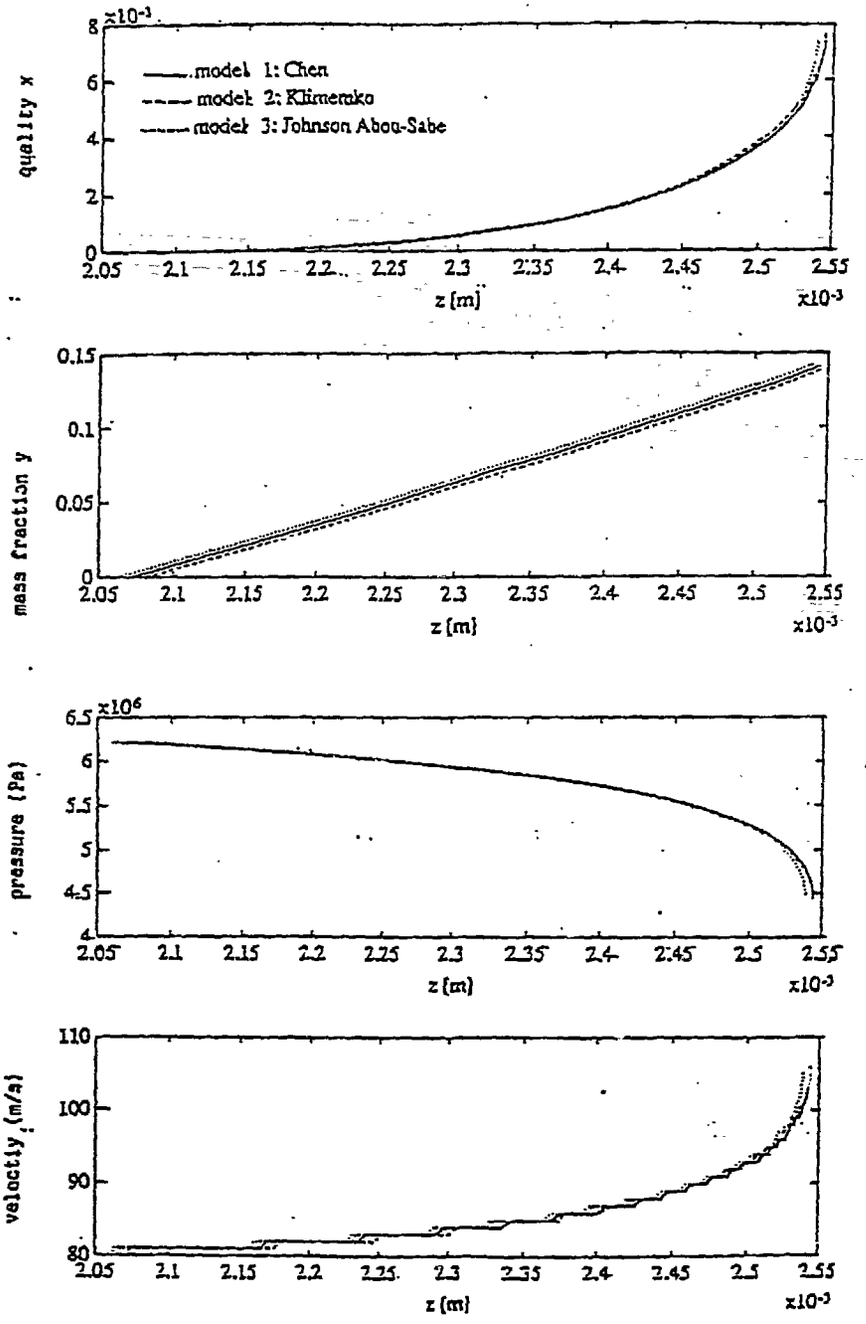

Figure 4. Results of a sensitivity analysis of the model with respect to the heat transfer correlations. The pressure losses are calculated according to the LOCHART MARTINELLI-RICHARDSON correlation.

