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# SPATIO-TEMPORAL CHAOS AND THERMAL NOISE IN JOSEPHSON JUNCTION SERIES ARRAYS

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## ABSTRACT

We study underdamped Josephson junction series arrays that are globally coupled through a resistive shunting load and driven by an rf bias current. We find that they can be an experimental realization of many phenomena currently studied in globally coupled logistic maps. Depending on the bias current the array can show Shapiro steps but also spatio-temporal chaos or "turbulence" in the IV characteristics. In the turbulent phase there is a saturation of the broad band noise for a large number of junctions. This corresponds to a break down of the law of large numbers as seen in globally coupled maps. We study this phenomenon as a function of thermal noise. We find that when increasing the temperature the broad band noise *decreases*.

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Josephson junction arrays are mesoscopic devices which are currently fabricated with modern photolitographic techniques [1]. In the last years they have been studied as a realization of nonlinear dynamical systems with many degrees of freedom [2, 3]. Moreover, they have potential applications as high frequency coherent power sources [4]. Spatio-temporal chaos has been investigated through prototype models as coupled logistics maps. In particular, globally coupled maps (GCM) have been studied as a mean field type extension of these models [5, 6]. The GCM exhibit coherent, ordered, partially ordered and turbulent regimes [5]. In the turbulent regime, even when there is full chaos in space and time, a subtle collective behavior emerges. This was seen as a violation of the law of large numbers [5] as a function of the number of logistic maps. Recently, we have studied a physical realization of the GCM in one-dimensional Josephson junction series arrays (JJSA) [7]. In this system, the role of the logistic maps is played by underdamped single Josephson junctions, which can show chaotic behavior under an rf bias current [8]. The global coupling is achieved by connecting this junctions in series and with a common resistive shunting load. We found that the break down of the law of large numbers can be observed in rf-driven underdamped JJSA, accompanied in this case by an emergence of novel pseudo Shapiro steps [7]. In this article we report the effects of thermal noise on these phenomena.

Let us consider an underdamped JJSA shunted by a resistive load [2], and subjected to an rf bias current $I_B(t) = I_{dc} + I_{rf}\sin(\omega_{rf}t)$. The dynamical equations of the JJSA [2] are

$$\ddot{\phi}_k + g\dot{\phi}_k + \sin\phi_k + (2\tilde{T}g)^{1/2}\eta_k(\tau) + i_L = i_{dc} + i_{rf}\sin(\Omega_{rf}\tau)\ , \tag{1}$$

$$i_L = \sigma v(\tau) = \frac{\sigma}{N}\sum_{j=1}^{N} g\dot{\phi}_j + \left(\frac{2\tilde{T}g\sigma}{N}\right)^{1/2}\eta_L(\tau)\ , \tag{2}$$

where $\phi_k$ is the superconducting phase difference across the junction $k$, and $k = 1,\ldots,N$. We use reduced units, with currents normalized by the critical current, $i = I/I_c$; time normalized by the plasma frequency $\omega_p t = \tau$, with $\omega_p = \sqrt{\frac{2eI_c}{\hbar C}}$ and $C$ the capacitance of the junctions; and voltages by $rI_c$, with $r$ the shunt resistance of the junctions. Here, $i_L$ is the current flowing through the resistive load; $g = (\frac{\hbar}{2eCr^2I_c})^{1/2}$; $v = V_{total}/N$ is the total voltage across the array per junction; $\sigma = \frac{rN}{R}$, with $R$ the resistance of the shunting load, represents the strength of the global coupling in the array; and the normalized rf frequency is $\Omega_{rf} = \omega_{rf}/\omega_p$. The thermal Johnson noise is given by the white noise terms $\eta(\tau)$, such that $\langle\eta_k(\tau)\rangle = 0$, $\langle\eta_k(\tau)\eta_{k'}(\tau')\rangle = \delta(\tau-\tau')\delta_{k,k'}$. Temperature is normalized such that $\tilde{T} = 2ekT/\hbar I_c$. The simplest attractor of the system is the coherent state for which $\phi_k(\tau) = \phi_j(\tau) = \phi_0(\tau)$. The equations reduce to the single junction, $\ddot{\phi}_0 + \tilde{g}\dot{\phi}_0 + \sin\phi_0 = i_{bias}(\tau)$, with $\tilde{g} = g(1+\sigma)$. The single junction can show chaos for $\tilde{g} < 2$, and $\Omega_{rf} < 1$ [8]. Here we take $\tilde{g} = 0.2$, $\Omega_{rf} = 0.8$, and $i_{rf} = 0.61$. We work with fixed $\tilde{g}$, instead of

$g$, to have always the same coherent attractor. We integrate the Eq. (2) with a second order Runge-Kutta method with step $\Delta\tau = T/160$, ($T = 2\pi/\Omega_{rf}$), for integration times $t = 1024T$, after discarding the first 256 periods. For each run we used different sets of random initial conditions $\{\phi_k(0), \dot\phi_k(0)\}$.