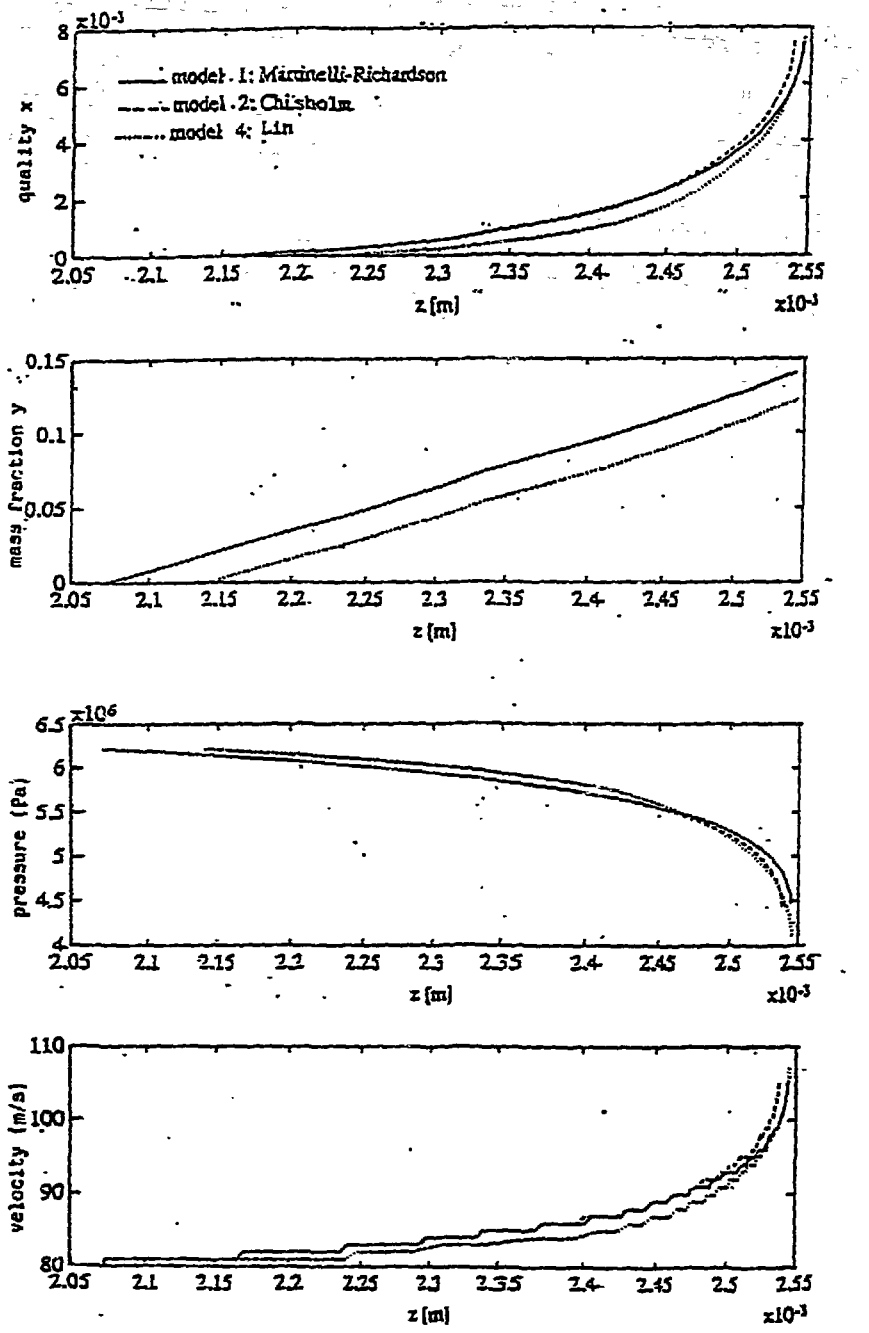

Figure 5. Results of a sensitivity analysis of the model with respect to the pressure drop correlations. The heat transfer is calculated according to CHEN's correlation.

also that the present model can fairly reproduce the flow rate decrease with the subcooled temperature decrease (fig.7).