We have studied the IV characteristics of the JJSA for $\tilde{T} = 0$ and we found a very rich dynamical behavior as a function of the bias current $i_{dc}$ [7]. We found that, depending on $i_{dc}$, there is (i) an ordered regime, which is periodic in time (it corresponds to Shapiro steps in the IV characteristics), and it is ordered in space in a finite number of "clusters" with the same phase, (ii) a coherent regime, with all the phases equal (iii) a partially ordered regime, and (iv) a turbulent regime, where there is chaos both in time and space (all the junction phases are different at a given time). We found that, as in GCM, the turbulent regime is the one that shows the most notable changes when increasing the number of junctions $N$. First of all, let us note that the voltage per junction $v^{(N)}(t) = \frac{1}{N}\sum_{j=1}^{N} g\dot\phi_j$ acts as a "mean field" in Eq. (2). Since in the turbulent phase the $\phi_j(t)$ take random values almost independently, one might expect that $v(t)$ will behave as an average noise. The power spectrum of $v(t)$ will be $S(\omega) = \frac{1}{N}|v_j(\omega)|^2 + \frac{1}{N^2}[\sum_{i\neq j} v_i(\omega)v_j^*(\omega)]$, with $v_j(\omega)$ the Fourier transform of $v_j(t) = g\dot\phi_j(t)$. If the $\dot\phi_j(t)$ are completly independent, the second term will vanish for low frequencies, $\omega \to 0$. Therefore $S_0^{(N)} \sim \frac{1}{N}S_0^{(1)}$, with $S_0^{(N)}$ the low frequency part of the power spectrum of a JJSA with $N$ junctions. This is the equivalent of the law of large numbers for a periodically driven system. However, we have found that within the turbulent phase $S_0$ saturates for large $N$, evidencing a break down of the law of large numbers [7], as observed in GCM [5, 6]. At the same time some pseudo Shapiro steps emerge in the IV characteristics [7].

Here we analyze how stable is this effect against thermal noise. In Fig. 1 we show the calculated values of $S_0$ as a function of $N$ for $\sigma = 0.4$ and for $i_{dc} = 0.124$ (which corresponds to the turbulent regime) for different temperatures. We see that for $\tilde{T} = 0$ $S_0$ saturates for large $N$. This effect is stable for small temperatures, and only after a critical $\tilde{T}_c \approx 4 \times 10^{-5}$ there is a crossover to a $1/N$ behavior. Similar phenomena has been found when adding a white noise term to GCM.

More interesting, from the experimental point of view, is the behavior of $S_0$ as a function of temperature for a given number of junctions. In the inset of Fig. 1 we show the results for bias $i_{dc} = 0.124$, $\sigma = 0.4$ and $N = 2048$ junctions and $N = 16384$ junctions. We see that when the temperature is increased, the broad band noise *decreases*. This counterintuitive behavior is a consequence of the fact that there is a breakdown of the law of large numbers at $\tilde{T} = 0$. The addition of thermal noise reduces the subtle coherence that made $S_0$ saturate for large $N$, favoring the $1/N$ behavior at large temperatures. This traduces into a decrease of $S_0$ when increasing $\tilde{T}$. Note that for $\tilde{T} > \tilde{T}_c$, $S_0$ reaches a constant value, coincident with the fulfillment of the $1/N$ law. JJSA like the one