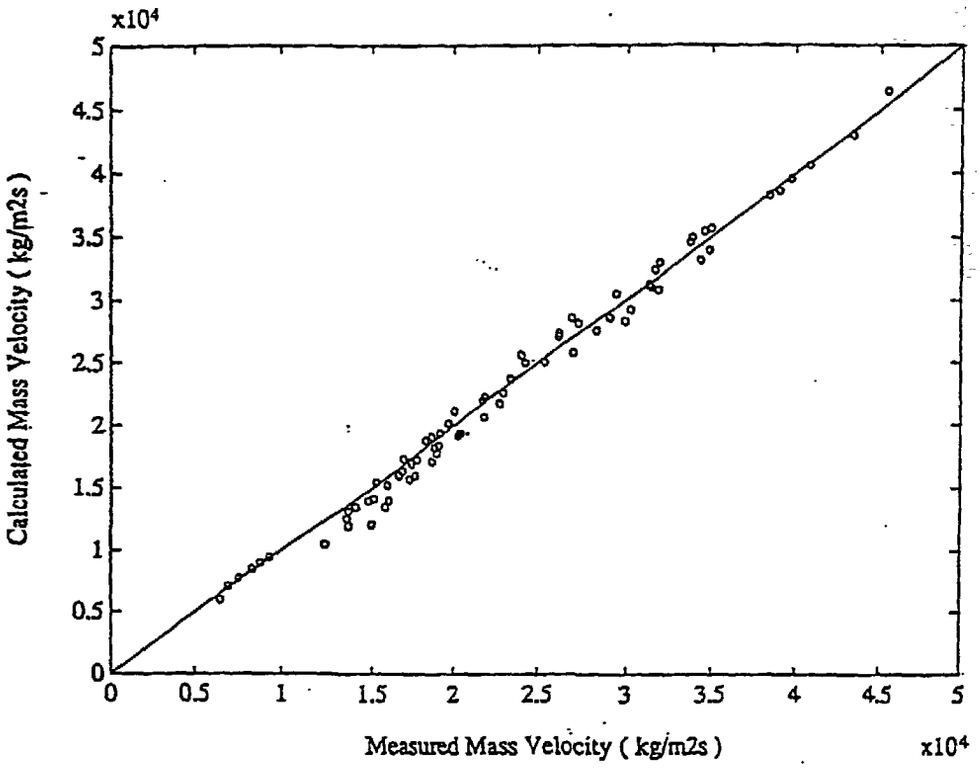

Figure 6. Comparison between the test results from John et al. and the ECREVISSE results.

4.2.2. SCHROCK ET AL. TESTS

The tests have been carried out with real cracks whose dimensions are bigger than those we are interested in.

The Schrock et al. cracks had a depth of about 19mm, a length varying between 1.6 and 28mm and a width ranging from 0.05 to 0.23mm. Unfortunately, no single-phase flow test has been made to determine the friction factor for each crack. So we have taken an average value constant for all the tests for each crack. The thermohydraulic parameters are the upstream pressure ranging from 40 to 96 bar and the upstream subcooled temperature ranging from 0 to 70°C.