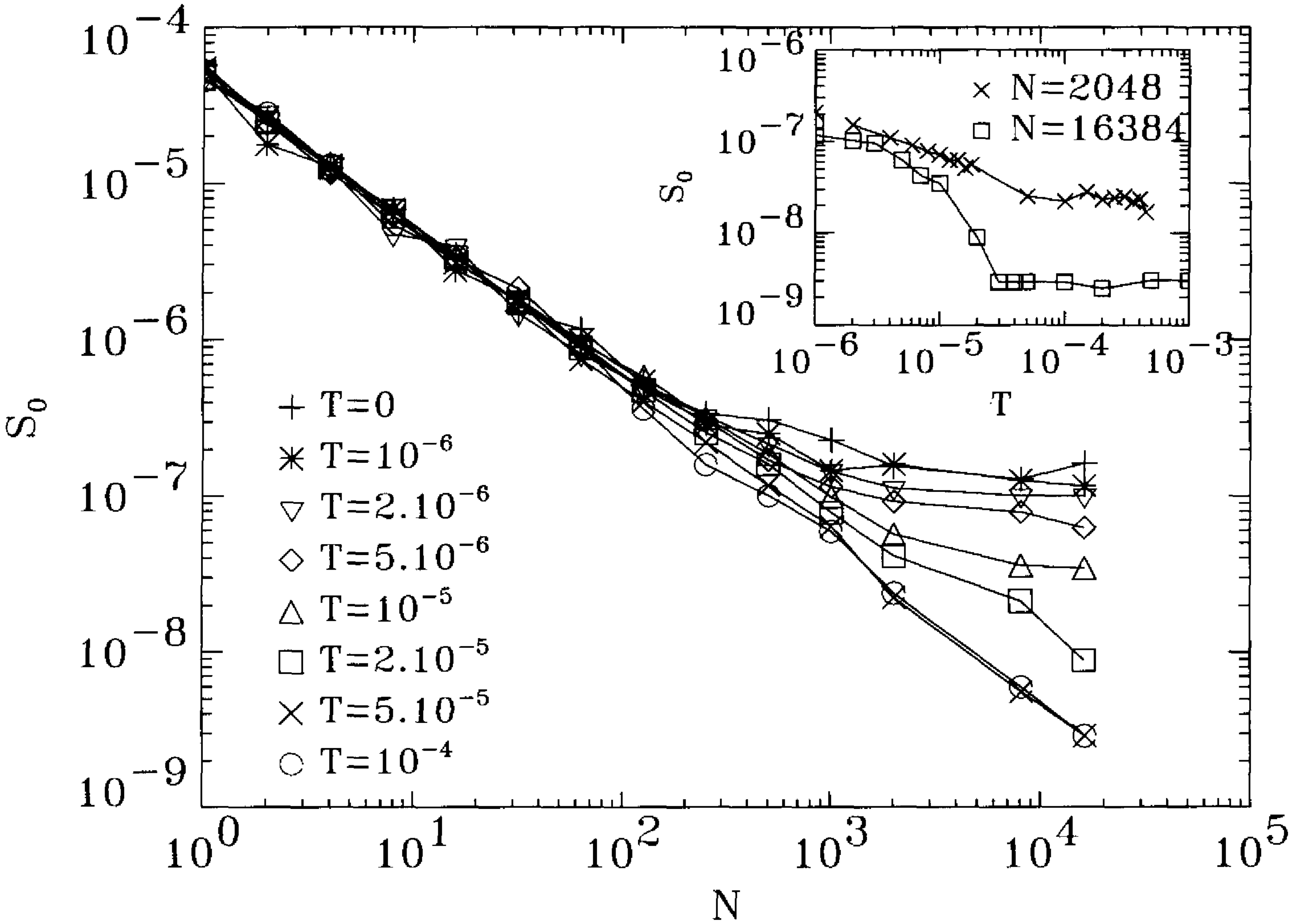


Figure 1: Low frequency limit of the power spectrum, $S_0 = \lim_{\omega\to 0} S(\omega)$, as a function of the size of the array $N$. For $\tilde{g} = 0.2$, $\Omega_{rf} = 0.8$, $i_{rf} = 0.61$, $i_{dc} = 0.124$, $\sigma = 0.4$ and different temperatures $\tilde{T}$. In the inset we show $S_0$, as a function of temperature for $N = 2048$ juncions and $N = 16384$.

discussed here can be fabricated with the present techniques [4]. In an experiment in a JJSA with a large number of junctions ($N \sim 1000$ to $10000$), in the turbulent regime in the IV characteristics, the broad band noise should increase sharply when decreasing the temperature below a certain $\tilde{T}_c$ (for junctions with $I_c = 1\mu A$, $\tilde{T}_c \approx 1mK$). This will be an indication of the break down of the law of large numbers in this system.

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