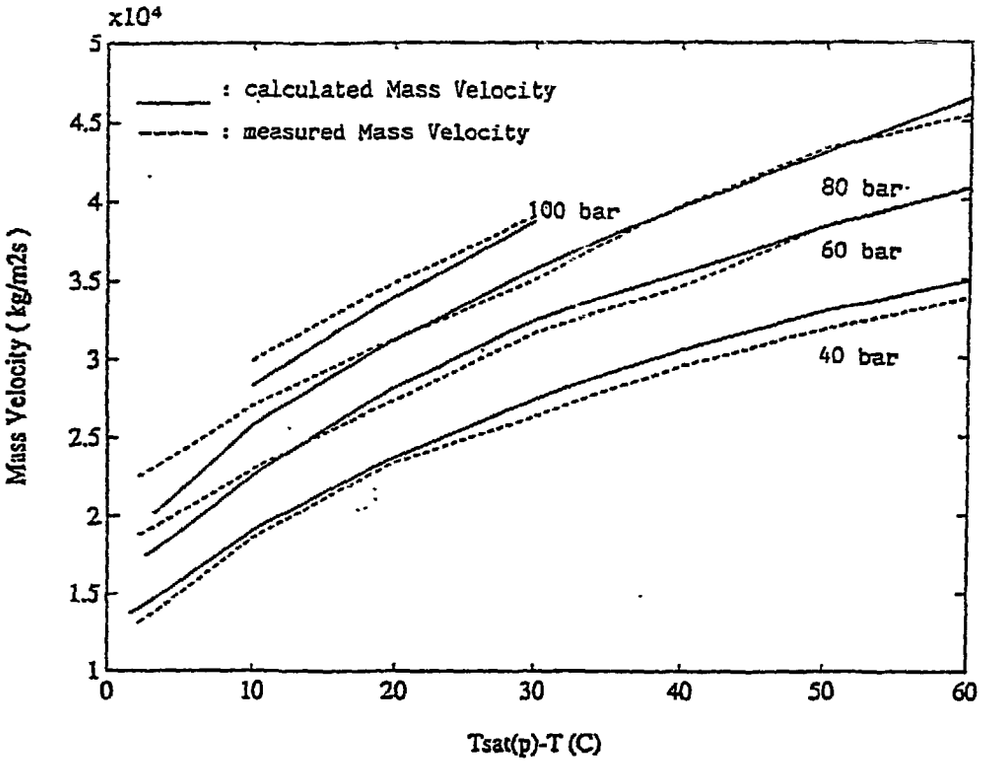

Figure 7. Critical mass velocity versus $T_{sat}(p)$-T at the entrance of the channel for different inlet pressures.

More than 30 data have been compared with ECREVISSE results. Figure 8 shows the agreement between the tests data and the corresponding ECREVISSE results.

CONCLUSIONS

A new model for two-phase choked flow through cracks is proposed. It takes into account the thermal non-equilibrium which appears when initially subcooled water is released in a piston-like steady state flow.

The crack is considered as a straight channel which can be uniform, convergent or divergent. Wall roughness is a priori determined by single-phase flow data. Wall heat flux and wall friction are calculated by classical single- and two-phase flow correlations.

The fluid is modeled as a three-phase mixture consisting of metastable liquid,

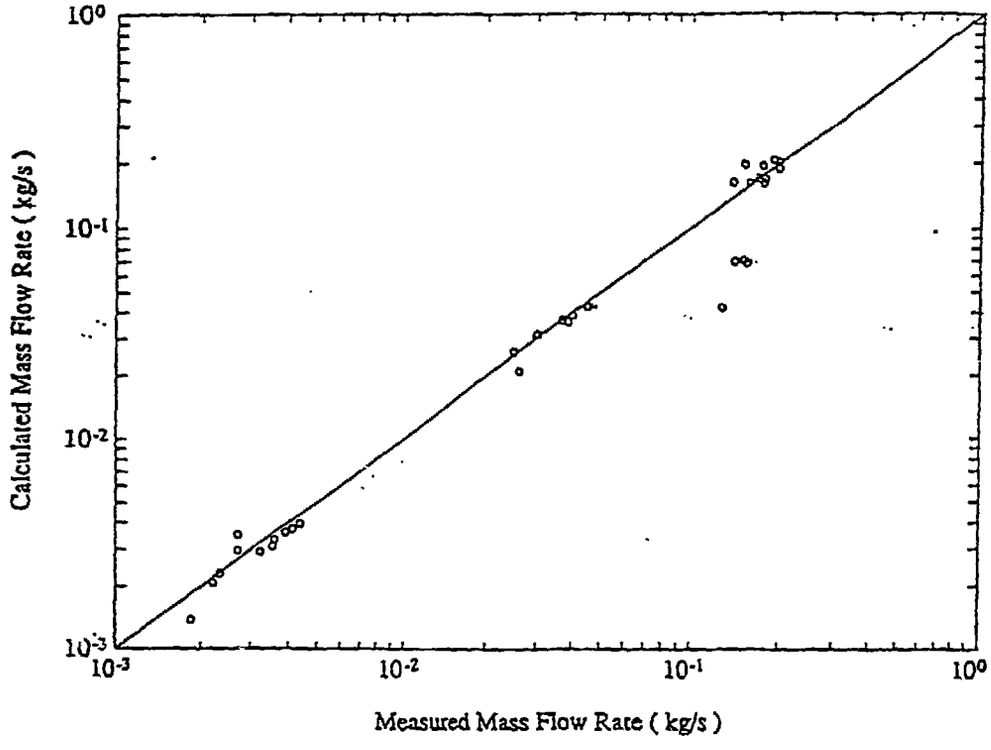

Figure 8. Comparison between the test results from Schrock et al. and the ECREVISSE results.

saturated liquid and saturated vapour. The slip between the phases is neglected. The set of balance equations is written and complemented by a closure law for the irreversible mass transfer between the two liquid phases. This law appears to be an extension of the relaxation equation (Bilicki and Kestin, 1990).

The critical flow condition associated with the compatibility condition are found to be formally similar to single compressible flow results.

The model is implemented in a computer code, called ECREVISSE, designed for the simulation of Steam Generator small leaks. The results have been compared with some data published by John et al. (1988), and Schrock et al. (1988). Good agreement has been found between the present results and the experimental data. The validation will proceed further for smaller cracks using a new test facility currently under construction at EDF.

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LIST OF SYMBOLS

A	cross section area
B	parameter in equation (28)
C_f	friction factor
c_p	specific heat
D_h	hydraulic diameter
F	parameter in equation (30)
h	enthalpy per unit mass
h_b	heat transfer coefficient in nucleate boiling

h_c	heat transfer coefficient in convection
h_{lv}	heat of vaporization per unit mass
k	heat conduction
k_1	constant in equation (1)
k_2	constant in equation (25)
$\dot{M}$	total mass flow rate
Nu_e	Nusselt number
n	Blasius' constant
P_h	heated perimeter
Pr	Prandtl number
P_w	wetted perimeter
p	pressure
$\dot{q}$	wall heat flux density
Re	Reynolds number
S	parameter in equation (30)
S_w	wall surface area
s	entropy per unit mass
T	temperature
V	volume
w	velocity
X	quality
$\overline{X}$	quality at equilibrium
X_{tt}	Martinelli's parameter
x	partial mass fraction of saturated vapour
y	mass fraction of saturated mixture
Z_G	parameter in equation (32)
z	abscissa
α	area fraction
Δ_{es}	sum of "external" entropy source, defined in equation (19)
Δ_{is}	factor of the "internal" entropy source, defined in equation (19)
Δh^*	Jakob number
θ	parameter in Bilicki's relaxation equation
μ	dynamic viscosity
ξ	head loss coefficient
ρ	density
τ	parameter in equation (28)
τ_w	wall shear-stress
Φ_L^2, Φ_{Lo}^2	ratio between pressure gradients in the Lockhart-Martinelli correlation
ψ	parameter in equation (32)

Subscripts

c	critical
G	vapour phase
L	liquid
LM	metastable liquid phase
LS	saturated liquid phase
m	mixture
0	onset of flashing
s or S	saturation
TP	three-phase
w	wall

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